E8 : 262

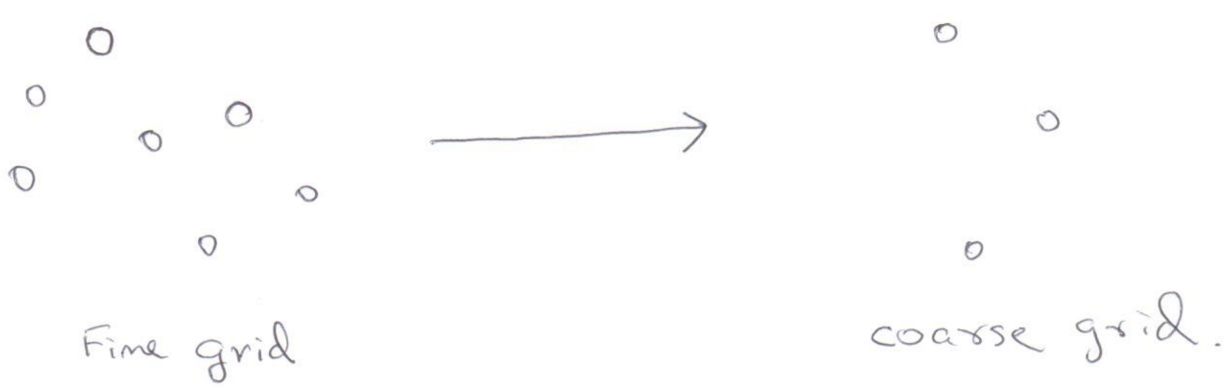
Lecture 15 : PDN DC Analysis

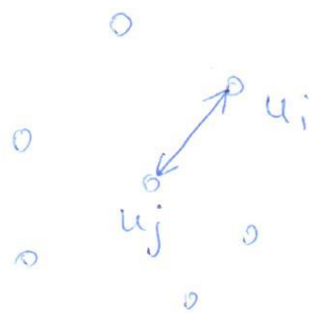
Multigrid based solvers

Algebraic Multigrid (AMG) :

1. Coarse grid selection
2. Interpolation and restriction
3. Coarse grid operator

Coarse grid selection





$\boxed{u_i \text{ depends strongly on } u_j}$

if $-a_{ij} \geq \theta \max_{k \neq i} \{-a_{ik}\}$

$$F \longrightarrow C$$

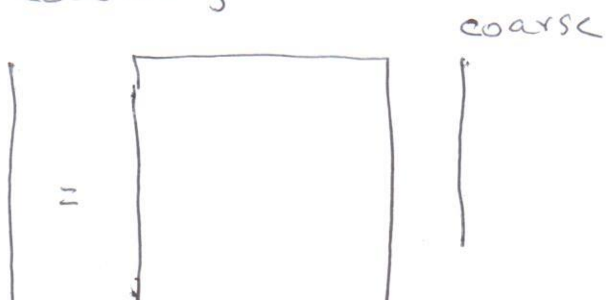
C is the maximal subset of all points with the property that no C-point strongly depends on another C-point.

Interpolation operator

$$I_{2h}^{h}(i,j) = \begin{cases} 1 & \text{if } i \in C \\ w_{ij} & \text{if } i \in F. \end{cases}$$

$h \equiv$ fine grid

$2h \equiv$ coarse grid.



$$w_{ij} = - \frac{a_{ij} + \sum_{m \in D_i^s} \left(\frac{a_{im}\, a_{mj}}{\sum_{k \in C_i} a_{mk}} \right)}{a_{ii} + \sum_{n \in D_i^w} a_{in}}$$

Restriction operator

$$I_h^{2h} = \left(I_{2h}^h \right)^T$$

Matrix operator in coarse grid

$$A^{2h} = I_h^{2h} A^h I_{2h}^h$$

Cycling algorithms

Case I :- Two grid cycle

$$v^h \leftarrow AMG(v^h, f^h)$$

Step 1 : Relax $A^h u^h = f^h$ with initial guess v^h (n ting)

Step 2 : $r_h = f^h - A^h v^h$; ~~The~~ Restrict to coarse

$$r^{2h} = I_h^{2h} r^h$$

Step 3 : $A^{2h} e^{2h} = r^{2h}$ solve..

Step 4 : Interpolate $e^h = I_{2h}^h e^{2h}$

Correct $v^h \leftarrow v^h + e^h$.

Step 5 : $A^h u^h = f^h$ Relax (n times)

V cycle :-

Step 1 :- Relax $A^h u^h = f^h$ n times initial guess v^h

Step 2 :- Restrict $f^{2h} = I_h^{2h} r^h$

Relax $A^{2h} u^{2h} = f^{2h}$ n times init=0

Step 3 :-

Restrict $f^{4h} = I_{2h}^{4h} r^{2h}$

Step 4 :-

Solve $A^{Lh} u^{Lh} = f^{Lh}$.

Step N :-

Interpolate $u^{L/2\,h} = I_{Lh}^{L/2\,h} u^{Lh}$

Step N+1 :-

Correct $v^{L/2\,h} \leftarrow v^{\frac{2}{2}h} + u^{L/2}$

Step N+2 :-

Relax $A^{L/2\,h} v^{L/2\,h} = f^{L/2\,h}$ init. $v^{L/2\,h}$

Step N+3 :-

Step :-

Compact recursive def^n :- V cycle.

$$v^h \leftarrow V^h(v^h, f^h)$$

1. Relax n times $A^h u^h = f^h$

2. If Ω^h = coarsest grid goto 4

else

$$f^{2h} = I_h^{2h}\left(f^h - A^h v^h\right)$$

$$v^{2h} \leftarrow 0$$

$$v^{2h} \leftarrow V^{2h}\left(v^{2h}, f^{2h}\right)$$

3. Correct $v^h \leftarrow v^h + I_{2h}^h v^{2h}$

4. ~~Relax~~ Solve $A^h u^h = f^h$.

μ cycle:-

$$v^h \leftarrow M\mu^h(v^h, f^h)$$

1. Relax n times $A^h u^h = f^h$ with v^h init

2. If Ω^h = coarsest grid go to 4.

else

$$f^{2h} \leftarrow I_h^{2h}(f^h - A^h v^h)$$

$$v^{2h} \leftarrow 0$$

$$v^{2h} \leftarrow M\mu^{2h}(v^{2h}, f^{2h}) \quad \boxed{m \text{ times}}$$

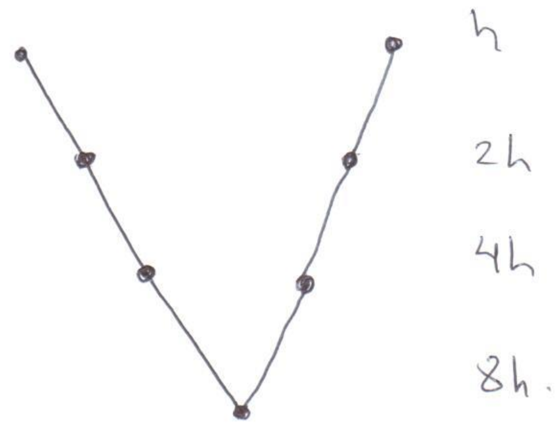
3. Correct $v^h \leftarrow v^h + I_{2h}^h v^{2h}$

4. Solve $A^h u^h = f^h$.

$m = 1$ V cycle.

$m = 2$ W cycle.

V cycle:-



W cycle:-

Full Multigrid V-cycle (FMG)

$$v^h \leftarrow FMG^h(f^h)$$

1. If Ω^h is coarsest grid set $v^h \leftarrow 0$ and go to 3

   else

$$f^{2h} \leftarrow I_h^{2h} f^h$$

$$v^{2h} \leftarrow FMG^{2h}(f^{2h})$$

2. Correct $v^h \leftarrow I_{2h}^h v^{2h}$

3. $v^h \leftarrow V^h(v^h, f^h)$ ν times.

Multigrid preconditioned CG

$$A x = b$$

Preconditioner.

$$P^{-1} A x = P^{-1} b.$$

$P^{-1} y \rightsquigarrow P$ is a good representation of A

$P \hat{x} = y \longrightarrow$ by Multigrid !!