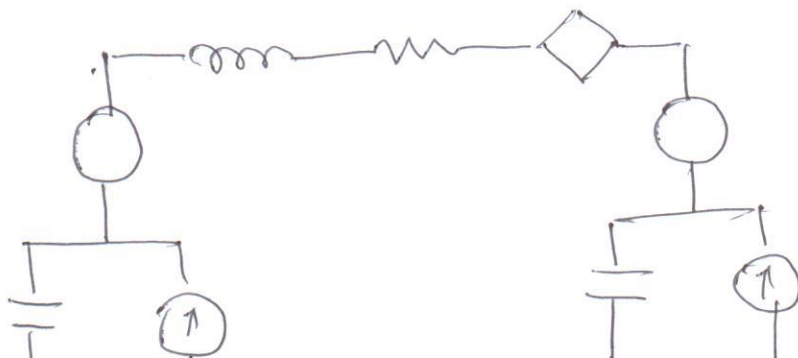


Advantages of PEEC

- ① Volumetric form^m ideal for on-chip cases (volumetric current flow)
- ② Better low freq stability.
- ③ "Stamps" provide interface between ckt and EM.
- ④ Multipurpose solvers (tran, ac, cap, ind)


$$E_0(r, t) = \frac{J(r, t)}{\delta} + \sum_{k=1}^K \frac{\partial}{\partial t} \left[\frac{\mu}{4\pi} \int G(r, r') J(r', t') dv' \right] + \sum_{k=1}^K \nabla \left[\frac{1}{4\pi\epsilon} \int G(r, r') \rho(r', t') dv' \right]$$



$$t' = t - \frac{R_{ij}}{v}$$

rPEEC: PEEC with retardation

$$U_i(\omega) = \sum_{j \neq i} \frac{P_{ij}}{P_{ji}} \phi_j'(\omega) e^{-jkR_{ij}}$$

Freq. domain 

Time domain.

ODE \longrightarrow DDE

delay differential eqⁿ.

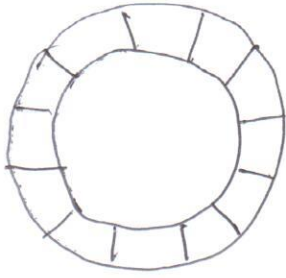
Maintain a history of v, i for ~~an~~ a window

if $t - t'_{ij} < \tau$ no delay.

$t - t'_{ij} > \tau$ interpolation. τ_{n-1} and τ_n .

τ is the time step

Storage in the form of circular buffers



$$\text{buffer size} = \frac{R_i^{\max}}{v\tau} + \frac{t_{\text{step}}^{\max}}{\tau}$$

↑
Why?

Dielectric PEEC

Recalled:-

$$\underline{E}_0(r,t) = \frac{\underline{J}(r,t)}{\sigma} + \frac{\partial A(r,t)}{\partial t} + \nabla\phi(r,t)$$

↙
 J_c

$$\begin{aligned} \nabla \times H &= \cancel{J^c} + \cancel{\frac{\partial D}{\partial t}} \\ &= \left[J^c + \epsilon_0(\epsilon_r - 1) \frac{\partial E}{\partial t} \right] + \epsilon_0 \frac{\partial E}{\partial t} \end{aligned}$$

↙
new J

implies: on a conductor.

$$\frac{J(r,t)}{\sigma} + \frac{\mu}{4\pi} \int G(r,r') \frac{\partial J}{\partial t} dv' + \frac{\nabla}{4\pi\epsilon_0} \int G(r,r') q^T(r',t') dv' = 0.$$

$$\Rightarrow \frac{J(r,t)}{\sigma} + \frac{\mu}{4\pi} \int G(r,r') \frac{\partial J_c}{\partial t} dv'$$

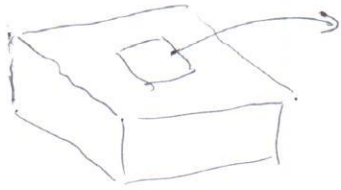
$$+ \epsilon_0 (\epsilon_r - 1) \frac{\mu}{4\pi} \int G(r,r') \frac{\partial^2 E(r',t')}{\partial t^2} dv'$$

$$+ \frac{\nabla}{4\pi\epsilon_0} \int G(r,r') q^T(r',t') dv' = 0.$$

on a dielectric

$$E(r,t) + \dots = 0.$$

Discretization:-



$$q_T = q_F + q_B$$

$q_T = q_F$ for conductor in air.

$q_T = q_F + q_B$ for conductor touching diel.

$q_T = q_B$ for dielectric ~~into~~ touching dielectric.

$$E_y + \epsilon_0 (\epsilon_r - 1) \frac{1}{4\pi} \int_V G_r(r, r') \frac{\partial^2 E_y}{\partial t^2} dv' + \dots = 0.$$

Testing:-

$$\frac{1}{a} \int_{V_c} E_y dv'_z = \frac{1}{a} \int_{a \times a} E_y da dl = V_c$$

Assume extra capacitance ϵ_s :-

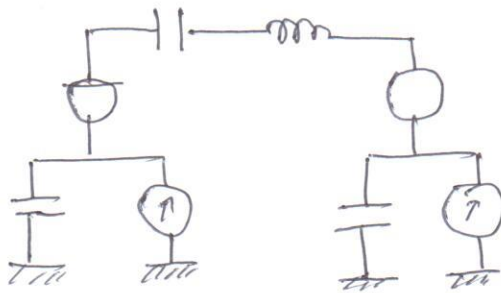
$$C_s = \frac{\epsilon_0 (\epsilon_r - 1) a}{l}$$

Now,

$$\epsilon_0 (\epsilon_r - 1) \frac{\mu}{4\pi} \int G(r, r') \frac{\partial^2 E_y(r', t_d)}{\partial t^2} dv'$$

$$i_c = C_r \frac{dv_c}{dt}$$

$$V_L = L_{pr1} \frac{di_c}{dt} = L_{pr1} C_r \frac{d^2 v_c}{dt^2}$$



Debye model for dielectric

$$C_e(s) = (\epsilon(s) - \epsilon_0) \frac{A}{d}$$

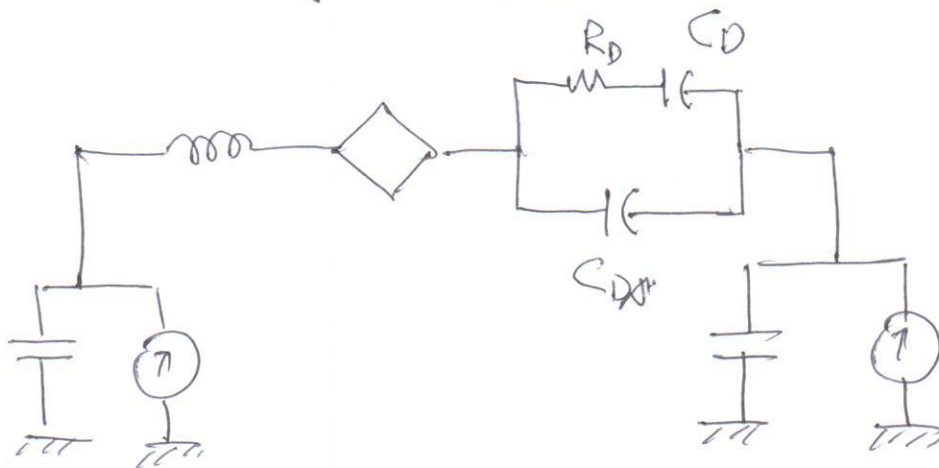
$$= \epsilon_0 \frac{A}{d} \left[(\epsilon_\infty - 1) + \frac{(\epsilon_s - \epsilon_\infty)}{1 + s\tau} \right]$$

$$sC_e(s) = \epsilon_0 \frac{A}{d} \left[s(\epsilon_\infty - 1) + \frac{s(\epsilon_s - \epsilon_\infty)}{1 + s\tau} \right]$$

$$sC_e(s) = sG_{D_\infty}(s) + Y_{RC}(s)$$

$$C_D = \frac{\epsilon_0 A}{d(\epsilon_s - \epsilon_\infty)}$$

$$R_D = \frac{\tau}{\left(\frac{\epsilon_0 A}{d(\epsilon_s - \epsilon_\infty)} \right)}$$



3 pole debye model

