E8-262 Lecture 11 D. Gope.

[Ref: ① C. Paul Chap 8
② Hall and Heck Chap 3]

$$\frac{\partial V(z,t)}{\partial z} = -l \frac{\partial I(z,t)}{\partial t}$$

$$\frac{\partial I(z,t)}{\partial z} = -c \frac{\partial V(z,t)}{\partial t}$$

$$\frac{\partial^2 V(z,t)}{\partial z^2} = lc \frac{\partial^2 V(z,t)}{\partial t^2}$$

$$\frac{\partial^2 I(z,t)}{\partial z^2} = lc \frac{\partial^2 I(z,t)}{\partial t^2}$$

$$V(z,t) = V^+\left(t - \frac{z}{v}\right) + V^-\left(t + \frac{z}{v}\right)$$

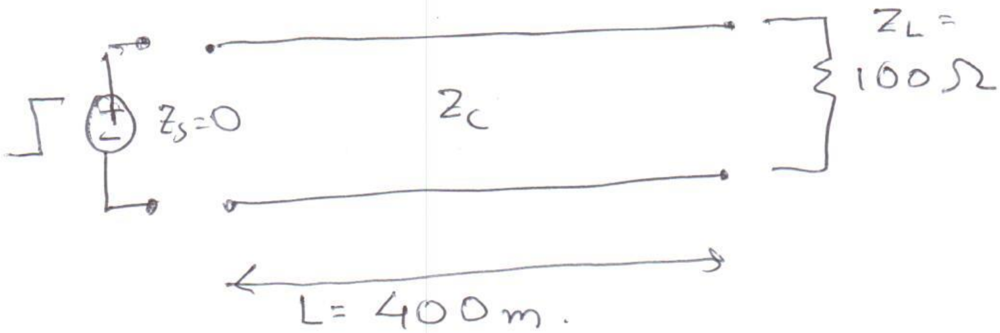
$$\boxed{v = \frac{1}{\sqrt{lc}}}$$

$$I(z,t) = I^+\left(t - \frac{z}{v}\right) + I^-\left(t + \frac{z}{v}\right)$$

$$= \frac{V^+}{Z_C}\left(t - \frac{z}{v}\right) - \frac{V^-}{Z_C}\left(t + \frac{z}{v}\right)$$

$$Z_C = \sqrt{\frac{l}{c}}$$

Time domain analysis: Wave tracing

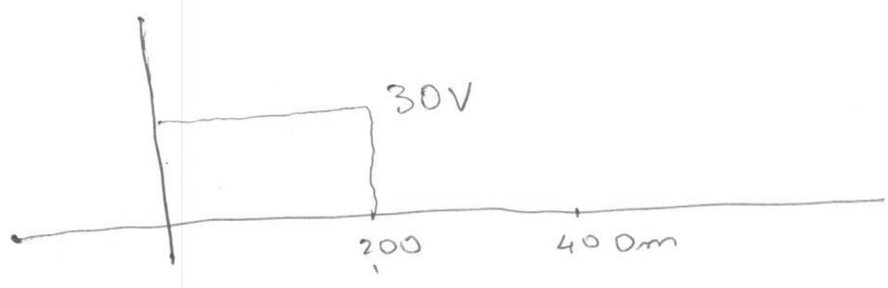


$$Z_c = 50$$

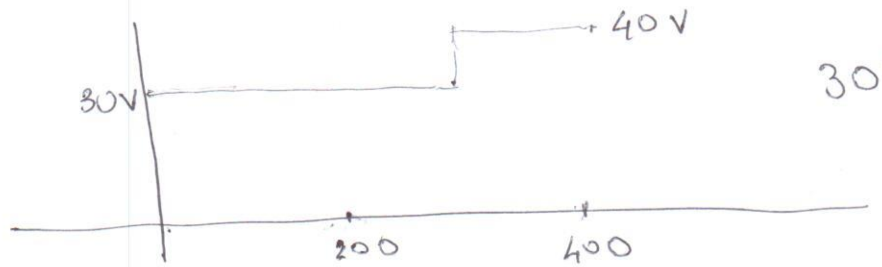
$$\Gamma_L = \frac{100-50}{100+50} = \frac{1}{3} \qquad \Gamma_s = \frac{0-50}{0+50} = -1$$

$$T_D = \frac{L}{v} = \frac{400\,m}{200e^{6}\,m/s} = 2\mu s.$$

At $t = 1\mu s$

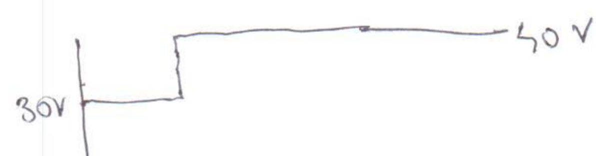


At $t = 2.5\mu s$



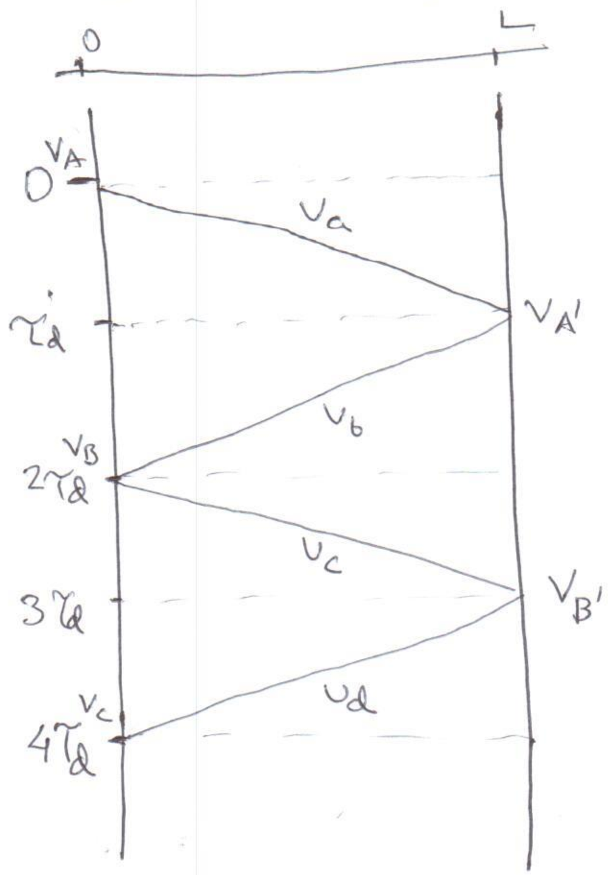
$$30\Gamma_L = 10$$

At $t = 4.5\mu s$



$$\Gamma_s 10 = -10$$

Lattice diagrams



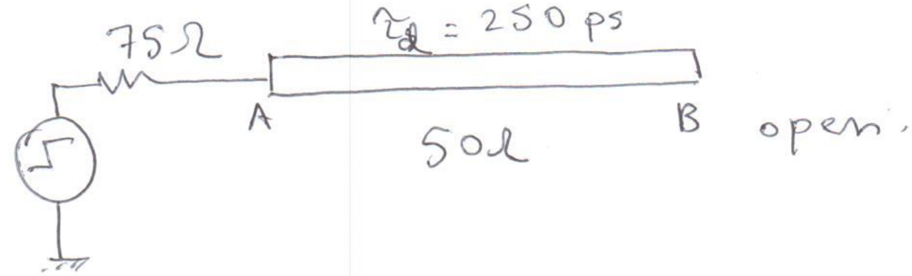
$V_A = v_a$ $\quad V_B = v_a + v_b + v_c$. $\quad V_C = v_a + v_b + v_c + v_d + v_e$.

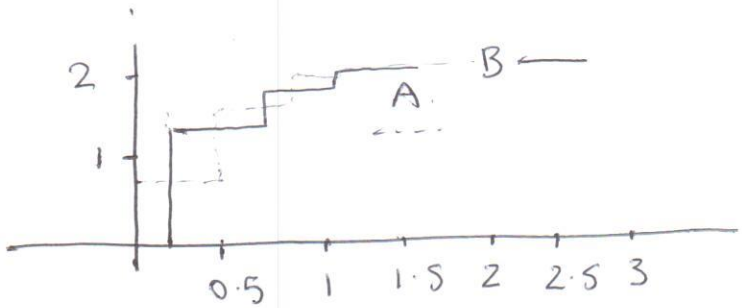
$$v_a = V_S \frac{Z_0}{Z_0 + R_S}$$

$$v_b = v_a \Gamma_L$$

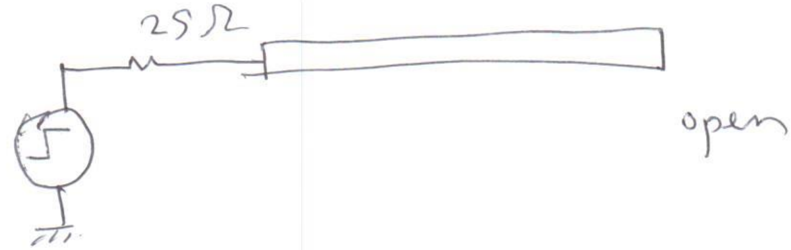
$$v_c = v_b \Gamma_0$$

Underdriven and overdriven line

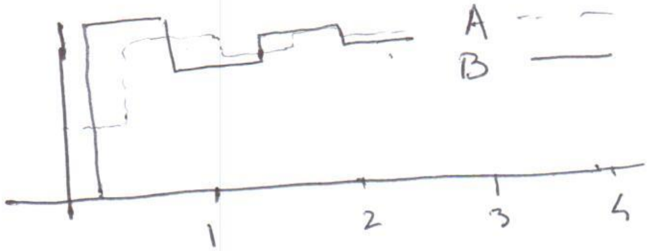




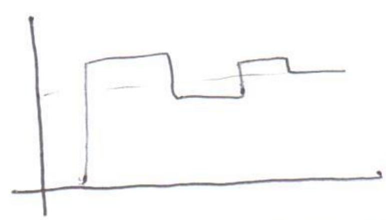
underdriven



overdriven

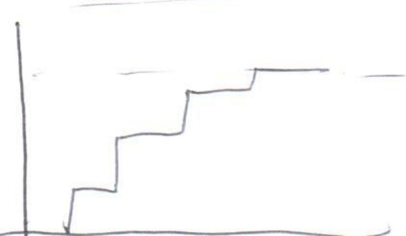


Γ_S	Γ_L
−	+
+	−



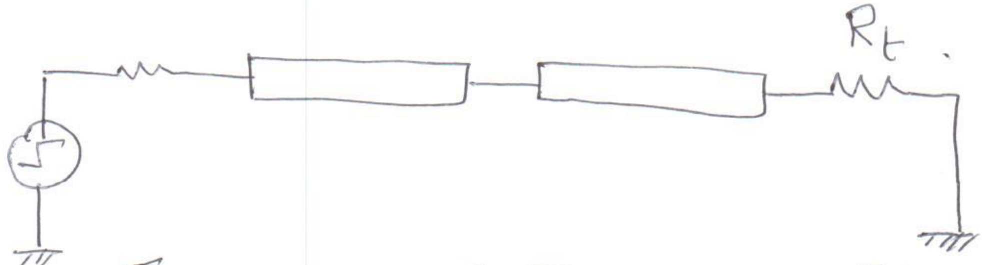
overdriven

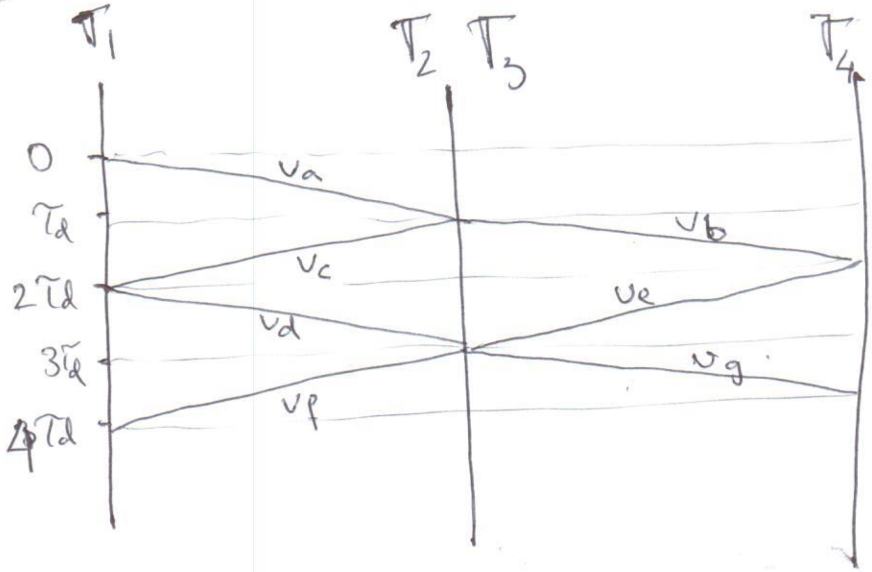
Γ_S	Γ_L
−	−
+	+



underdrive

Cascaded Tx lines



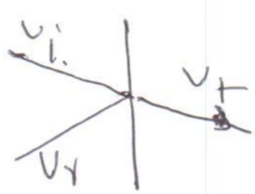


$$v_a = v_s \frac{Z_{01}}{Z_{01} + R_S}$$

$$v_b = v_a T_2$$

$$v_c = v_a \Gamma_2 \quad \ldots\ldots$$

What is T?

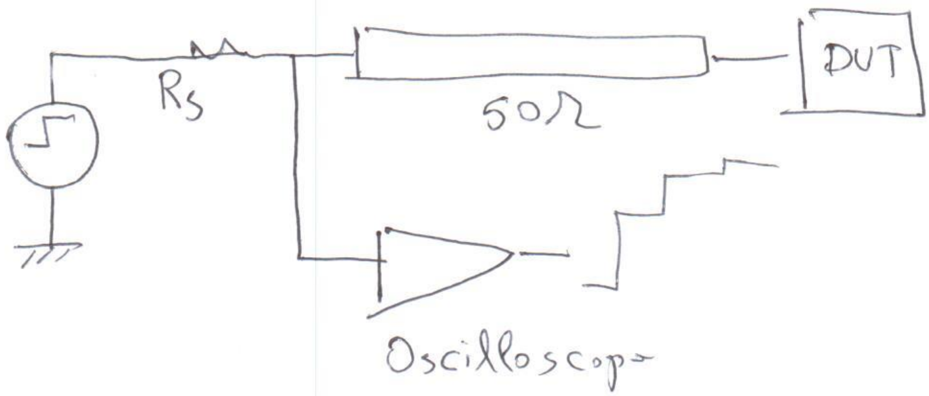


$$v_i = V_i^+ e^{-j\beta_1 z} \qquad v_r = \Gamma v_i$$

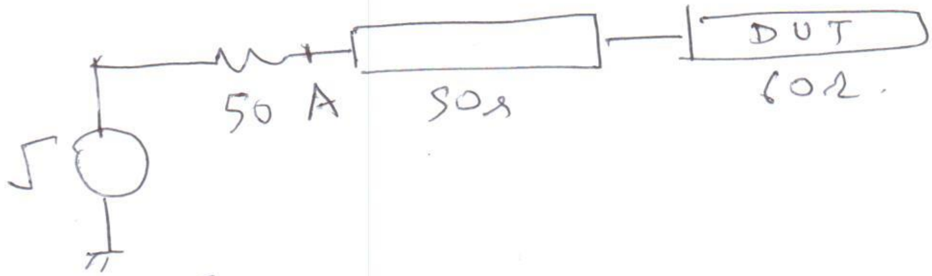
$$v_r = V_r^- e^{+j\beta_1 z} \qquad v_t = v_i + v_r.$$

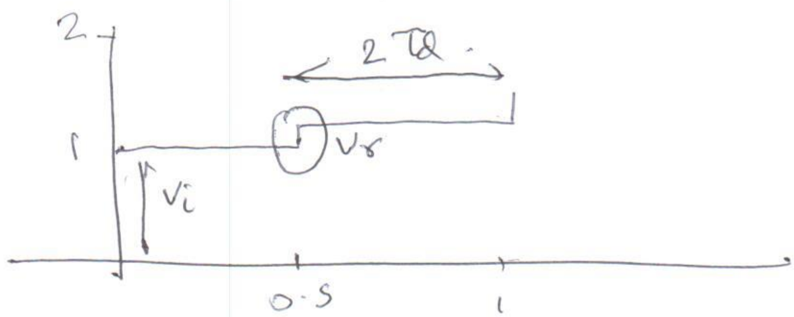
$j\beta_2 z.$

Time domain reflectometry (TDR)



Output — Z_0 and T_d.





$$\Gamma = \frac{V_r}{V_i} = \frac{Z_{DUT} - Z_0}{Z_{DUT} + Z_0}$$

$$Z_{DUT} = Z_0 \frac{V_i + V_r}{V_i - V_r}$$

$$Z_{DUT} = 50 \frac{1 + 0.091}{1 - 0.091} = 60\Omega$$

SPICE simulation

Method of characteristics or Branin's method

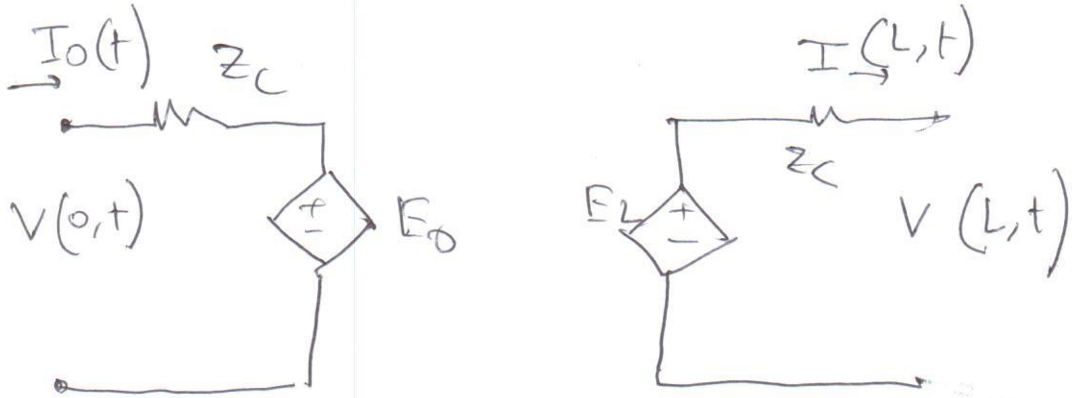
$$\begin{cases} V(z,t) = V^+\left(t - \frac{z}{v}\right) + V^-\left(t + \frac{z}{v}\right) \\ Z_c I(z,t) = V^+\left(t - \frac{z}{v}\right) - V^-\left(t + \frac{z}{v}\right) \end{cases}$$

$$\begin{cases} V(z,t) + Z_c I(z,t) = 2V^+\left(t - \frac{z}{v}\right) \\ V(z,t) - Z_c I(z,t) = 2V^-\left(t + \frac{z}{v}\right) \end{cases}$$

$$\begin{cases} V(0,t) + Z_c I(0,t) = 2V^+(t) \\ V(0,t) - Z_c I(0,t) = 2V^-(t) \\ V(L,t) + Z_c I(L,t) = 2V^+(t - T_D) = 2D^{-1}V^+(t) \\ V(L,t) - Z_c I(L,t) = 2V^-(t + T_D) = 2D\,V^-(t) \end{cases}$$

$$V(0,t) - Z_C I(0,t) = V(L, t-T_D) - Z_C I(L,t-T_D)$$

$$V(L,t) + Z_C I(L,t) = V(0,t-T_D) + Z_C I(0,t-T_D)$$



$$E_0 = V(L,t-T_D) - Z_C I(L,t-T_D)$$

$$E_L = V(0,t-T_D) + Z_C I(0,t-T_D)$$

Stamps?

Freq dependant :- lossy Tx line.

$$\frac{dV(z,\omega)}{dz} = -\left[r(\omega) - j\omega l_i(\omega)\right] I(z,\omega) - j\omega l I(z,\omega)$$

$$\frac{\partial V(z,t)}{\partial z} = -z_i * I(z,t) - l\frac{\partial I(z,t)}{\partial t}$$

$$z_i * I(z,t) = \int_0^t z_i(\tau)\, I(z, t-\tau)\, d\tau.$$

1. How to get $z(\tau)$
2. convolution.