E8 262 Lecture 17

Partial Element Equivalent Ckt (PEEC)

[Ref: Ruehli 1974, 1996, 1999.]

3D Modeling

(a) Quasistatic Conductor PEEC.
(b) Fullwave Conductor PEEC
(c) Dielectric PEEC (d) Nonorthogonal.
(e) Surface PEEC
(f) Fast compressed PEEC.

$$E_0(r,t) = \frac{J(r,t)}{\sigma} + \frac{\partial A(r,t)}{\partial t} + \nabla\phi(r,t)$$

$$A(r,t) = \sum_{k=1}^{K} \frac{\mu}{4\pi} \int G(r,r')\, J(r',t')\, dv'$$

$$t' = t - \frac{|r - r'|}{c}\sqrt{\epsilon_r \mu_r}$$

$$G(r,r') = \frac{1}{|r - r'|}$$

$$\phi(r,t) = \sum_{k=1}^{K} \frac{1}{4\pi\epsilon} \int G(r,r')\, q(r',t')\, dv'$$

$$E_0(r,t) = \frac{J(r,t)}{\sigma} + \sum_{k=1}^{K} \frac{\partial}{\partial t}\left[\frac{\mu}{4\pi}\int G(r,r')\, J(r',t')\, dv'\right.$$
$$+ \sum_{k=1}^{K} \nabla\left[\frac{1}{4\pi\epsilon}\int G(r,r')\, q(r',t')\, dv'.\right.$$

PEEC discretization

$$J = J_x\hat{x} + J_y\hat{y} + J_z\hat{z}.$$

Rectangular volume pulse basis fn.

$$P_{\gamma nk} = 1$$

$\gamma \to x$ or y or z.
$n \to n^{th}$ element on
$k \to k^{th}$ conductor.

$$J_{\gamma k}(t') = \sum_{n=1}^{N_{\gamma k}} P_{\gamma nk}\, J_{\gamma nk}(t_n)$$

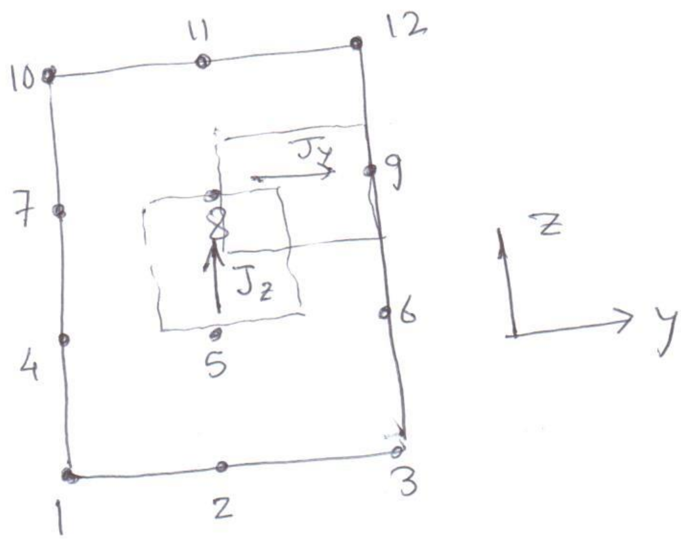
$$t_n = t - \frac{|r - r'|}{c}\left(\sqrt{\epsilon_r\mu_r}\right).$$

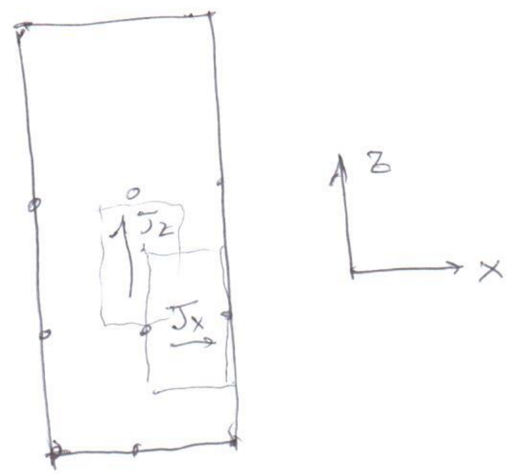
$$E_{0\gamma}(r,t) = \frac{J_\gamma(r,t)}{\sigma} + \sum_{k=1}^{K}\sum_{n=1}^{N_{\gamma k}} \frac{\mu}{4\pi}\left[\int G(r,r')\, dv'\right]\frac{\partial J_{\gamma nk}(t)}{\partial t}$$
$$+ \sum_{k=1}^{K} \frac{\partial}{\partial \gamma}\left[\frac{1}{4\pi\epsilon}\int G(r,r')\, q(r',t')\, dv'\right]$$

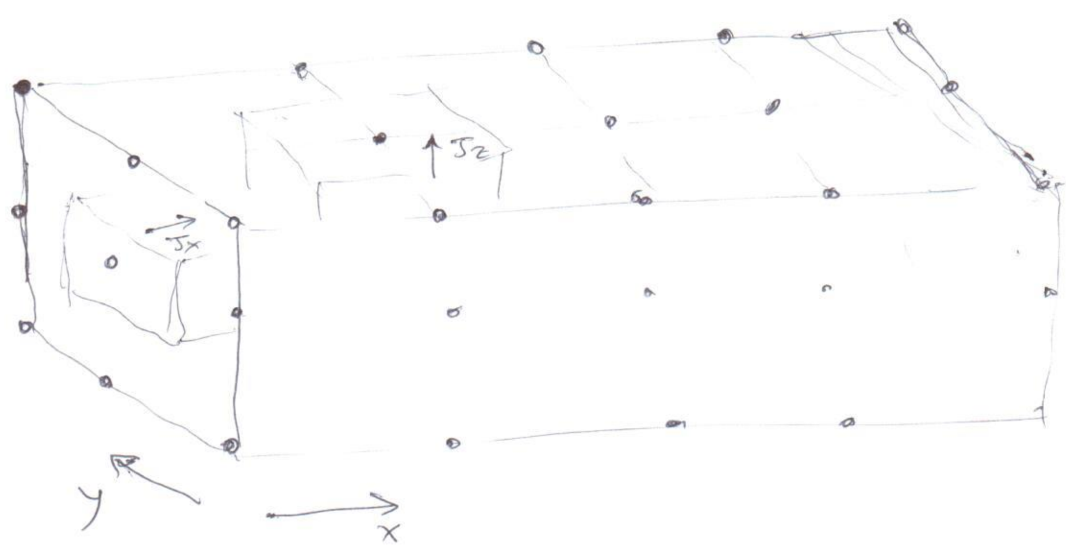
$$q_k(t_m) = \sum_{m=1}^{M_k} p_{mk}\, q_{mk}(t_m)$$

$M_k \longrightarrow$ surface cells.

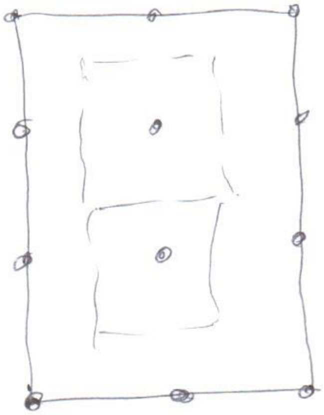
$p_{mk} = 1$ on m^{th} cell and 0 elsewhere.







Inductive Resistive partitions.



Capacitive partitions.

$$\frac{1}{\sigma}\int_{v_l} J_r(r,t)\,dv_l + \sum_{k=1}^{K}\sum_{n=1}^{N_{rk}} \frac{\mu}{4\pi}\left[\int_{v_l}\int_{v_{nk}} G(r,r')\,dv'\,dv_l\right]\frac{\partial J_{rnk}(t)}{\partial t}$$

$$+ \sum_{k=1}^{K}\frac{1}{4\pi\epsilon}\int_{v_l}\frac{\partial}{\partial r}\left[\int_{S_k} G(r,r')\,q(r',t')\,ds'\right]dv_l = 0$$

E_0

$\frac{1}{a_l}\times$ Eqn $= 0$

$$\boxed{V_r + V_l + V_c = 0}$$

Resistive term:-

$$V_r = \frac{1}{\sigma a_l}\int_{v_l} J_r(r,t)\,dv_l.$$

$$= \frac{1}{\sigma}\frac{1}{a_l}\frac{I_r}{\cancel{a_l}}\cancel{a_l}\,l$$

$$= \rho l \; I_r$$

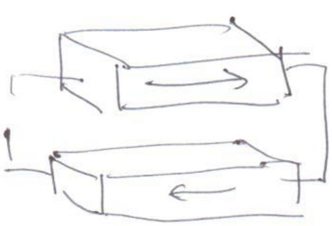
Inductive term

$$V_L = \sum_{k=1}^{K} \sum_{n=1}^{M_{\gamma_k}} \left[\frac{\mu}{4\pi} \frac{1}{a_l \, a_{m_{nk}}} \int_{v_l} \int_{V_{nk}} G(r, r') \, dv' \, dv_l \right] \frac{\partial I_{\gamma_{nk}}(t_{nk})}{\partial t}.$$

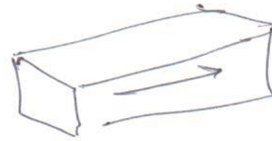
$$V_L = \sum_{k=1}^{K} \sum_{n=1}^{N_{\gamma_k}} L_{p_{l,\gamma_{nk}}} \frac{d}{dt} I_{\gamma_{nk}}(t_{nk})$$

partial inductance

vs. loop inductance



loop inductance



return through infinity.

or return has no EM effect

Capacitive term

$$F(r) = \int_{S_{mk}} G(r, r')\, q(r', t')\, ds'$$

$$\int_{V_\ell} \frac{\partial}{\partial r} F(r)\, dv_\ell = a_\ell \left[F\left(r + \frac{\Delta r}{2}\right) - F\left(r - \frac{\Delta r}{2}\right) \right]$$

shifted cap cells !

$$V_c = \sum_{k=1}^{K} \sum_{m=1}^{M_k} \left[q_{mk}(t_{mk}) \frac{1}{4\pi\epsilon} \int_{S_{mk}} G(r^+, r')\, ds' \right.$$

$$\left. - q_{mk}(t_{mk}) \frac{1}{4\pi\epsilon} \int_{S_{mk}} G(r^-, r')\, ds' \right]$$

$$V_c = \sum_{k=1}^{K} \sum_{m=1}^{M_k} Q_{mk}(t_{mk}) \left[PP_i(mk)^+ - PP_i(mk)^- \right]$$

$$PP_{ij} = \frac{1}{a_j} \int_{S_j} G(r, r')\, ds'$$

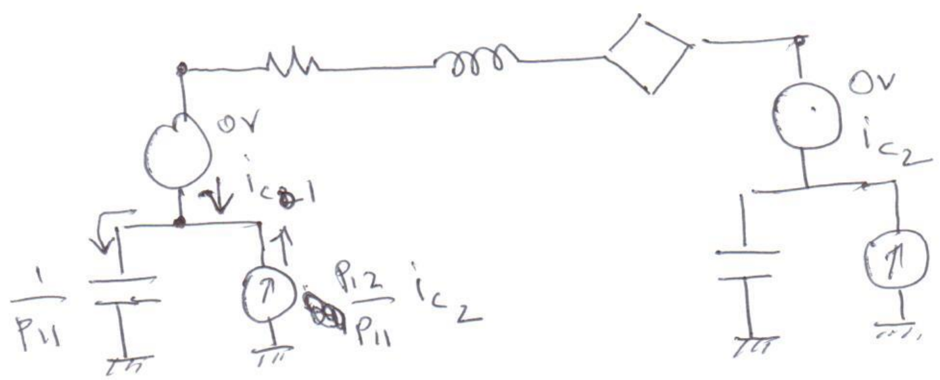
$$v_c = \phi_i - \phi_j$$

$$\phi_1 = P_{11} Q_1 + P_{12} Q_2$$

$$\frac{1}{P_{11}} \phi_1 = Q_1 + \frac{P_{12}}{P_{11}} Q_2$$

$$\frac{1}{P_{11}} \frac{d\phi_1}{dt} = \frac{dQ_1}{dt} + \frac{P_{12}}{P_{11}} \frac{dQ_2}{dt}$$

$$= i_{c_1} + \frac{P_{12}}{P_{11}} i_{c_2}$$



Equivalent ckt for 1 element.