E2 209 – Topics in Information Theory and Statistical Learning
Homework #2 **Jan-Apr 2018**

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(1) *Coins with multiplicatively separated biases*
Establish the sample complexity of distinguishing a coin with bias p from a coin with bias $(1-\varepsilon)p$.

(2) *Three birthdays*
How many samples from a uniform distribution over $[k]$ are needed to get three identical samples with a constant probability?

(3) *χ^2-distance of product distributions*
The χ^2-distance between two distributions P and Q on $\mathcal{X}$ is given by

$$\chi^2(P,Q) = \sum_x \frac{(P(x)-Q(x))^2}{P(x)}.$$

Show that for $P^n = P_1 \times ... \times P_n$ and $Q^n = Q_1 \times ... \times Q_n$,

$$\chi^2(P^n,Q^n) = \prod_{i=1}^{n} \left(1+\chi^2(P_i,Q_i)\right) - 1.$$

(4) *Probability assignment for k-ary alphabet*
Show that

$$\overline{r}(k,n) = r(k,n) = \frac{k-1}{2}\log n + O_k(1).$$

We showed this result for $k=2$ in class. Almost the same proof goes through.

(5) *Multiplicative Weight Update*
Consider the prediction problem discussed in the class with k experts and time horizon n. Let $L_1, ..., L_n$ denote the sequence of losses incurred by the multiplicative weight update algorithm. Show that $n = O(\frac{k}{\varepsilon^2}\sqrt{\log\frac{1}{\delta}})$ attempts suffice to get

$$\mathbb{P}\left(\frac{1}{n}\sum_{t=1}^{n} L_t - \min_{i\in[k]} \frac{1}{n}\sum_{t=1}^{n} l_i(t) > \varepsilon\right) \le \delta.$$

for some constant $c > 0$.

(6) *Multiarmed Bandit with Noisy Observations*
Consider the modified version of the coin-toss multiarmed bandit problem discussed in the class where you observe the output of your chosen coin flipped with probability δ. Analyse the EXP3 algorithm for this case and provide a minimax regret bound as a function of the number of coins k, the time-horizon n, and flip-over probability δ.