A Framework for Designing Multihop Energy Harvesting Sensor Networks

Albert Sunny and Joy Kuri

Department of Electronic Systems Engineering

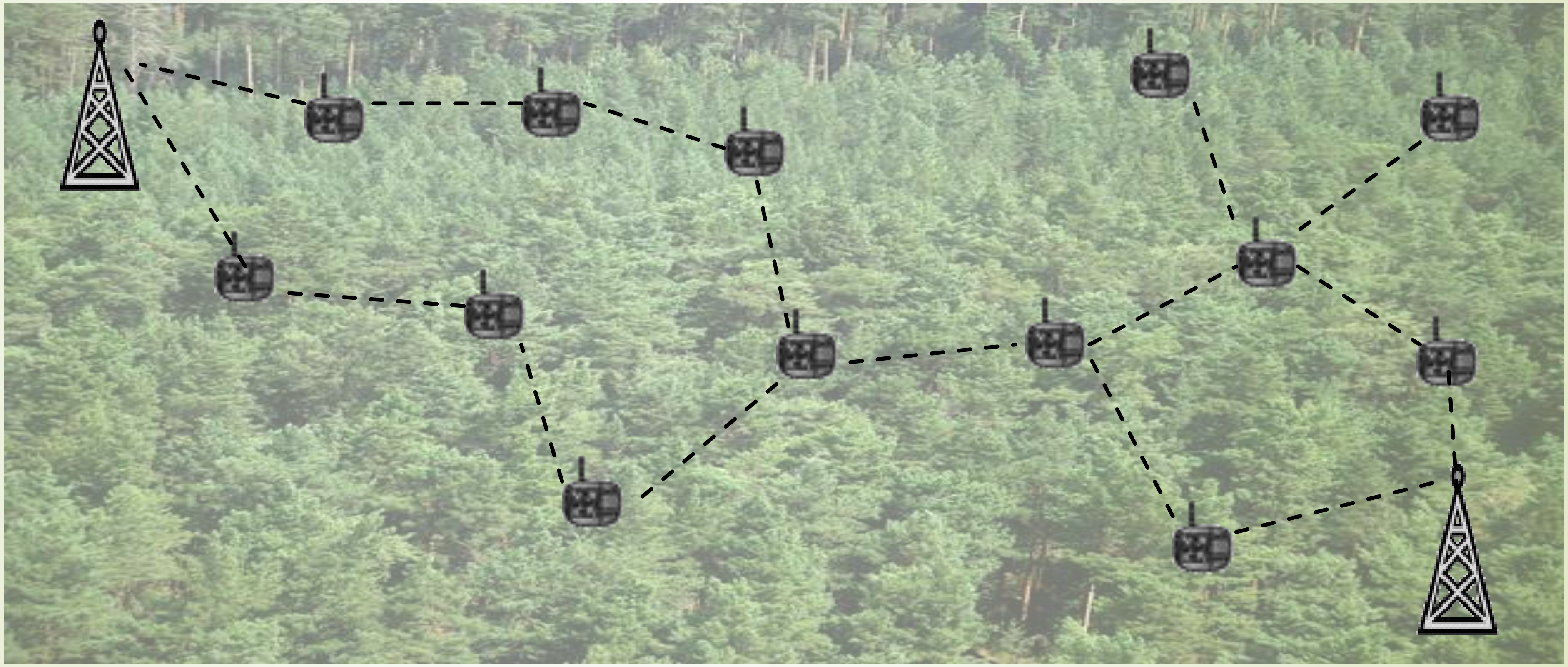
Indian Institute of Science

Use case scenarios

As an integral part of cyber-physical systems, for example

- Urban sensing systems
- Integrated environment monitoring
- Industrial automation
- Civilian surveillance

A typical multihop sensor network deployment



Ingredients of multihop sensor networks

- **Sensor nodes:** equipped with multiple sensor modules, finite energy, finite storage, and typically has a single antenna radio interface.
- **Gateway nodes:** larger nodes equipped with a wireless interface for communications with the WSN, and a wired interface for communications with the controlling station.
- **Ad hoc architecture:** offer a range of benefits, including reliability, robustness, quick and easy network deployment, energy efficient network operations etc.

Issues

- Quality of sensing
- Network flows: Link capacities, routing and scheduling
- Evolving energy levels: Consumption and harvesting

Quality of Sensing

A utility function

- Time is divided into slots of length σ.
- $d_i(t)$: fraction of time sensor node is sensing the environment in the t^{th} slot.
- Let us define $\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T} d_i(t) = d_i$ - fraction of time sensor node i senses. We define the utility as $\sum_{i\in\mathcal{N}} U_i(d_i)$; $\mathrm{U_i}$'s are increasing concave utility function.
- We use this utility function to compare and contrast different deployment scenarios

Network Flows

Evolution of the data queue

- The nodes have finite buffer size of q_{max} units
- In the t^{th} slot, sensor node i produces $r^s \cdot d_i(t)$ units of data.
- The evolution of the data queue is given as follows:

$$q_i(t+1) = \min\{q_{max}, q_i(t) + r^s \cdot d_i(t) + \underbrace{\sum_{k\in\mathcal{N}} \sum_{l\in\mathcal{I}(i)} y_{kl}(t)} - \underbrace{\sum_{k\in\mathcal{N}} \sum_{l\in\mathcal{O}(i)} y_{kl}(t)}\}$$

Data from sensing

Net inflow of data from other nodes

Net outflow of data

Capacity and scheduling constraints

- Let $c_l(t)$ be the capacity of link l, in the t^{th} slot. Then, we have

$$\sum_{k \in \mathcal{N}} y_{kl}(t) \leq c_l(t)$$

- We have assumed the node-exclude interference model. Conflicting links can be scheduled simultaneously. This can be captured using the notion of **maximal independent sets (MIS)**.. Let $a_I(t)$ be the fraction of time MIS I is active, in the t^{th} slot. Then, we have

$$\sum_I a_I(t) \leq 1$$

Energy

Evolution of the battery level

- Let $b_i(t)$ denote the battery level at node i, at the beginning of the t^{th} slot.
- Let $e_i(t)$ be the amount of energy harvested by node i in the t^{th} slot.
- e^s and e denote the energy consumed for sensing and active radio.
- b_{min} be the minimum battery level
- $y_{kl}(t)$ part of flow from node k that is sent over link l in the t^{th} slot
- Then, the evolution of the battery level is given as follows:

$$b_i(t+1) = \min\{b_{max}, b_i(t) + e_i(t) - e^s \cdot d_i(t) - \sum_{k \in \mathcal{N}} e \cdot (\sum_{l \in \mathcal{O}(i)} y_{kl}(t) + \sum_{l \in \mathcal{I}(i)} y_{kl}(t))\}$$

Harvested energy

Energy consumed due to sensing

Energy consumed due to wireless data transfer

Constraints

Energy, Flow, Capacity, Scheduling

Long –term time-averaged system

- In such WSN, the goal is to come up with optimal decision rules for $\{d(t), Y(t), a(t), t \geq 1\}$; usually posed as ***Markov decision process (MDP).***
- However, in our setting, the reward depends on the long-term time-averaged quantities $\{d_1, d_2, \cdots, d_n\}$.
- This enables us to look at the long-term time-averaged system.
- It can be shown that the long-term time-averaged system under consideration should satisfy the following equations

$$e^s \cdot d_j + \sum_{k \in \mathcal{N}} e \cdot \left(\sum_{l \in \mathcal{O}(i)} y_{kl} + \sum_{l \in \mathcal{I}(i)} y_{kl}\right) \leq e_i$$

rate of energy consumption ≤ rate of energy harvesting

Long-term time-averaged system

$$\sum_{l \in \mathcal{O}(i)} y_{il} = r^s \cdot d_i, \sum_{l \in \mathcal{I}(i)} y_{il} = 0$$

→ no accumulation at source

$$\sum_{s \in \mathcal{S}} \sum_{l \in \mathcal{I}(s)} y_{il} = r^s \cdot d_i \quad \forall i \in \mathcal{N}$$

$$\sum_{k \in \mathcal{I}(k)} y_{kl} = \sum_{k \in \mathcal{O}(k)} y_{kl} \quad \forall k \neq i$$

→ no packet drops

flow conservation

$$\sum_{k \in \mathcal{N}} y_{kl} \leq (\mathbf{M} \cdot \mathbf{a})_l$$

rate of flow on a link ≤ effective link capacity

$$\sum_I a_I \leq 1$$

two different MIS cannot be active simultaneously

An optimization problem

$$P_1 : \max_{\mathbf{d},\mathbf{Y},\mathbf{a}} \sum_{j \in \mathcal{N}} U_i(d_i)$$

Subject to:

$$\sum_{l \in \mathcal{O}(i)} y_{il} = r^s \cdot d_i, \sum_{l \in \mathcal{I}(i)} y_{il} = 0$$

$$\sum_{s \in \mathcal{S}} \sum_{l \in \mathcal{I}(s)} y_{il} = r^s \cdot d_i \quad \forall i \in \mathcal{N}$$

$$\sum_{k \in \mathcal{I}(k)} y_{kl} = \sum_{k \in \mathcal{O}(k)} y_{kl} \quad \forall k \neq i$$

$$e^s \cdot d_j + \sum_{k \in \mathcal{N}} e \cdot \left(\sum_{l \in \mathcal{O}(i)} y_{kl} + \sum_{l \in \mathcal{I}(i)} y_{kl}\right) \leq e_i \quad \forall i \in \mathcal{N}$$

$$\sum_{k \in \mathcal{N}} y_{kl} \leq (\mathbf{M} \cdot \mathbf{a})_l$$

$$\sum_I a_I \leq 1$$

computation of matrix M does not scale well with the network size

How to handle the complexity of computing matrix **M**?

- Replace the MIS constraints with the following clique constraints.

$$\mathbf{Fc} \leq \mathbf{1}$$

here $\boldsymbol{F}$ is the contention matrix.

- For the node-exclusive interference model, the clique constraints can be written as

$$\sum_{l \in \mathcal{O}(i) \cup \mathcal{I}(i)} \left(\frac{\sum_{k \in \mathcal{N}} y_{kl}}{c_l^0} \right) \leq 1 \quad \forall i \in \mathcal{N}$$

- We note that for the node-exclusive interference model, $\boldsymbol{F}$ has a computational complexity of $\mathcal{O}(|\mathcal{L}|)$. Clique constraints are computationally scalable, however, they are necessary but not sufficient.

A solution approach

- After replacing the MIS constraint with computationally tractable Clique constraints, we obtain a new optimization problem (problem P_2)
- We can solve problem P_2 by relaxing the energy and the capacity constraints.
- Once these constraints are relaxed, we obtain the dual of problem P_2. The Lagrange multipliers in the dual can be interpreted as prices.
- Following standard approaches, this dual can be decomposed into two sub-problems that can be solved independent of each other

An alternate optimization problem

$$P_1 : \max_{\mathbf{d},\mathbf{Y},\mathbf{a}} \sum_{j \in \mathcal{N}} U_i(d_i)$$

Subject to:

$$\sum\nolimits_{l \in \mathcal{O}(i)} y_{il} = r^s \cdot d_i, \sum\nolimits_{l \in \mathcal{I}(i)} y_{il} = 0$$

$$\sum\nolimits_{s \in \mathcal{S}} \sum\nolimits_{l \in \mathcal{I}(s)} y_{il} = r^s \cdot d_i \quad \forall i \in \mathcal{N}$$

$$\sum\nolimits_{l \in \mathcal{I}(i)} y_{kl} = \sum\nolimits_{l \in \mathcal{O}(i)} y_{kl} \quad \forall k \neq i$$

$$e^s \cdot d_j + \sum\nolimits_{k \in \mathcal{N}} e \cdot (\sum\nolimits_{l \in \mathcal{O}(i)} y_{kl} + \sum\nolimits_{l \in \mathcal{I}(i)} y_{kl}) \leq e_i \quad \forall i \in \mathcal{N}$$

$$\sum\nolimits_{k \in \mathcal{N}} y_{kl} \leq c_l \quad \forall l$$

$$\mathbf{Fc} \leq \mathbf{1}$$

Relaxation of some constraints

And A Path-based Approach

A solution approach

- Joint sensing fraction allocation and routing subproblem

$$d_i(\beta,\gamma) = \left[U'^{-1}\left(\beta_i \cdot e_s + r^s \cdot c_i^{lcp}(\beta,\gamma)\right)\right]^*$$

where c_i^{lcp} is the cost of least-cost path and is given as

$$c_i^{lcp}(\beta,\gamma) = \min_{s\in\mathcal{S}} \min_{P\in\mathcal{P}_{js}} \left(\sum_{l\in P\cap\mathcal{L}} \gamma_l + 2e \cdot \sum_{k\in P\cap\mathcal{N}} \beta_k\right)$$

- Scheduling subproblem is given by the following linear program

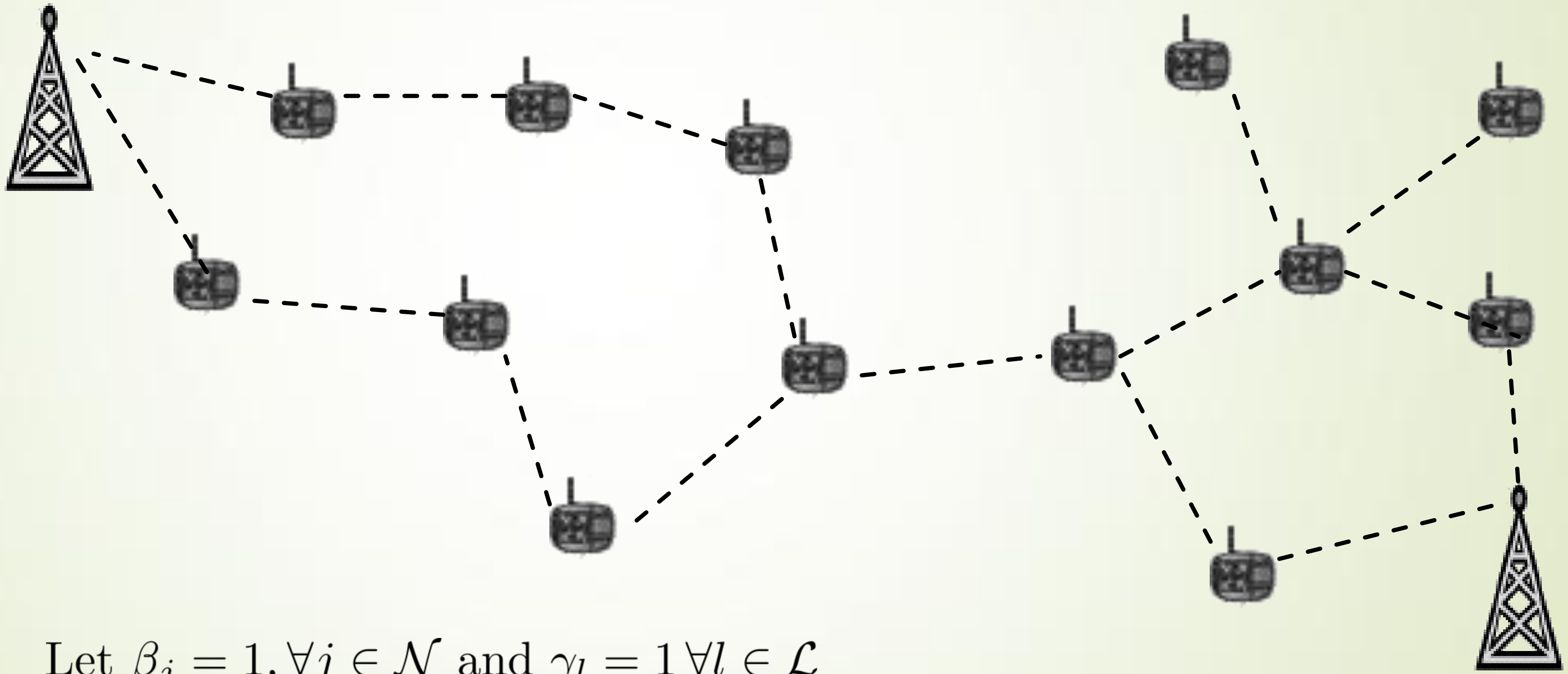
$$\max_{\mathbf{c}\geq 0} \gamma^T \mathbf{c} \quad \text{subject to} \quad \mathbf{F}\mathbf{c} \leq 1$$

- Let $\boldsymbol{p} = [\boldsymbol{\beta},\boldsymbol{\gamma}]^T$ denote the price vector. Then, the price vector can be updated using the projected subgradient method as follows

$$\mathbf{p}(k+1) = [\mathbf{p}(k) - \delta \cdot \mathbf{g}(\mathbf{p}(k))]^+$$

A solution approach

Observation: The collection of least cost paths forms a forest



Let $\beta_j = 1, \forall j \in \mathcal{N}$ and $\gamma_l = 1 \, \forall l \in \mathcal{L}$

Sufficiency of the clique constraints with respect to the optimal utility

- While the clique constraints may not be sufficient to ensure conflict free schedules, we show that the are sufficient to optimally solve our initial resource allocation problem. As a consequence of this, we have the following propositions

Proposition 1: The optimal values of problems P_1 and P_2 are equal.

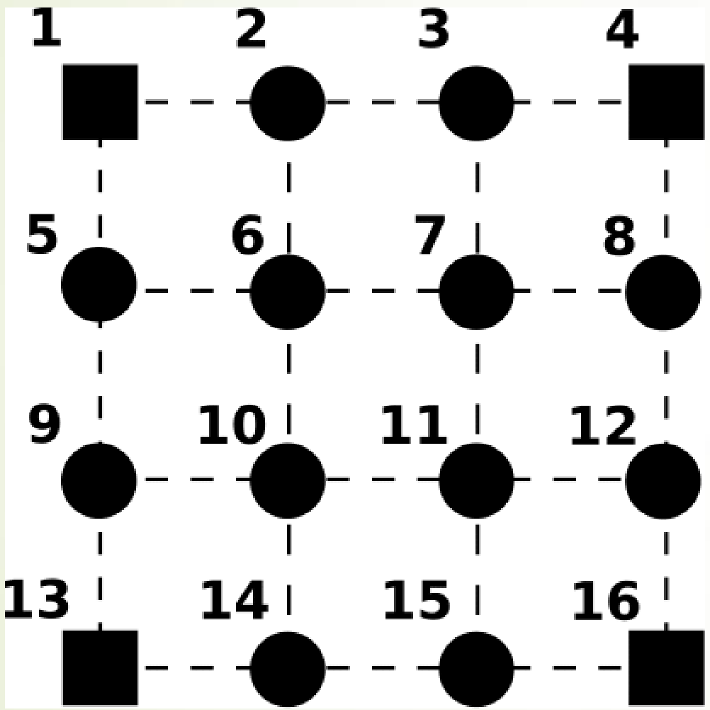
Proposition 2: The projected subgradient method can be made to converges to an ϵ-band around the optimal solution of problem P_1.

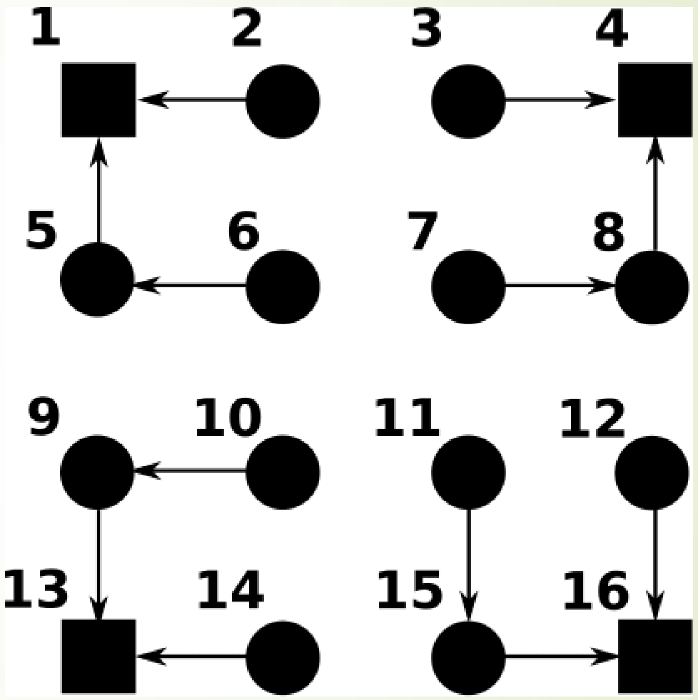
Outline of proof of Proposition 1

- Since clique constraints are not sufficient, we have $P_1^{opt} \leq P_2^{opt}$.
- To show that $P_1^{opt} \geq P_2^{opt}$, consider an optimal price vector $[\beta^*, \gamma^*]$.
- Consider the collection of least cost path from the nodes to the sink nodes
- Remove every other links in the network (these are not part of the optimal route).
- The resulting network is a forest - a perfect graph.
- For a perfect graph, clique constraints are sufficient to ensure conflict-free schedules. Therefore, one can find valid schedules on the reduced network.
- The schedules in the reduced network remain valid in the original network.

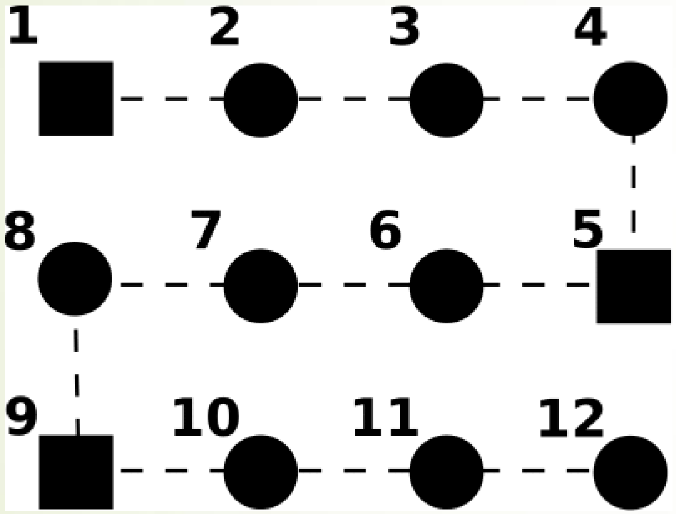
Results

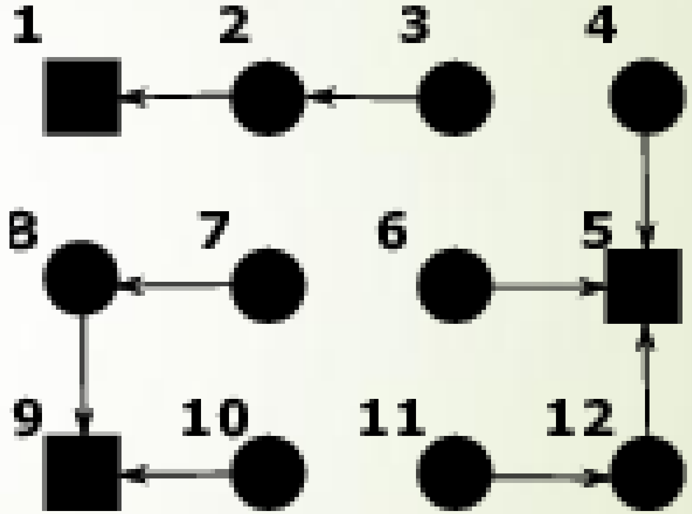
Numerical evaluations





Numerical evaluations





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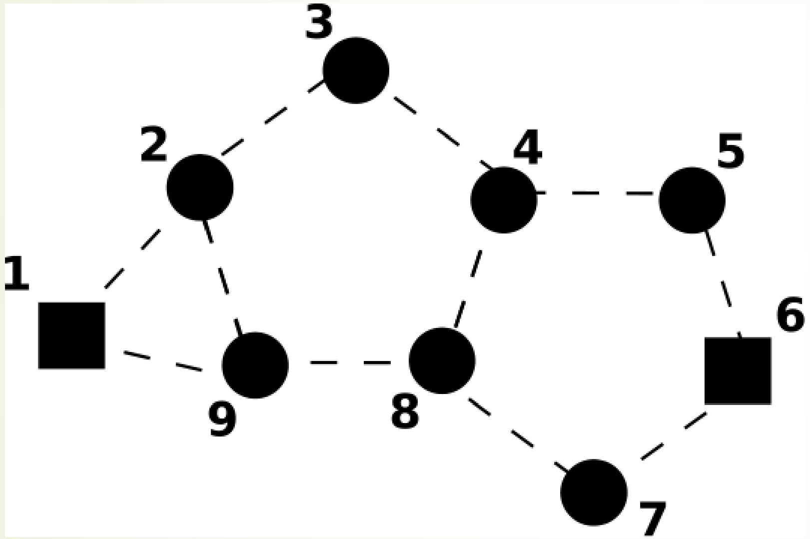

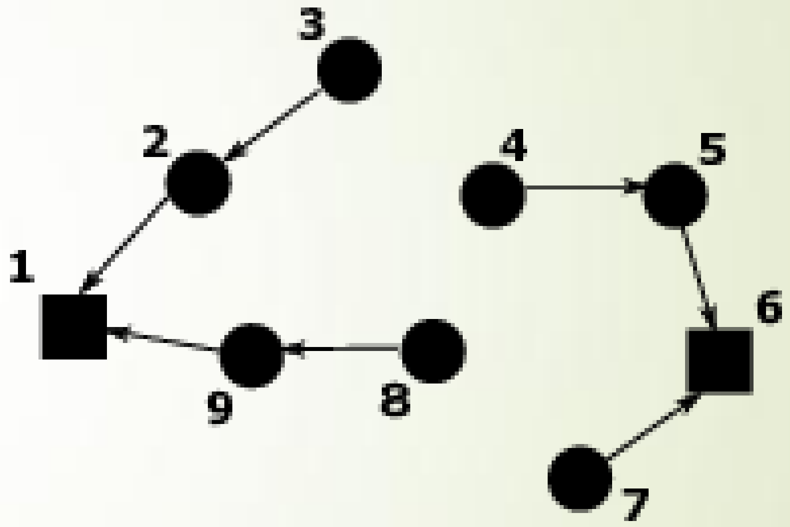

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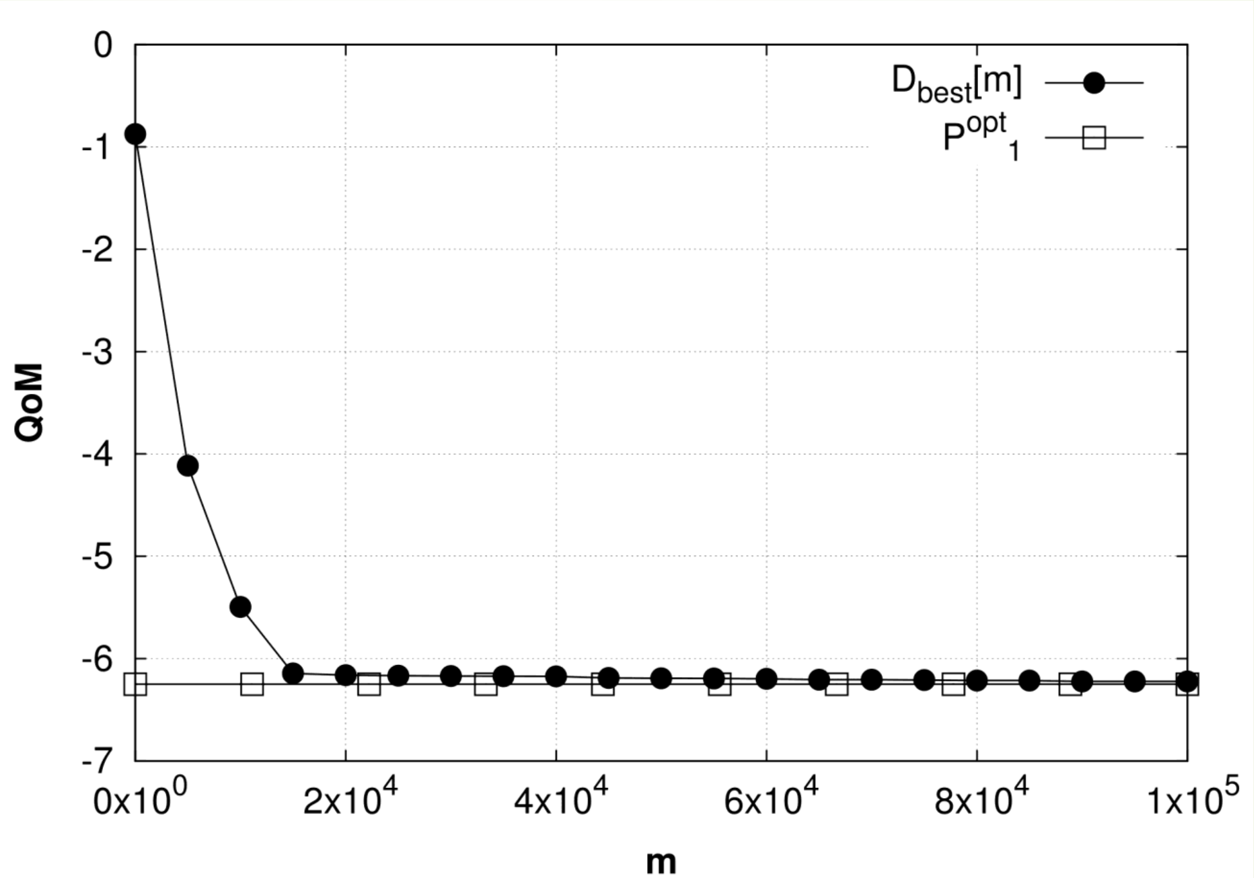

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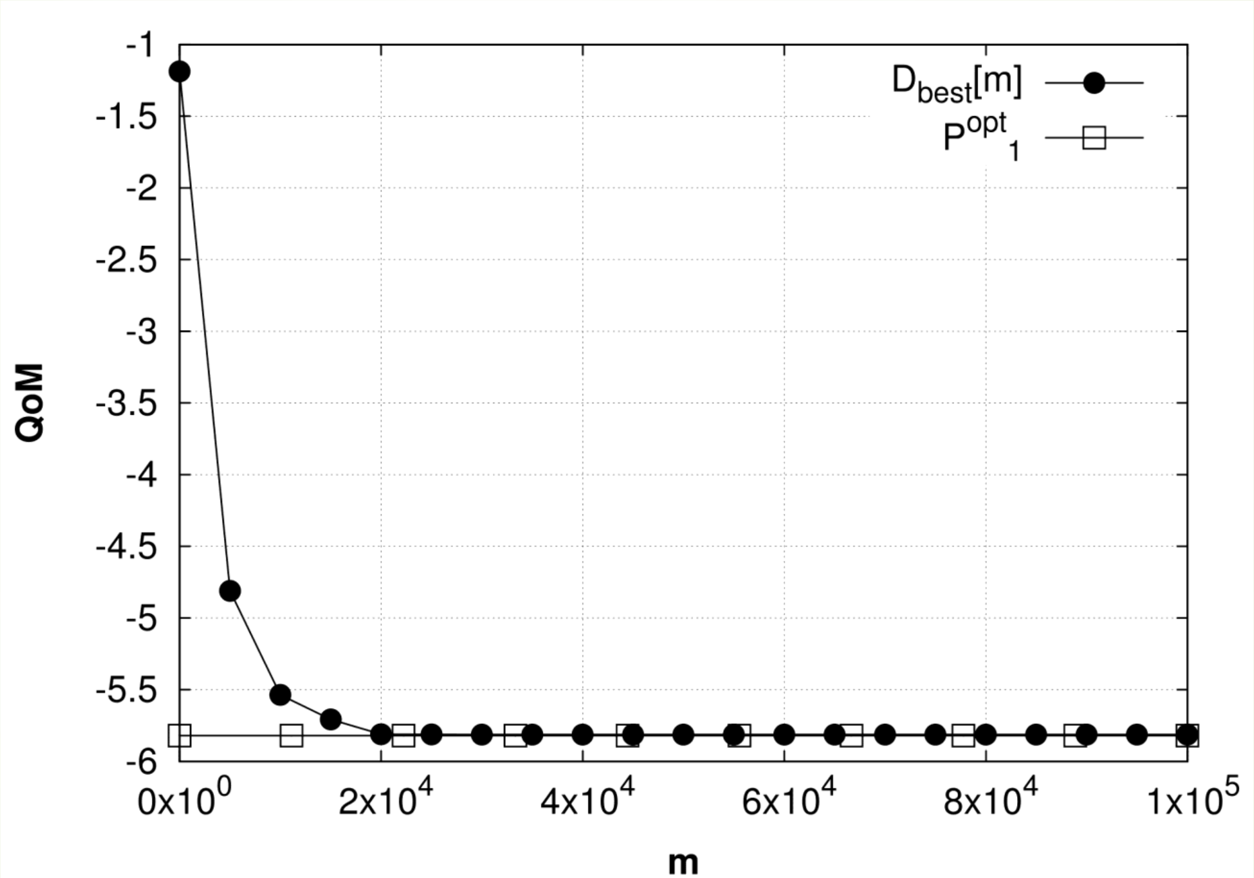

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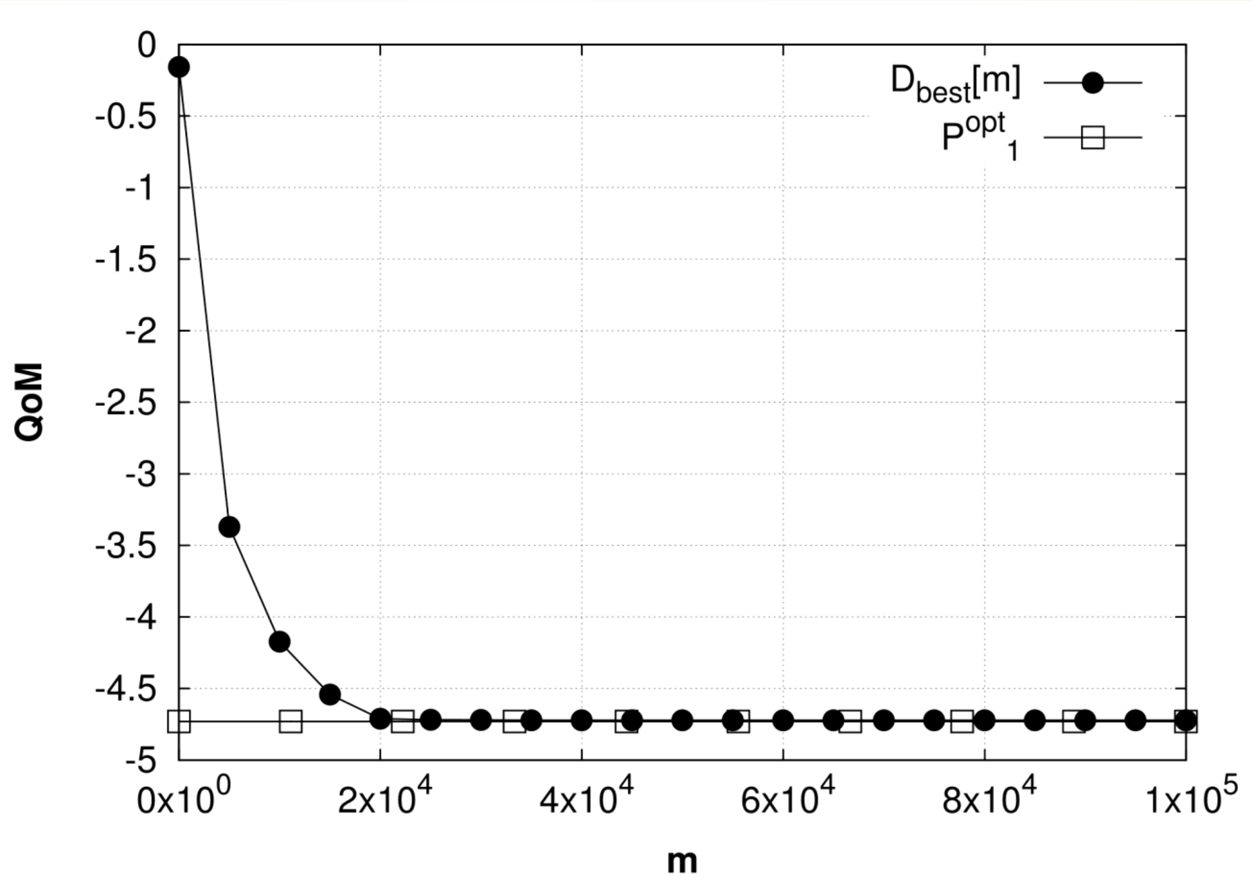

Summary

A few post-deployment challenges

- **Sensing rate allocation**
- **Wireless link scheduling**
- **Routing**
- **Energy management**

Literature survey

- Mao et al. (2012) investigate the problem of maximizing the long-term time averaged sensing rate of WSNs with replenishment under certain QoS constraints for the data and battery queues. But, they have not factored the energy required for sensing into the dynamics of the battery level

- Zussman et al. (2014) consider max-min fair rate allocation and routing in energy harvesting networks. While they consider different routing and multihop topologies, they do not consider capacity and scheduling constraints that are inherent to wireless networks.

- Tan et al. (2015) do not consider any optimization problem, but model the behaviour of the sensor nodes as a potential game where the high harvesting power nodes cooperate with the low harvesting power to ensure that the network remains connected.


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