E8: 262 Lecture 13

Freq and Time domain analysis of MTL : [Chap 7 and 9 C. Paul].

$$\frac{\partial \bar{V}(z,t)}{\partial z} = -\bar{\bar{R}}\,\bar{I}(z,t) - \bar{\bar{L}}\,\frac{\partial \bar{I}(z,t)}{\partial t}$$

$$\frac{\partial \bar{I}(z,t)}{\partial z} = -\bar{\bar{G}}\,\bar{V}(z,t) - \bar{\bar{C}}\,\frac{\partial \bar{V}(z,t)}{\partial t}$$

In freq domain :-

$$\frac{dV(z)}{dz} = -\left(\bar{\bar{R}} + j\omega \bar{\bar{L}}\right) I(z)$$

$$\frac{dI(z)}{dz} = -\left(\bar{\bar{G}} + j\omega \bar{\bar{C}}\right) V(z)$$

$$\frac{d\bar{V}(z)}{dz} = -\bar{\bar{Z}}\,\bar{I}(z) \; ; \quad \frac{dI(z)}{dz} = -\bar{\bar{Y}}\,V(z)$$

Second order

$$\frac{d^2 V(z)}{dz^2} = \bar{\bar{Z}}\,\bar{\bar{Y}}\,V(z)$$

$$\frac{d^2 I(z)}{dz^2} = \bar{\bar{Y}}\,\bar{\bar{Z}}\,I(z)$$

Remember the 2 line case :-

$$\frac{d^2 V(z)}{dz^2} = \underbrace{\gamma^2}_{\text{scalar}} V(z)$$

$\downarrow$

define V^+ and V^-

$\downarrow$

find β, Z_C, Z_{in}, Γ

$$Z_{in}(0) = Z_C \frac{Z_L + jZ_C \tan(\beta L)}{Z_C + jZ_L \tan(\beta L)}$$

MTL eq^{n}s are coupled!

$$\frac{d^2 \bar{V}(z)}{dz^2} = \bar{\bar{Z}}\bar{\bar{Y}} \bar{V}(z) \; ; \quad \frac{d^2 \bar{I}(z)}{dz^2} = \bar{\bar{Y}}\bar{\bar{Z}} \bar{I}(z).$$

$$\bar{\bar{Z}}\bar{\bar{Y}} \neq \bar{\bar{Y}}\bar{\bar{Z}}$$

Modal decomposition to diagonalize $\bar{\bar{Z}}\bar{\bar{Y}}$:

$$\bar{V}(z) = \bar{\bar{T}}_V \bar{V}_m(z)$$

$$\bar{I}(z) = \bar{\bar{T}}_I \bar{V}_I(z)$$

$$\frac{d^2}{dz^2} V_m(z) = \bar{\bar{T}}_V^{-1} \bar{\bar{Z}}\bar{\bar{Y}}\bar{\bar{T}}_V \bar{V}_m(z)$$

$$= \bar{\bar{\gamma}}^2 V_m(z).$$

How to find $\bar{\bar{T}}_V$ and $\bar{\bar{T}}_I$

Classic eigenvalue problem :-

$$T^{-1} M T = \Lambda$$

$$M T - T \Lambda = 0$$

$$\Rightarrow \left(M - \lambda_i I_{n \times n} \right) T_i = 0$$

columns of $\bar{\bar{T}}_V$ are eigenvectors of $\bar{\bar{Z}}\bar{\bar{Y}}$

γ_i^2 are eigenvalues of $\bar{\bar{Z}}\bar{\bar{Y}}$

Also: $$\bar{\bar{T}}_V^{\,t} \bar{\bar{Y}} \bar{\bar{Z}} \left(\bar{\bar{T}}_V^{-1} \right)^t = \bar{\bar{\gamma}}^2 = \bar{\bar{T}}_I^{-1} \bar{\bar{Y}} \bar{\bar{Z}} \bar{\bar{T}}_I$$

$$\Rightarrow \bar{\bar{T}}_V^{\,t} = \bar{\bar{T}}_I^{-1} \text{ and } \bar{\bar{T}}_I^{\,t} = \bar{\bar{T}}_V^{-1}$$

$$\bar{V}_m(z) = e^{-\gamma z}\,\bar{V}_m^{+} + e^{+\gamma z}\,\bar{V}_m^{-}$$

$$I_m(z) = e^{-\gamma z}\,\bar{I}_m^{+} - e^{+\gamma z}\,\bar{I}_m^{-}$$

$$V(z) = \bar{\bar{T}}_V\left(e^{-\gamma z} V_m^{+} + e^{\gamma z} V_m^{-}\right)$$

$$I(z) = \bar{\bar{T}}_I\left(e^{-\gamma z} I_m^{+} - e^{+\gamma z} I_m^{-}\right)$$

Reduce $4n$ unknowns to $2n$:

$$V(z) = -\bar{\bar{Y}}^{-1}\frac{d}{dz} I(z)$$

$$= +\bar{\bar{Y}}^{-1}\,\bar{\bar{T}}_I\,\gamma\left(e^{-\gamma z} I_m^{+} + e^{\gamma z} I_m^{-}\right)$$

$$= \left[\bar{\bar{Y}}^{-1}\,\bar{\bar{T}}_I\,\gamma\,\bar{\bar{T}}_I^{-1}\right]\bar{\bar{T}}_I\left(e^{-\gamma z} I_m^{+} + e^{\gamma z} I_m^{-}\right)$$

$$= Z_C\,\bar{\bar{T}}_I\left(e^{-\gamma z} I_m^{+} + e^{\gamma z} I_m^{-}\right)$$

$$Z_C = \bar{\bar{Y}}^{-1}\,\bar{\bar{T}}_I\,\gamma\,\bar{\bar{T}}_I^{-1}$$

or $$Z_C = \bar{\bar{Z}}\,\bar{\bar{T}}_I\,\gamma^{-1}\,\bar{\bar{T}}_I^{-1}$$

Case 1 :- Lossless cond.

Lossy Homogenous diel.

$$\bar{\bar{Y}}\bar{\bar{Z}} = (G + j\omega C)(j\omega L)$$

$$= j\omega GL - \omega^2 CL$$

Homogenous implies :-

$$GL = LG = \mu\sigma I_{n\times n}$$

$$CL = LC = \mu\epsilon I_{n\times n}.$$

Already diagonal !

$$Z_C = \frac{j\omega \bar{\bar{L}}}{\bar{\bar{\gamma}}}$$

if $\sigma = 0$ $\quad \gamma = j\omega\sqrt{\mu\epsilon}$

$$\bar{\bar{Z}}_C = \frac{j\omega L}{j\beta} = \frac{\omega L}{\omega\sqrt{\mu\epsilon}} = vL$$

Case 2 :- Lossy conductor.
Lossy homogenous diel.

$$\bar{\bar{Y}}\,\bar{\bar{Z}} = (G + j\omega C)(R + j\omega L)$$

$$= GR + j\omega CR + j\omega GL - \omega^2 CL$$

$$= GR + j\omega CR + (j\omega\mu\sigma - \omega^2\mu\epsilon)\, I_{mn}$$

$$= \left(\frac{\sigma}{\epsilon} + j\omega\right) CR + (j\omega\mu\sigma - \omega^2\mu\epsilon)\, I_{mn}$$

$$\downarrow$$

diagonalize.

Case 3:- Lossless conductor.
Lossless Inhomogenous diel.

$$\bar{\bar{Y}}\,\bar{\bar{Z}} = -\omega^2 CL$$

$$\downarrow$$

diagonalize

Case 4 :- General case

Lossy conductor.

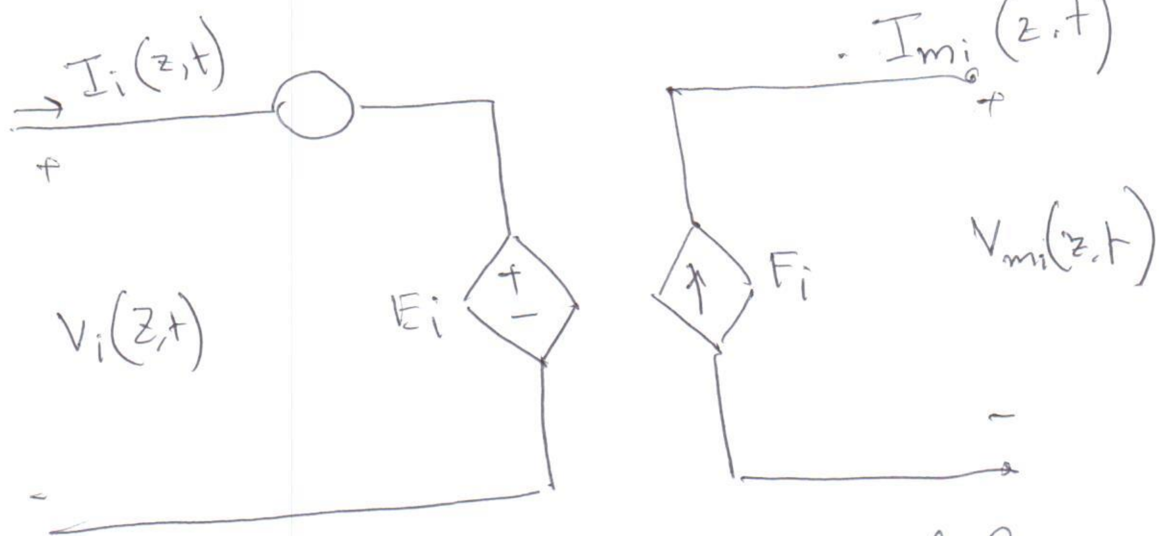
Lossy Inhomogenous diel.

$$\bar{\bar{Y}}\,\bar{\bar{Z}} = (G + j\omega C)(R + j\omega L)$$

freq dependent T_V and T_I.

Incorporating in TD SPICE

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} T_{v_{11}} & & \\ & \cdots & \\ & & \end{bmatrix} \begin{bmatrix} V_{m1} \\ V_{m2} \\ \vdots \\ V_{mn} \end{bmatrix}$$



actual

modal

$$E_i = \sum_{k=1}^{n} T_V(i,k)\, V_m(k)$$

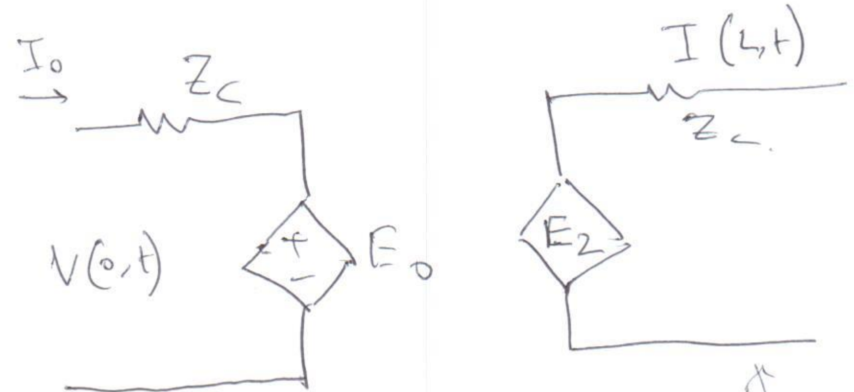
$$F_i = \sum_{k=1}^{n} [T_I]^{-1}(i,k)\, I(k)$$

Recap 2cond SPICE

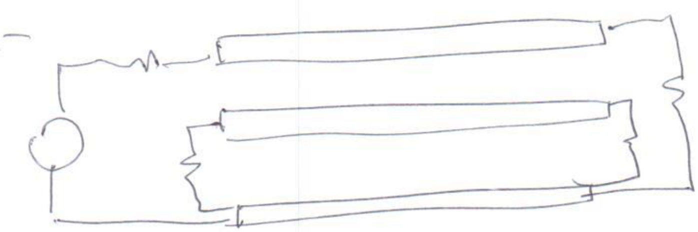
Method of characteristics:-

$$V(0,t) - Z_c I(0,t) = V(L, t - T_D) - Z_c I(L, t - T_D)$$

$$V(L,t) + Z_c I(L,t) = V(0, t - T_D) + Z_c I(0, t - T_D)$$



Now:-



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