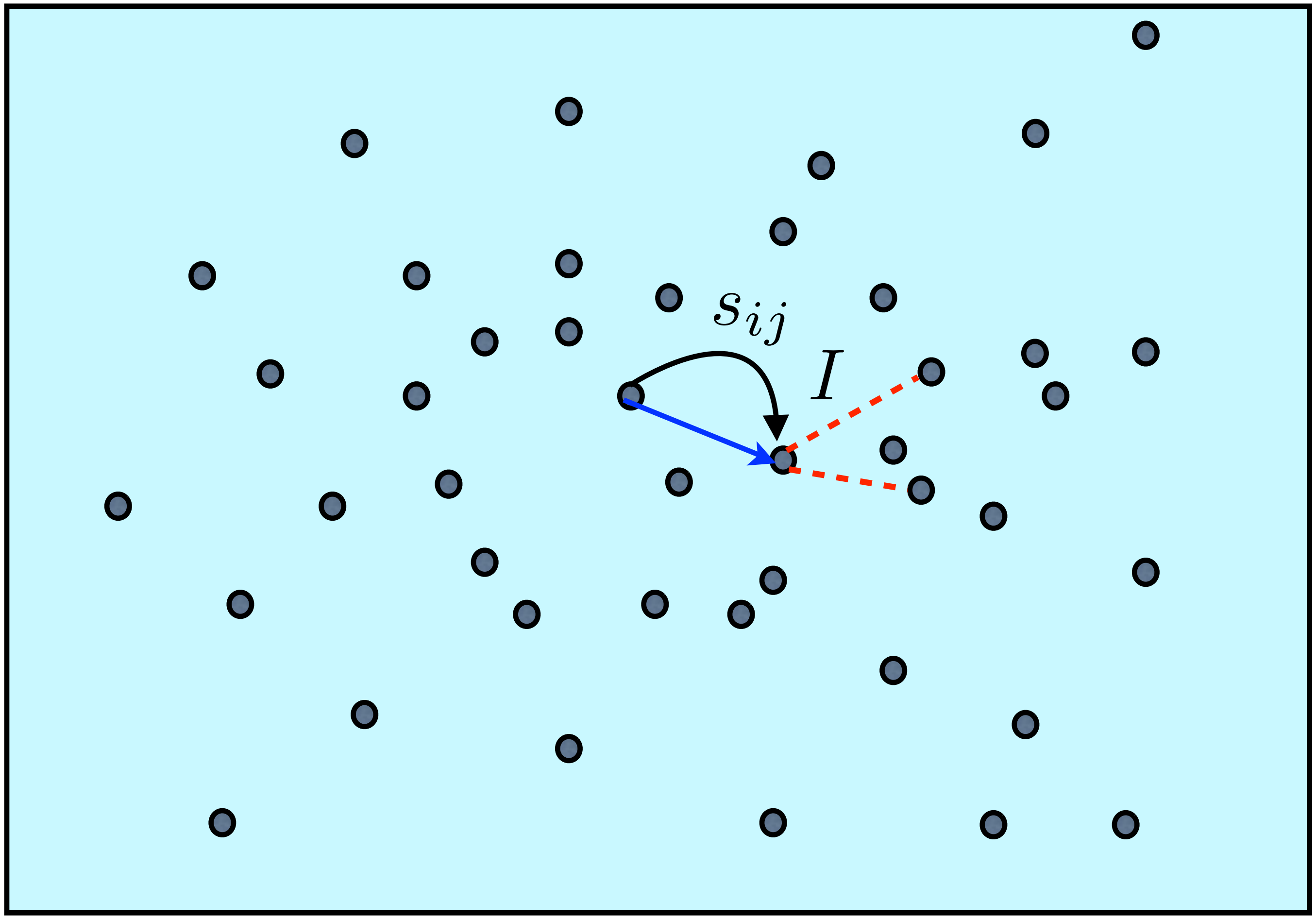
Escaping the *Poisson* Packet Black Hole

Rahul Vaze and Srikanth Iyer

Wireless Network



PPP distributed nodes

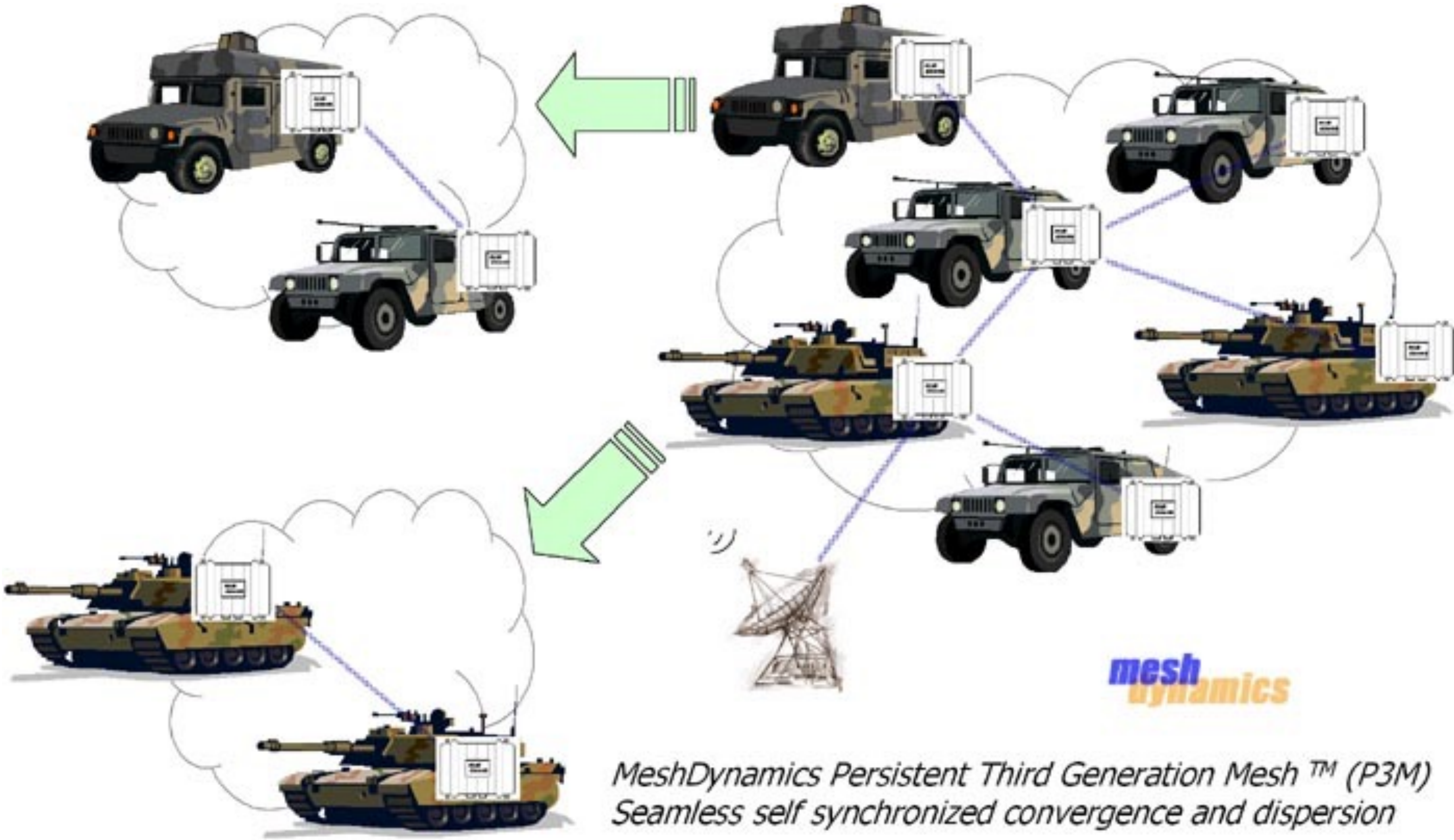
$$s_{ij} = h_{ij}\ell(d_{ij})$$

$$I = \sum_{k \in \Phi \setminus \{i\}} h_{kj}\ell(d_{kj})$$

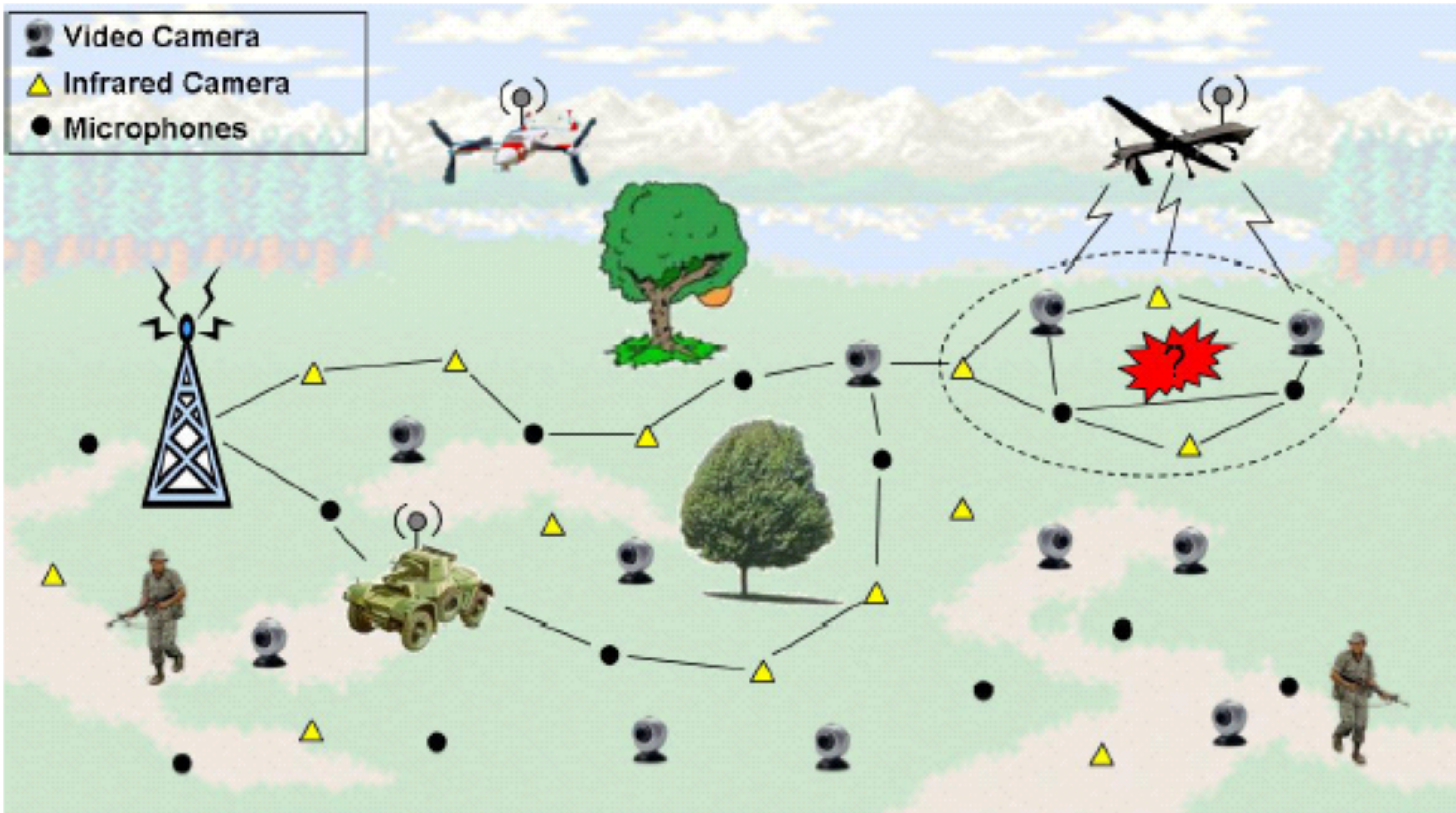
$$SINR_{ij} = \frac{s_{ij}}{I + N}$$

$$SINR_{ij} > \beta$$

Military Networks

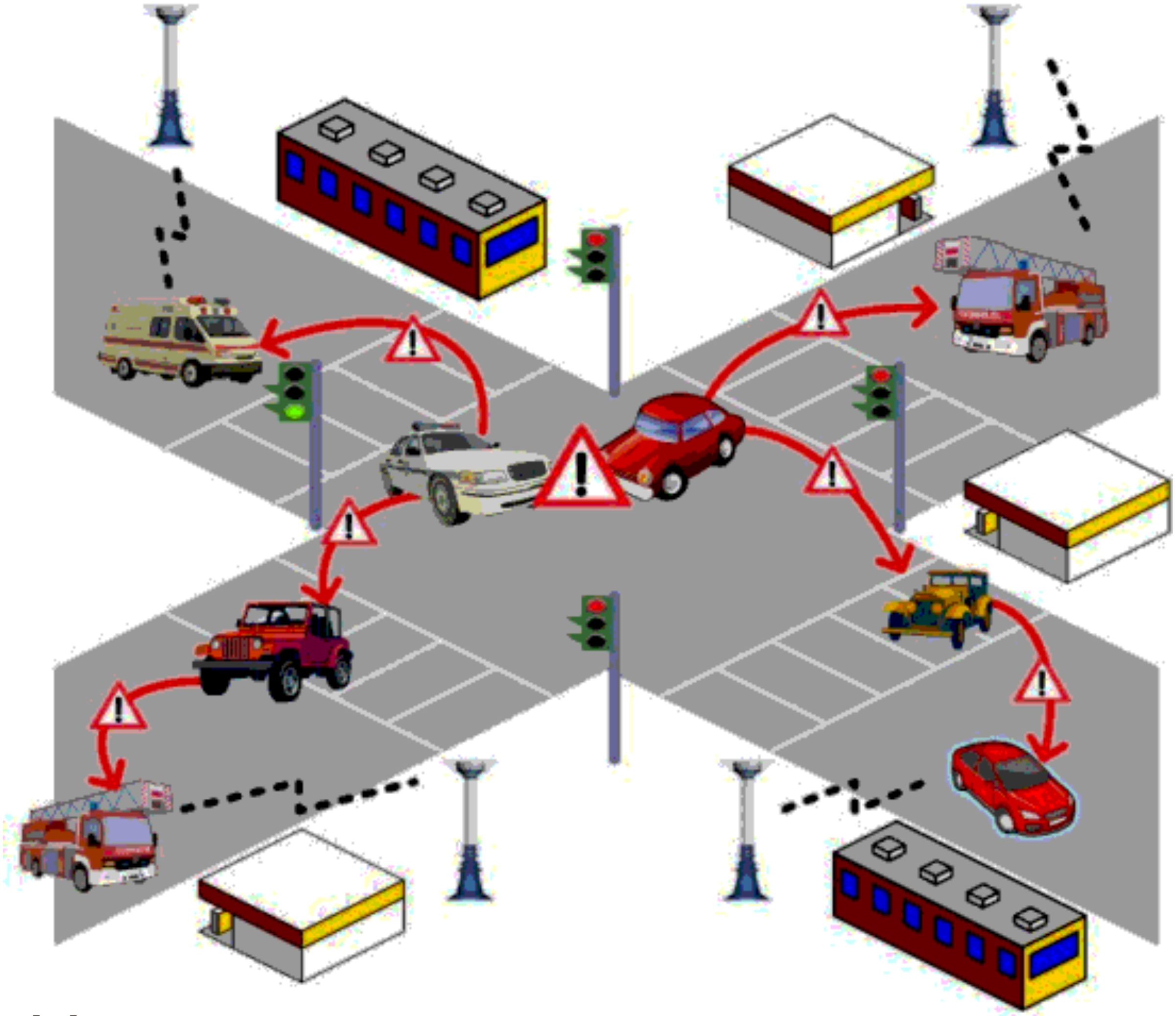


Sensor Network



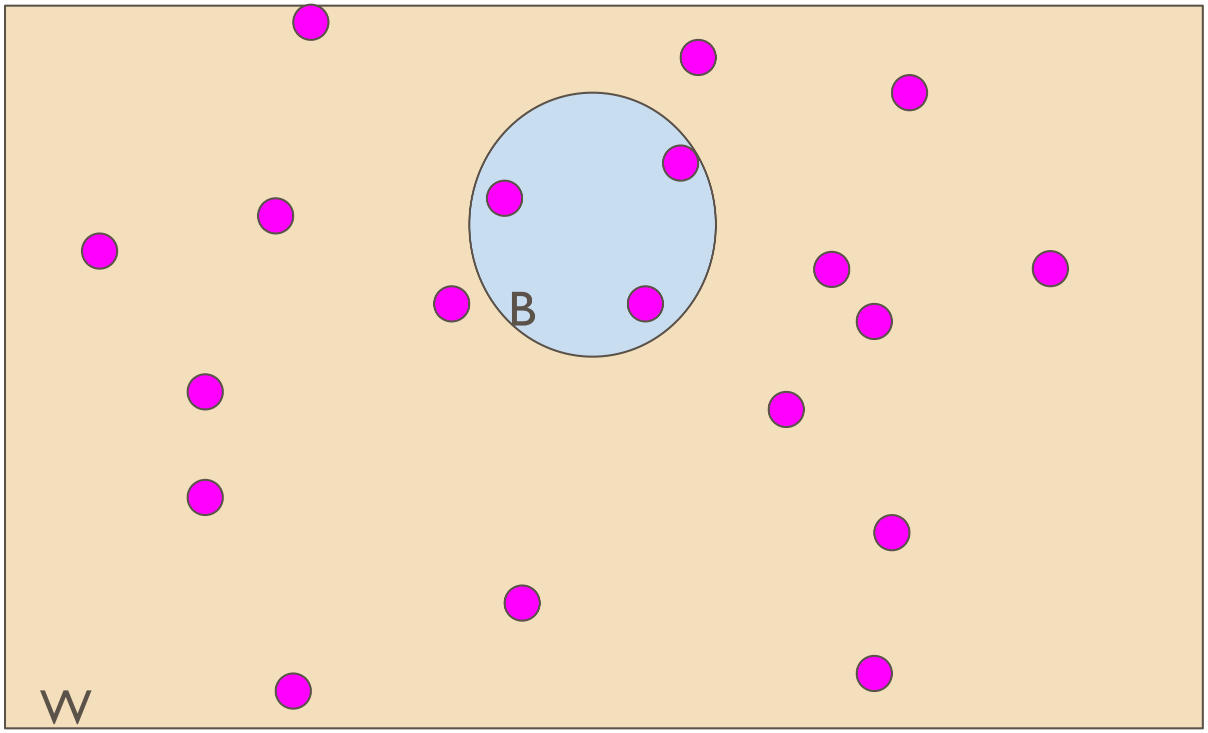
- Surveillance
- Data collection

Vehicular Network



- Traffic Management
- Road Safety

Why Poisson ?



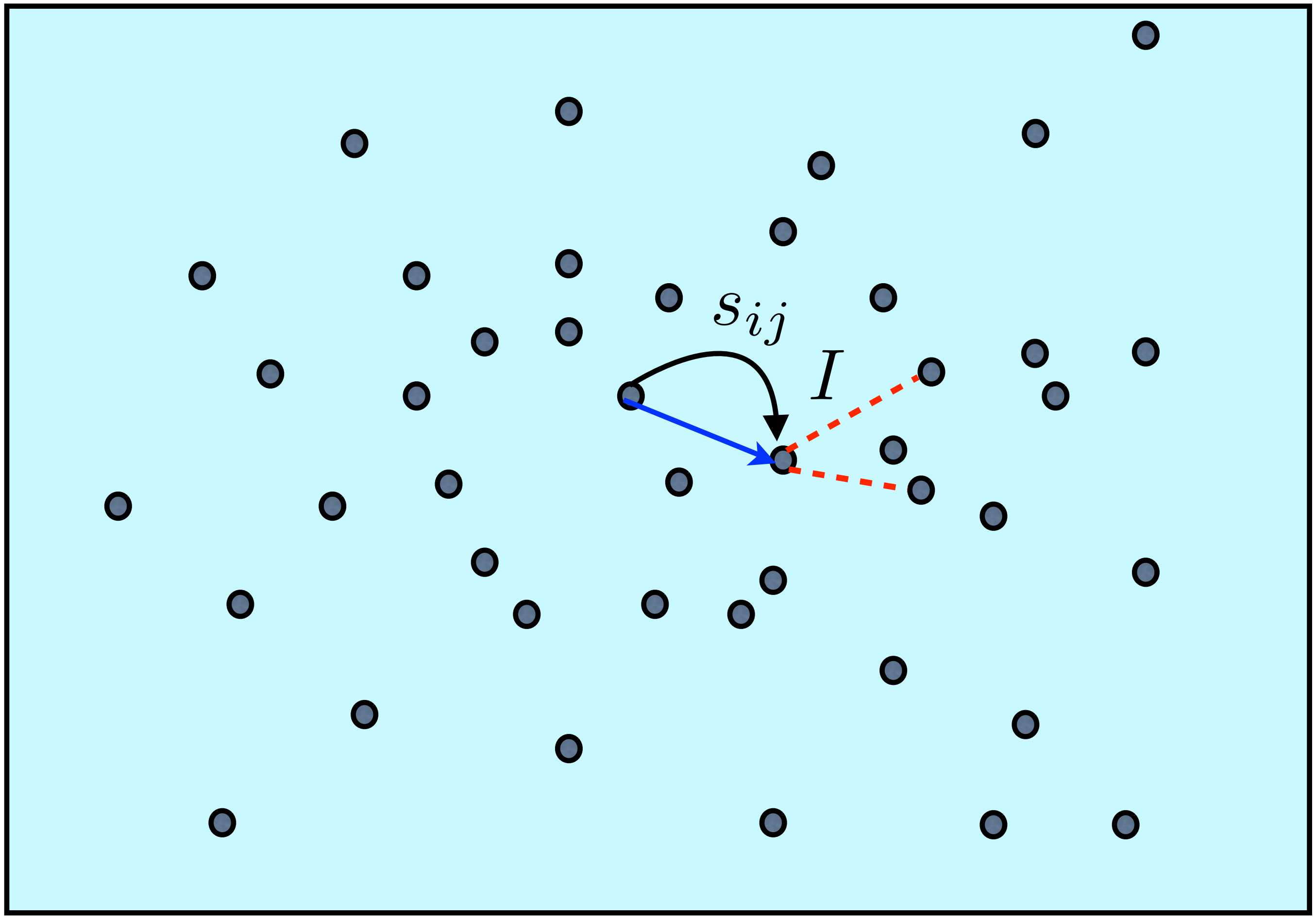
- The number of points falling in B is *Binomial* with

$$P\left(\#(B)=k\right)=\binom{n}{k}p^k(1-p)^{n-k} \quad p=\frac{\nu(B)}{\nu(W)}$$

- As $n \to \infty$ while fixing $\frac{n}{\nu(W)}=\lambda$

$$P\left(\#(B)=k\right) \sim Poisson\left(\lambda\nu(B)\right)$$

Wireless Network



PPP distributed nodes

$$s_{ij} = h_{ij}\ell(d_{ij})$$

$$I = \sum_{k \in \Phi \setminus \{i\}} h_{kj}\ell(d_{kj})$$

$$SINR_{ij} = \frac{s_{ij}}{I + N}$$

$$SINR_{ij} > \beta$$

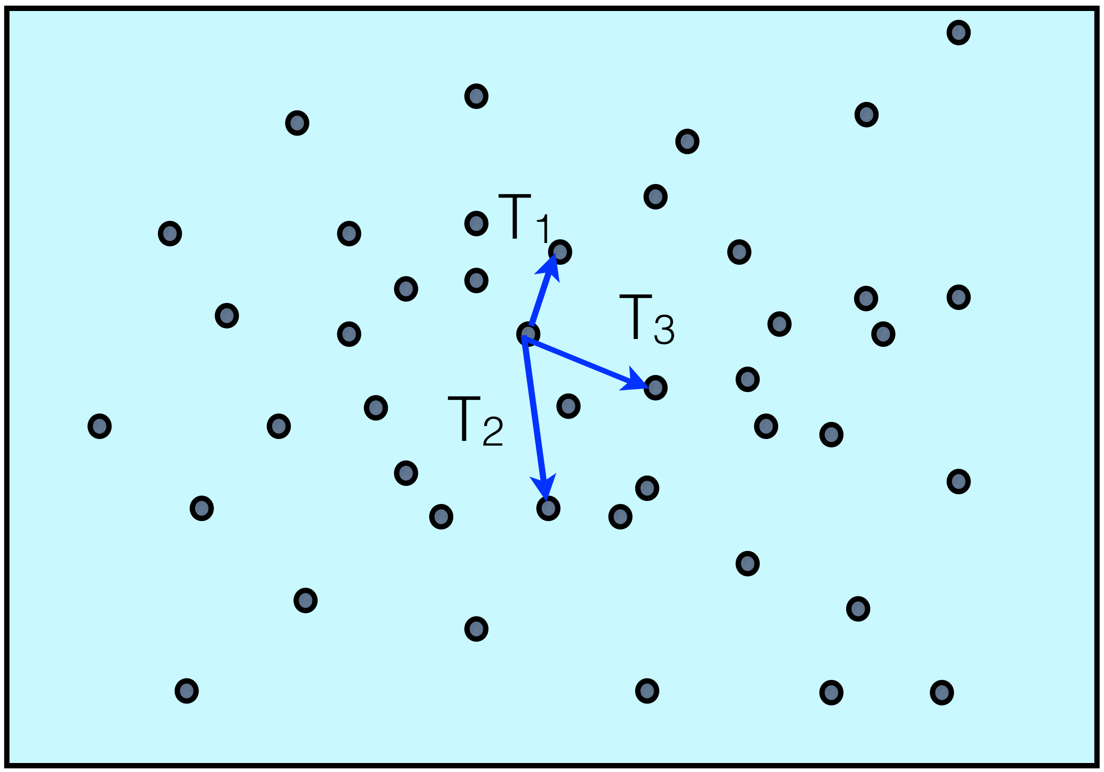
SINR is a random variable: multiple attempts required

PPP network

Negative result

- Expected delay to any node is infinite

[Baccelli, Blacszczyszyn, Mirsadeghi, Adv. App. Prob. 2010]

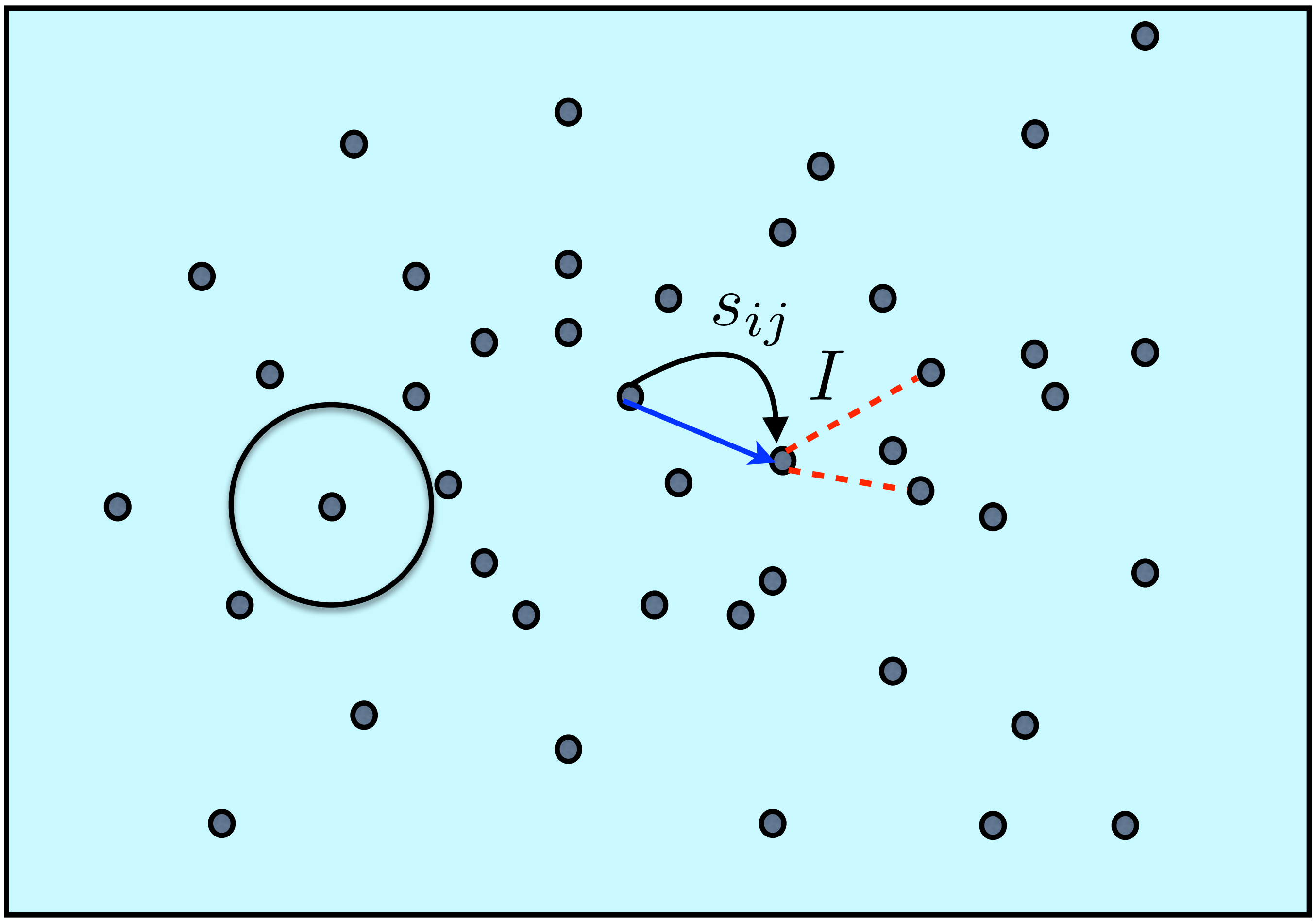


min delay has infinite expectation

Negative Result

Even in the absence of interference (AWGN is sufficient)

Unbounded sized holes in PPP



More Bad News.

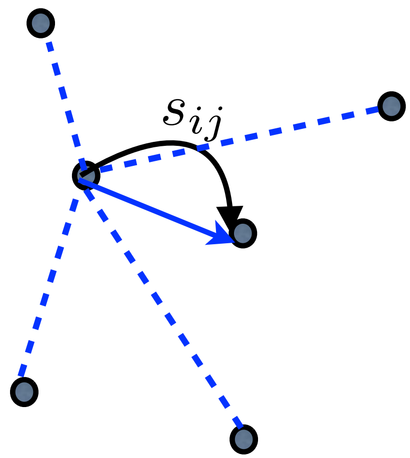
Let $d(t)$ be the successful distance traveled by a tagged particle from origin in time t

Then speed or information velocity is

$$\mathcal{V} = \lim_{t \to \infty} \frac{d(t)}{t}$$

In a PPP network $\mathcal{V} = 0$ [Baccelli, et al, 2010]

Negative Result Primer



$$s_{ij} = P_i h_{ij} \ell(d_{ij})$$

T Time to exit to any other node

Drop the Interference $SINR_{ij} = \frac{s_{ij}}{I_j + N}$ $SNR_{ij} = \frac{s_{ij}}{N}$

$$\mathbf{P}(\mathsf{T} > k) = \mathbf{P}(SNR_{ij}(t) < \beta, \forall\ j \in \Phi, t = 1, \dots, k)$$

Conditioned over location of nodes, SNRs are independent

$$P(\mathsf{T} > k|\Phi) = \prod_{j \in \Phi} P(SNR_{ij}(t) > \beta)^k$$

After unconditioning $\mathbf{P}(\mathsf{T} > k) > \frac{1}{k}$

Main Tool

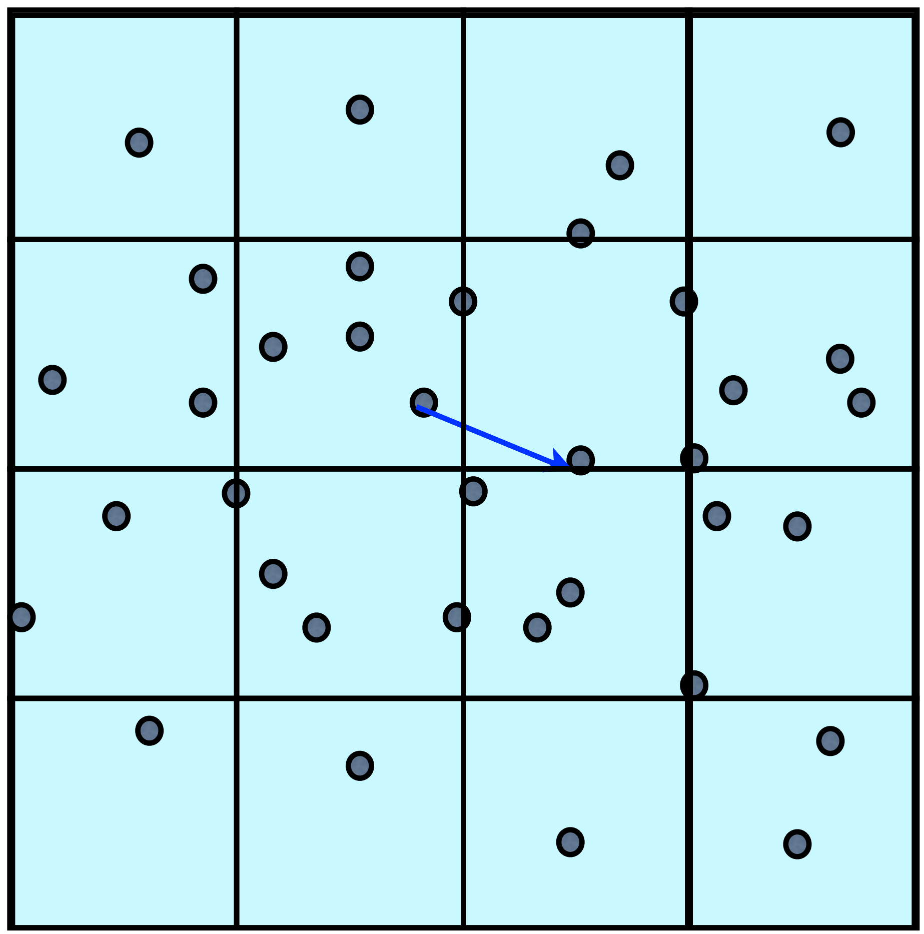
$$\text{PGF}(f) = \mathbb{E}\left\{\prod_{x_n \in \Phi} f(x_n)\right\}$$

For a PPP Φ with density λ

$$\text{PGF}(f) = \exp^{-\lambda \int (1 - f(x))dx}$$

Remedy

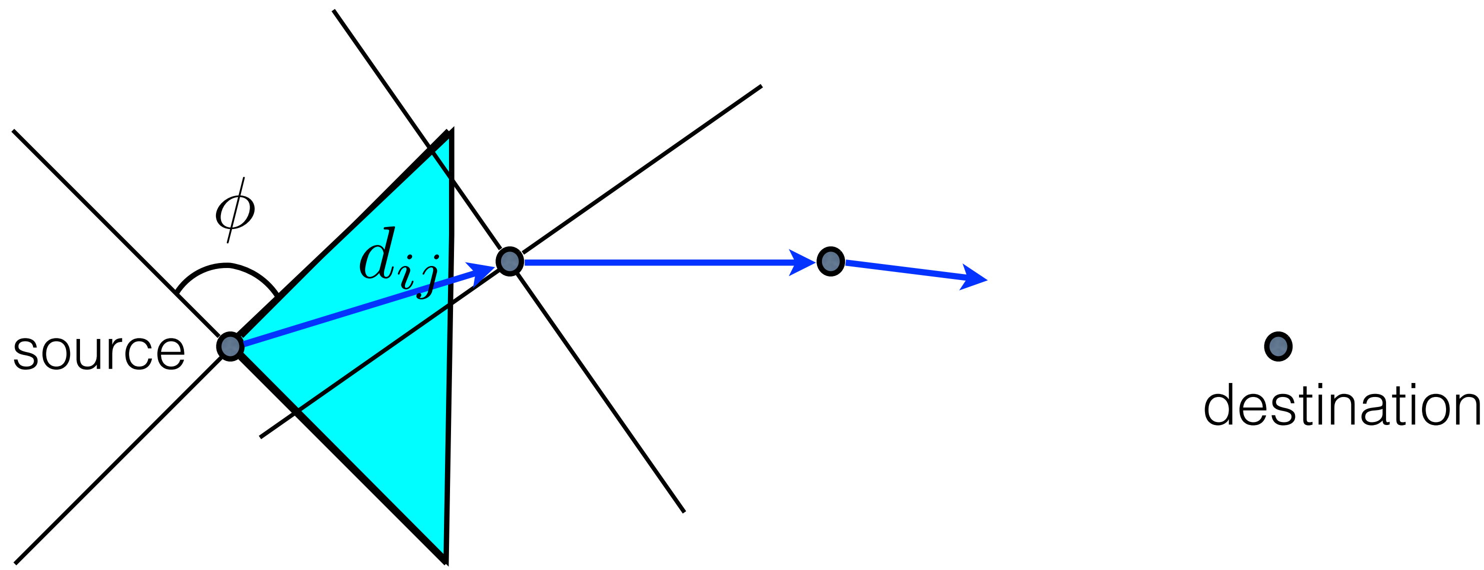
- Add a regular square grid [Baccelli, et al, 2010]



A more practical solution

$$s_{ij} = h_{ij}\ell(d_{ij})$$

$$SINR_{ij} = \frac{s_{ij}}{I + N}$$



Power Control : power $P_i = c\ell(d_{ij})^{-1}$ with prob. $p_i = MP_i^{-1}$

Transmit with higher power

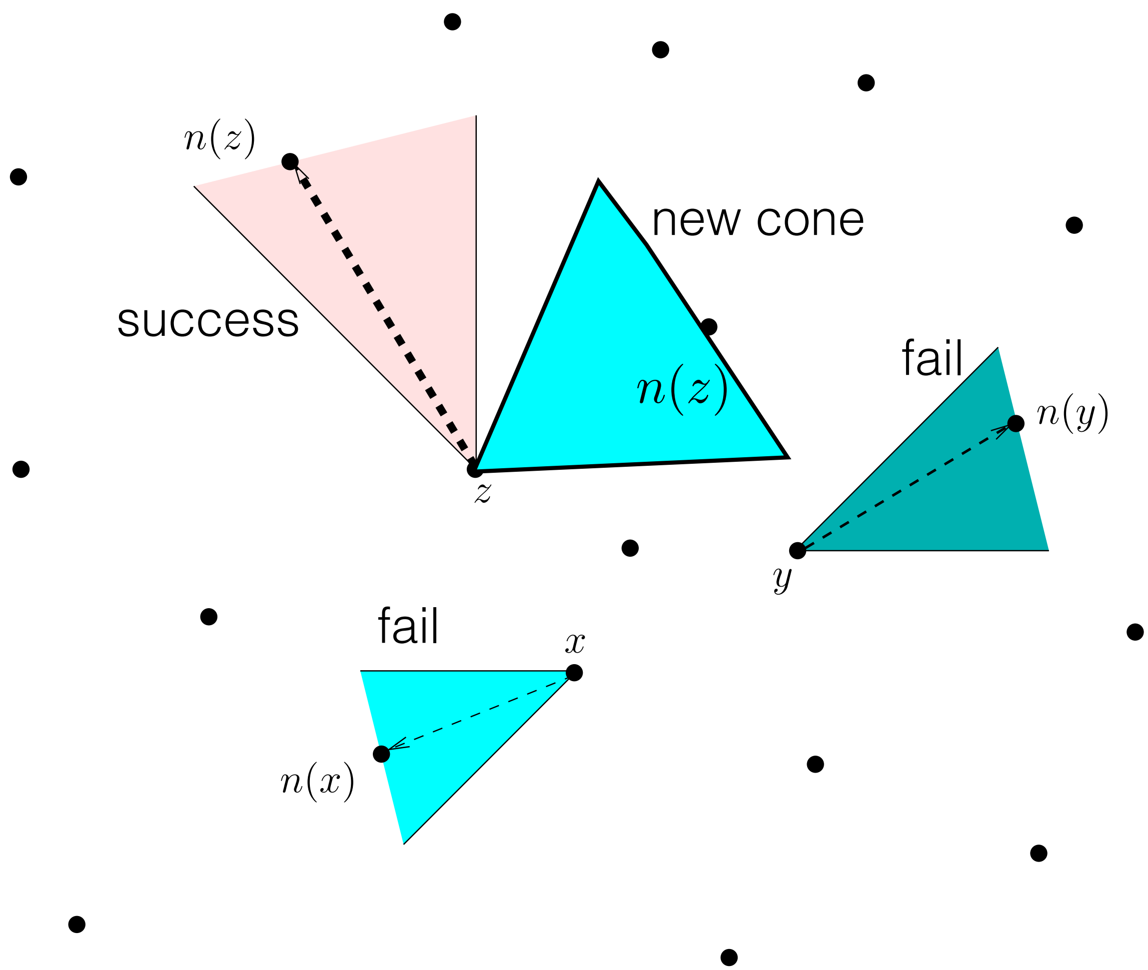
Less frequently

Results

- Expected delay to nearest neighbor is finite
- Information velocity is positive

Issues with New Policy

Choice of cones at any time are correlated



Analysis Ideas - Power Control

$$\mathbf{P}(\mathsf{T} > k) = \mathbf{P}(SINR_{on(o)}(t) < \beta, t = 1, \ldots, k)$$

Earlier it was sufficient to condition over Φ

Not any more

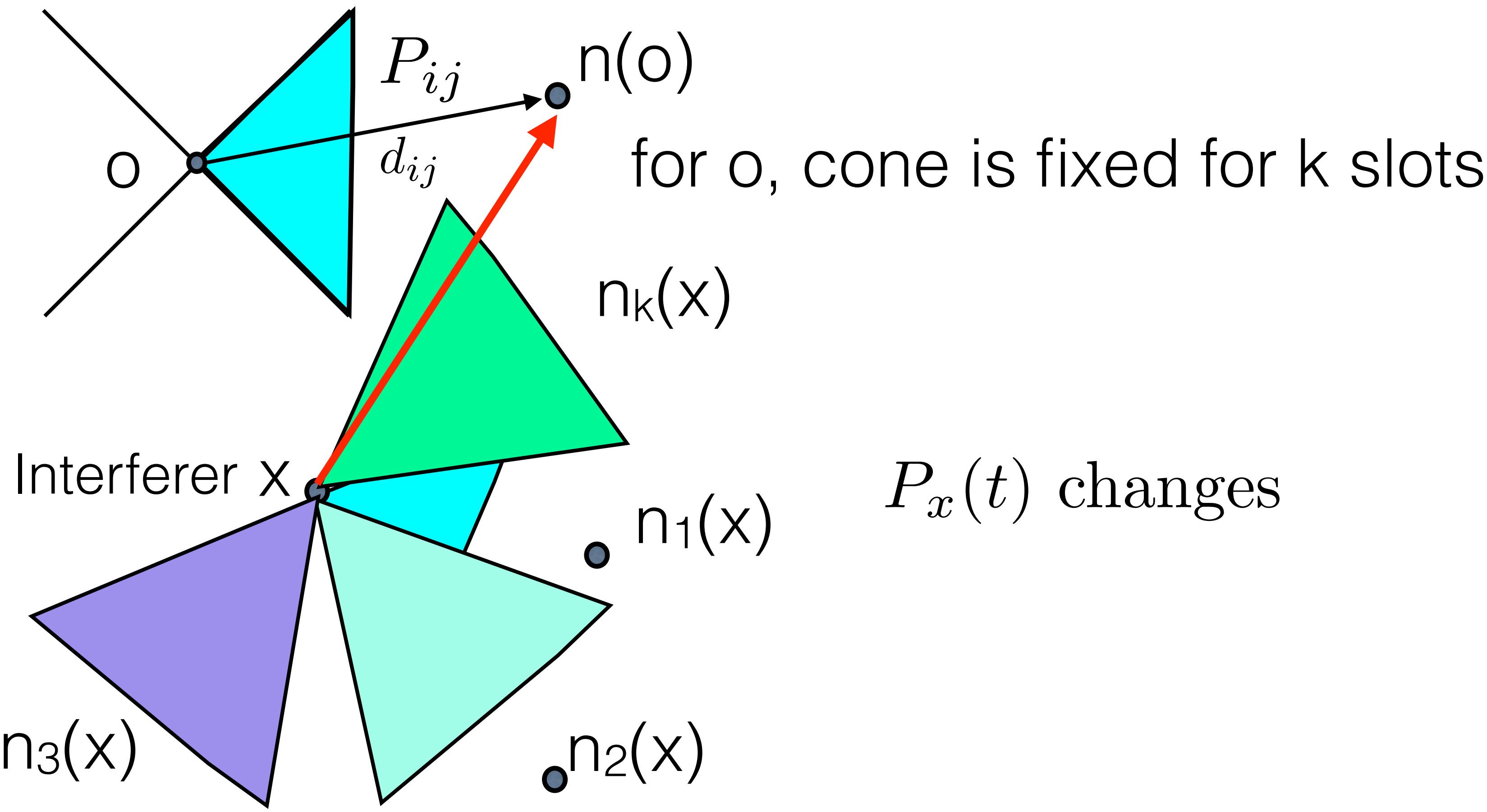
With correlated choice of cones across time slots

condition over sigma field $\mathcal{G}_k$ generated by Φ and choice of cones

$$\mathbf{P}(\mathsf{T} > k) = \mathbf{E}\left\{\prod_{t=1}^{k} \mathbf{P}(SINR_{on(o)}(t) < \beta | \mathcal{G}_k)\right\}$$

Analysis Ideas - Power Control

$$\mathbf{P}(\mathsf{T} > k) = \mathbf{E}\left\{\prod_{t=1}^{k} \mathbf{P}(SINR_{on(o)}(t) < \beta | \mathcal{G}_k)\right\}$$



For any interferer z, use the choice of cone that maximizes the interference seen at n(o) at time t=1,…,k

Analysis Ideas - Power Control

$$\mathbf{E}\{\mathsf{T}\} = \sum_{k=0}^{\infty} \mathbf{P}(\mathsf{T} > k) \qquad \mathbf{P}(\mathsf{T} > k) = \mathbf{E}\left\{\prod_{t=1}^{k} \mathbf{P}(SINR_{on(o)}(t) < \beta | \mathcal{G}_k)\right\}$$

Power Control : power $P_i = c\ell(d_{ij})^{-1}$ with prob. $\mathsf{p}_i = MP_i^{-1}$

For any interferer z, the average interference is bounded

$$\mathsf{p}_z P_z \leq M$$

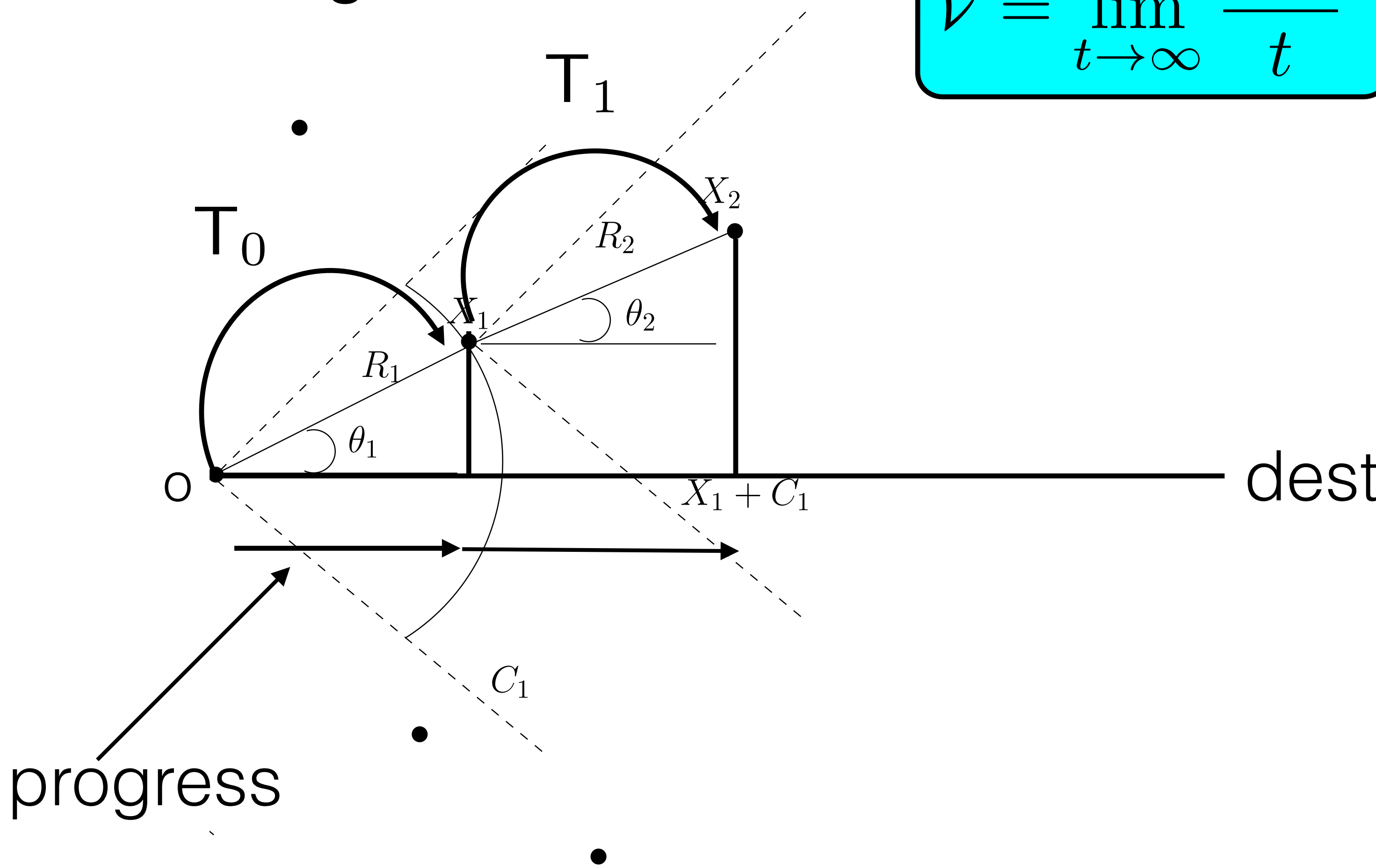
Thus, if $$\int \ell(|x|)dx < \infty$$

using Campbell's Theorem we can show that expected delay is finite

Information Velocity

Look at tagged particle at origin

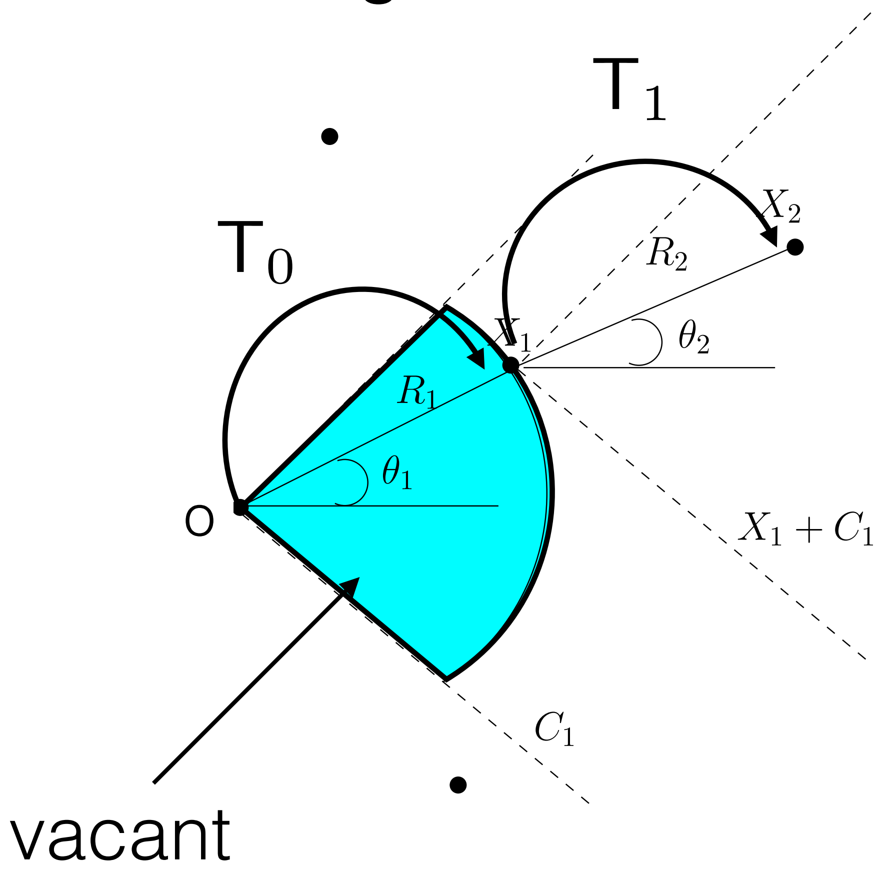
$$\mathcal{V} = \lim_{t \to \infty} \frac{d(t)}{t}$$



First Guess $$\nu = \frac{E\{R\cos(\theta)\}}{E\{T_i\}}$$

Information Velocity

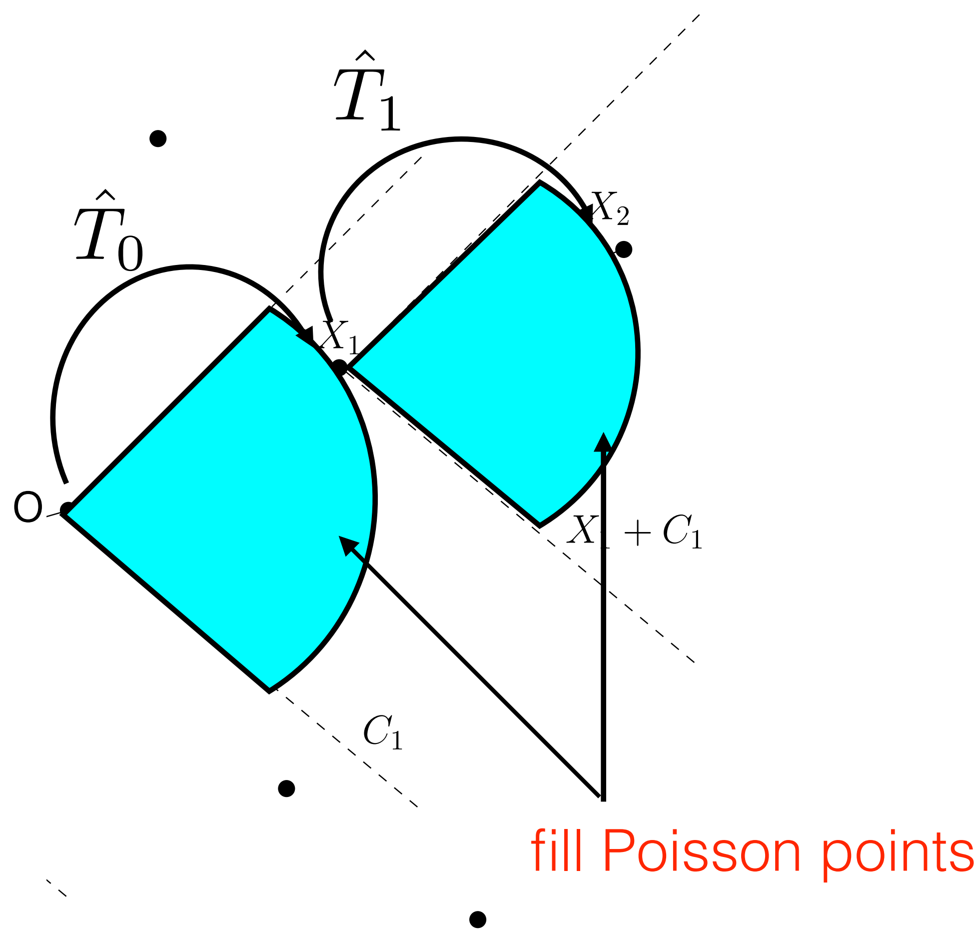
Look at tagged particle at origin



T_0, T_1 are not identically distributed

Information Velocity

Dominate the delay by adding an infinite chain of points



same distribution as R and θ

$\hat{\mathsf{T}}_0, \hat{\mathsf{T}}_1, \ldots$ are identically distributed

Information Velocity

$\hat{T}_0, \hat{T}_1, \ldots$ are stationary

similar to before $E\{\hat{T}_i\} < \infty$

Birkoff's Ergodic Theorem $$\lim_{n\to\infty} \frac{1}{n} \sum_{k=0}^{n-1} \hat{T}_k = \hat{T}$$

where $\hat{T}$ is a rv with mean $E\{\hat{T}_i\}$

$$\nu \geq \lim_{t\to\infty} \frac{\sum_{k=1}^{N(t)} R_k \cos(\theta_k)}{\sum_{k=1}^{N(t)+1} \hat{T}_k} = \frac{\mathbf{E}[R\cos(\theta)]}{\hat{T}}$$

effective distance progress

Conclusions

- Good Old Power Control works !
- Cone angle to be optimized
- Best Upper bound on expected delay ?
- Fundamental Lower Bound on delay and velocity ?

Rahul Vaze teaches at the School of Technology and Computer Science, Tata Institute of Fundamental Research, Mumbai. He obtained his PhD from the University of Texas at Austin. His research interests are in the areas of multiple antenna communication, ad hoc networks, and combinatorial resource allocation. He is the recipient of the Indian National Science Academy's young scientist award for the year 2013.

This book discusses the theoretical limits of information transfer in random wireless networks or ad hoc networks, where nodes are distributed uniformly random in space and there is no centralized control. Examples of ad hoc networks include sensor networks, military networks, and vehicular networks that have widespread applications. Decentralized nature of these networks makes them easily configurable, scalable, and inherently robust.

The author provides a detailed analysis of the two relevant notions of capacity for random wireless networks – transmission capacity and throughput capacity. The book starts with the transmission capacity framework that is first presented for the single-hop model and later extended to the multi-hop model with retransmissions. By reusing some of the tools developed for analysis of transmission capacity, few key long-standing questions about the performance analysis of cellular networks are also addressed for the benefit of students. To complete the throughput capacity characterization, the author finally discusses the concept of hierarchical cooperation that allows the throughput capacity to scale linearly with the number of nodes.

Rahul Vaze teaches at the School of Technology and Computer Science, Tata Institute of Fundamental Research, Mumbai

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Random Wireless Networks

An Information Theoretic Perspective

Rahul Vaze

The optimal role of multiple antennas, ARQ protocols, and scheduling protocols in random wireless networks is identified using the transmission capacity paradigm. This book provides a holistic view of all relevant tools and concepts used to analyse random wireless networks. A conscious attempt is made to bring out the connections between transmission and throughput capacity, between percolation theory and throughput capacity, and stochastic geometry and cellular networks. For effective understanding, an extensive effort is made to explain the physical interpretation of all results.