E8-262 Lecture 14

Power delivery and 2.5D Modeling

PDN modeling

DC	AC
- IR drop analysis	- RLGC models
- Resistance only model	- Z parameter
- Sparse direct or iterative solvers.	- decoupling caps

DC analysis :-

Input netlist (R, V, I) $\longrightarrow$ Voltage at nodes, Current through branches

DC IR drop solver

- Direct.
  - LU
  - Cholesky
- Iterative
  - Stationary
    - Gauss Seidel
    - Jacobi
    - SOR
  - Krylov subspace -based
    - Conjugate Gradient
    - GMRES

Positive definite Systems

[Ref: ① Golub Van Loan
② Briggs]

$$x^T A x > 0$$

for $x = (1, 0)^T \Rightarrow a_{11} > 0$

$x = (1, -1)^T \Rightarrow |a_{12}| \leq (a_{11} + a_{22})/2$

The resistance only matrix is Symmetric Positive definite (SPD)

Cholesky factorization:

$A = G G^T$ where A is SPD matrix
G is lower triangular

$$A(i,j) = \sum_{k=1}^{n} G(i,k)\, G^T(k,j)$$

$$= \sum_{k=1}^{n} G(i,k)\, G(j,k)$$

$$\Rightarrow A(:,j) = \sum_{k=1}^{n} G(:,k)\, G(j,k)$$

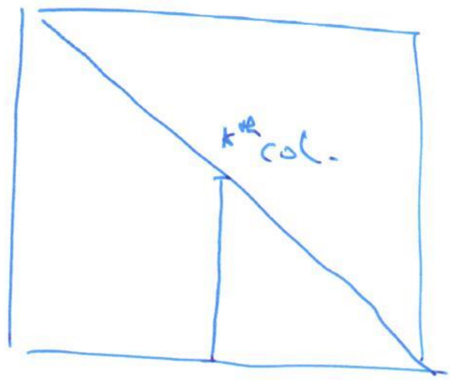
$$\Rightarrow G(j,j)\, G(:,j) = A(:,j) - \sum_{k=1}^{j-1} G(j,k)\, G(:,k)$$

$$= v$$

gaxpy cholesky

for $j = 1 : n$
    $v(j:n) = A(j:n, j)$
    for $k = 1 : j-1$
        $v(j:n) = v(j:n) - G(j,k)\, G(j:n,k)$
    end
    $G(j:n, j) = v(j:n) / \sqrt{v(j)}$
end

Complexity analysis (dense)



$(k-1)$ column subtraction; $(N-k)$ elements in k^{th} column.

$$\Rightarrow \quad (k-1)(N-k)$$

So all operations: $\displaystyle\sum_{k=1}^{N} (k-1)(N-k)$

$$= \sum_{k=1}^{N} Nk + \sum_{k=1}^{N} k - \sum_{k=1}^{N} N - \sum_{k=1}^{N} k^2$$

$$= N\frac{N(N+1)}{2} + \cdots\cdots$$

Outer product cholesky:

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for k = 1:n
    A(k,k) = sqrt(A(k,k))
    A(k+1:n, k) = A(k+1:n, k) / A(k,k)
    for j = k+1:n
        A(j:n, j) = A(j:n, j) - A(j:n, k) A(j, k)
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Stationary Iterative methods

$$Au = f.$$

$u \downarrow$ sol^n

$$Av = f_1$$

$r = f - Av$; $e = u - v$

$r \downarrow$ residual $e \downarrow$ error

$$Au = f$$
$$Av = f - r$$

$$\Rightarrow Ae = r$$

Jacobi $A = D - L - U$

$$\Rightarrow Du = (L+U)u + f.$$

$$\Rightarrow u = D^{-1}(L+U)u + D^{-1}f.$$

$$\Rightarrow v^{(1)} = \underbrace{D^{-1}(L+U)}_{R_J} v^{(0)} + D^{-1}f$$

Weighted Jacobi :-

$$v^{(1)} = \left[(1-\omega)I + \omega R_J\right] v^{(0)} + \omega D^{-1} f.$$

$$R_\omega = (1-\omega)I + \omega R_J$$

Gauss - Seidel :-

$$(D-L)u = Uu + f.$$

$$u = (D-L)^{-1} U u + (D-L)^{-1} f.$$

Multigrid based method

V Cycle.

- Relax $A^h u^h = f^h$ n_1 times with initial v^h
- Interpolate $f^{2h} = I_h^{2h} r^h$
    - Relax $A^{2h} u^{2h} = f^{2h}$ n_1 times with initial = 0
    - Interpolate $f^{4h} = I_{2h}^{4h} r^{2h}$.
        - Solve $A^{Lh} u^{Lh} = f^{Lh}$
    - Correct $v^{4h} \leftarrow v^{4h} + I_{8h}^{4h} v^{8h}$.
    - Relax $A^{4h} u^{4h} = f^{4h}$ with guess v^{4h}.