Lecture 7 EE-262

Transmission line basics (recap)

$$\nabla_t \times E_t + \nabla_z \times E_t = -\mu \frac{\partial H_t}{\partial t}$$

$$\nabla_t \times H_t + \nabla_z \times H_t = \epsilon \frac{\partial E_t}{\partial t} + J$$

Can define V on the cross section

$$\frac{\partial E_t}{\partial z} = \mu \left[a_z \times \frac{\partial H_t}{\partial t} \right]$$

$$-\frac{\partial H_t}{\partial z} = \epsilon \left[a_z \times \frac{\partial E_t}{\partial t} \right] + \sigma \left[a_z \times E_t \right]$$

$$\frac{\partial^2 E_t}{\partial z^2} = \mu \sigma \frac{\partial E_t}{\partial t} + \mu \epsilon \frac{\partial^2 E_t}{\partial t^2}$$

$$\frac{\partial^2 H_t}{\partial z^2} = \mu \sigma \frac{\partial H_t}{\partial t} + \mu \epsilon \frac{\partial^2 H_t}{\partial t^2}$$

Time domain wave eq^n.

$$E(x,y,z,t) = e(z,t) \nabla \phi(x,y)$$

Freq domain wave eq^n.

$$e(z,t) = Re\left\{\hat{e}(z)e^{j\omega t}\right\}$$

$$\frac{d^2\hat{e}(z)}{dt^2} = -\omega^2\mu\epsilon\,\hat{e}(z)$$

$$\hat{e}(z) = \hat{e}^+e^{-j\beta z} + \hat{e}^-e^{+j\beta z}$$

$$\beta = \omega\sqrt{\mu\epsilon}$$

$\downarrow$

phase constant rad/m.

For lossy diel:

$$\frac{d^2\hat{e}(z)}{dz^2} = j\omega\mu\sigma\,\hat{e}(z) + (j\omega)^2\mu\epsilon\,\hat{e}(z)$$

$$= \gamma^2\hat{e}(z)$$

$$\gamma = j\omega\mu(\sigma + j\omega\epsilon) = \alpha + j\beta$$

$$\hat{e}(z) = \hat{e}^+e^{-\alpha z}e^{-j\beta z} + \hat{e}^-e^{+\alpha z}e^{+j\beta z}$$

TEM wave eq^n $\rightarrow$ TEM Tx line eq^n

$$-\frac{\partial E_t}{\partial z} = -\mu \left[\vec{a}_z \times \frac{\partial H_t}{\partial t} \right]$$

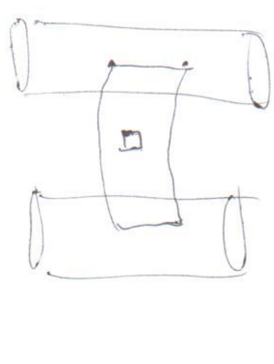
$$\int_c -\frac{\partial E_t}{\partial z} \cdot dl = -\mu \int_c \left(\vec{a}_z \times \frac{\partial H_t}{\partial t} \right) \cdot dl$$

$$\Rightarrow \frac{\partial V(z,t)}{\partial z} = -\mu \frac{\partial}{\partial t} \int (a_z \times H_t) \cdot dl$$

$\boxed{A \cdot B \times C = C \cdot A \times B}$

$$= -\mu \frac{\partial}{\partial t} \left[-H_t \cdot (a_z \times dl) \right]$$

$$= \mu \frac{\partial}{\partial t} \int H_t \cdot \vec{a}_n \, dl$$

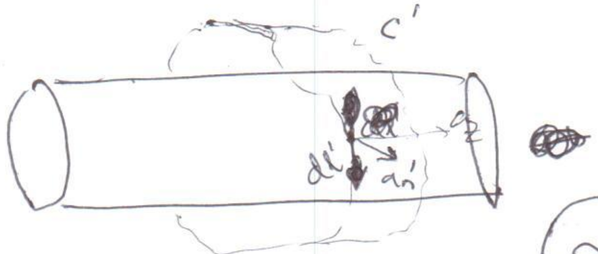


$$l \Delta z = \frac{\Psi}{I(z,t)} = \frac{\int \mu H \cdot \vec{a}_n \, ds}{I(z,t)}$$

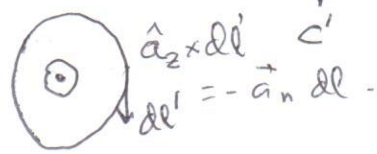
$$= \Delta z \int \mu H \cdot \vec{a}_n \, dl$$

$$\boxed{\frac{\partial V(z,t)}{\partial z} = -\lambda \frac{\partial I(z,t)}{\partial t}}.$$

$$-\frac{\partial \vec{H}_t}{\partial z} = \sigma\left(\hat{a}_z \times E_t\right) + \epsilon\left[\hat{a}_z \times \frac{\partial \vec{E}_t}{\partial t}\right]$$



$$I = \int_{c'} H_t \cdot dl'$$



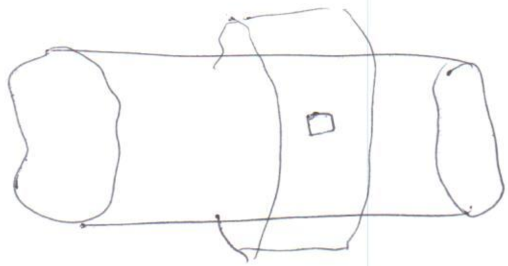
$$-\oint_{c'} \frac{\partial \vec{H}_t}{\partial z} \cdot dl' = \sigma \oint_{c'} \left(\vec{a}_z \times E_t\right) \cdot dl' + \epsilon \oint_{c'} \left[\hat{a}_z \times \frac{\partial E}{\partial t}\right] \cdot dl'$$

$$\left(\hat{a}_z \times E_t\right) \cdot dl' \quad \cancel{= -\left(E_t \times a_z\right) \cdot dl'}$$

$$= -E_t \cdot \left(\hat{a}_z \times dl'\right) \quad \cancel{= E_t \cdot -\hat{a}_n'\, dl'}$$

$$= E_t \cdot a_n'\, dl. \quad \cancel{= E_t}$$

$$-\frac{\partial I(z,t)}{\partial z} = \sigma \cdot E_t \cdot a_n' dl + \epsilon \frac{\partial E}{\partial t} \cdot a_n\, dl$$



$$C\Delta z = \frac{Q}{V} = \frac{\epsilon \oint_{s'} \vec{\mathcal{E}}_t \cdot \vec{a}_n{}' \, ds}{V(z,t)}$$

$$= \frac{\Delta z \, \epsilon \oint \mathcal{E}_t \cdot \vec{a}_n \cdot dl}{V(z,t)}$$

$$g\Delta z = \frac{\sigma \oint_{s'} E_t \cdot a_n \, ds'}{V(z,t)} = \frac{\Delta z \, \sigma \oint E_t \cdot a_n \, dl'}{V(z,t)}$$

$$\boxed{\frac{\partial I(z,t)}{\partial z} = - g V(z,t) - c \frac{\partial V(z,t)}{\partial t}}$$

$$\frac{\partial^2 V(z,t)}{\partial z^2} = gl \frac{\partial V(z,t)}{\partial t} + lc \frac{\partial^2 V(z,t)}{\partial t^2}$$

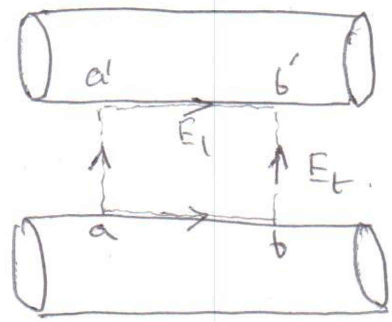
$$\frac{\partial^2 I(z,t)}{\partial z^2} = gl \frac{\partial I(z,t)}{\partial t} + lc \frac{\partial^2 I(z,t)}{\partial t^2}$$

Compare with TEM wave eq^n.

$$g l = \mu \sigma$$

$$\rho c = \mu \epsilon .$$

Two conductor transmission line



$$\int E \cdot dl = -\mu \frac{d}{dt} \int H \cdot ds$$

$$\Rightarrow \int_a^{a'} E_t \cdot dl + \int_{a'}^{b'} E_1 \cdot dl + \int_{b'}^{b} E_1 \cdot dl + \int_b^{a} E_t \cdot dl = \mu \frac{d}{dt} \int H \cdot a_n$$

$$-V(z,t) \qquad r_1 \Delta z\, I(z,t) \qquad V(z+\Delta z, t) \qquad r_0 \Delta z\, I(z,t)$$

$$\Rightarrow \frac{V(z+\Delta z, t) - V(z,t)}{\Delta z} = -(r_1 + r_0)\, I(z,t) + \frac{\mu \frac{d}{dt} \int H \cdot a_n\, ds}{\Delta z}$$

$$\psi = -\mu \lim_{\Delta z \to 0} \frac{1}{\Delta z} \int \vec{H}_t \cdot \vec{a}_n\, ds$$

$$= l\, I$$

$$\Rightarrow \partial V \quad = -r I - l\, \partial I$$

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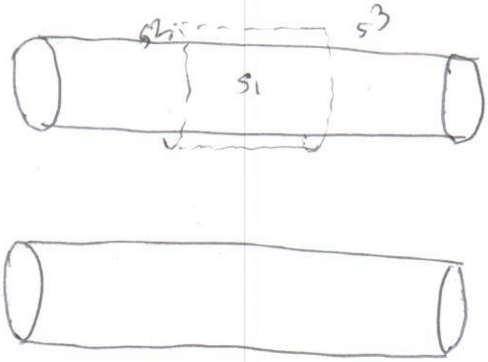

$$\oint\!\!\oint J\cdot ds = -\frac{\partial Q_v}{dt}$$

$$\Rightarrow \oint\!\!\oint_{s_1} J\cdot ds + \oint\!\!\oint_{s_2} J\cdot ds + \oint\!\!\oint_{s_3} J\cdot ds = -\frac{dQ_v}{dt}$$

$$-I(z,t) \quad + \quad I(z+\Delta z, t)$$

$$\sigma \oint\!\!\oint_{s_1} E_t \cdot ds \qquad\qquad c\,V(z,t)\,\Delta z$$

$$g\,V(z,t)\,\Delta z$$

$$\Rightarrow \frac{\partial I(z,t)}{\partial z} = -g\,V(z,t) - c\,\frac{\partial V(z,t)}{\partial t}$$