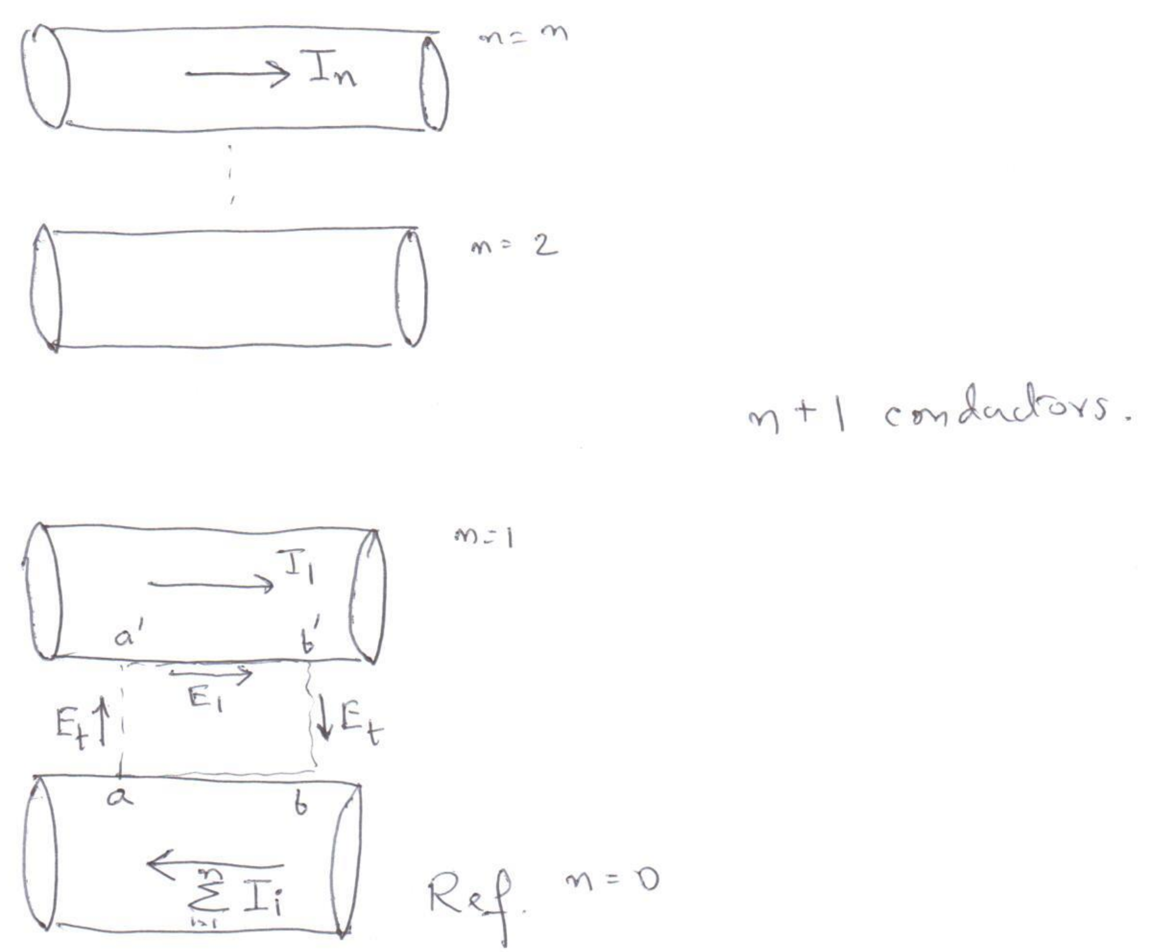
E8 : 262 Lecture 12

Multiconductor Tx lines

[Ref: C Paul Chap 3, 5, 7]

a) Tx line eqn.
b) pul parameters
c) Freq domain analysis.



$$\int_a^{a'} E_t \cdot dl + \int_{a'}^{b'} E_l \cdot dl + \int_{b'}^{b} E_t \cdot dl + \int_{b}^{a} E_l \cdot dl = \mu \frac{d}{dt} \int H_t \cdot a_n ds$$

(1) (2) (3) (4) (5)

Term 1:

$$\int_{a}^{a'} E_t \cdot dl = -V(z,t)$$

Term 3:-

$$\int_{b'}^{b} E_t \cdot dl = -\int_{b}^{b'} E_t \cdot dl = -\left(-V_i(z+\Delta z,t)\right)$$

Term 2:

$$\int_{a'}^{b'} E_1 \cdot dl = r_i \, \Delta z \, I_i(z,t)$$

Term 4:

$$\int_{b}^{a} E_1 \cdot dl = r_0 \, \Delta z \sum_{k=1}^{N} I_k(z,t).$$

Term 5:-

$$\mu \frac{d}{dt} \int H_t \cdot a_n ds = -\Psi_i$$

$$= -l_{i1} I_1 - l_{i2} I_2 \cdots + l_{in} I_n$$

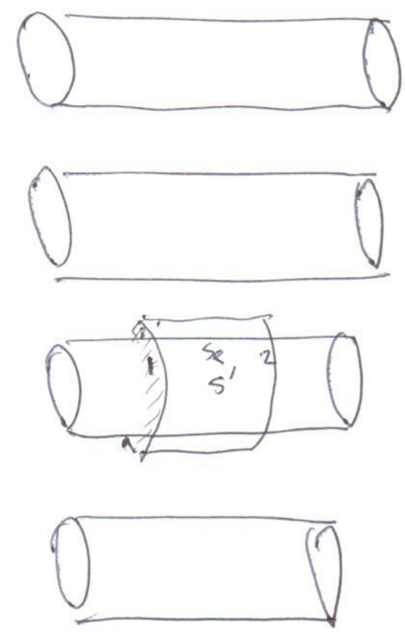
$$\frac{\partial V_i(z,t)}{\partial z} = -r_0 I_1(z,t) - r_0 I_2(z,t) - \cdots - (r_0 + r_i) I_i(z,t)$$
$$- r_0 I_n(z,t) - l_{i1} \frac{\partial I_1(z,t)}{\partial t} \cdots - l_{in} \frac{\partial I_n(z,t)}{\partial t}$$

$$\frac{\partial \bar{V}(z,t)}{\partial z} = - \bar{\bar{R}} \bar{I}(z,t) - \bar{\bar{L}} \frac{\partial \bar{I}}{\partial t}(z,t).$$

$$\bar{V}(z,t) = \begin{bmatrix} V_1(z,t) \\ \\ \\ V_n(z,t) \end{bmatrix} \qquad \bar{I}(z,t) = \begin{bmatrix} I_1(z,t) \\ \\ \\ I_n(z,t) \end{bmatrix}$$

$$\bar{\bar{L}} = \begin{bmatrix} L_{11} & & L_{1n} \\ & & \\ & & \\ L_{n1} & & L_{nn} \end{bmatrix}$$

$$\bar{\bar{R}} = \begin{bmatrix} r_1 & & & \\ & r_2 & & \\ & & & \\ & & & r_n \end{bmatrix} + r_0 \times \mathrm{ones}(n, n).$$



$$\oint\!\!\!\oint_{s'} J \cdot ds' = -\frac{d Q_{enc}}{dt}$$

$$\Rightarrow \iint_1 J \cdot ds^1 + \iint_2 J \cdot ds^2 + \iint_{se} J \cdot ds_e = -\frac{d Q_{enc}}{dt}.$$

$$-I_i(z,t) + I_i(z+\Delta z, t) + \text{Term 1} = \text{Term 2}$$

Term 1:

$$\iint_{se} J \cdot ds_e = \sigma \iint_{se} E_t \cdot ds_e .$$

$$= \Delta z \Big[g_{i1}(V_i - V_1) + g_{i2}(V_i - V_2) + \cdots g_{ii} V_i + \cdots$$

Term 2:

$$-\frac{d Q_{enc}}{dt} = -\Delta z \frac{d}{dt}\Big[c_{i1}(V_i - V_1) + c_{i2}(V_i - V_2) \cdots c_{ii} V_i + \cdots \Big]$$

$$\frac{\partial}{\partial z}\bar{I}(z,t) = -\bar{\bar{G}}V(z,t) - \bar{\bar{C}}\frac{\partial}{\partial t}V(z,t)$$

$$\bar{\bar{G}} = \begin{bmatrix} \sum_{k=1}^{n} g_{1k} & -g_{12} & -g_{1n} \\ & & \\ -g_{1n} & & \sum_{i=1}^{n} g_{nk} \end{bmatrix}$$

$$\bar{\bar{C}} = \begin{bmatrix} \sum_{k=1}^{n} c_{1k} & -c_{12} & -c_{1n} \\ & & \\ -c_{1n} & & \sum_{k=1}^{n} c_{nk} \end{bmatrix}$$

MTL eq^n

$$\frac{\partial}{\partial z}\begin{bmatrix} V(z,t) \\ I(z,t) \end{bmatrix} = -\begin{bmatrix} 0 & L \\ C & 0 \end{bmatrix}\frac{\partial}{\partial t}\begin{bmatrix} V(z,t) \\ I(z,t) \end{bmatrix}$$

$$\boxed{\begin{aligned} LC &= CL = \mu\epsilon I_{n\times n} \\ LG &= GL = \mu\sigma I_{n\times n} \end{aligned}}$$

Per unit length parameters

$$\bar{\Psi} = \bar{\bar{L}}\,\bar{I} \qquad \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_n \end{bmatrix} = \begin{bmatrix} l_{11} & & l_{1n} \\ & & \\ l_{n1} & & l_{nn} \end{bmatrix} \begin{bmatrix} I_1 \\ \\ I_n \end{bmatrix}$$

$$\bar{Q} = \bar{\bar{C}}\,\bar{V} \qquad \begin{bmatrix} q_1 \\ q_2 \\ q_n \end{bmatrix} = \begin{bmatrix} \sum_{k=1}^{n} c_{1k} & & -c_{1n} \\ & & \\ -c_{n1} & & \sum_{k=1}^{n} c_{nk} \end{bmatrix} \begin{bmatrix} V_1 \\ \\ V_n \end{bmatrix}$$

$$\bar{I}_t = \bar{\bar{G}}\,\bar{V} \qquad \begin{bmatrix} I_{t_1} \\ \\ I_{t_n} \end{bmatrix} = \begin{bmatrix} \sum_{k=1}^{n} g_{1k} & & -g_{1n} \\ & & \\ -g_{n1} & & \sum_{k=1}^{n} g_{nk} \end{bmatrix} \begin{bmatrix} V_1 \\ \\ V_n \end{bmatrix}$$

Method of moments 2D pul extraction.

$$\nabla \cdot D = \rho \implies \nabla \cdot \epsilon E = \rho.$$

Poisson's eqn: $\nabla^2 \phi(r) = -\frac{\rho(r')}{\epsilon}$

Green's fn: $\nabla^2 g(r, r') = -\frac{\delta(r - r')}{\epsilon}$

$$g(r, r') = \frac{C_1}{r} + C_2$$

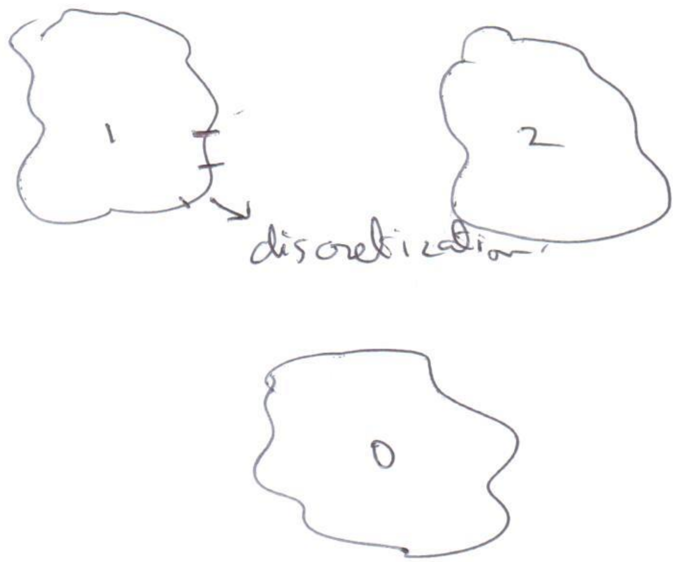
pt. charge $g(r, r') = \frac{1}{4\pi\epsilon_0} \frac{1}{r} \longrightarrow$ 3D

line charge  $g_{2D}(r, r') = \cancel{\frac{1}{4\pi\epsilon_0}\frac{1}{r}} - \frac{1}{2\pi\epsilon_0} \ln|r - r'|$

Does this satisfy poisson in 2D?

Does this satisfy:

$$\int_V \nabla^2 G(r, r')\, dv = -\frac{1}{\epsilon_0}$$



$$
\begin{bmatrix} v_0 \\ \\ \\ \\ \\ \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & \\ & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} q_0 \\ \\ \\ \\ q_N \end{bmatrix}
$$

$$
Z_{ij} = \int_{dl} G_{2D}(r, r')\, dl .
$$

$$
C_{ij} = \sum_{m \in i} q_m \quad \text{when} \quad v_m = 1 \;\; m \in j
$$
$$
v_m = 0 \;\; m \notin j .
$$

$$\begin{bmatrix} q_0 \\ q_1 \\ \vdots \\ q_n \end{bmatrix} = \begin{bmatrix} c_{00}' & \cdots & \\ & \ddots & \\ & & \end{bmatrix} \begin{bmatrix} \phi_0 \\ \phi_1 \\ \vdots \\ \phi_n \end{bmatrix}$$

But we need :-

$$\begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix} = \begin{bmatrix} c_{11} & \cdots & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_n \end{bmatrix}$$

Convert $(n+1) \times (n \times 1)$ C matrix to $n \times n$.

$$q_0 = C'_{00}\phi_0 + c'_{01}\phi_1 \cdots$$

$$q_0 = c'_{00}\phi_0 + c'_{01}V_1 + \cdots \sum_{m=1}^{n} c'_{0m}\phi_0$$

$$= c'_{01}V_1 + \cdots \sum_{m=0}^{n} c'_{0m}\phi_0$$

$$q_1 = c'_{10}\phi_0 + c'_{11}\phi_1 + \cdots$$

$$= \cancel{c'_{10}\phi_0} \cdot c'_{11}V_1 + c'_{12}V_2 \cdots + \sum_{m=0}^{n} c'_{1m}\phi_0$$

$$q_n = c'_{n1}V_1 + c'_{n2}V_2 + \cdots \sum_{m=0}^{n} c'_{nm}\phi_0$$

Adding:

$$0 = \sum_{m=0}^{n} c'_{m1}V_1 + \sum_{m=0}^{n} c'_{m2}V_2 + \cdots$$

$$+ \sum_{m=0}^{n}\sum_{l=0}^{n} c'_{lm}\phi_0 .$$

$$\Rightarrow \phi_0 = -\left(\frac{\sum_{k=1}^{n}\sum_{m=0}^{n} c'_{mk}V_k}{\sum_{l=0}^{n}\sum_{m=0}^{n} c'_{lm}}\right)$$

~~Also~~

Recap:

$$q_1 = c_{11}' V_1 + c_{12}' V_2 + \ldots \sum_{m=0}^{n} C_{1m}' \phi_0 .$$

$$\Rightarrow C_{11} = c_{11}' - \frac{\left(\sum_{m=0}^{n} C_{mj}'\right)\left(\sum_{m=0}^{n} C_{1m}\right)}{\left(\sum_{m=0}^{n}\sum_{l=0}^{n} C_{lm}\right)}$$

$$\Rightarrow C_{ij} = C_{ij}' - \frac{\left(\sum_{m=0}^{n} C_{mj}'\right)\left(\sum_{m=0}^{n} C_{im}\right)}{\left(\text{———}\right)}.$$