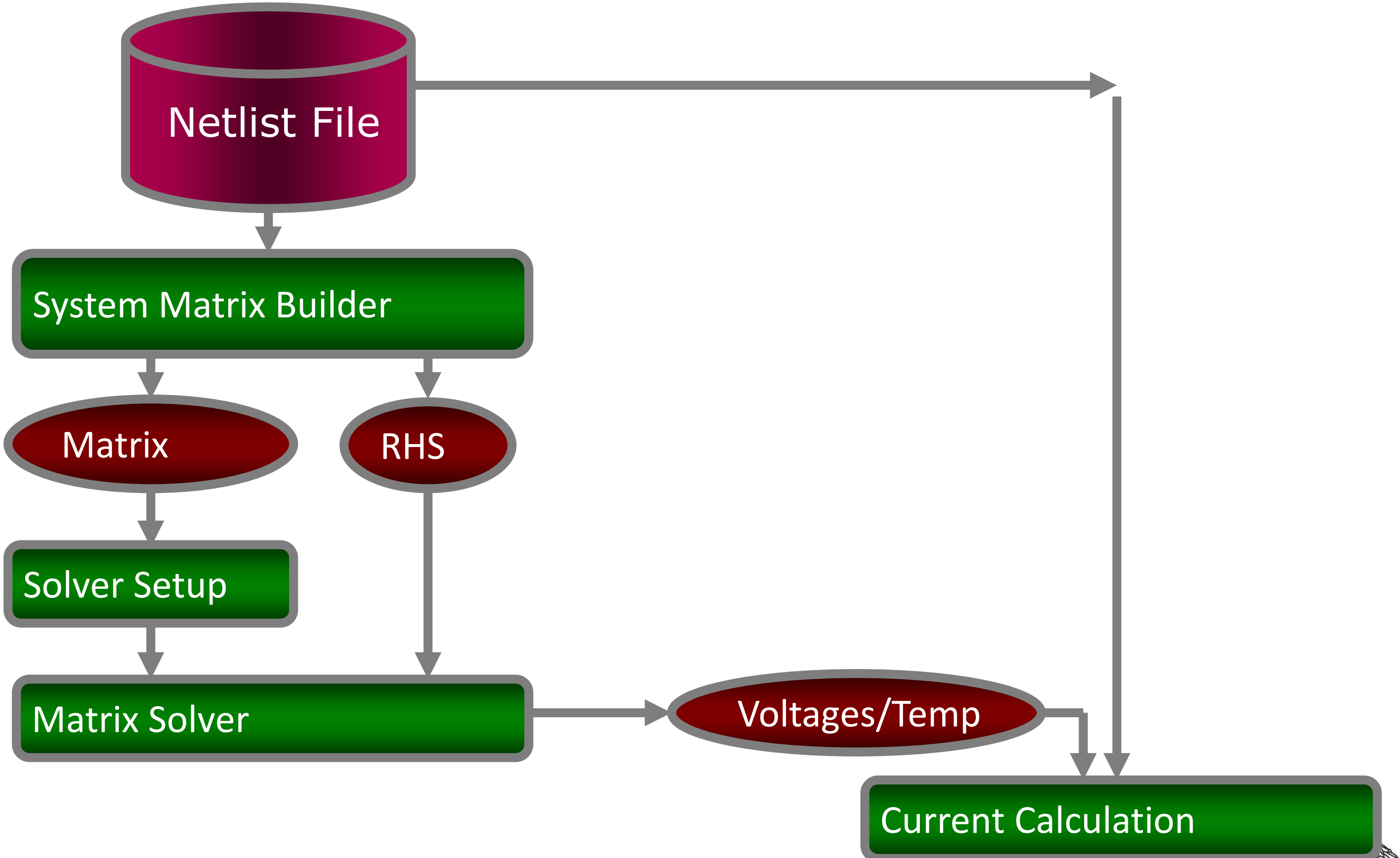
Multigrid-based DC Power Integrity Analysis

E8-262: CAD for High-Speed Chip-Package-Systems

Flow



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MGCG: Prior Art and Motivation

Reduce Number of Iterations in Iterative Solution

- Stand-alone Multigrid (Ref: Briggs *et al*. 1982) and then…
- Multigrid preconditioned Conjugate Gradient (Ref: Tatabe 1996)

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Stand-Alone Multigrid Basics

Basic Essence: Hierarchy

Grid Hierarchy Is Necessary

- Basic Principle is Based on Two Complimentary Ideas
  - Relaxation (GS, Jacobi) iterations: Effective when error is oscillatory
  - Accurate interpolation to coarse grid: Effective when error is smooth

Grid Hierarchy Is Possible

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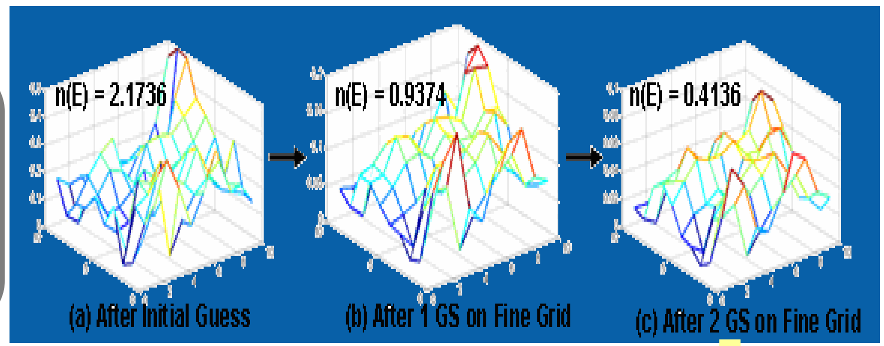
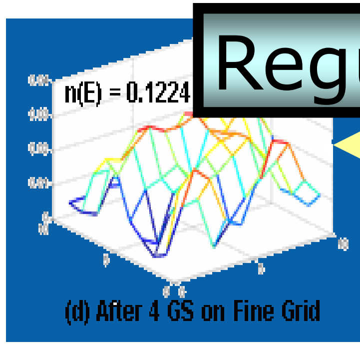
Relaxation Property: Smoothens error



Relaxation Property: Slows on smooth error

Top-Grid Smooth Error: Slows Relaxation



Regular

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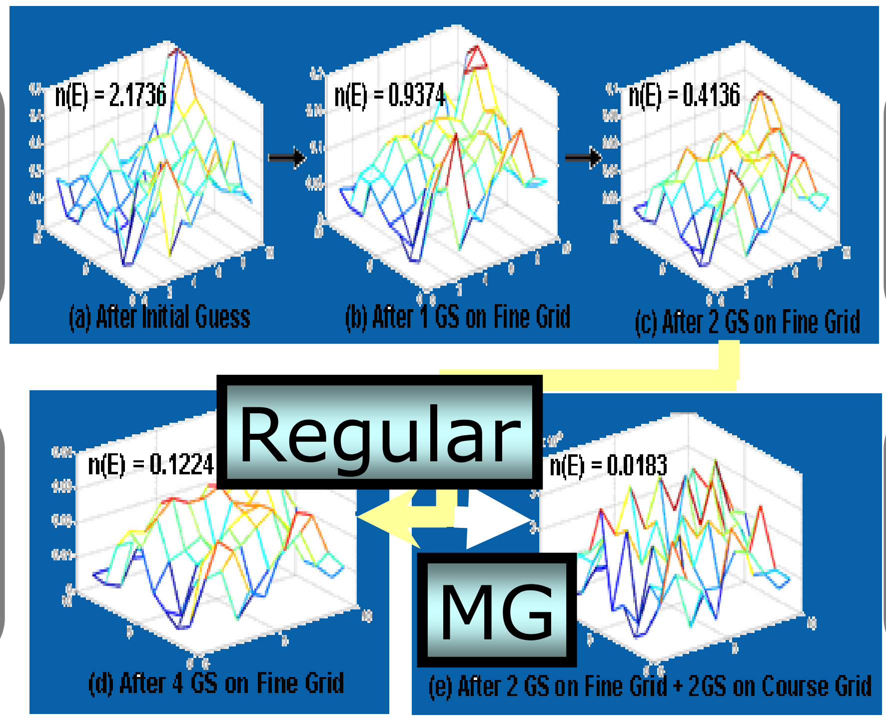


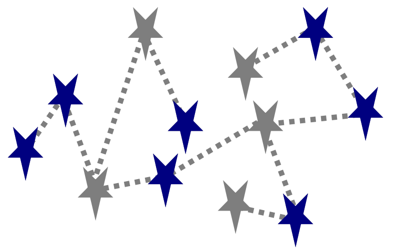
Relaxation Property: Slows on smooth error

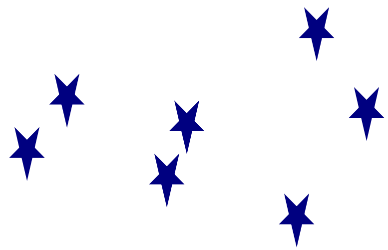
Top-Grid Smooth Error: Slows Relaxation

Top-Grid Smooth Error: Helps Interpolation
Fine-grid Oscillatory Error: Helps Relaxation

Stand-Alone Multigrid Ingredients

- Coarse Grid Selection
  - Way to select nodes to form the coarse level
- Interpolation Operator
  - Dependence relation between levels
- Coarse Grid Matrix Operator
  - Dependence relation





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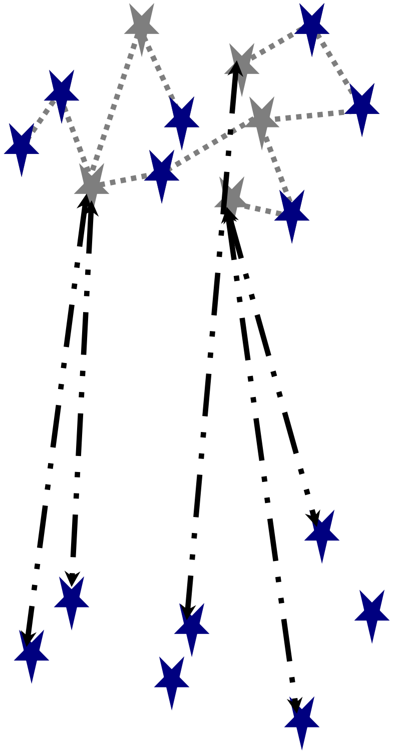

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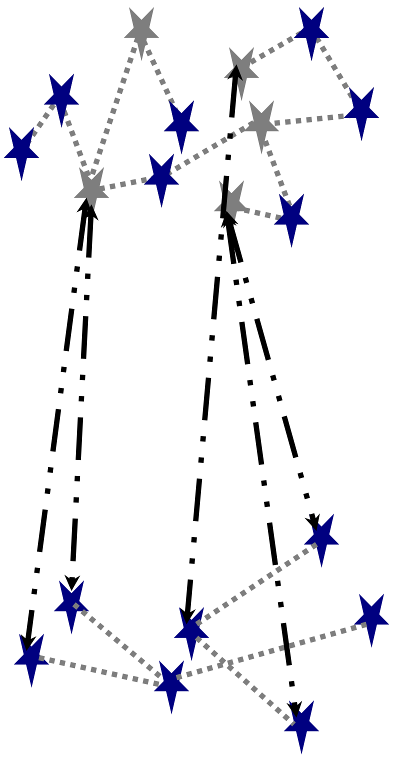

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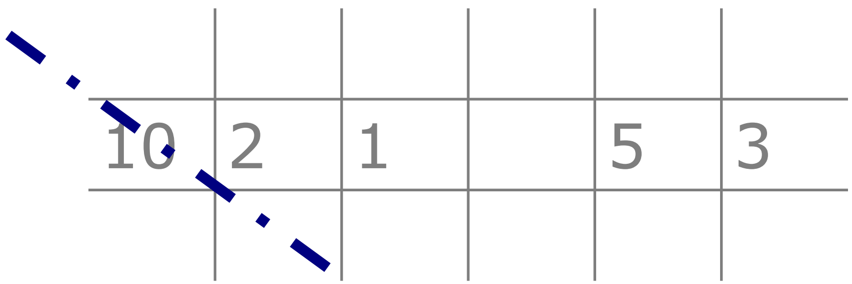
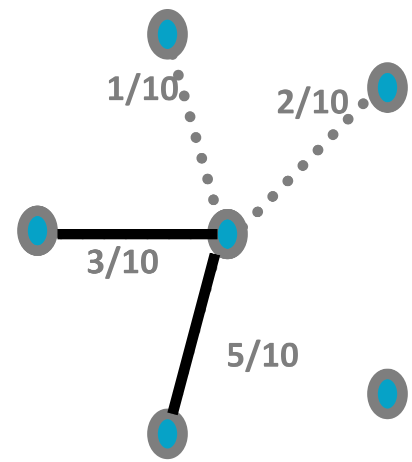
Coarse Grid Selection: Algebraic

- Strong/Weak Dependence Enumerated from Matrix Elements





- Select Subset of Nodes (C) such that:
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  - Minimum possible number of nodes are chosen for (C)

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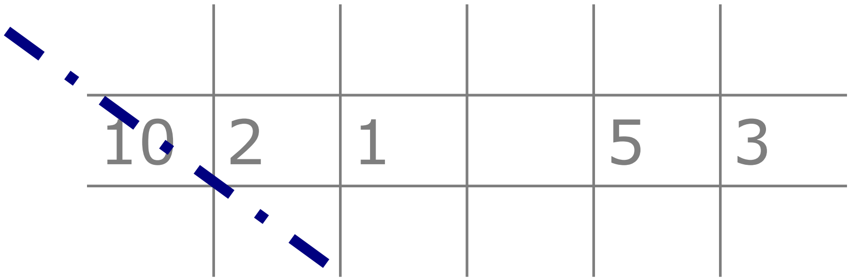

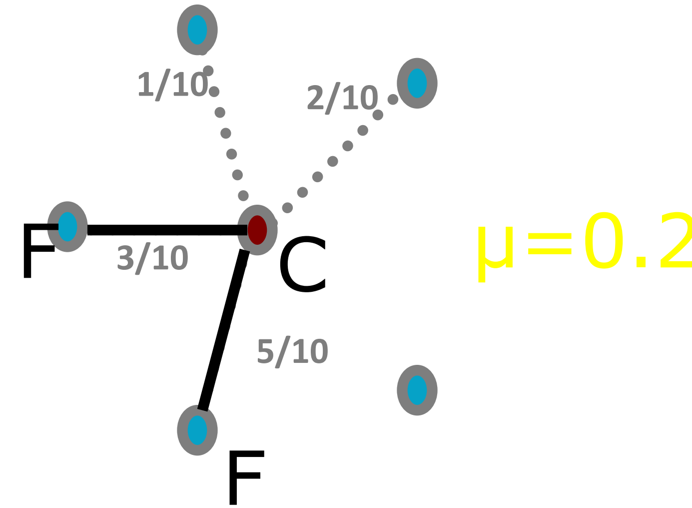

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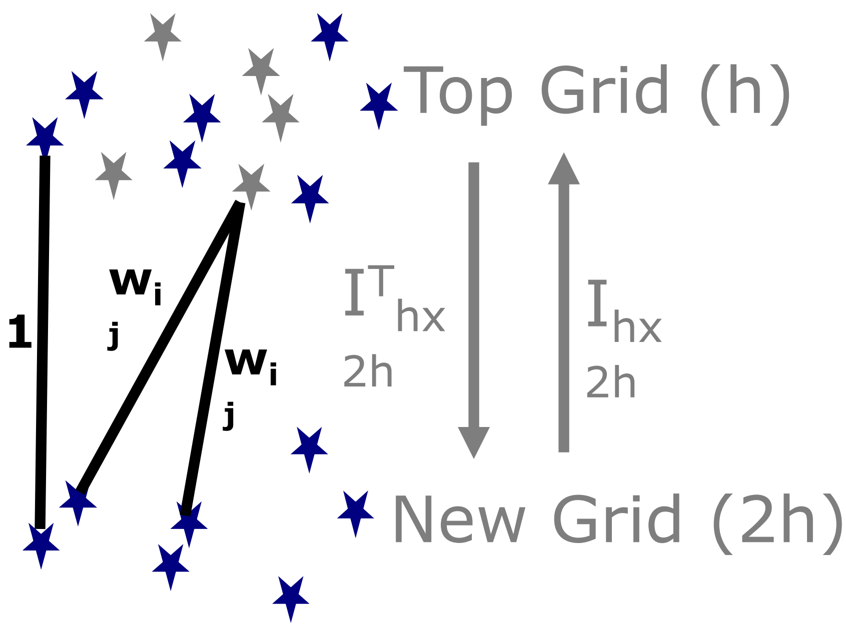
Interpolation Operator

- Coarse Grid To Top Grid – Interpolation (2h->h)
- Top Grid To Coarse Grid – Restriction (h->2h)

★ C points
★ F points



$$I_{h\times 2h}(i,j) = \begin{cases} 1 & \textbf{if } (i \in C) \,\&\, (u_j = u_i) \\ w_{ij} & \textbf{if } (i \in F) \,\&\, (u_j \in Cs_i) \\ 0 & \textbf{otherwise} \end{cases}$$

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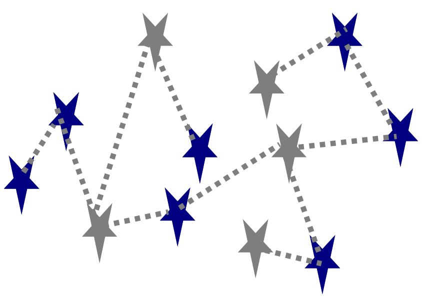
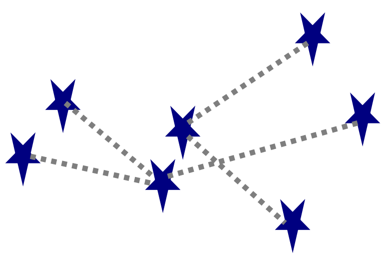
Coarse Grid Operator

- The 0^{th} Level Operator is the Starting Matrix
- Subsequent Level Operators/Matrices are obtained as follows:

$$A_{L+1} = {I_{L+1 \to L}}^T \times A_L \times I_{L+1 \to L}$$





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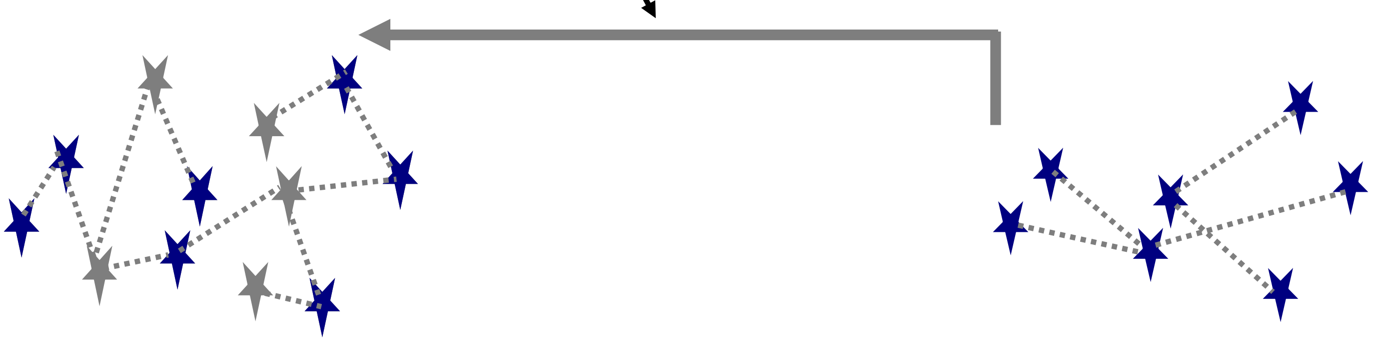

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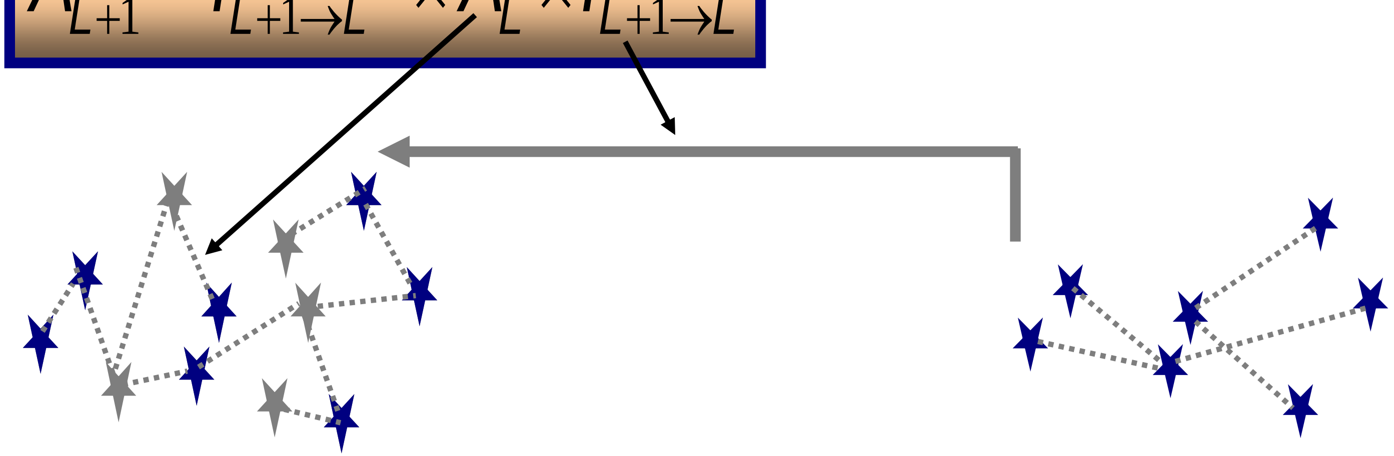

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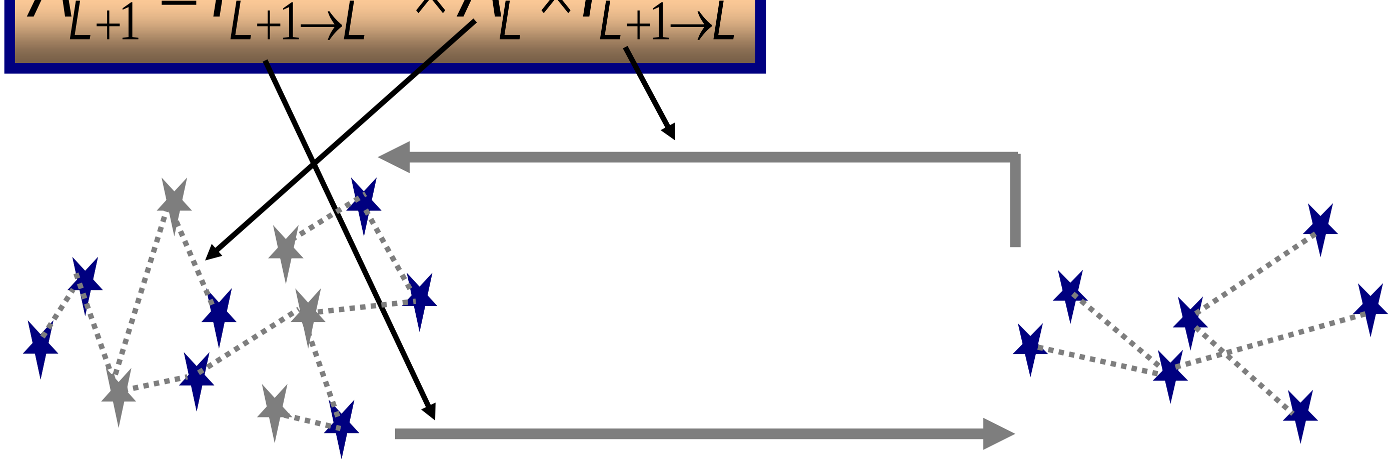

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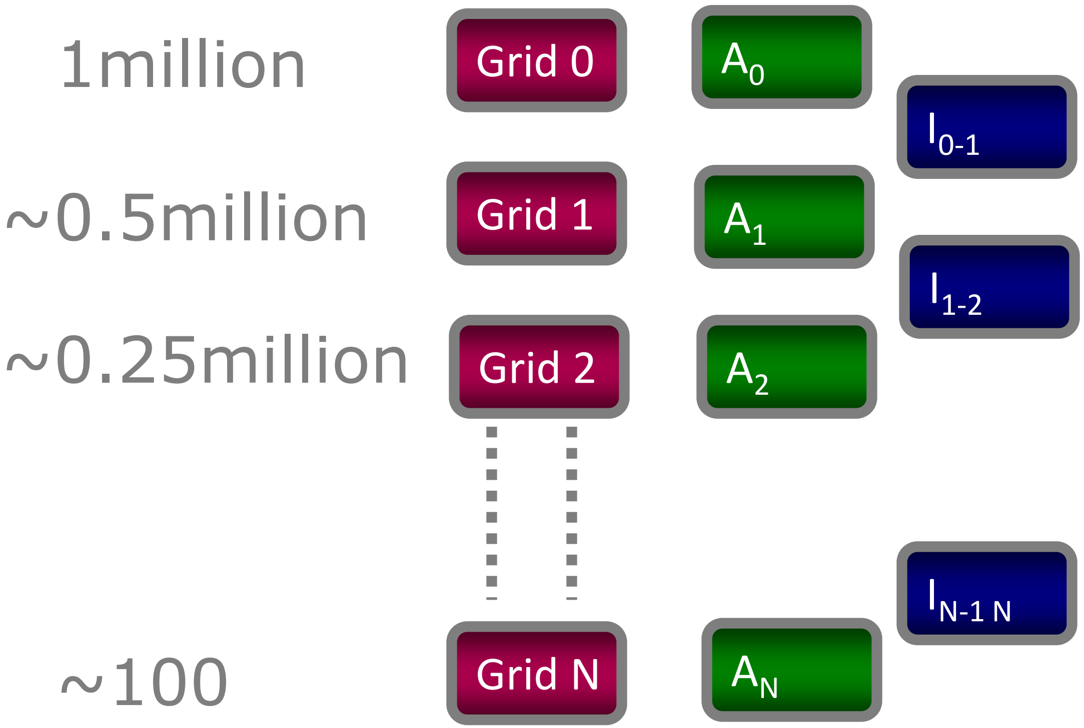
Multigrid Setup: One Time



- Storage is Dominated by Matrices at All Levels
- If A_0 is Symmetric, then A_i is Symmetric

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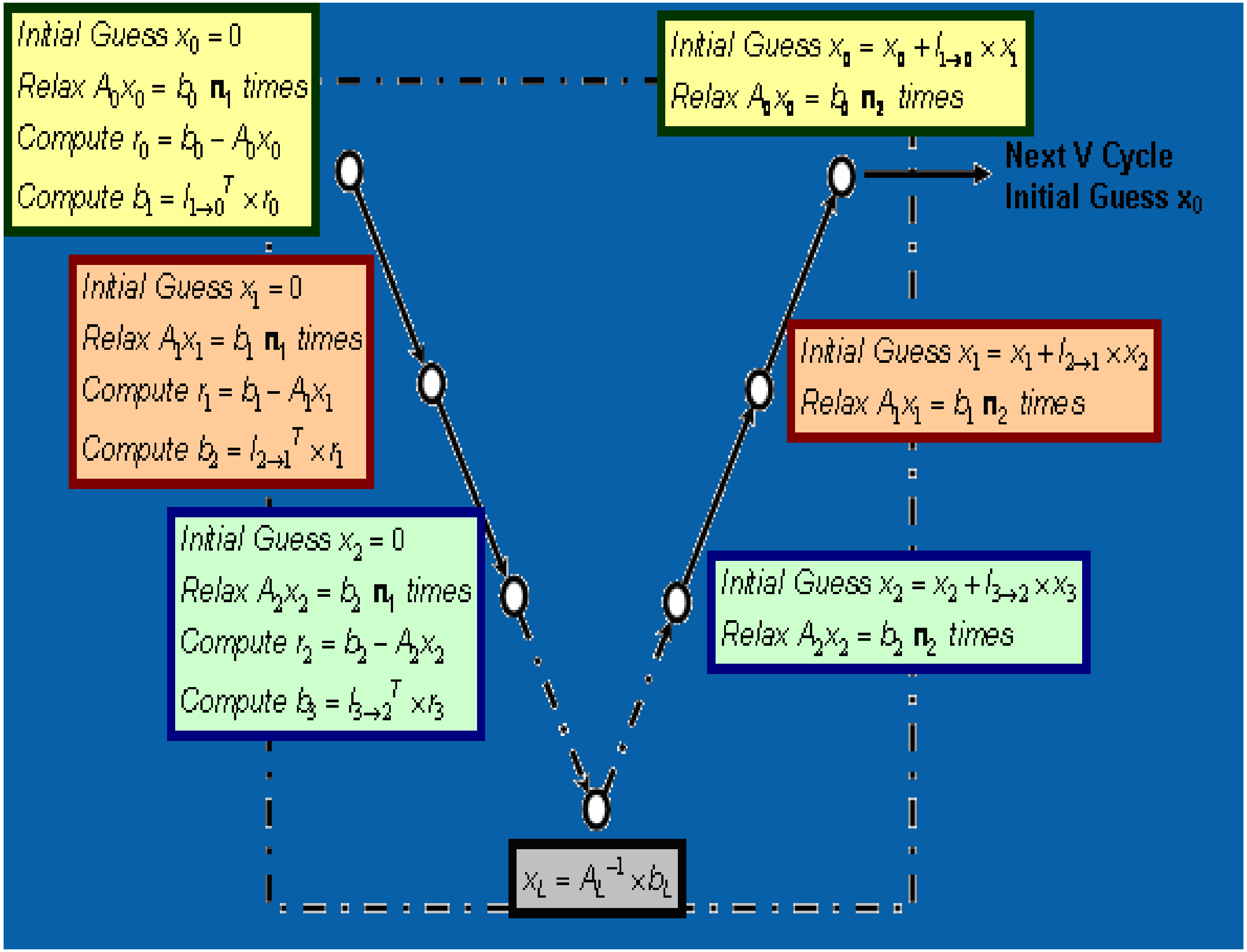
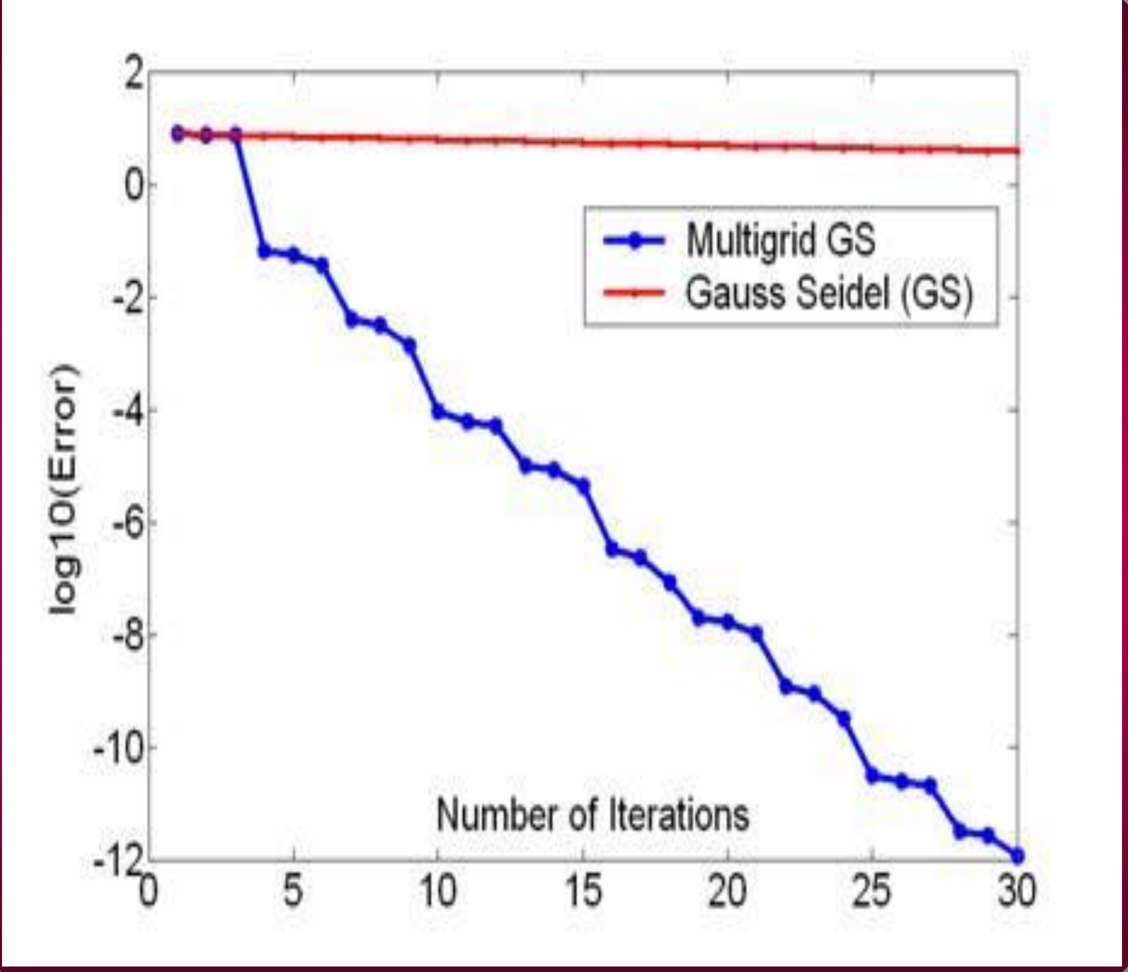
Multigrid Solve: V Cycle



Stand-Alone Multigrid Convergence



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Multigrid Preconditioned Conjugate Gradient

- Direct Solver: $x=A^{-1}b$
- Iterative Solver: $Ax_0-b\ldots\ Ax_1-b\ldots\ Ax_n-b$

Conjugate Gradient

Convergence Depends on Distribution of Eigen Values of Matrix A

- Preconditioning: $PAx=Pb\ldots$ P is close to A^{-1}

One V-Cycle of Stand-Alone Multigrid is Employed as P

MGCG Convergence

MGCG Convergence is Often Superior to Stand-Alone MultiGrid