E2 206: Information and Communication Complexity (2017)
Homework 2

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Homework Questions Q4 and Q7 are slightly difficult.

Q1 Suppose pairs of random variables (X_1, Y_1) and (X_2, Y_2) are independent. Show that (X_1, Y_1) and (X_2, Y_2) remain independent conditioned on $(f(X_1, X_2), g(Y_1, Y_2), X_1, Y_2)$ for any functions f and g.

Q2 Show that $R_{1/3}(\texttt{EQ}_n) = \theta(\log n)$.

Q3 Suppose that $\mathrm{P}_{XY}\left(f^{-1}(z)\right) \geq 1/2$ for a $z \in \mathcal{Z}$ and there exists $\alpha > 0, \delta \in (0, 1)$ such that

$$\delta \geq \max\left\{\mathrm{P}_{XY}\left(R \cap f^{-1}(z)\right) : \alpha \mathrm{P}_{XY}\left(R \cap f^{-1}(z)\right) > \mathrm{P}_{XY}\left(R \setminus f^{-1}(z)\right), \forall R \in \mathcal{R}(\mathcal{X} \times \mathcal{Y})\right\}.$$

Show that

$$D_\epsilon(f|\mathrm{P}_{XY}) \geq \log\frac{1}{\delta} - \log\left(\frac{\alpha}{\alpha(0.5-\epsilon)-\epsilon}\right).$$

Q4 Show that $D_\epsilon(\texttt{DISJ}_n|\mathrm{P}_X\mathrm{P}_Y) = O\sqrt{n}$, for every independent distribution $\mathrm{P}_X\mathrm{P}_Y$ on the inputs.

Q5 Let $D_{JS}(\mathrm{P}, \mathrm{Q})$ denote the Jenson-Shannon divergence between P and Q, given by

$$D_{JS}(\mathrm{P}, \mathrm{Q}) = \frac{1}{2}\left(D(\mathrm{P}\|\mathrm{Q}) + D(\mathrm{Q}\|\mathrm{P})\right)..$$

Further, let $h^2(\mathrm{P}, \mathrm{Q})$ denote the squared Hellinger distance between P and Q, given by

$$h^2(\mathrm{P}, \mathrm{Q}) = \frac{1}{2}\sum_x(\sqrt{\mathrm{P}(x)} - \sqrt{\mathrm{Q}(x)})^2.$$

Show that

(i) $D_{JS}(\mathrm{P}, \mathrm{Q}) \geq h^2(\mathrm{P}, \mathrm{Q})$.

(ii) $h^2(\mathrm{P}, \mathrm{Q}) \geq 1 - (1 - d_{TV}^2(\mathrm{P}, \mathrm{Q}))^{\frac{1}{2}}$.

Q6 Complete the proof of BBCR direct sum theorem; in particular, show that the expected number of disagreements in the paths simulated by the parties is no more than $\sqrt{\frac{1}{2}|\pi| IC(\pi|\mathrm{P}_{XY})}$.

Q7 **(Braverman '12).** Given a private coin protocol π with inputs from $\mathcal{X} \times \mathcal{Y}$, show that when the inputs are generated by P_{XY}, π can be ϵ-simulated using no more than $2^{\tilde{O}_\epsilon(IC(\pi|\mathrm{P}_{XY}))}$ bits of communication.