E8 202 Class 2

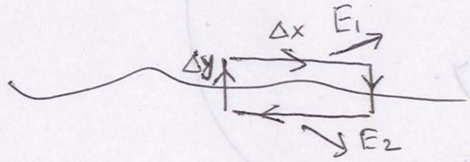
$$\nabla \times E = -j\omega B$$

$$\nabla \times H = j\omega D + J_{ic}$$

$$\nabla \cdot D = \rho$$

$$\nabla \cdot B = 0$$

$$\nabla \cdot J = -j\omega \rho$$

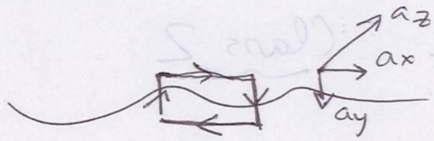


$$\oint E \cdot dl = \iint B \cdot ds$$

$$E_1 \cdot \hat{a}_x \Delta x + E_2 \cdot (-\hat{a}_x)\, \Delta x = 0$$

$$\boxed{(E_1 - E_2) \cdot \hat{a}_x} \, \Delta x = 0$$

$$\hat{n} \times (E_1 - E_2) = 0$$



$$\oint_C H \cdot dl = \iint D \cdot ds + \iint J_{ic} \cdot ds$$

$$H_1 \cdot \hat{a}_x \Delta x - H_2 \cdot \hat{a}_x \Delta x$$

$$\Rightarrow \quad (H_1 - H_2) \cdot \hat{a}_x \, \Delta x$$

$$J_{ic} \cdot \hat{a}_z \, \Delta x \, \Delta y.$$

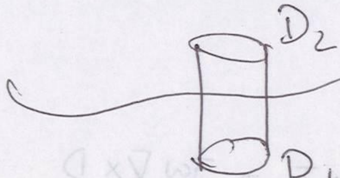
$$\Rightarrow \; J_s \cdot \hat{a}_z \, \Delta x$$

$$(H_1 - H_2) \cdot (\hat{a}_y \times \hat{a}_z)$$

$$A \cdot B \times C = C \cdot A \times B$$

$$\hat{a}_z \cdot \left[(H_1 - H_2) \times \hat{a}_y \right]$$

$$\left[\hat{a}_y \times (H_1 - H_2) - J_s \right] \cdot \hat{a}_z = 0$$



$$\oiint D \cdot ds = \iiint \rho \, dV$$

$$\hat{n} \cdot (D_2 - D_1) A_0 = \rho A_0 \Delta z$$
$$= \rho_s A_0 .$$

Wave eq^n:-

$$\nabla \times E = -j\omega B$$

$$\nabla \times \nabla \times E = -j\omega \nabla \times B = -j\omega\mu \nabla \times H$$
$$= -j\omega\mu (j\omega D + J)$$
$$= \omega^2 \mu\epsilon E - j\omega\mu\sigma E$$

$$\nabla(\nabla \cdot E) - \nabla^2 E = \omega^2 \mu\epsilon E - j\omega\mu\sigma E$$

$$\nabla^2 E - \frac{\nabla\rho}{\epsilon} + \omega^2 \mu\epsilon E - j\omega\mu\sigma E = 0$$

$$\nabla^2 E - \frac{\nabla\rho}{\epsilon} + \omega^2 \mu E \left(\epsilon - \frac{j\sigma}{\omega}\right) = 0$$

$$\boxed{\nabla^2 E - \frac{\nabla\rho}{\epsilon} + \omega^2 \mu \epsilon_c E = 0}$$

$$\nabla \times H = J_{ic} + j\omega D$$

$$\nabla \times \nabla \times H = \nabla \times J_i + \nabla \times J_c + j\omega \nabla \times D$$

$$= \nabla \times J_i + \nabla \times \sigma E + j\omega\epsilon \nabla \times E$$

$$= \nabla \times J_i + j\omega \nabla \times E \left(\epsilon - \frac{j\sigma}{\omega} \right)$$

$$= \nabla \times J_i + j\omega (-j\omega B) \epsilon_c$$

$$\nabla(\nabla \cdot H) - \nabla^2 H = \nabla \times J_i + \omega^2 \mu\epsilon H$$

$$\boxed{\nabla^2 H + \nabla \times J_i + \omega^2 \mu\epsilon H = 0}$$

~~Source-free~~ ✓ Homogenous
Linear
Isotropic.
Dispersive.

Source-free.

$\downarrow$

$$\nabla^2 E + k^2 E = 0$$
$$\nabla^2 H + k^2 H = 0.$$

Laplace

$$\nabla^2 \phi = 0$$

$\downarrow$

$$\Delta \phi$$

$\to$ Laplace operator.

Poisson

$$\nabla^2 \phi = f$$

Helmholtz

$$(\nabla^2 + k^2)\phi = 0$$

Sol^n to Wave eq^n

(a) 1D Plane Wave.

$$\nabla^2 E + \omega^2 \mu \epsilon E = 0$$

$k^2 = \omega^2 \mu \epsilon$

$\gamma^2 = -\omega^2 \mu \epsilon$

$\gamma = \alpha - j\beta$

$k \longrightarrow$ wave number.

$\beta \longrightarrow$ phase constant

$\alpha \longrightarrow$ attenuation const.

$\gamma \longrightarrow$ propagation const.

$k = \omega\sqrt{\mu\epsilon} = \frac{2\pi f}{c} = \frac{2\pi}{\lambda}$

$$E_z = Ae^{-jkz} + Be^{+jkz}$$

$$\mathcal{E}_z(z,t) = \left(Ae^{-jkz} + Be^{+jkz}\right)e^{j\omega t}$$

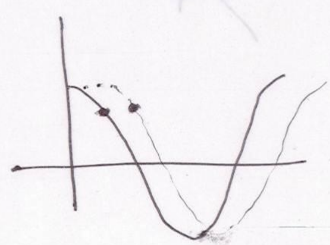
Phase vel. → vel of equiphase pts

gr vel →

$\omega t - kz = \text{const.}$

$\omega - k\frac{dz_p}{dt} = 0$

$v_p = \frac{\omega}{k}$



(b) Rectangular waveguide

$$\nabla^2 E + k^2 E = 0$$

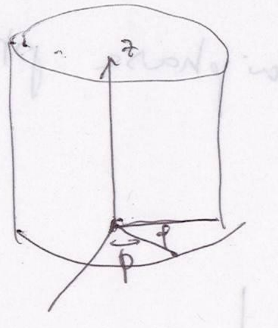
$$\nabla^2 \left(\hat{a}_x E_x + \hat{a}_y E_y + \hat{a}_z E_z \right) + k^2 \left(\hat{a}_x E_x + \hat{a}_y E_y + \hat{a}_z E_z \right) = 0$$

$$\nabla^2 E_x + k^2 E_x = 0 .$$

$$E_x = f(x)\, g(y)\, h(z) .$$

$$\frac{1}{f}\frac{d^2 f}{dx^2} + \frac{1}{g}\frac{d^2 g}{dy^2} + \frac{1}{h}\frac{d^2 h}{dz^2} + \beta^2 = 0 .$$

(c) Cylindrical coordinates :



$$h_m^{(1)}$$

(1) → kind

m → order