E8-262 Lecture 19 + 20

Surface PEEC

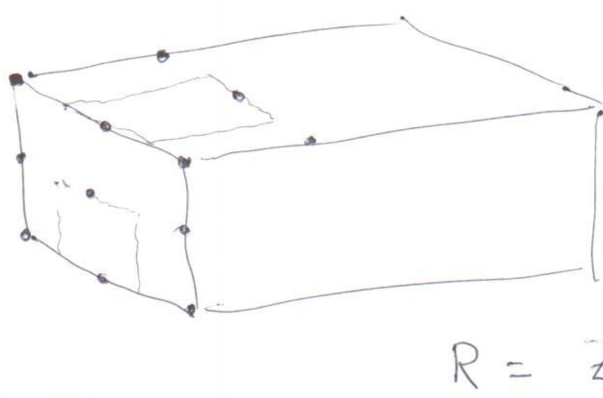
- Conductor
- Dielectric
- Freq stability vs. MPIE

Conductor surface PEEC :-

$$E = Z_s J + \frac{\partial A}{\partial t} + \nabla\phi$$

$$\frac{1}{l}\int E \cdot ds = \frac{Z_s}{l}\int J \cdot ds + \frac{1}{l}\int \frac{\partial A}{\partial t} ds + \frac{1}{l}\int \nabla\phi \, ds$$

$$0 = V_R + V_L + V_C$$



$$z_s = (1+j)\sqrt{\frac{\omega\mu}{2\sigma}}$$

$$R = Z_s \frac{b}{l}.$$

$$F(r) = \int G(r, r')\, q(r')\, ds'$$

$$\int_{a_\ell} \frac{\partial F(r)}{\partial r}\, ds = \ell \left[F\left(r + \frac{\Delta r}{2}\right) - F\left(r - \frac{\Delta r}{2}\right) \right]$$

$$v_c = q \frac{1}{4\pi\epsilon} \int G(r^+, r')\, ds' - q \frac{1}{4\pi\epsilon} \int G(r^-, r')\, ds'$$

$$= Q \left[PP_i^+ - PP_i^- \right]$$

where $PP_{ij} = \frac{1}{a_j} \int_{s_j} G(r, r')\, ds$

Dielectric

Surface PEEC based on PMCHWT.

[Ref: Gope, Ruehli MTT 2006]

[Ref: Poggio Miller in Computer Tech. for EM, Mittra]

$$\nabla \times E = -j\omega B - M$$
$$\nabla \times H = j\omega D + J$$
$$\nabla \cdot D = \rho$$
$$\nabla \cdot B = \rho_m$$

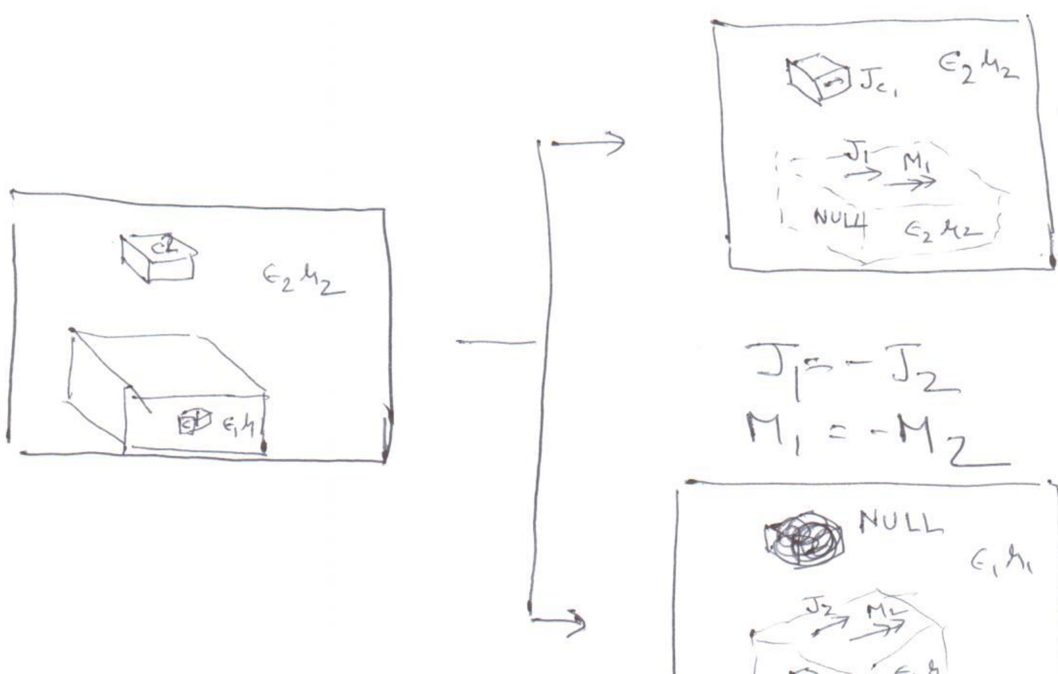
eq^n

$$E = -j\omega A - \nabla\phi - \frac{1}{\epsilon}\nabla \times F$$

$$H = -j\omega F - \nabla\phi_m + \frac{1}{\mu}\nabla \times A.$$

B.C.

$$n \times E_1 = n \times E_2 + n\, E^{inc}$$
$$n \times H_1 = n \times H_2$$



Exterior Problem

$$J_1 = -J_2$$
$$M_1 = -M_2$$

Interior Probl

$$E_{1,2}(r) = -j\omega A_{1,2}(r) - \nabla \phi_{1,2}(r) - \frac{1}{\epsilon_{1,2}} \nabla \times F_{1,2}(r)$$

$$H_{1,2}(r) = -j\omega F_{1,2}(r) - \nabla \phi^m_{1,2}(r) + \frac{1}{\mu_{1,2}} \nabla \times A_{1,2}(r)$$

$$A_{1,2}(r) = \mu_{1,2} \int G_{1,2}(r, r') \, [+,-] \, J_1(r) \, dr'$$

$$F_{1,2}(r) = \epsilon_{1,2} \int G_{1,2}(r, r') \, [+,-] \, M_1(r) \, dr'$$

$$\phi_{1,2}(r) = \frac{1}{\epsilon_{1,2}} \int G_{1,2}(r, r') \, [+,-] \, q_1 \, dr'$$

$$\phi^m_{1,2}(r) = \frac{1}{\mu_{1,2}} \int G_{1,2}(r, r') \, [+,-] \, q^m_1 \, dr'$$

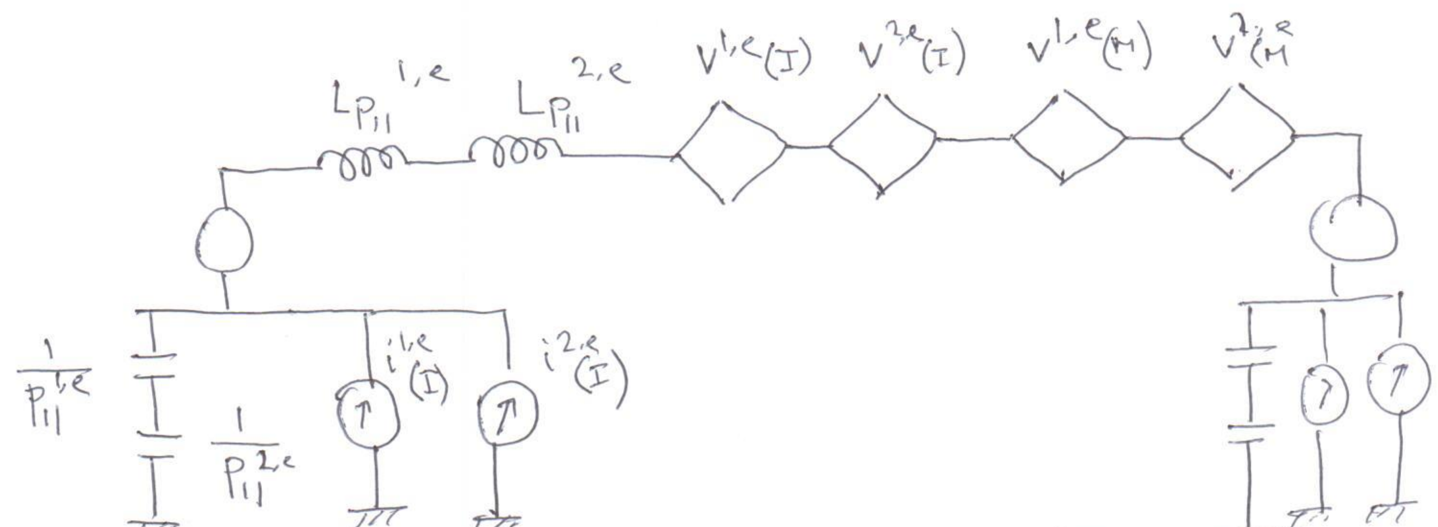
$$n \times E^{inc}(r) = n \times \left[j\omega \left(A_1(r) - A_2(r)\right) + \nabla \left(\phi_1(r) - \phi_2(r)\right) + \frac{1}{\epsilon} \nabla \times F_1(r) - \frac{1}{\epsilon_2} \nabla \times F_2(r) \right]$$

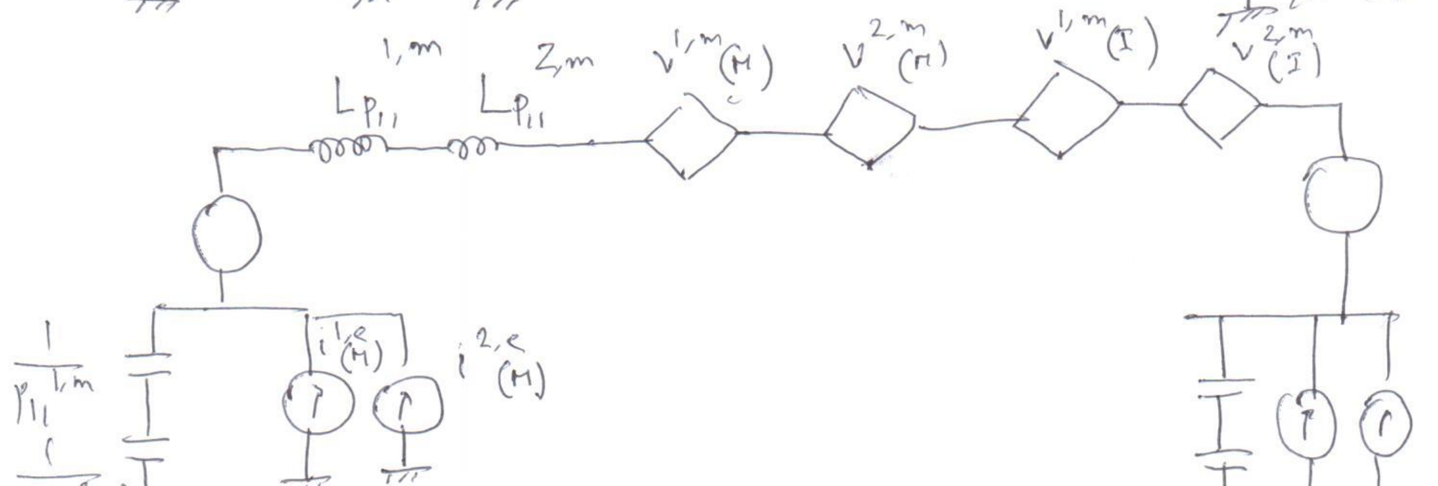
$$0 = n \times \left[j\omega \left(F_1(r) - F_2(r)\right) + \nabla \left(\phi^m_1(r) - \phi^m_2(r)\right) - \frac{1}{\mu_1} \nabla \times A_1(r) - \frac{1}{\mu_2} \nabla \times A_2(r) \right]$$

$$v_L = j\omega \frac{1}{l} \int_{s'} \left(A_1(r) - A_2(r) \right) ds'$$

$$= j\omega \frac{1}{l} \int_{s'} G J_1 \, ds' - j\omega \frac{1}{l} \int G J_2 \, ds'$$

$$= \underbrace{j\omega \frac{1}{l} \int_{s'} G J_1 \, ds'}_{\text{Interior contribution}} + \underbrace{j\omega \frac{1}{l} \int G J_1 \, ds'}_{\text{Exterior contribution.}}$$



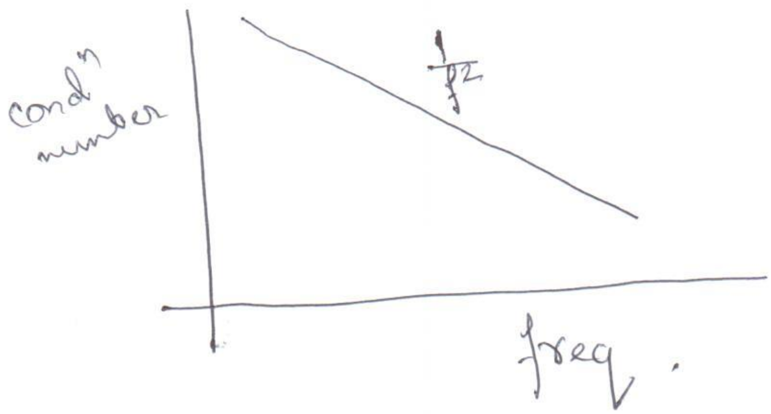


Freq dependence in PEEC

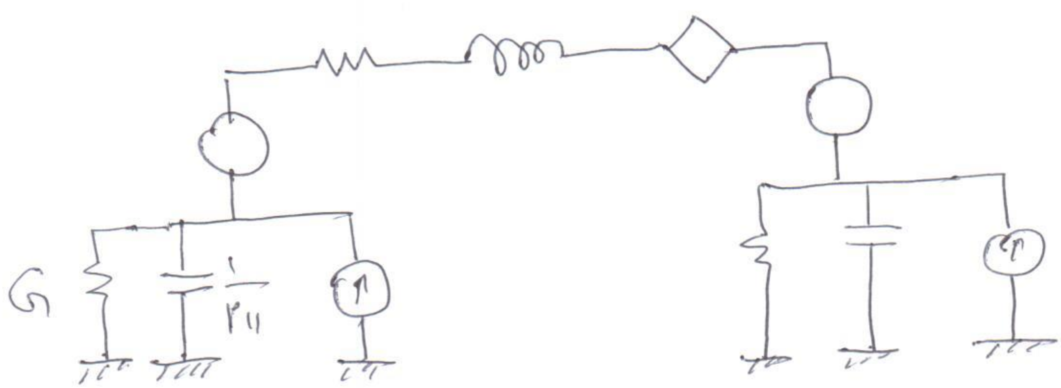
[Ref: Gope, Ruehli et al. TADVP]

Recollect in MPIE EFIE :-

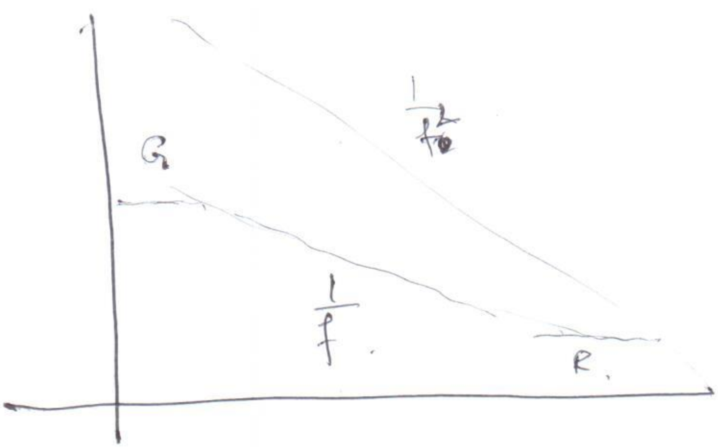
$$M = j\omega L + \frac{APA^T}{j\omega}$$



Now in PEEC :-



$$
\begin{bmatrix}
 & & & & 1 & -1 & \\
 & G+\frac{s}{p_{11}} & & & & -1 & -\frac{p_{12}}{p_{11}} \\
 & & & & -1 & & 1 \\
 & & & G+\frac{s}{p_{22}} & & -\frac{p_{12}}{p_{22}} & -1 \\
1 & & -1 & & R+j\omega L & & \\
1 & -1 & & & & & \\
 & & 1 & -1 & & &
\end{bmatrix}
\begin{bmatrix}
V_{C_1} \\ \Phi_1 \\ V_{C_2} \\ \Phi_2 \\ I_L \\ I_{C_1} \\ I_{C_2}
\end{bmatrix}
$$



Applied to EFIE (SPIE)

$$\begin{bmatrix} j\omega L & -A \\ P \times A^T & D \end{bmatrix} \begin{bmatrix} J \\ V \end{bmatrix} = \begin{bmatrix} \langle t, E_{inc} \rangle \\ \ \end{bmatrix}$$

$$M_{N_e \times N_e} \times J_{N_e \times 1} = \left[j\omega L_{N_e \times N_e} + A_{N_e \times N_p} D^{-1}_{N_p \times N_p} P_{N_p \times N_p} A^T_{N_p \times N_e} \right] \times J_{N_e \times 1}$$

$$= b_{N_e \times 1}$$

if:

$$\Rightarrow \cancel{j\omega L_{N_e \times N_e}} - V = D^{-1}_{N_p \times N_p} P_{N_p \times N_p} A^T_{N_p \times N_e} J_{N_e \times 1}$$

$$D_{ii} = \frac{S}{P_{ii}}$$

$\rightsquigarrow$ Augmented EFIE: Chew