$\boxed{E8262}$ Lecture 9 :- C. Paul
Chap 8

Two conductor Tx line

Freq domain analysis.

$\boxed{Eq^n}$

$$\frac{dV(z)}{dz} = -(r + j\omega l)\,I(z)$$

$$\frac{dI(z)}{dz} = -(g + j\omega c)\,V(z)$$

Lossless :- both $r = 0$ & $g = 0$.

$\boxed{\beta, v, Z_c}$ $\frac{dV(z)}{dz} = -j\omega l\,I(z)$; $\frac{dI(z)}{dz} = -j\omega c\,V(z)$.

$$\frac{d^2V}{dz^2} = -\omega^2 lc\,V \Rightarrow V(z) = V^+ e^{-j\beta z} + V^- e^{+j\beta z}$$

phase const. $\beta = \omega\sqrt{lc}$

phase vel $v = \frac{1}{\sqrt{lc}}$

$$I(z) = I^+ e^{-j\beta z} + I^- e^{+j\beta z}.$$

$I^+ = \frac{V^+}{Z_c}$ $\quad I^- = -\frac{V^-}{Z_c}$

$$Z_c = \sqrt{\frac{L}{C}} \quad \text{how?}$$

$\downarrow$

characteristic impedance.

$\boxed{\Gamma \text{ and } Z_{in}.}$

$$\Gamma(z) = \frac{V^- e^{j\beta z}}{V^+ e^{-j\beta z}} = \frac{V^-}{V^+} e^{j2\beta z}.$$

$$V(z) = V^+ e^{-j\beta z}\left[1 + \Gamma(z)\right]$$

$$I(z) = \frac{V^+}{Z_C} e^{-j\beta z}\left[1 - \Gamma(z)\right]$$

At $z = L$ $\quad \cancel{\Gamma_L = \frac{V^-}{V^+} e^{j2\beta L}}$

$$Z_L = Z_C \left[\frac{1 + \Gamma(L)}{1 - \Gamma(L)}\right]$$

$$\Gamma_L = \frac{Z_L - Z_C}{Z_L + Z_C}$$

$$\Gamma(z) = \Gamma(L)\, e^{j2\beta(z-L)}$$

$$\Rightarrow \quad V(z) = V^+ e^{-j\beta z}\left[1 + \Gamma_L\, e^{j2\beta(z-L)}\right]$$

$$I(z) = \frac{V^+}{Z_c} e^{-j\beta z}\left[1 - \Gamma_L\, e^{j2\beta(z-L)}\right]$$

$$Z_{in} = \frac{V(z)}{I(z)} = Z_c \frac{\left[1 + \Gamma(z)\right]}{\left[1 - \Gamma(z)\right]}$$

$$Z_{in}(0) = Z_c \frac{\left[1 + \Gamma(0)\right]}{\left[1 - \Gamma(0)\right]}$$

$$= Z_c \left[\frac{1 + \Gamma_L e^{-j2\beta L}}{1 - \Gamma_L e^{-j2\beta L}}\right]$$

Use $\Gamma_L e^{-j2\beta L} = \dfrac{\Gamma e^{-j\beta L}}{e^{+j\beta L}} = \dfrac{(Z_L - Z_c)\, e^{-j\beta L}}{(Z_L + Z_c)\, e^{+j\beta L}}$

$$\boxed{Z_{in}(0) = Z_c \frac{Z_L + jZ_c \tan(\beta L)}{Z_c + jZ_L \tan(\beta L)}}$$

$$L = \lambda/4$$

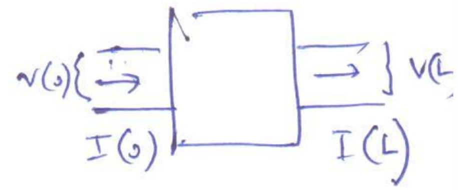
$$Z_{in}(0) = \frac{Z_c^2}{Z_L}$$

$$L = n\lambda/2$$

$$Z_{in}(0) = Z_L$$

$$Z_{in}(z \pm n\lambda/2) = Z_{in}(z).$$

ABCD, Z, Y parameters



$$\begin{bmatrix} V(L) \\ I(L) \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V(0) \\ I(0) \end{bmatrix}$$

$$\begin{bmatrix} V(0) \\ I(0) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \frac{1}{Z_C} & -\frac{1}{Z_C} \end{bmatrix} \begin{bmatrix} V^+ \\ V^- \end{bmatrix}$$

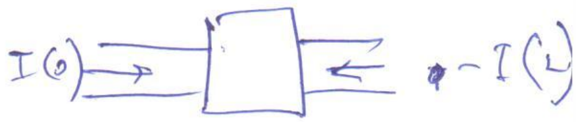
$$\begin{bmatrix} V(L) \\ I(L) \end{bmatrix} = \begin{bmatrix} e^{-j\beta L} & e^{j\beta L} \\ \frac{1}{Z_C} e^{-j\beta L} & -\frac{1}{Z_C} e^{j\beta L} \end{bmatrix} \begin{bmatrix} V^+ \\ V^- \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} V(L) \\ I(L) \end{bmatrix} = \begin{bmatrix} e^{-j\beta L} & e^{j\beta L} \\ \frac{1}{Z_C} e^{-j\beta L} & -\frac{1}{Z_C} e^{j\beta L} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ \frac{1}{Z_C} & -\frac{1}{Z_C} \end{bmatrix}^{-1} \begin{bmatrix} V(0) \\ I(0) \end{bmatrix}$$

$$= \begin{bmatrix} e^{-j\beta L} & e^{j\beta L} \\ \frac{1}{Z_C} e^{-j\beta L} & -\frac{1}{Z_C} e^{j\beta L} \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1 & Z_C \\ 1 & -Z_C \end{bmatrix} \begin{bmatrix} V(0) \\ I(0) \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\beta L) & -j Z_C \sin(\beta L) \\ -j \frac{\sin(\beta L)}{Z_C} & \cos(\beta L) \end{bmatrix} \begin{bmatrix} V(0) \\ I(0) \end{bmatrix}$$

Z parameters:



$$\begin{bmatrix} V(0) \\ V(L) \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I(0) \\ -I(L) \end{bmatrix}$$

$$V(L) = \cos(\beta L)\, V(0) - j Z_c \sin(\beta L)\, I(0)$$

$$I(L) = -j \frac{\sin \beta L}{Z_c} V(0) + \cos(\beta L)\, I(0)$$

$$\Rightarrow V(L) = \cos(\beta L) \left[-\frac{Z_c}{j \sin \beta L} I(L) + \frac{Z_c \cos \beta L}{j \sin \beta L} I(0) \right] - j Z_c \sin \beta L\, I(0)$$

$$\Rightarrow V(L) = \frac{Z_c \cos(\beta L)}{j \sin(\beta L)} \left[-I(L) \right] + \frac{Z_c \left(\cos^2 \beta L + \sin^2 \beta L \right)}{j \sin \beta L} I(0).$$

$$Z = \begin{bmatrix} \frac{Z_c \cos(\beta L)}{j \sin(\beta L)} & \frac{Z_c}{j \sin(\beta L)} \\ \frac{Z_c}{j \sin(\beta L)} & \frac{Z_c \cos(\beta L)}{j \sin(\beta L)} \end{bmatrix}$$

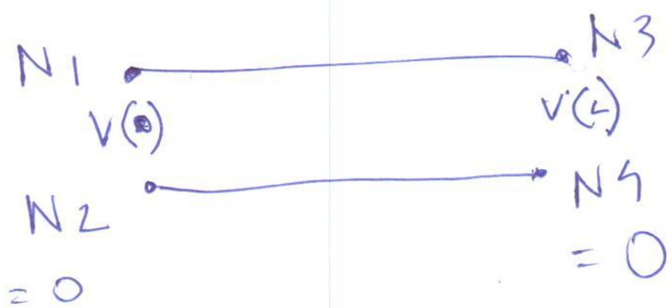
SPICE simulation. lossless

TX N1 N2 N3 N4 $Z = Z_C$ $T_D = TD$

Stamp of spice matrix $\equiv$ Z or Y.

Z parameters depend on. (βL) and Z_c

$$\beta L = \omega \sqrt{lc}\, L = \frac{\omega}{v} L = \omega TD.$$



$$\begin{pmatrix} I(0) \\ -I(L) \end{pmatrix} = Y \begin{pmatrix} V_0 \\ V_L \end{pmatrix}$$

lossy line :-

$$\frac{d\,V(z)}{dz} = -z\,I(z)$$

$$\frac{d\,I(z)}{dz} = -y\,V(z)$$

$$\frac{d^2V(z)}{dz^2} = zy\,V(z) \; ; \quad \frac{d^2I(z)}{dz^2} = yz\,I(z)$$

$$\gamma = \sqrt{zy} = \sqrt{(r+j\omega L)(g+j\omega c)}$$

$$= \alpha + j\beta .$$

$$V(z) = V^+e^{-\gamma z} + V^-e^{+\gamma z},$$

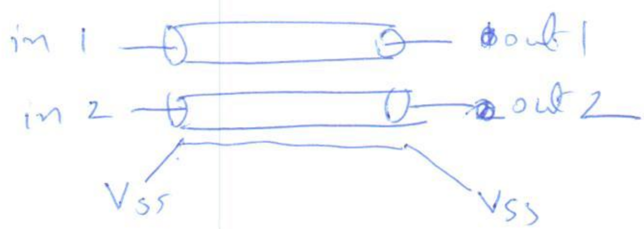
$$Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{r+j\omega L}{g+j\omega C}}$$

SPICE lossy transmission line

W element modeling

Wline1 in1 in2 Vss out1 out2 Vss

+ RLGCmodel = 'modelname' N=2 L=0.1



$\boxed{\text{Include GDimag = yes}}$

.MODEL modelname ModelType = RLGC, N=1

+ Lo = 3.8e-7

+ Co = 1.3e-10

+ Ro = 2.74e0

+ G0 = 0

+ Rs = 1e-3

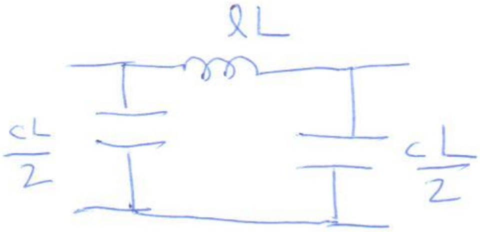
+ Gd = 0.07

+ wp = 0.07

$$R(f) = R_0 + \sqrt{f}\,(1+j)\,R_S$$

$$G(\omega) = G_0 + \sum_k G_{dk} \frac{j\omega}{j\omega + \omega_{pk}}$$

Pi model



Tmodel