E 8262 CAD for high speed circuits

Lecture 6 [Ref: C. Paul Chap 1]

Transmission line basics and TEM approx.

Maxwell's eqn

$$\nabla \times E = -j\omega B \qquad \nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times H = j\omega D + J \quad \longleftrightarrow \quad \nabla \times H = \frac{\partial D}{\partial t} + J$$

$$\nabla \cdot D = \rho \qquad \nabla \cdot D = \rho$$

$$\nabla \cdot B = 0 \qquad \nabla \cdot B = 0$$

Freq domain — Time domain

~~[illegible]~~

PDE ODE

Distributed Lumped

Differential form.

Stokes theorem

$$\iint_S (\vec{\nabla} \times \vec{V}) \cdot ds = \oint_C \vec{V} \cdot dl$$

Gauss divergence theorem

$$\cancel{\oiint A \cdot ds} \quad \iiint_V \nabla \cdot \vec{A} \, dv = \oiint_S \vec{A} \cdot ds .$$

$$\oint_{\partial l} E \cdot dl = \iint -\frac{\partial B}{\partial t} \, ds$$

$$\oint H \cdot dl = \iint \frac{\partial D}{\partial t} \, ds + \iint J \, ds$$

$$\oiint D \cdot ds = \iiint \rho \, dv$$

$$\oiint B \cdot ds = 0$$

Integral form time domain

Constitutive relⁿ

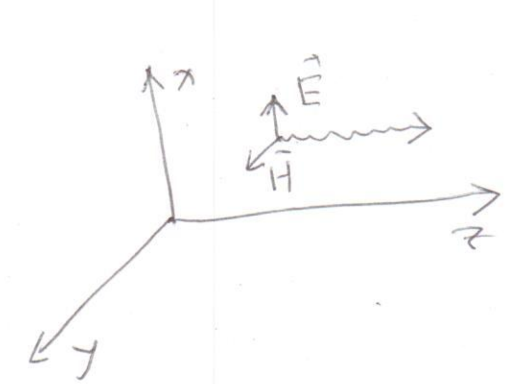
$$D = \epsilon E$$

$$B = \mu H$$

$$J = \sigma E$$

TEM assumption/approximation.

Defⁿ



Transverse Electromagneti

Maxwell's eqⁿ under TEM

$$(\nabla_t + \nabla_z)\times \vec{E}_t = \underbrace{\nabla_t \times \vec{E}_t}_{\text{z dir}} + \underbrace{\nabla_z \times \vec{E}_t}_{\text{on t plane}} = -\mu \frac{\partial H_t}{\partial t}$$

implies.

$$\boxed{\begin{aligned} \nabla_t \times \vec{E}_t &= 0 \\ \nabla_z \times \bar{E}_t &= -\mu \frac{\partial H_t}{\partial t} \end{aligned}} \quad — (1)$$

Similarly.

$$\nabla \times H = \frac{\partial D}{\partial t} + J$$

$$\nabla_t \times H_t + \nabla_z \times H_z = \epsilon \frac{\partial \vec{E}_t}{\partial t} + J$$

$$\boxed{\begin{aligned} \nabla_t \times H_t &= 0 \\ \nabla_z \times H_t &= \epsilon \frac{\partial E_t}{\partial t} + \sigma E_t \end{aligned}} \quad — (2)$$

Now since: $\nabla_t \times \nabla_t f(x,y) = 0$.

$$\vec{E}_t = e(z,t)\, \nabla_t \phi(x,y)$$

E can be expressed as gradient of a scalar quality → lets say ϕ.

Now:-

$$\nabla_t \cdot \vec{E}_t = 0$$

$$e(z,t)\, \nabla_t \cdot \nabla_t \phi(x,y) = 0$$

$$\Rightarrow \nabla_t^2 \phi(x,y) = 0$$

Laplace eq^n → static distribution
→ it is possible to define V, I.

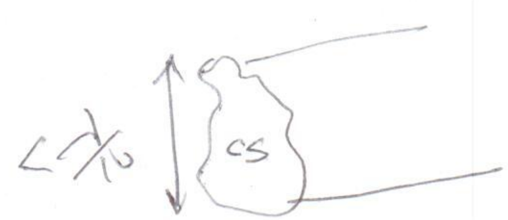
Properties of TEM:

(1) E_t and H_t are $\perp$

(2) We can define V, I in transverse ~~direction~~ plane.

Necessary for TEM:- (0) >1 conductor.

(1) cross-section is electrically small



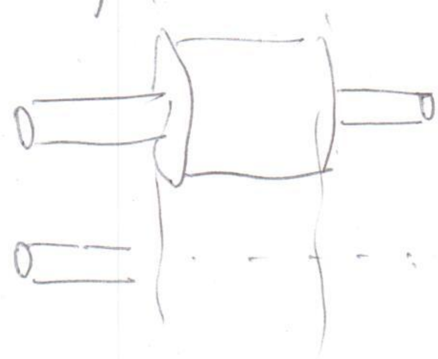
(2) PEC. $\sigma \cdot \sigma \rightarrow E_z$ exists (3)

(3) H...

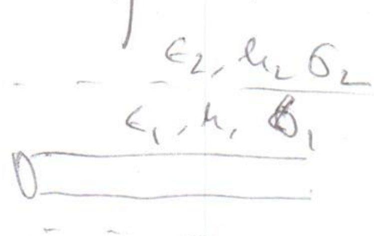
Quasi-TEM mode

(1) for finite conductivity. (lossy cond)

(2) Non uniform cond.



(3) Non uniform diel :-



Higher order modes $\rightarrow$ cutoff freq.

Wave eq^n for TEM mode

$$\boxed{\begin{aligned} \nabla_z \times \vec{E}_t &= -\mu \frac{\partial H_t}{\partial t} \\ \nabla_z \times H_t &= \frac{\partial D}{\partial t} + J. \end{aligned}}$$

$$\Rightarrow \quad a_z \times \frac{\partial E_t}{\partial z} = -\mu \frac{\partial H}{\partial t}$$

$$a_z \times a_z \times \frac{\partial E_t}{\partial z} = -a_z \times \mu \frac{\partial H}{\partial t}$$

$$\Rightarrow \quad -\frac{\partial E_t}{\partial z} = -a_z \times \mu \frac{\partial H}{\partial t}$$

$$\Rightarrow \quad \frac{\partial E_t}{\partial z} = \mu \left[a_z \times \frac{\partial H}{\partial t} \right]$$

Also, $\hat{a}_z \times \frac{\partial H}{\partial z} = \epsilon \frac{\partial E}{\partial t} + \sigma E_t$

$$\frac{\partial^2 E_t}{\partial z^2} = \mu \frac{\partial}{\partial z} \left[a_z \times \frac{\partial H}{\partial t} \right]$$

$$\frac{\partial}{\partial t} \left(a_z \times \frac{\partial H}{\partial z} \right) = \epsilon \frac{\partial^2 E}{\partial t^2} + \sigma \frac{\partial E}{\partial t}$$

$$\Rightarrow \quad \frac{\partial^2 E}{\partial z^2} = \mu\sigma \frac{\partial E}{\partial t} + \mu\epsilon \frac{\partial^2 E}{\partial t^2}$$

Similarly:-

$$\frac{\partial^2 H}{\partial z^2} = \mu\sigma \frac{\partial H}{\partial t} + \mu\epsilon \frac{\partial^2 H}{\partial t^2}$$

$$E(x, y, z, t) = e(z,t)\, \nabla_t \phi(x, y)$$

$$\frac{\partial^2 e(z,t)}{\partial z^2} = \mu\sigma \frac{\partial e(z,t)}{\partial t} + \mu\epsilon \frac{\partial^2 e(z,t)}{\partial t^2}$$

If $\sigma = 0$ Lossless diel

$$\frac{\partial^2 e}{\partial z^2} = \mu\epsilon \frac{\partial^2 e}{\partial t^2}$$

$$e(z,t) = e^+\left(t - \frac{z}{v}\right) + e^-\left(t + \frac{z}{v}\right)$$

$$v = \frac{1}{\sqrt{\mu\epsilon}}$$