mmWave systems: Introduction

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This lecture

- Why mm wave is exciting
  - Key applications
- Check that it is not just hype
  - Physical feasibility

Check that the area is interesting and scientifically plausible before commencing work on it

Acknowledgements (Funding)

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Acknowledgements

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  - Partner in crime for the past decade, esp LoS MIMO
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  - early WPAN collaborator
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  - Bridge to CS systems community
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  - mm-wave radar, LoS MIMO
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  - Co-PIs on NSF GigaNets project
- Sanjai Kohli and the Facebook Terragraph team

Students/postdocs

- Eric Torkildson, Munkyo Seo, Colin Sheldon
  - LoS MIMO
- Hong Zhang
  - Diversity on sparse multipath channels
- Dinesh Ramasamy, Sriram Venkateswaran
  - Compressive estimation; training very large arrays
- Babak Mamandipoor
  - Mm-wave radar, super-resolution algorithms
- Zhinus Marzi
  - Picocellular interference analysis, compressive architectures
- Maryam Eslami Rasekh, Yanzi Zhu
  - Experiments with Facebook Terragraph nodes
- Anant Gupta
  - Mm-wave radar

20 good years for wireless

- Digital cellular started in the 1990s
  - 6B mobile phone subscribers today!
  - Connects the most remote locations to the global economy
- WiFi is no slouch either
  - Huge growth in carrier and enterprise markets
  - Huge potential in residential markets in developing nations
- Technology ~~is~~ was converging
  - MIMO, OFDM part of all modern standards

mmWave represents a fundamental disruption

mmWave: what's different?

- System goals: multiGbps wireless
- Bandwidth no longer a constraint
- Channel characteristics
  - Sparse rather than rich scattering
- The nature of MIMO
  - Beamforming, diversity, multiplexing all different at tiny wavelengths
- Signal processing at multiGbps speeds
  - ADC is a bottleneck, OFDM may not be the best choice
- Networking with highly directional links

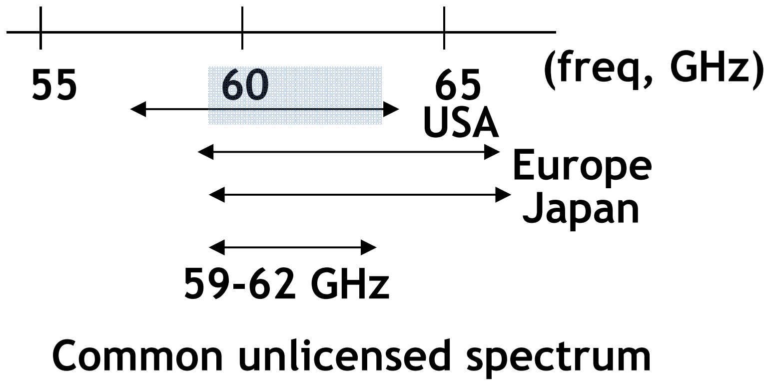
So really, everything is different!

Why the interest in mmWave?

A few marketing slides

The end of spectral hunger (at short ranges)

60 GHz: 7 GHz of unlicensed spectrum in US, Europe, Japan

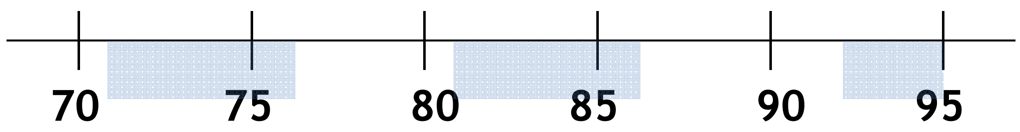


Oxygen absorption band

Ideal for short-haul multihop

(reduced interference)

E/W bands: 13 GHz of spectrum in US with minimal licensing/registration



Avoids oxygen absorption

Good for long-haul P2P

Bands beyond 100 GHz becoming accessible as RFIC and packaging technology advances

Initial industry focus: indoor 60 GHz networks

- WiGig spec/IEEE 802.11ad standard: up to 7 Gbps
- Support for moderately directional links
- 32 element antennas that can steer around obstacles

www.technologyreview.com

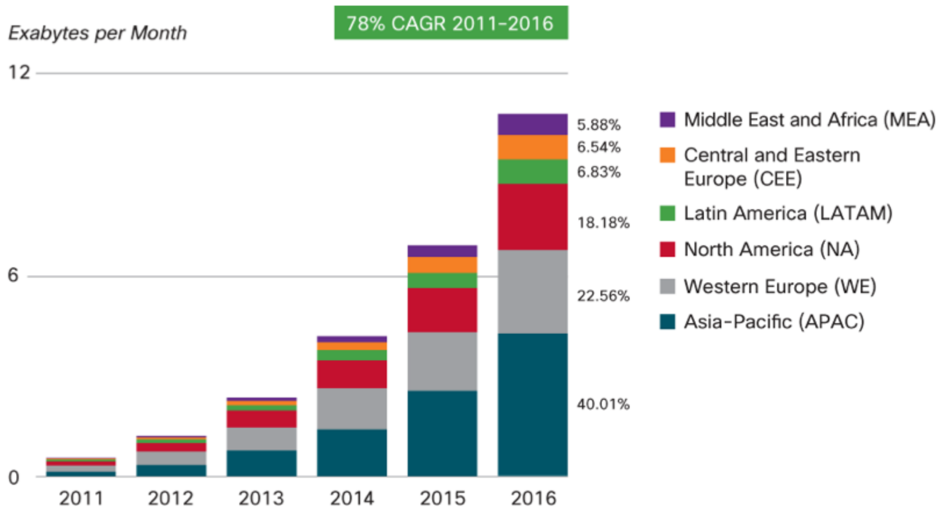
Progress due to push for WiGig

- 60 GHz CMOS RFICs ✓
  - WiFi-like economies of scale if and when market takes off
- Antenna array in package (32 elements) ✓
  - Good enough for indoor consumer electronics applications
- MAC protocol supporting directional links ✓
  - Good enough for quasi-static environments
  - Does not provide interference suppression
  - Does not scale to very large number of elements
- Gigabit PHY ✓
  - Standard OFDM and singlecarrier approaches
  - Does not scale to 10 Gbps at reasonable power consumption (ADC bottleneck)

Current focus: mmWave for cellular

Driven by exponential growth in cellular data demand

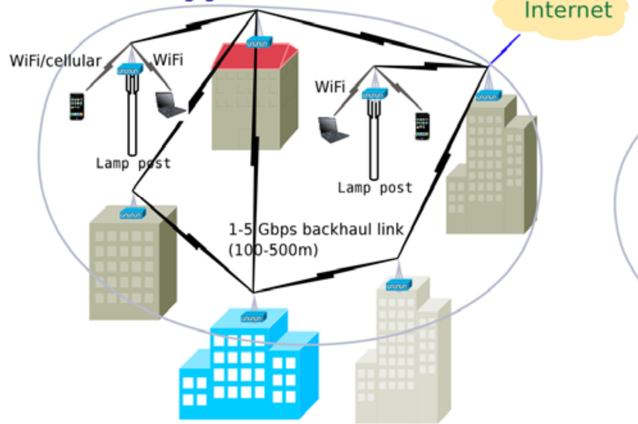
Figure 2. Global Mobile Data Traffic Forecast by Region

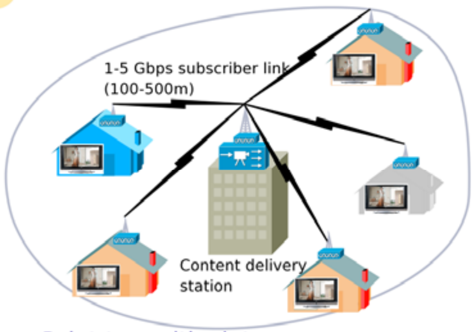


Source: Cisco VNI Mobile, 2012

NEED EXPONENTIAL INCREASE IN CELLULAR NETWORK CAPACITY (without breaking the bank)

mmWave Picocells and broadband

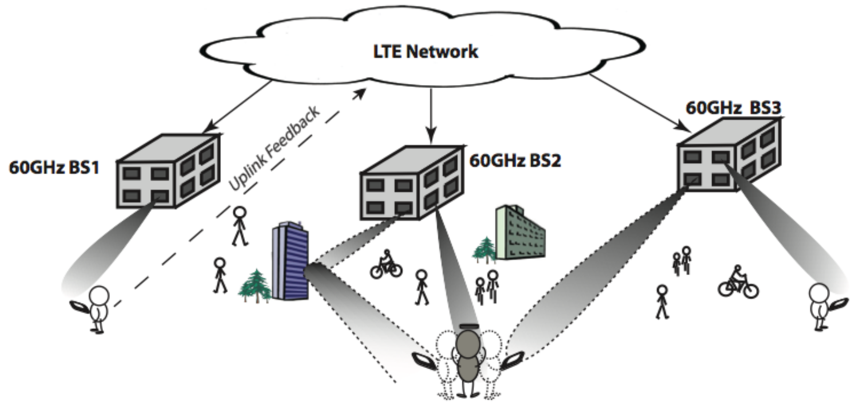




Point-to-multipoint
content distribution infrastructure

Terragraph project
(60 GHz, repurposing WiGig)

MultiGigabit mesh networks are happening now



mmWave to the mobile will happen soon

Industry consensus on the need for Cellular 1000X

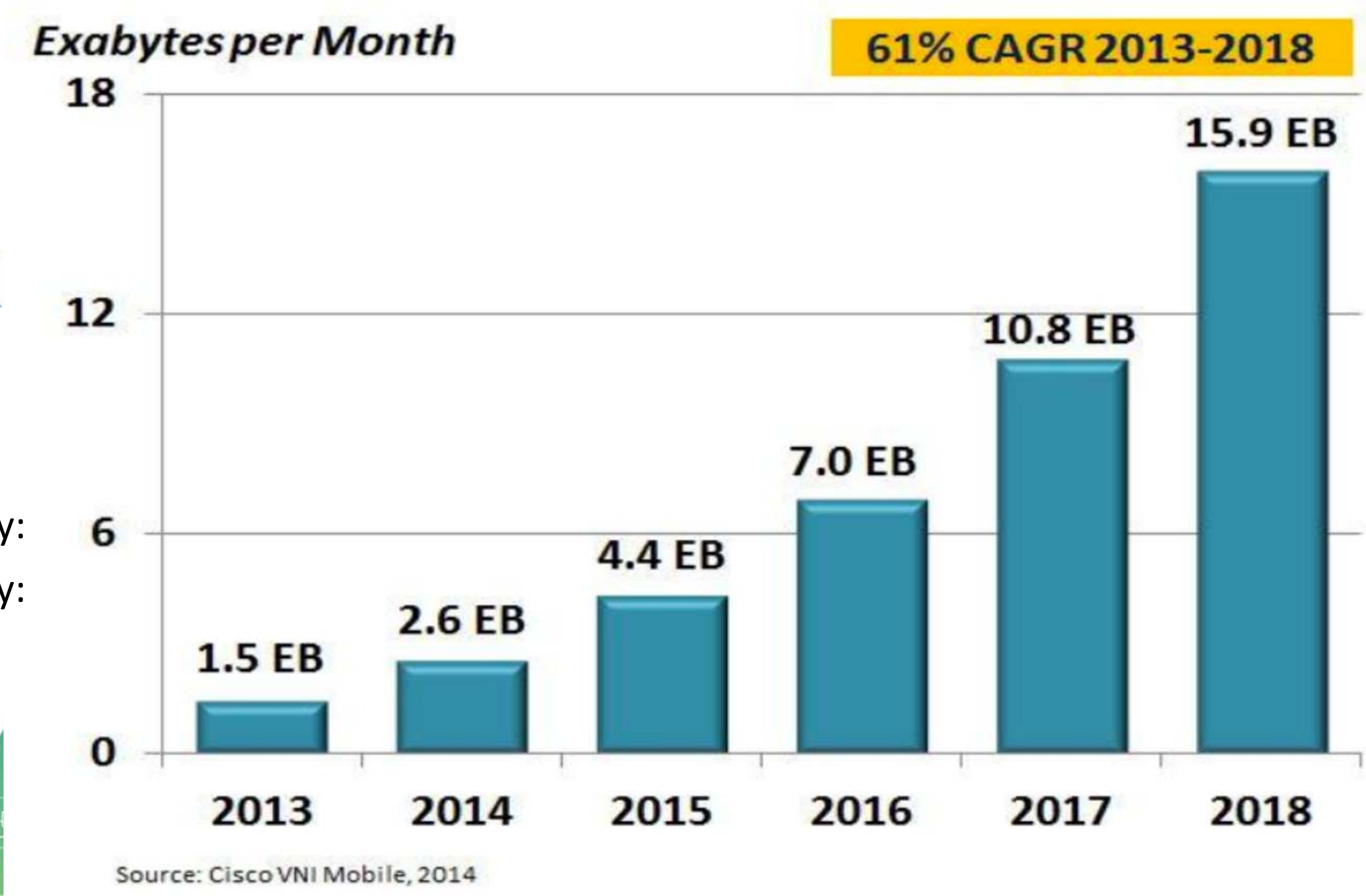


Figure 2.1. Global Mobile Data Traffic growth 2013 to 2018 (Cisco VNI).

NOK

Enhan

Figure courtesy: Nokia

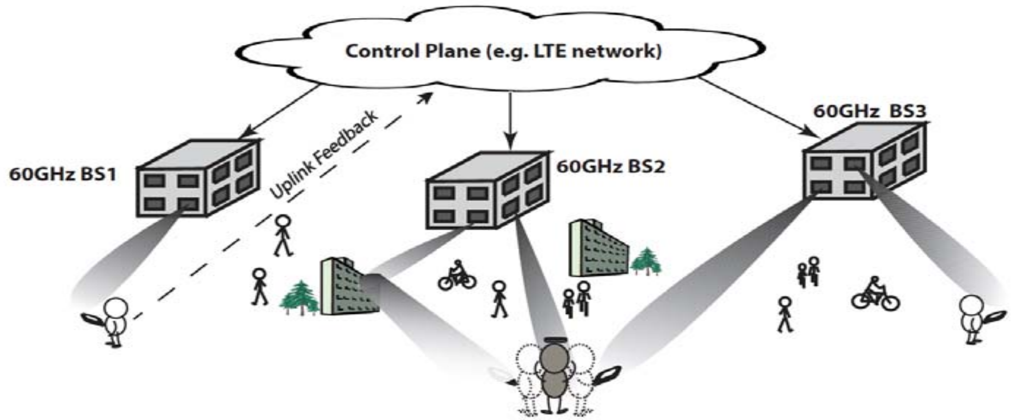
Figure courtesy: Qualcomm

Figure courtesy: Cisco

Mm-wave enables aggressive spatial reuse

- Large arrays in small form factors
- Directive links
- Limited interference
- Dense cells / much higher spatial reuse

32 x 32
8 x 8 cm^2

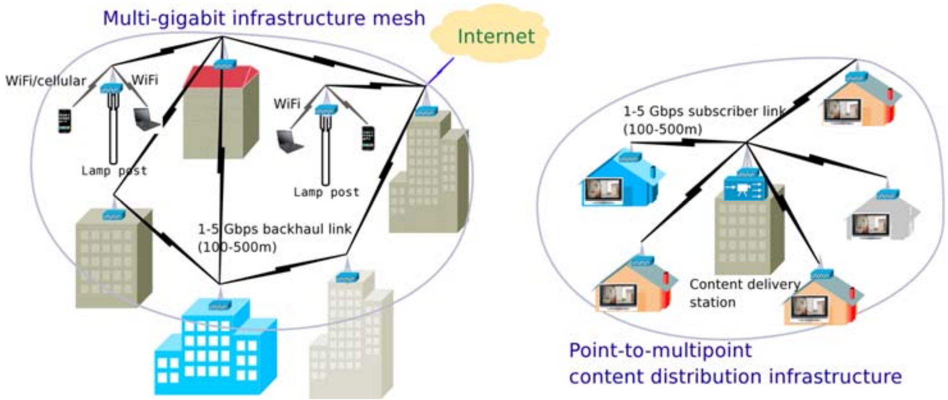


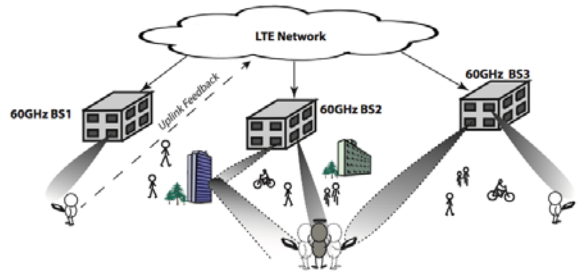
mmWave for the under-served

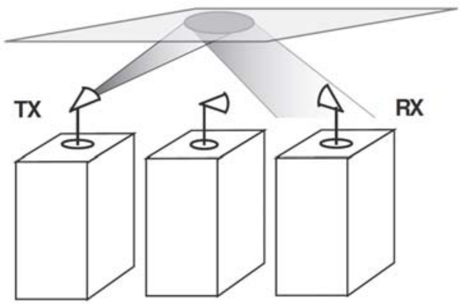
'loons and drones

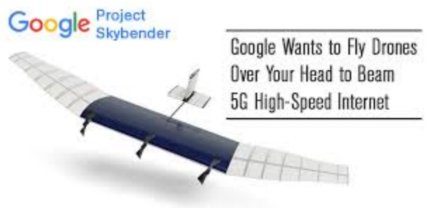
MOUNTAIN VIEW, Calif.—As of March 17, the FCC has granted permission to Google to perform airborne and terrestrial millimeter wave testing throughout the U.S., according to Android Headlines and other sources. The testing frequencies cover the 71-76 GHz and 81-86 GHz range. Google's window for testing will come to a close on April 1.

In short: mmWave is the future of communications



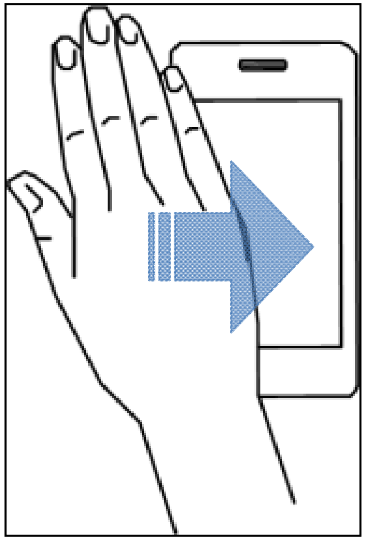






In addition…mmWave commodity radar

Vehicular situational awareness



Gesture recognition

Designs constrained by cost, complexity and geometry
Very different from classical long-range military radar

Concept Systems → Research Opportunities

- Revisiting MIMO
  - For tiny wavelengths
- Revisiting signal processing architectures
  - The ADC bottleneck
- Revisiting networking
  - Highly directional links change MAC design considerations
  - Multi-band operation (e.g., 1-5 GHz and 60 GHz)
- Revisiting radar
  - Short-range geometry and hardware constraints
- Inherently cross-layer even at the level of comm and estimation theory
  - Node form factor, hardware constraints, propagation geometry

Example research in our group

- Established that blockage is not a dealbreaker
  - Even in cluttered indoor environments
- LoS MIMO: theory and prototype
  - The road to “wireless fiber”
- Diversity for sparse multipath
  - Five 9s wireless backhaul is possible
- Mesh networking with highly directional links
  - Trade off deafness against lower interference
  - Routing and resource allocation for mm-wave backhaul
- Mm wave picocellular networks
  - Need to adapt large arrays ➔ theory of compressive estimation, super-resolution algorithms
  - Compressive network architecture
  - Interference analysis showing Cellular 1000-10000X is feasible
  - Experimental results
- ADC-limited communication
  - Fundamental limits, time-interleaved ADC, analog multiband
- Mm wave radar
  - Fundamentals of short-range radar
  - New target models and algorithms

Now that we are motivated…

The Plan

- The mmWave channel
- MIMO concepts revisited
  - Spatial multiplexing and diversity for sparse channels
- Steering large arrays: theory and algorithms
  - Compressive estimation, super-resolution
- Networking with highly directional links: mesh networks, picocells
- Signal processing at high bandwidths
- Short-range mmWave radar

Step 0: can we close the link?

Link Budgets

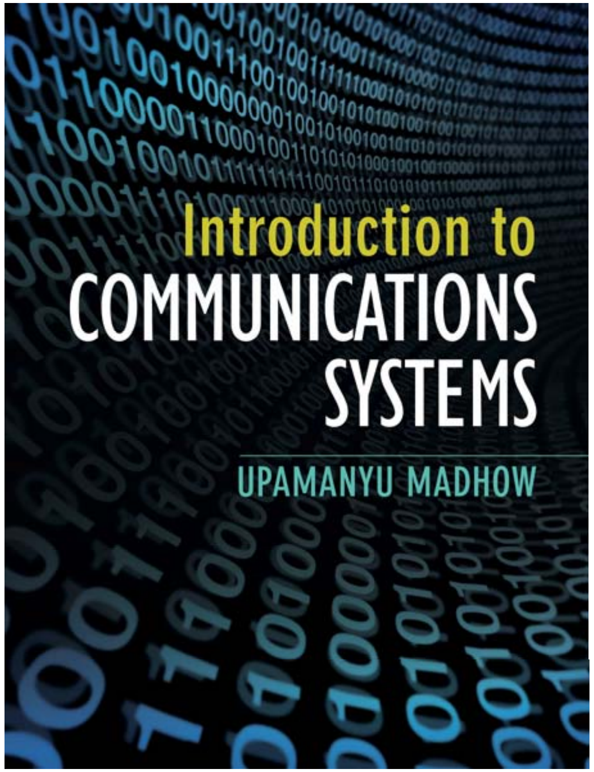
Is propagation on our side?

- Can we attain the kind of system specs we want with technology compatible with the mass market?
  - Link budget for indoor links
  - Link budget for outdoor links (oxygen absorption)
- CMOS power amps: sweet spot 0-10 dBm
- SiGe power amps can go higher
- With power pooling and beamforming gain from antenna arrays, can we go far enough so it is interesting?

UWB researchers should have asked such questions

Link budget analysis

Undergraduate level book-keeping
that we should all know about



Example link budgets done on board

Link budget analysis: a review

Basic comm theory maps modulation & coding scheme to E_b/N_0 requirement; we then need to map to received power needed

Receiver sensitivity: minimum received power required to attain a desired error probability
(depends on the modulation scheme, bit rate, channel model, receiver noise figure)

We can now design the physical link parameters: transmit and receive antennas, transmit power, link range

Link budget: Once we know the receiver sensitivity, we can work backward and figure out the physical link parameters required to deliver the required received power (plus a margin of safety)

Receiver sensitivity

$$SNR_{reqd} = \left(\frac{E_b}{N_0}\right)_{reqd} \frac{R_b}{B}$$

$$P_n = N_0 B = N_{0,nom} 10^{F/10} B$$ (noise power)

Receiver sensitivity (minimum receive power needed)

$$P_{RX}(\min) = SNR_{reqd} P_n = \left(\frac{E_b}{N_0}\right)_{reqd} \frac{R_b}{B} N_0 B = \left(\frac{E_b}{N_0}\right)_{reqd} R_b N_{0,nom} 10^{F/10}$$

Receiver sensitivity in dBm

At room temperature and for a bandwidth of 1 Hz, the noise power equals -174 dBm

$$kT_{room} \times 1\text{ Hz} = 4 \times 10^{-21}\text{ W} = 4 \times 10^{-18}\text{ mW}$$

$$\Rightarrow \text{-174 dBm}$$

$$\boxed{\text{Noise power over 1 Hz} = -174 + F \text{ dBm}}$$

(for noise figure of F dB)

We therefore obtain

$$\boxed{P_{RX,\mathrm{dBm}}(\min) = \left(\frac{E_b}{N_0}\right)_{reqd,dB} + 10\log_{10} R_b - 174 + F}$$

How should we design the system to attain the desired RX sensitivity?

Need to relate transmit power to received power

Numerical value of noise PSD

$$\text{What is the value of } N_0\text{?}$$

White noise arising from many devices in the receiver can be summarized into a single quantity
Noise figure: tells us how big the PSD is with respect to a benchmark
Benchmark: thermal noise of a resistor with matched impedance at "room temperature"

$$\text{Benchmark noise power} = kTB$$

$$k = 1.38 \times 10^{-23} \text{ Joules/Kelvin (Boltzmann's constant)}$$

$$T \text{ is temperature, } B \text{ is bandwidth}$$

$$\text{Noise figure } F = \frac{N_0}{kT_{room}}$$

$$T_{room} = 290 \text{ Kelvin}$$

Noise figure usually expressed in dB, and noise power can be computed as follows:

$$P_n = N_0 B = kT_{room} 10^{F(dB)/10} B$$

Power often expressed in dBm

$$\text{Power (dBm)} = 10 \log_{10}(\text{Power (milliwatts)})$$

Free space propagation

The simplest model for how transmit power translates to received power

Isotropic transmission ➔ at range *R*, the power is distributed over the surface of a sphere of radius *R*

Receiver antenna provides an aperture with an effective area for catching a fraction of this power

$$P_{RX} = \frac{P_{TX}}{4\pi R^2} A_{RX}$$

Receive antenna aperture

If the transmitter uses a directional antenna:

$$P_{RX} = \frac{P_{TX}}{4\pi R^2} G_{TX} A_{RX}$$

Transmit antenna gain

Relating gain to aperture

Antenna gain = ratio of aperture to that of an isotropic antenna

$$G = \frac{A}{\lambda^2/(4\pi)} = \frac{4\pi A}{\lambda^2}$$

Aperture for an
"isotropic" antenna

Remarks

--For given aperture, gain decreases with wavelength

--Aperture roughly related to area → at lower carrier frequencies (larger wavelengths) we need larger form factors to achieve a given antenna gain

Friis' formula for free space propagation

Given the antenna gains:

$$P_{RX} = P_{TX}\, G_{TX}\, G_{RX}\, \frac{\lambda^2}{16\pi^2 R^2}$$

For fixed antenna gains, the larger the wavelength the better

Given the antenna apertures:

$$P_{RX} = P_{TX}\, \frac{A_{TX} A_{RX}}{\lambda^2 R^2}$$

For fixed antenna apertures (roughly equivalent to fixed form factors), the smaller the wavelength the better, provided we can point the transmitter and receiver at each other

Applying Friis' formula

Going to the dB domain:

$$P_{RX,\mathrm{dBm}} = P_{TX,\mathrm{dBm}} + G_{TX,\mathrm{dBi}} + G_{RX,\mathrm{dBi}} + 10\log_{10}\frac{\lambda^2}{16\pi^2R^2}$$

More generally:

$$P_{RX,\mathrm{dBm}} = P_{TX,\mathrm{dBm}} + G_{TX,\mathrm{dBi}} + G_{RX,\mathrm{dBi}} - L_{pathloss,\mathrm{dB}}(R)$$

Plug in your favorite model for path loss

Free space path loss model gives us back the first formula:

$$L_{pathloss,\mathrm{dB}}(R) = 10\log_{10}\frac{16\pi^2R^2}{\lambda^2}$$

Link budget

Given a desired receiver sensitivity,

what is the required transmit power to attain a desired range?

OR

what is the attainable range for a given transmit power?

Must account for transmit and receive directivities, path loss, and add on a link margin (for unmodeled, unforseen contingencies)

$$P_{TX,\mathrm{dBm}} = P_{RX,\mathrm{dBm}}(\mathrm{min}) - G_{TX,\mathrm{dBi}} - G_{RX,\mathrm{dBi}} + L_{pathloss,\mathrm{dB}}(R) + L_{margin,\mathrm{dB}}$$

Example 60 GHz indoor link budget

2.5 Gbps link using QPSK and rate 13/16 code operating 2 dB from Shannon limit

$$(E_b/N_0)_{reqd} \approx 2.5dB$$

Noise figure 6 dB

Receiver sensitivity = -71.5 dBm

4x4 antenna array at each end, 2 dBi gain per element
➔ 14 dBi gain at each end

10 m range ➔ free-space path loss is about 88 dB

Transmit power with 10 dB link margin is only about -1.5 dBm!
(➔ can use less directive antennas)

Role of channel coding

Semi-powerful code leads to Eb/N0 requirement of 2.5 dB
Uncoded QPSK with 10^{-6} BER would require Eb/N0 of 10 dB

Not that important for indoor link
(can easily bump up TX power by 10 dB)
But can make a big difference outdoors

Example 100 m outdoor 60 GHz link (backhaul, base-to-mobile)

Using 10 m indoor link budget as reference

Free space propagation loss increases by 20 dB

Oxygen absorption (16 dB/km) leads to 1.6 dB additional loss

Rain margin (25 dB/km for 2 inches/hr): 2.5 dB

Required transmit power goes up to **22.6 dBm**
For 4x4 array, TX power per element is **10.6 dBm**
(doable with CMOS, easy with SiGe)
EIRP = 22.6 dBm + 14 dBi = **36.6 dBm** < FCC EIRP limit of 40 dBm

Coding gain plays an important role here: why?

What the link budgets tell us

- 60 GHz is well matched to indoor networking and to picocellular networks
  - Oxygen absorption has limited impact at moderate ranges
  - Heavy rain can be accommodated in link budget
  - Moderate directivity suffices
  - Electronically steerable links give flexibility in networking
  - Low-cost silicon implementations are possible
- For truly long range, need to avoid 60 GHz
  - 71-76, 81-86 GHz as candidates
  - Bands above 100 GHz
  - Need very high directivity (can we steer effectively?)

Now we can pursue novel system concepts

Systems to be explored

- LoS MIMO: wireless links at optical speeds
- MultiGigabit mesh backhaul
- MultiGigabit picocellular networks: cellular 1000X

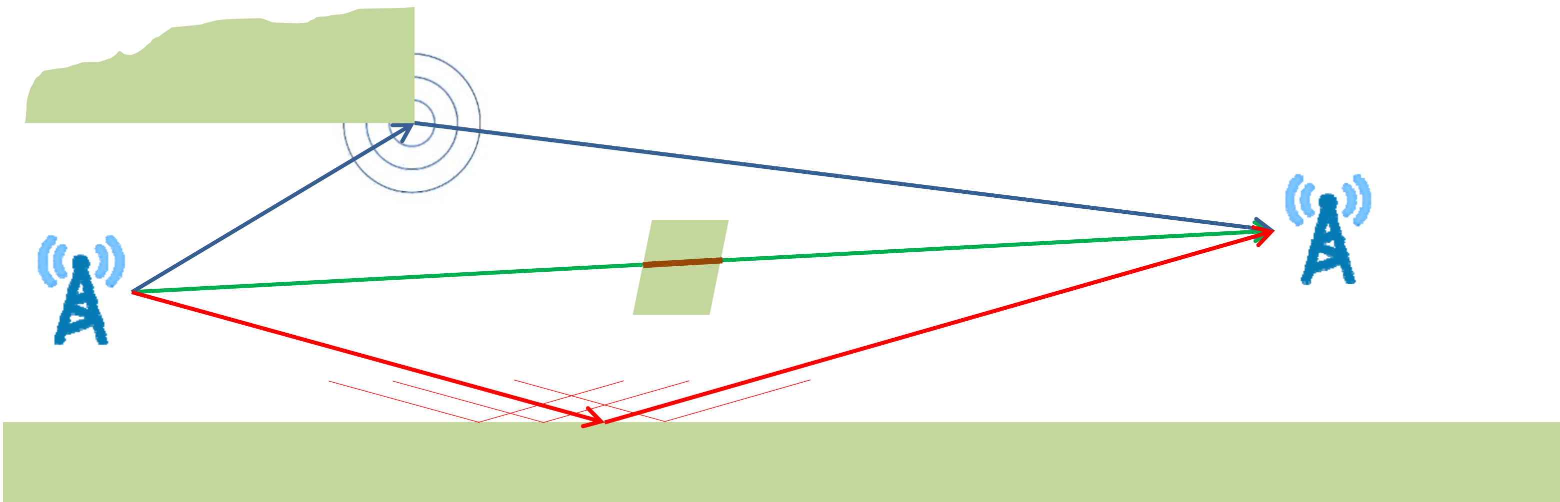
Mm wave channel modeling

Maryam Eslami Rasekh
(presented by U. Madhow)

MR2

Basics of channel modeling

- Sum of propagation paths
  - Free space propagation (LOS)
  - Specular reflection
  - Propagation through dielectric obstacles
  - Diffraction and scattering



All these components are strong in conventional lower frequency bands (<6GHz)

but in mmwave..?

Slide 2

MR2	not sure how you wanted this slide, here are 3 versions. I think the second one makes most sense since the text says channel is "combination of" different paths Maryam Rasekh, 09-06-2016

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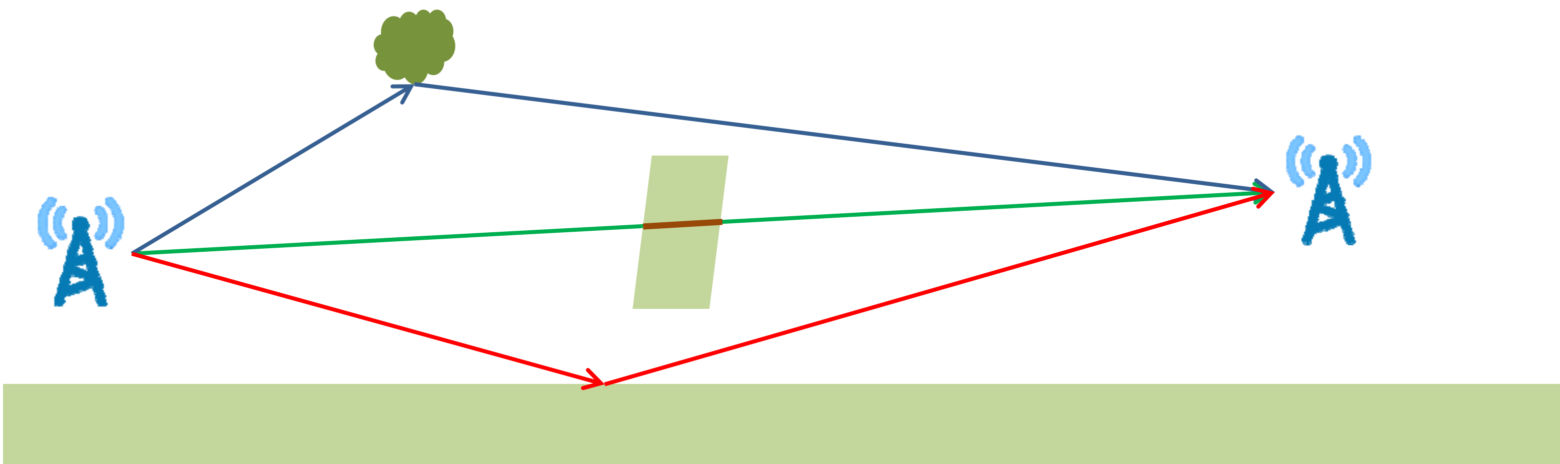

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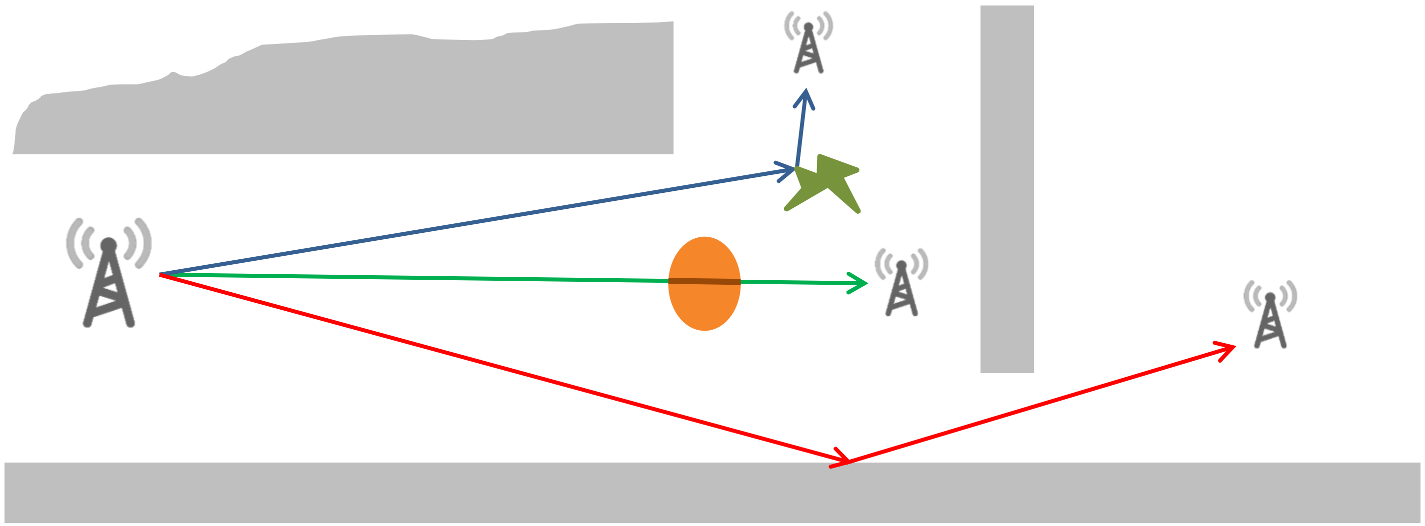

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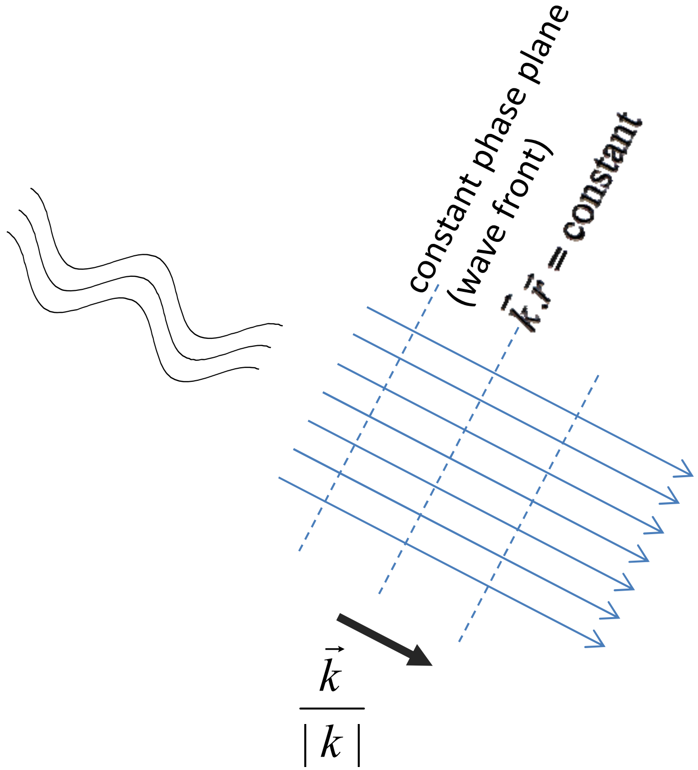
Reflection

- Plane wave traveling in homogenous environment

$$E(\vec{r},t) = |\vec{E}_0| \cos(2\pi f t - \vec{k}.\vec{r} + \angle\vec{E}_0)$$



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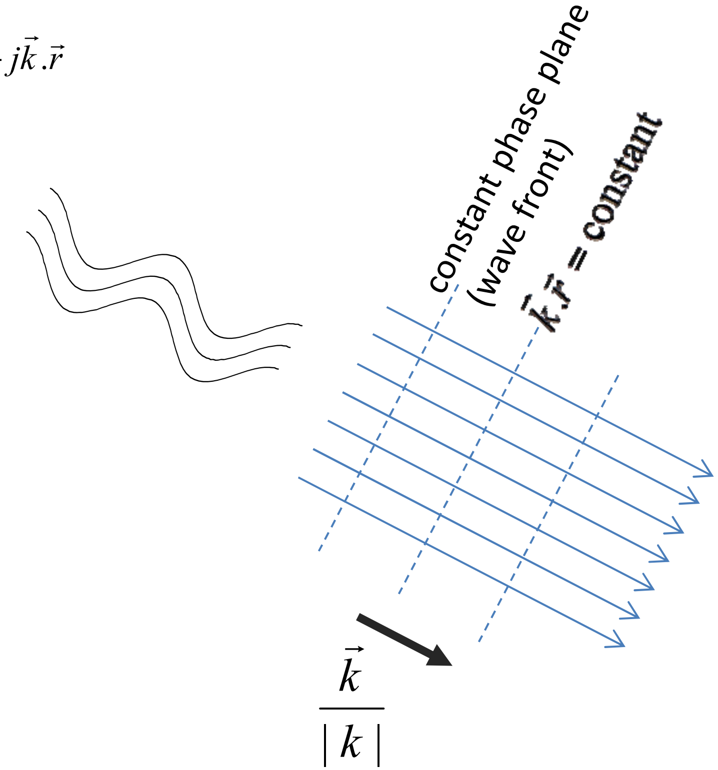

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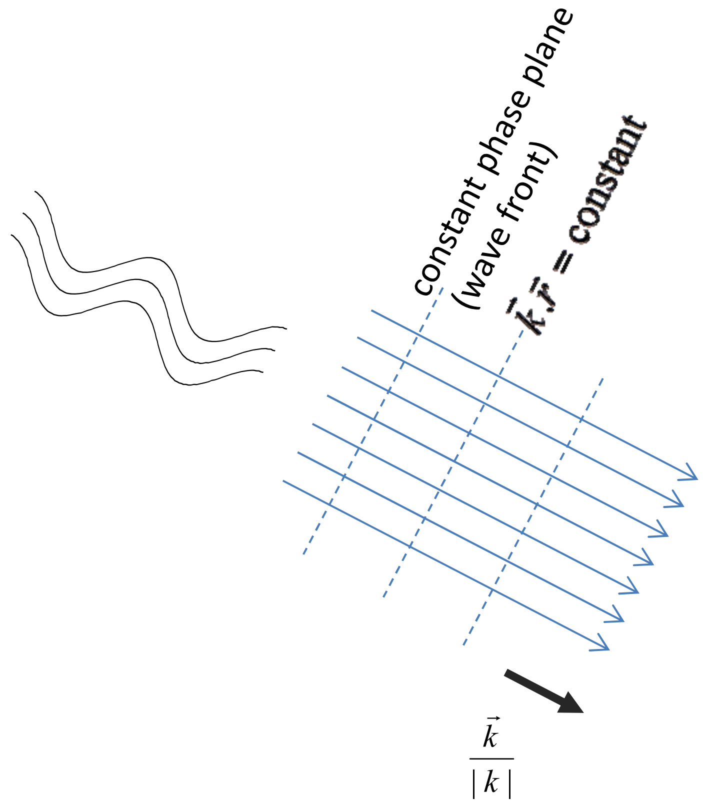
$$|k| = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} = 2\pi f \sqrt{\mu\varepsilon}$$

wave number



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$$|k| = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} = 2\pi f\sqrt{\mu\varepsilon}$$

Magnetic permeability $\mu = \mu_r \mu_0$

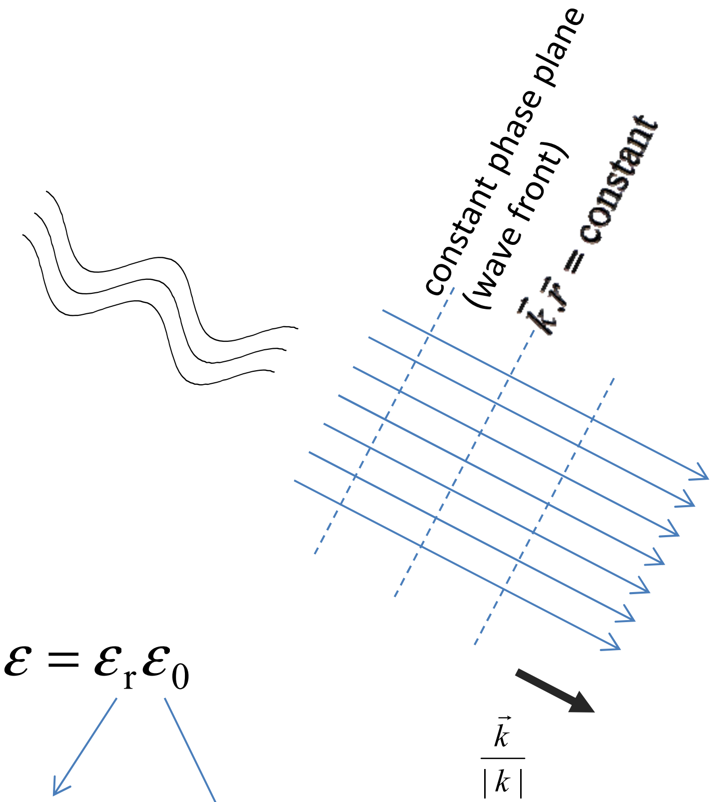
Electric permittivity $\varepsilon = \varepsilon_r \varepsilon_0$

Relative permeability

Magnetic permeability of vacuum

Relative permittivity

Electric permittivity of vacuum



Reflection

- Plane wave traveling in homogenous environment

$$\vec{E}(\vec{r},t) = |\vec{E}_0| \cos(2\pi f t - \vec{k}.\vec{r} + \angle\vec{E}_0)$$

Most substances are dielectrics with no magnetic properties:

$\mu_r = 1$

$\varepsilon_r \geq 1$

Magnetic permeability $\mu = \mu_r \mu_0$

Relative permeability

Magnetic permeability of vacuum

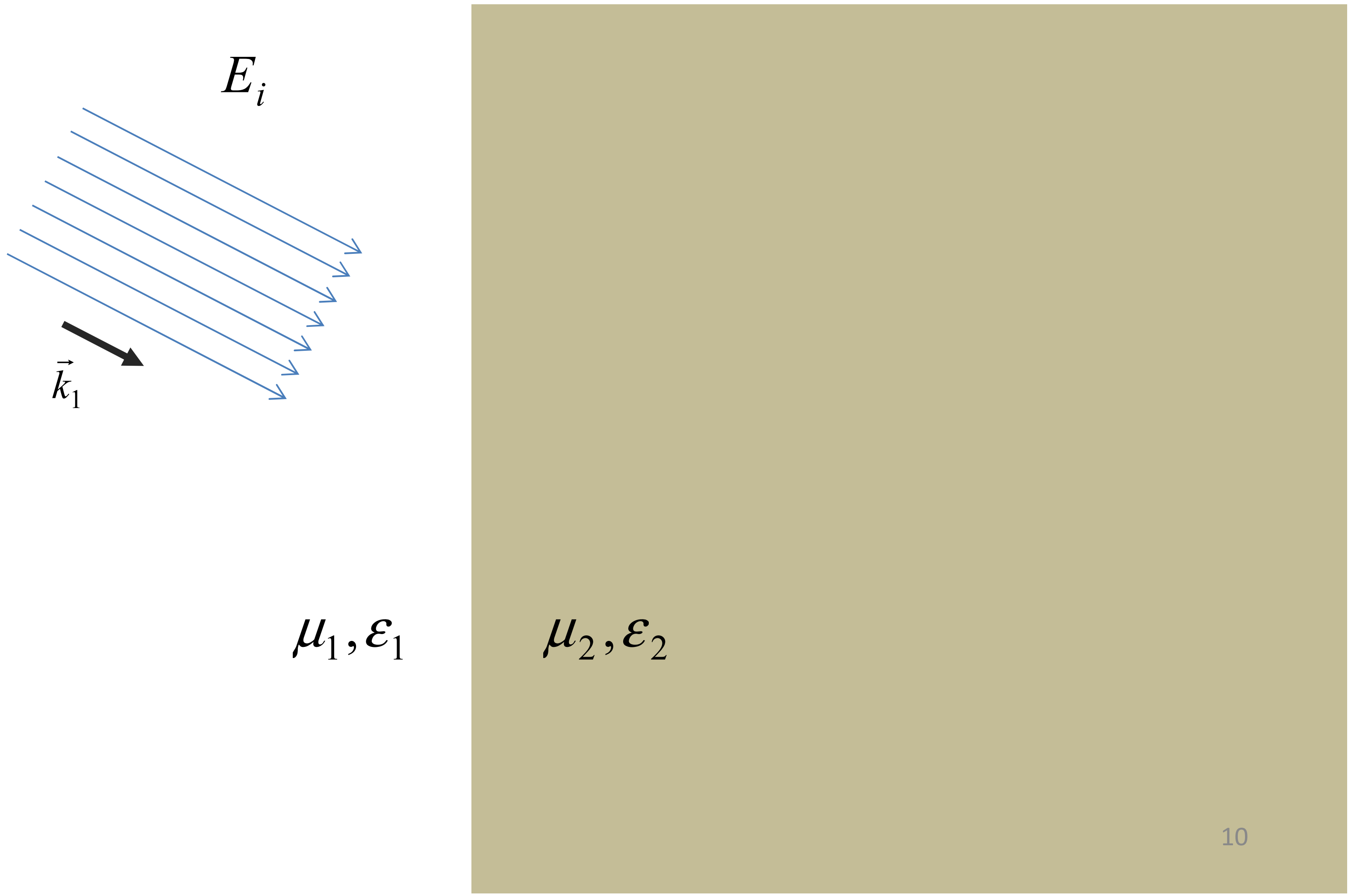
Electric permittivity $\varepsilon = \varepsilon_r \varepsilon_0$

Relative permittivity

Electric permittivity of vacuum

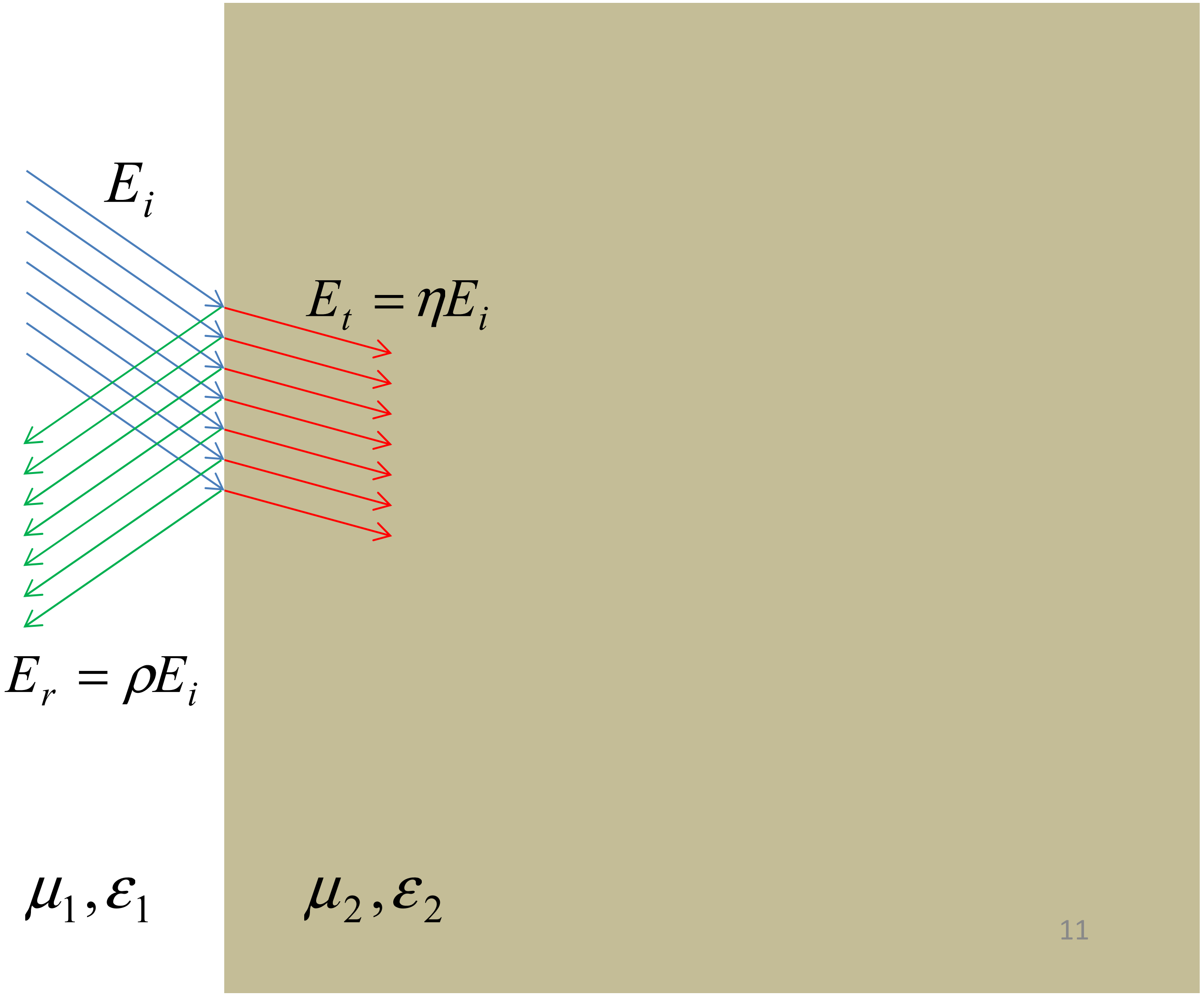
Reflection

- What happens at intersect of different environments?



Reflection

- Specular reflection (like from a mirror)



Reflection

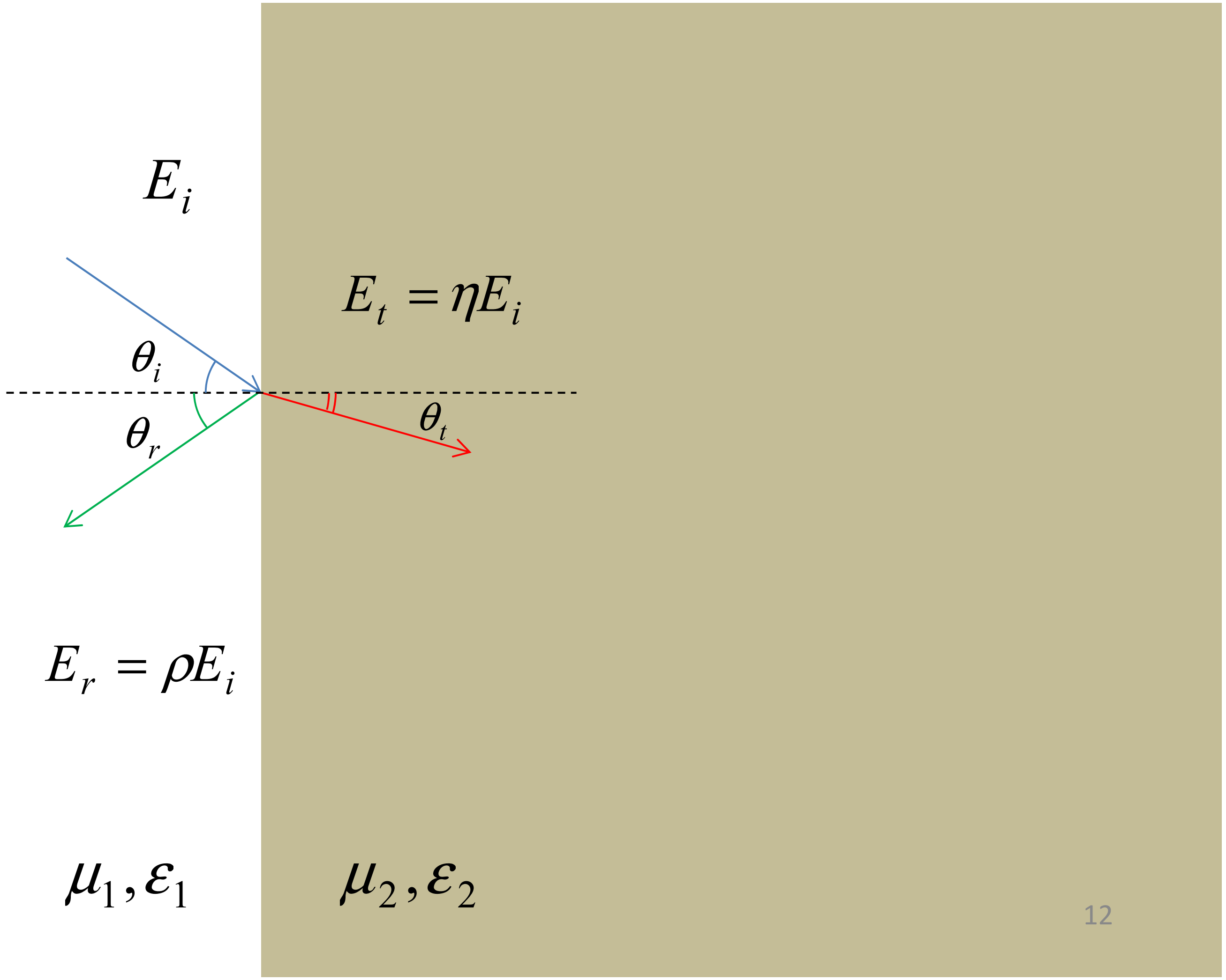
- Plane wave reflection and transition: Snell's law

$$\theta_r = \theta_i$$

$$\frac{\sin\theta_i}{v_1} = \frac{\sin\theta_t}{v_2}$$

$$v = \frac{c}{n} = \frac{1}{\sqrt{\mu\varepsilon}}$$

Speed of light in substance



Reflection

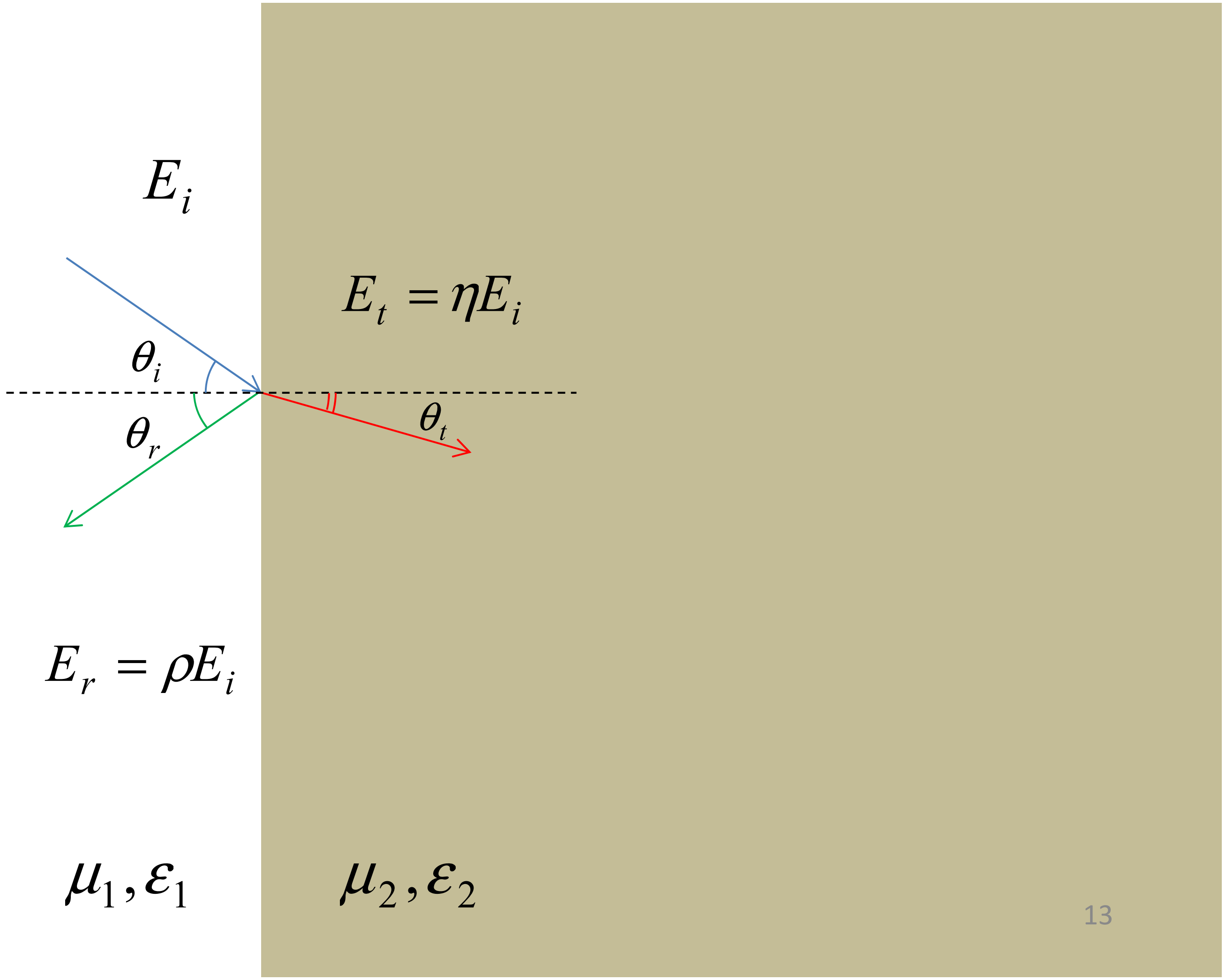
- Plane wave reflection and transition: Snell's law

$$\theta_r = \theta_i$$

$$\sqrt{\mu_1 \varepsilon_1}\,\sin\theta_i = \sqrt{\mu_2 \varepsilon_2}\,\sin\theta_t$$

$$v = \frac{c}{n} = \frac{1}{\sqrt{\mu\varepsilon}}$$

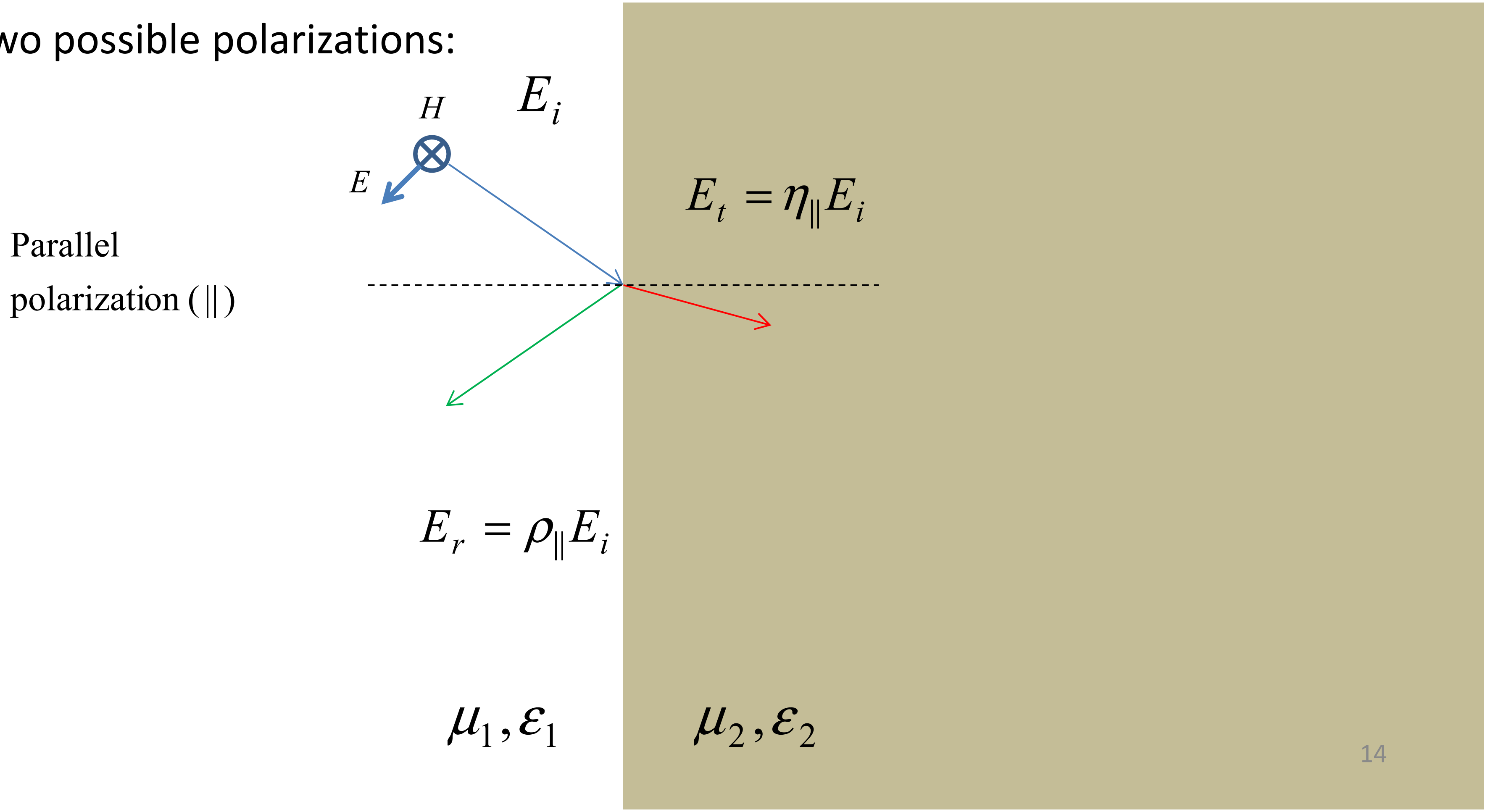
Speed of light in substance



Reflection

- **Wave propagation:** electric field vector, magnetic field vector, and direction of propagation are perpendicular to each other and form a right corner

Two possible polarizations:

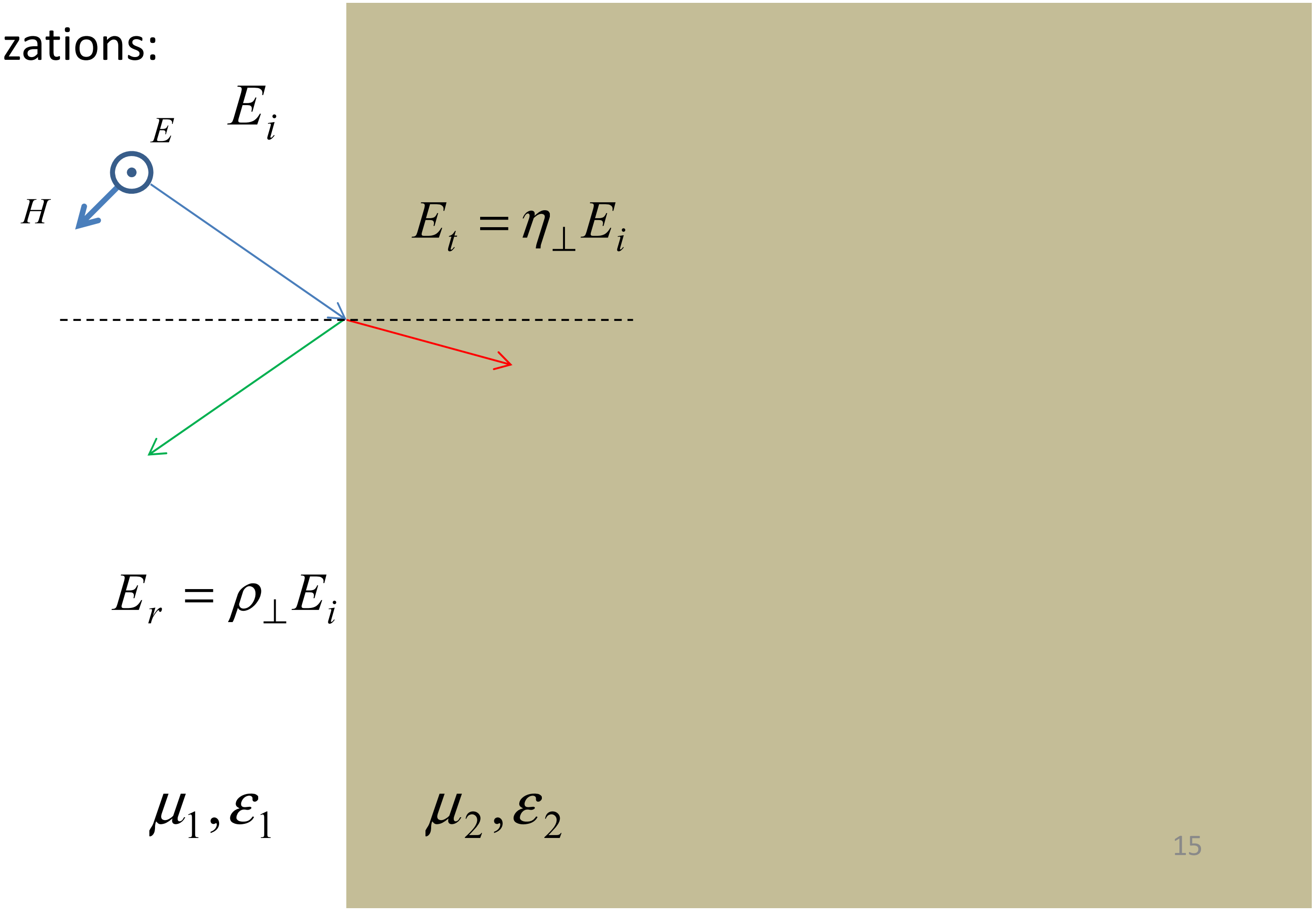


Reflection

- Wave propagation: electric field vector, magnetic field vector, and direction of propagation are perpendicular to each other and form a right corner

Two possible polarizations:

Perpendicular polarization ($\perp$)

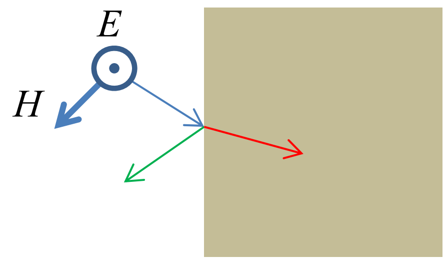


Reflection

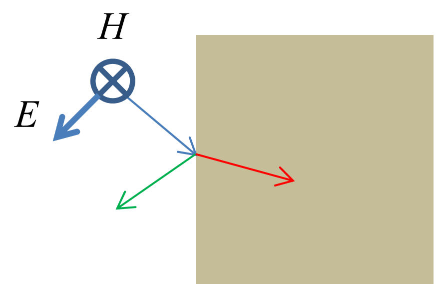
- Reflection loss of plane wave

  Fresnel formula derived from Maxwell's equations

$$\rho_{\perp} = \frac{\sqrt{\frac{\mu_2}{\varepsilon_2}}\cos\theta_i - \sqrt{\frac{\mu_1}{\varepsilon_1}}\cos\theta_t}{\sqrt{\frac{\mu_2}{\varepsilon_2}}\cos\theta_i + \sqrt{\frac{\mu_1}{\varepsilon_1}}\cos\theta_t}$$



$$\rho_{\parallel} = \frac{\sqrt{\frac{\mu_2}{\varepsilon_2}}\cos\theta_t - \sqrt{\frac{\mu_1}{\varepsilon_1}}\cos\theta_i}{\sqrt{\frac{\mu_2}{\varepsilon_2}}\cos\theta_t + \sqrt{\frac{\mu_1}{\varepsilon_1}}\cos\theta_i}$$



Reflection

$$\rho_{\perp} = \frac{\sqrt{\frac{\mu_2}{\varepsilon_2}}\cos\theta_i - \sqrt{\frac{\mu_1}{\varepsilon_1}}\cos\theta_t}{\sqrt{\frac{\mu_2}{\varepsilon_2}}\cos\theta_i + \sqrt{\frac{\mu_1}{\varepsilon_1}}\cos\theta_t} \qquad \rho_{\parallel} = \frac{\sqrt{\frac{\mu_2}{\varepsilon_2}}\cos\theta_t - \sqrt{\frac{\mu_1}{\varepsilon_1}}\cos\theta_i}{\sqrt{\frac{\mu_2}{\varepsilon_2}}\cos\theta_t + \sqrt{\frac{\mu_1}{\varepsilon_1}}\cos\theta_i}$$

Note: for normal surfaces perpendicular reflection coefficient is negative

$$\varepsilon_2 > \varepsilon_1, \quad \mu_2 = \mu_1 = \mu_0$$

$$\Rightarrow \begin{cases} \theta_t < \theta_i \Rightarrow \cos(\theta_t) > \cos(\theta_i) \\ \dfrac{1}{\sqrt{\varepsilon_2}} < \dfrac{1}{\sqrt{\varepsilon_1}} \end{cases} \Rightarrow \boxed{\rho_{\perp} < 0}$$

Reflection

$$\rho_\perp = \frac{\sqrt{\frac{\mu_2}{\varepsilon_2}}\cos\theta_i - \sqrt{\frac{\mu_1}{\varepsilon_1}}\cos\theta_t}{\sqrt{\frac{\mu_2}{\varepsilon_2}}\cos\theta_i + \sqrt{\frac{\mu_1}{\varepsilon_1}}\cos\theta_t} \qquad \rho_\parallel = \frac{\sqrt{\frac{\mu_2}{\varepsilon_2}}\cos\theta_t - \sqrt{\frac{\mu_1}{\varepsilon_1}}\cos\theta_i}{\sqrt{\frac{\mu_2}{\varepsilon_2}}\cos\theta_t + \sqrt{\frac{\mu_1}{\varepsilon_1}}\cos\theta_i}$$

Note: for normal surfaces perpendicular reflection coefficient is negative

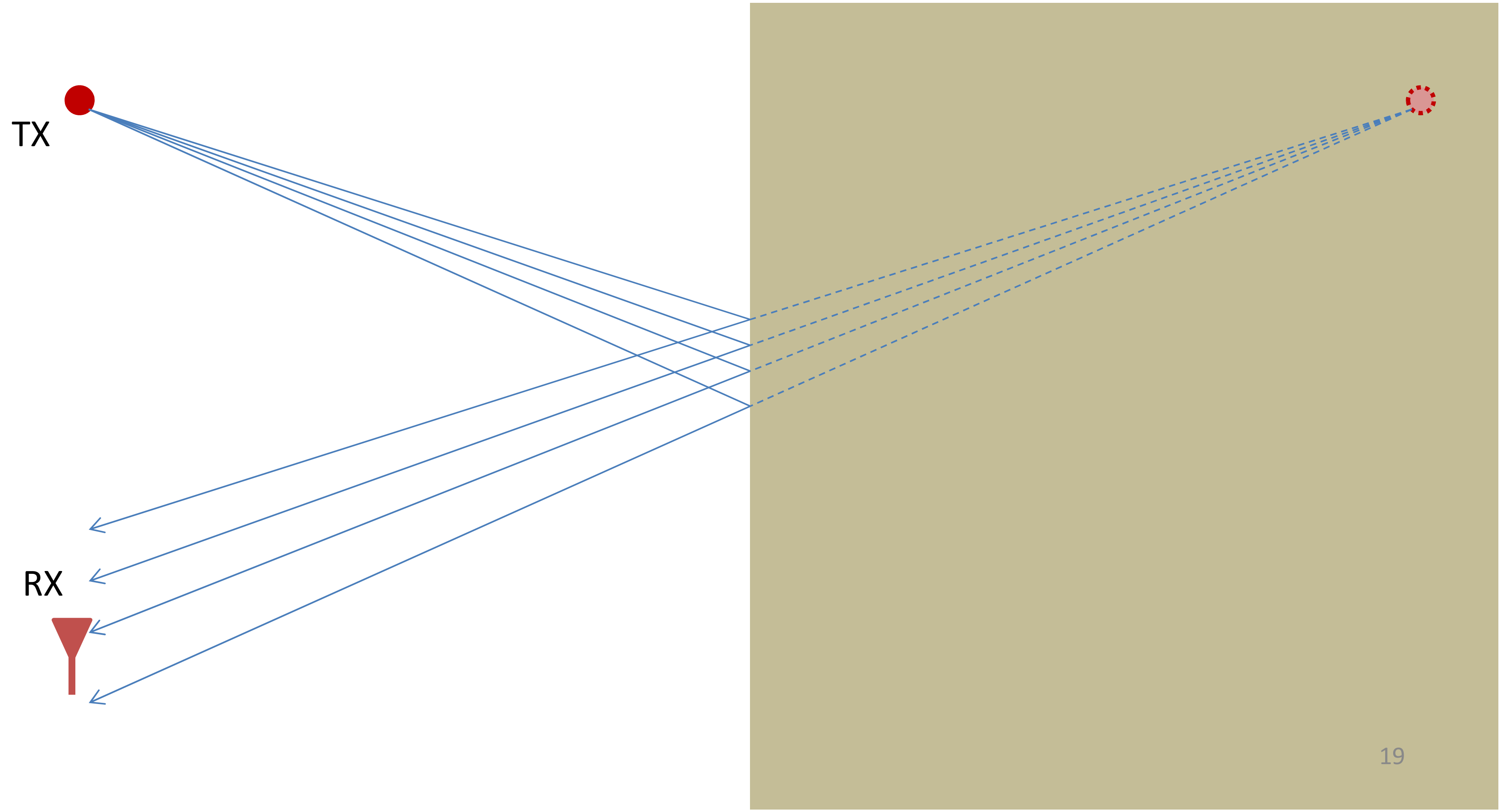
$$\varepsilon_2 > \varepsilon_1, \quad \mu_2 = \mu_1 = \mu_0 \Rightarrow \quad \boxed{\rho_\perp < 0}$$

and for parallel reflection we have:

$$\varepsilon_2 > \varepsilon_1, \quad \mu_2 = \mu_1 = \mu_0 \Rightarrow \begin{cases} \theta_i < \sin^{-1}\sqrt{\dfrac{\varepsilon_2}{\varepsilon_1 + \varepsilon_2}} \Rightarrow \rho_\parallel < 0 \\ \\ \theta_i > \sin^{-1}\sqrt{\dfrac{\varepsilon_2}{\varepsilon_1 + \varepsilon_2}} \Rightarrow \rho_\parallel > 0 \end{cases}$$

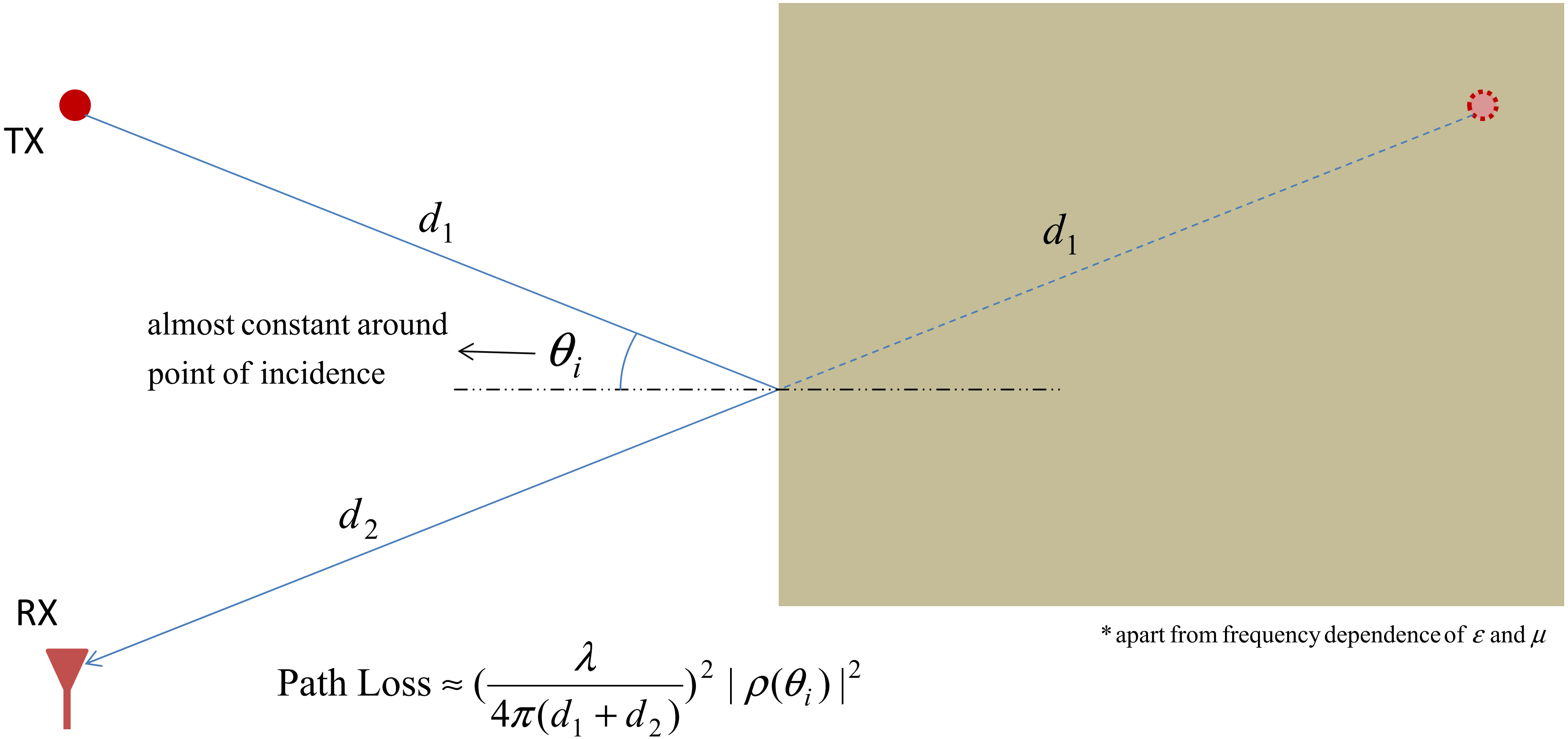
Reflection

- Quasi-plane wave:



Reflection

- Quasi-plane wave:



$$\text{Path Loss} \approx \left(\frac{\lambda}{4\pi(d_1+d_2)}\right)^2 |\rho(\theta_i)|^2$$

$$\text{Excess Loss} \approx |\rho(\theta_i)|^2 \;\Rightarrow\; \text{independent of frequency}\,*$$

* apart from frequency dependence of ε and μ

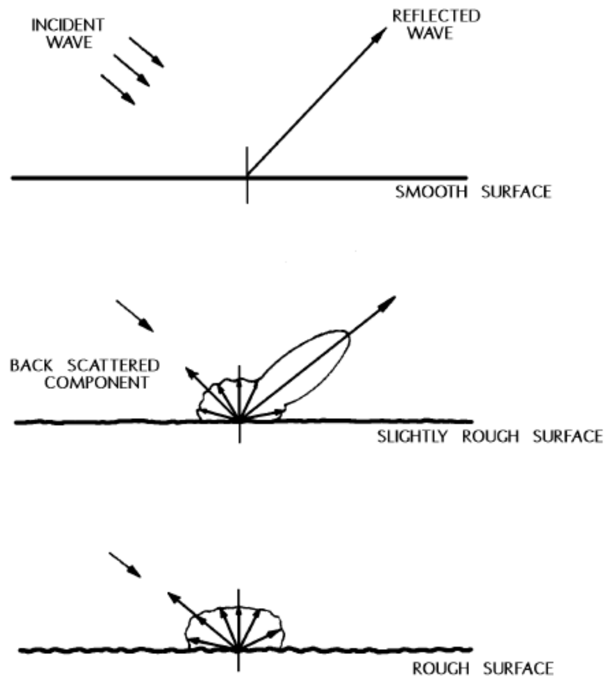
Reflection

- Reflection from **rough** surfaces:
  part of wave energy is **scattered**

$$\text{Path Loss} \approx \left(\frac{\lambda}{4\pi(d_1+d_2)}\right)^2 |\rho(\theta_i)|^2 \ \exp\left(-\frac{1}{2}\left(\frac{4\pi h_s \cos\theta_i}{\lambda}\right)^2\right)$$

h_s = std deviation of surface height *

$$\text{Excess Loss} \approx |\rho(\theta_i)|^2 \ \exp\left(-\frac{1}{2}\left(\frac{4\pi h_s \cos\theta_i}{\lambda}\right)^2\right)$$



$\Rightarrow$ Higher loss at higher frequencies (exponential)

(surfaces are *rougher* at shorter wavelengths)

* assuming Gaussian distribution of surface heights without sharp edge and shadowing effects

Reflection

- Surface roughness std deviation varies from 0 (e.g. glass) to a few mm
- At low frequencies (f < 6 GHz, λ > 5 cm) most surfaces are smooth

$$h_s < 2\,\text{mm},\quad \lambda > 5\,\text{cm} \Rightarrow \exp(-8(\frac{\pi h_s \cos\theta_i}{\lambda})^2) \geq 0.88$$

$$\text{roughness loss} \leq 0.55\,\text{dB}$$

- At 60 GHz a surface with 0.6 mm roughness causes 5 dB of excess loss

$$h_s = 0.6\,\text{mm},\quad \lambda = 5\,\text{mm} \Rightarrow \exp(-8(\frac{\pi h_s \cos\theta_i}{\lambda})^2) = 0.32$$

$$\text{roughness loss} = 4.95\,\text{dB}$$

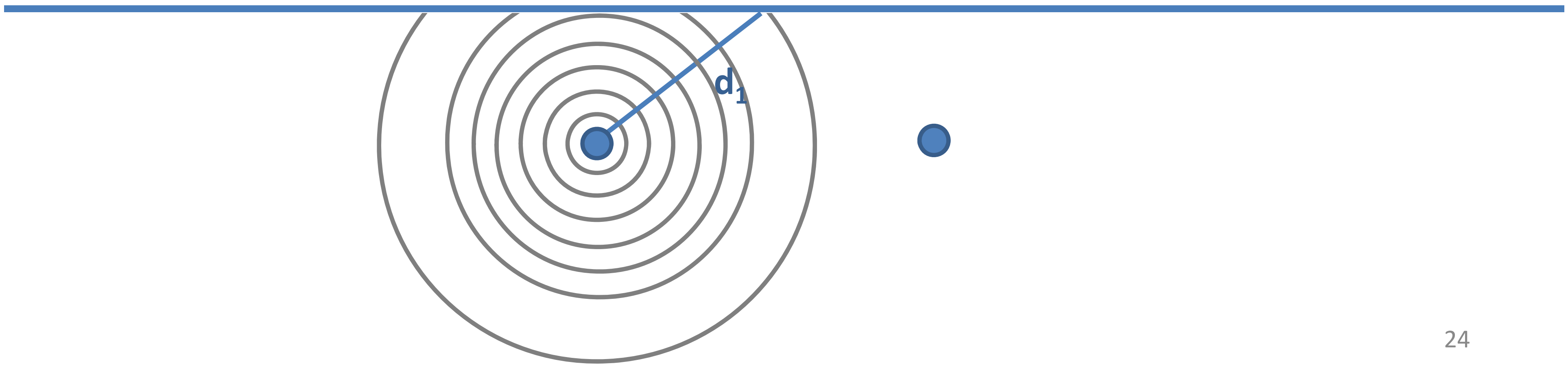
Scattering and diffraction

- Wave incident to irregular surfaces are scattered, e.g. objects with sharp edges (diffraction) or curvatures of radius smaller than or in order of wavelength
- One example is reflection from rough surface – part of wave power is scattered
- Scattered waves from finite objects can be modeled as fields caused by excited currents on surface of object

  $\Rightarrow$ **modeled by secondary sources**
- Fundamental difference with *specular reflection*: two independent expansions of wavefront

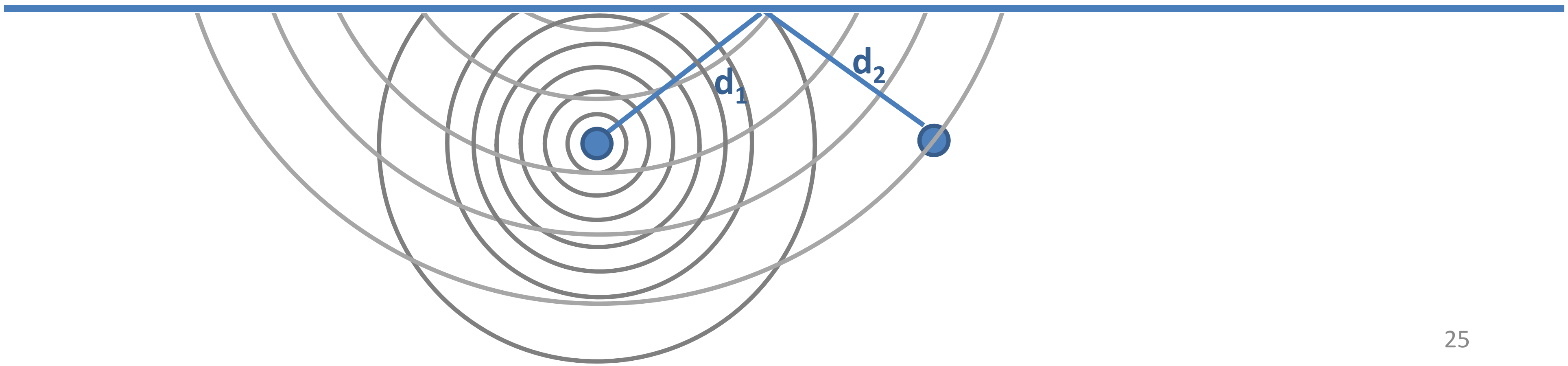
Scattering and diffraction

- For comparison: consider reflection



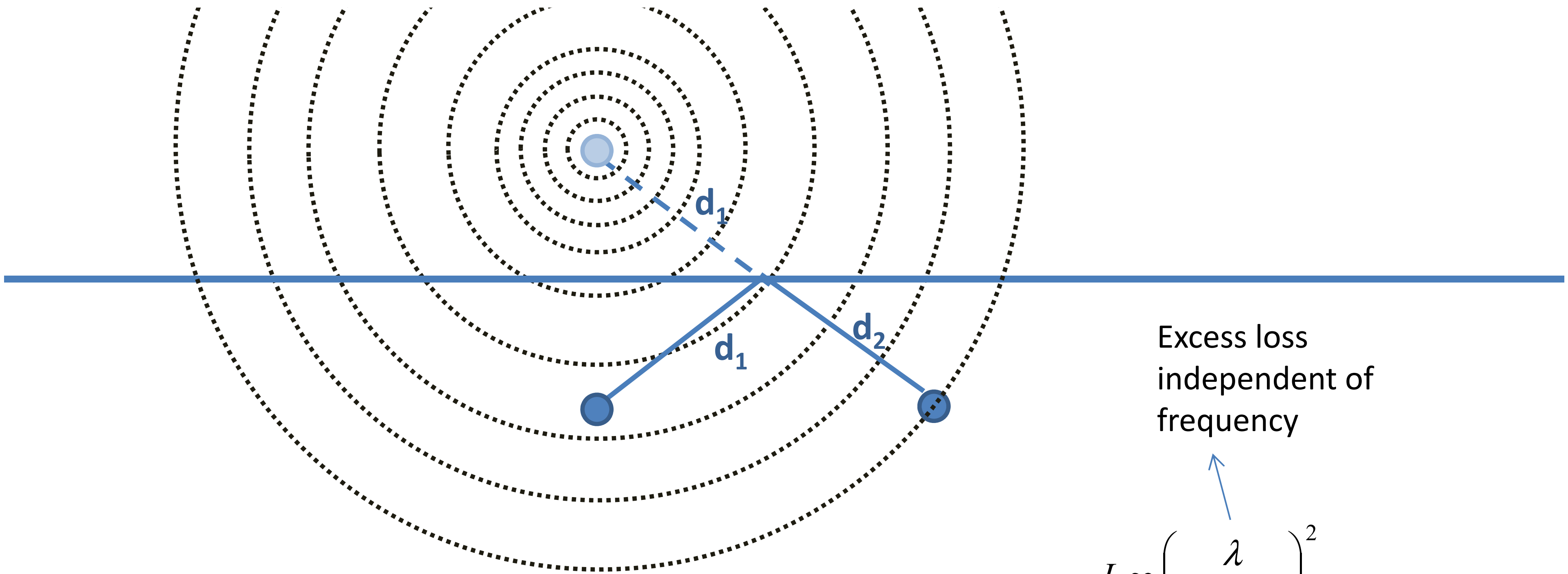
Scattering and diffraction

- Specular reflection means wave front continues its initial expansion from original source



Scattering and diffraction

- Specular reflection means wave front continues its initial expansion from original source
- Equivalent to one expansion from image source

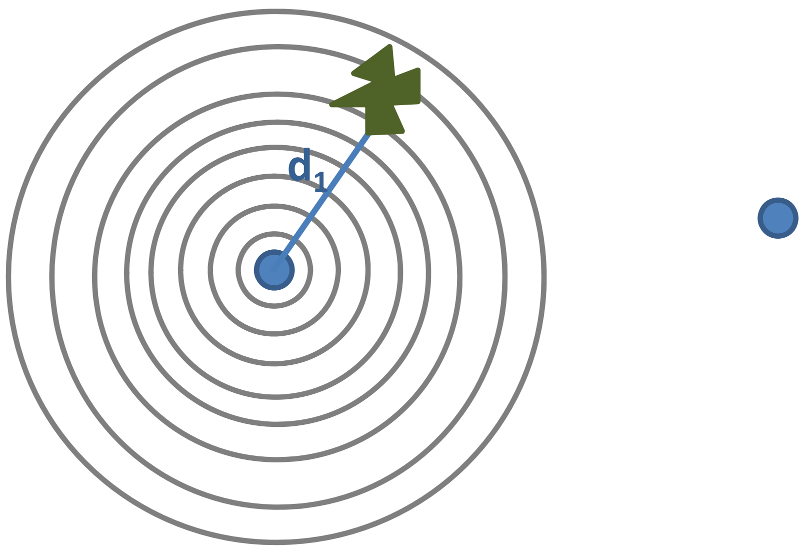


Excess loss independent of frequency

$$L \propto \left(\frac{\lambda}{d_1 + d_2} \right)^2$$

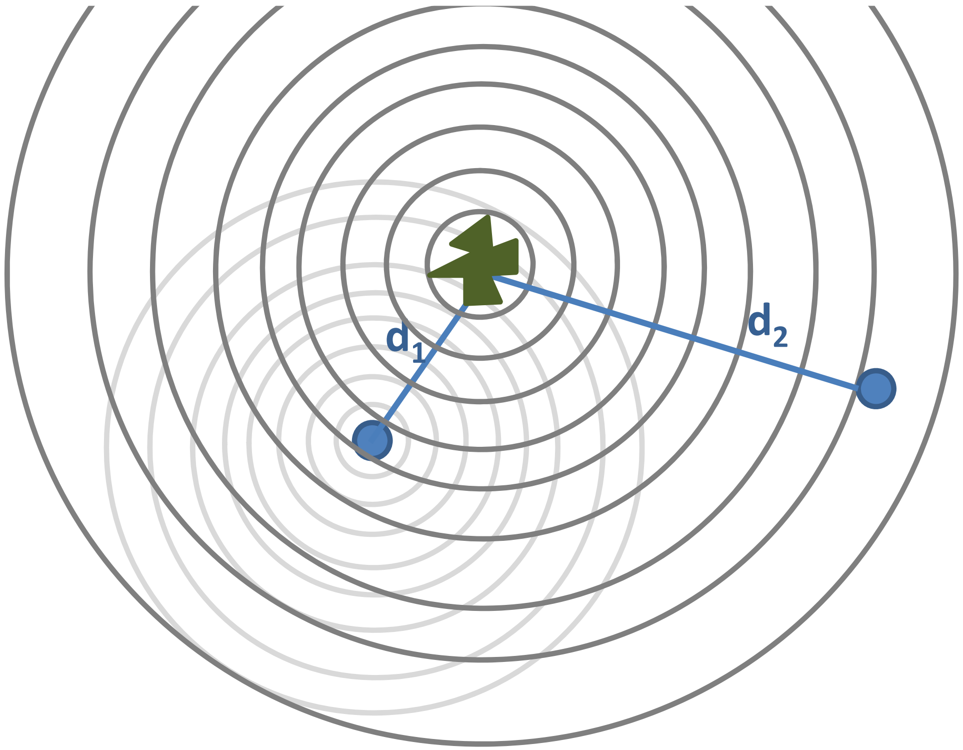
Scattering and diffraction

- Scattering involves a finite object being illuminated by original source



Scattering and diffraction

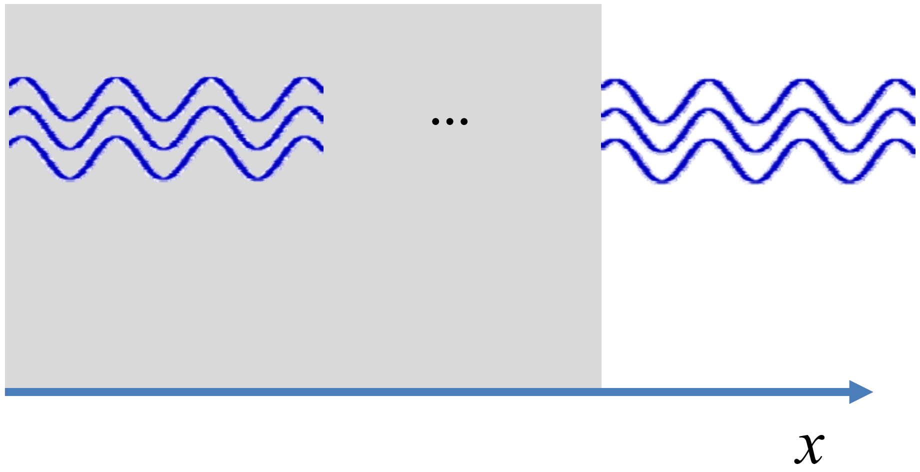
- Scattering involves a finite object being illuminated by original source
- Power captured by object is radiated in a second expansion



Excess loss proportional to squared frequency

$$L \propto \left(\frac{\lambda}{d_1}\right)^2 \left(\frac{\lambda}{d_2}\right)^2$$

Propagation through dielectrics



$$E(x) = E_0 e^{-jkx}$$

$$k = 2\pi f\sqrt{\mu\varepsilon} \qquad \varepsilon = \varepsilon' - j\varepsilon'' \quad (\varepsilon'' << \varepsilon')^*$$

$$k \approx 2\pi f\sqrt{\mu\varepsilon'}(1 - j\frac{\varepsilon''}{2\varepsilon'})$$

Attenuation

Phase change

$$P(x)/P_0 = |E(x)/E_0|^2 \approx \exp(-2\pi f\sqrt{\mu\varepsilon'}\frac{\varepsilon''}{\varepsilon'}x)$$

$\Rightarrow$ Penetration loss increases exponentially with depth and frequency

* for substances with conductivity σ, effectively $\varepsilon = \varepsilon' - j\varepsilon'' - j\dfrac{\sigma}{2\pi f}$

Propagation through dielectrics

- Dielectric loss is result of resonance of particles with electromagnetic field – varies with frequency
- Oxygen absorption (e.g. at 60 GHz) result of such resonance
- Exponential decay with distance

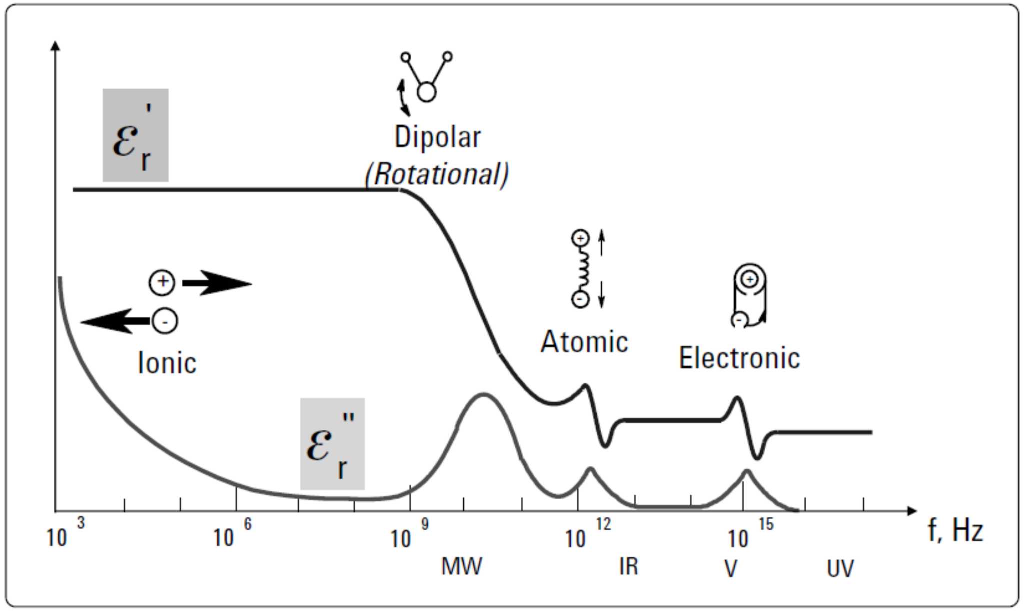


Figure 7. Frequency response of dielectric mechanisms

Propagation through dielectrics

Example: relative permittivity of concrete

$$P(x)/P_0 = \exp(-2\pi f \sqrt{\mu\varepsilon'}\,\frac{\varepsilon''}{\varepsilon'}\,x)$$

@ 5 GHz: ε_r = 4.8 – j0.6

$$\sqrt{\mu_0\varepsilon_0} = \frac{1}{c} = 0.33\times10^{-8} \ \ \text{(s/m)}$$

@ 60 GHz: ε_r = 3.3 – j0.38

$\Rightarrow$ Loss of a 3 cm thick slab of concrete

@ 5 GHz:

@ 60 GHz:

How thick can a slab of concrete be for <10dB attenuation?

@ 5 GHz:

@ 60 GHz:

Propagation through dielectrics

Example: relative permittivity of concrete

$$P(x)/P_0 = \exp(-2\pi f\sqrt{\mu\varepsilon'}\frac{\varepsilon''}{\varepsilon'}x)$$

@ 5 GHz: ε_r = 4.8 – j0.6

@ 60 GHz: ε_r = 3.3 – j0.38

$$\sqrt{\mu_0\varepsilon_0} = \frac{1}{c} = 0.33\times10^{-8} \text{ (s/m)}$$

$$\text{Loss} = \exp(-2\pi f\sqrt{\mu_0\varepsilon_0}\sqrt{\varepsilon_r'}\frac{\varepsilon_r''}{\varepsilon_r'}x)$$

$\Rightarrow$ Loss of a 3 cm thick slab of concrete

@ 5 GHz: $\text{Loss} = \exp(-2\pi\times5\times10^9\times0.33\times10^{-8}\times\sqrt{4.8}\times\frac{0.6}{4.8}\times0.03) = 3.7\text{ dB}$

@ 60 GHz: $\text{Loss} = \exp(-2\pi\times60\times10^9\times0.33\times10^{-8}\times\sqrt{3.3}\times\frac{0.38}{3.3}\times0.03) = 34\text{ dB}$

How thick can a slab of concrete be for <10dB attenuation?

@ 5 GHz: 8.04 cm

@ 60 GHz: 8.8 mm

Blockage

- Signal attenuation due to presence of obstacle in signal path
- Blockage is severe at mmwave frequencies
  - No propagation through obstacles
  - No diffraction around obstacles
- Can also be explained by Huygens' principle
- Objects are bigger at mmwave -- smaller objects can block wave

Blockage

Huygens' principle

Each point on a primary wavefront can be considered to be a new source of a secondary spherical wave, and a secondary wavefront can be constructed as the envelope of these secondary spherical waves

$\Rightarrow$ Received signal is the superposition of secondary sources that are not within the obstacles

Huygens' principle

MR1

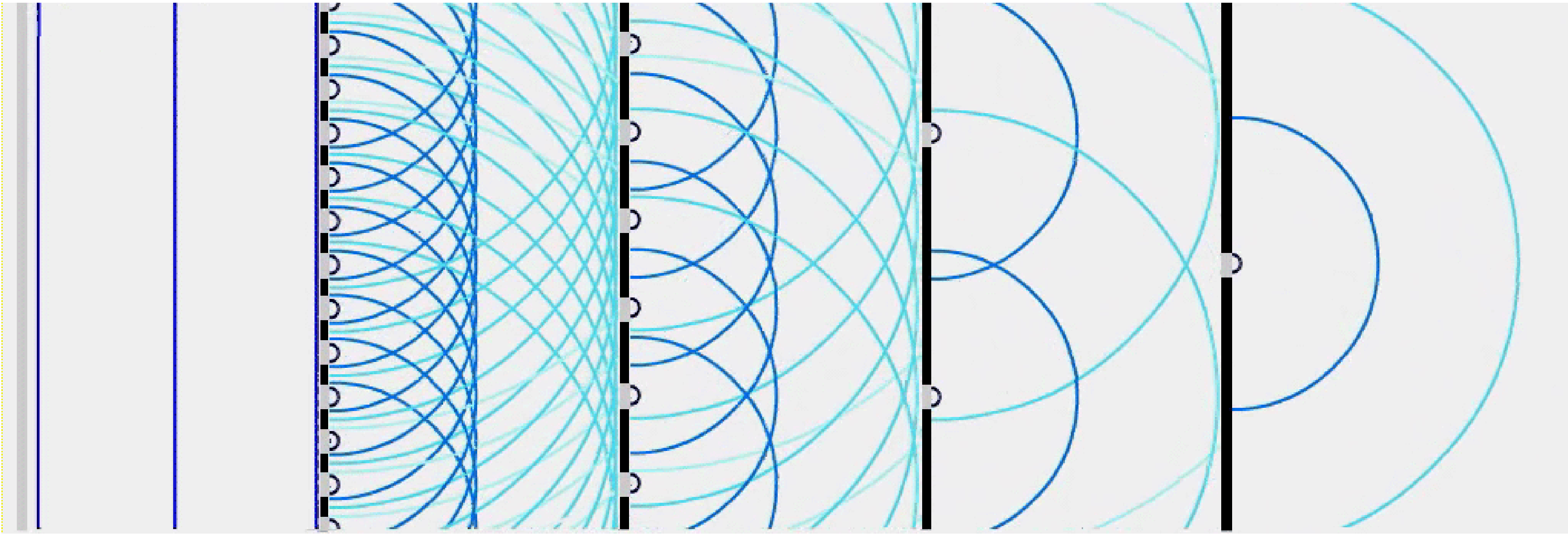


Image courtesy of: Toda Akihiko
http://home.hiroshima-u.ac.jp/atoda/index_e.html

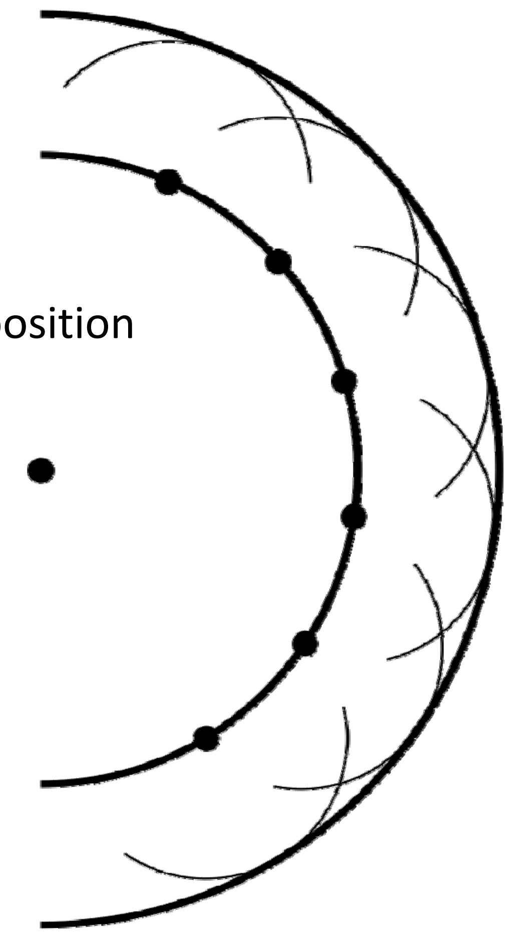
Slide 35

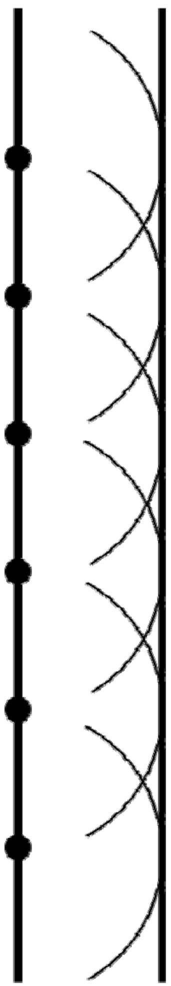
MR1 I don't think this is showing the huygen's principal

Maryam Rasekh, 06-06-2016

Huygens' principle

wavefront decomposition
for point source

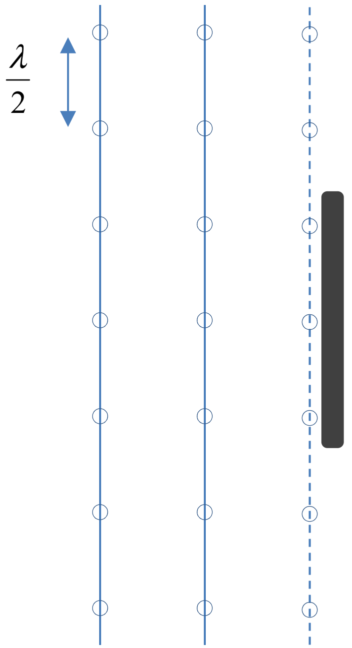


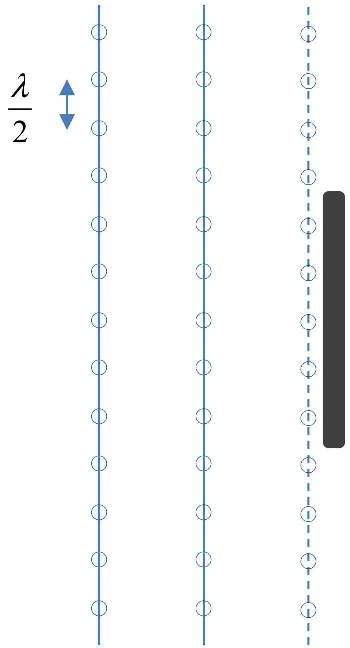


wavefront decomposition
for plane wave

Huygens' principle

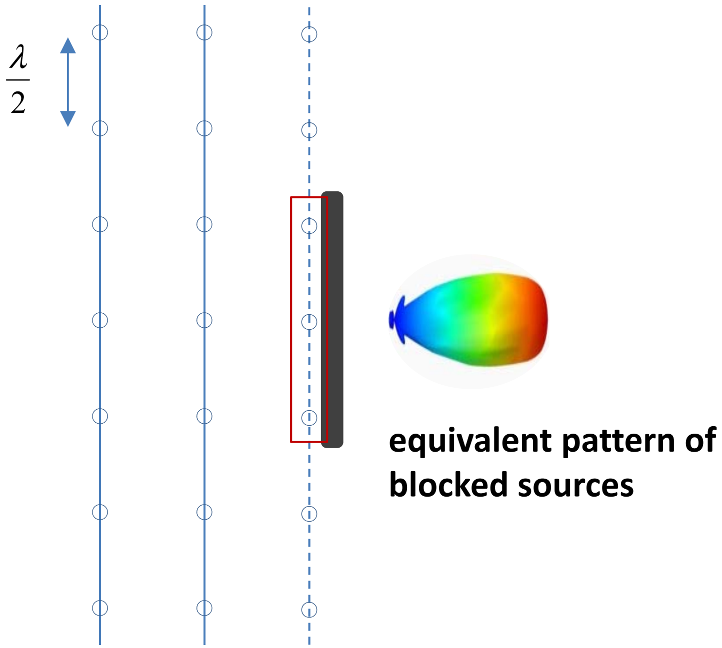
- If part of wavefront is blocked contribution of that portion is lost

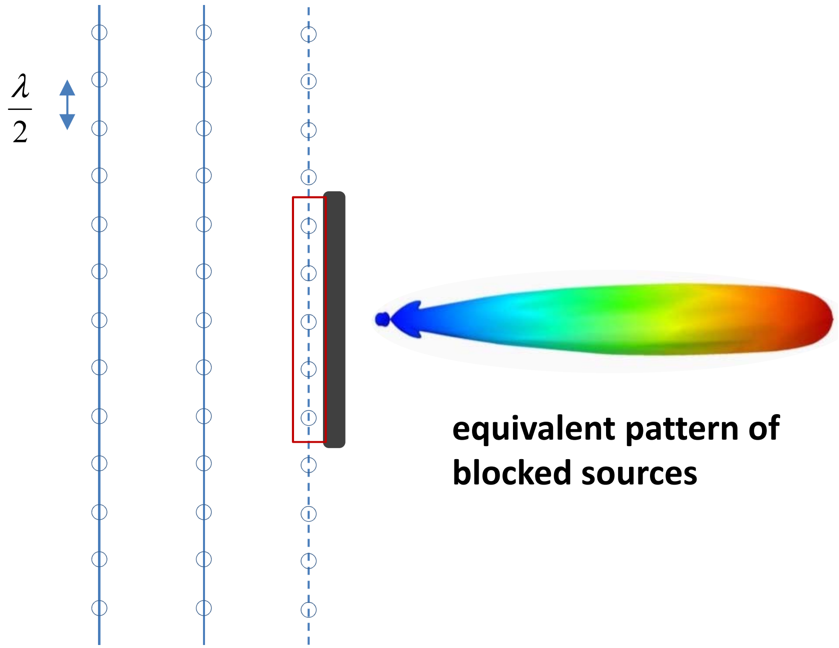




Huygens' principle

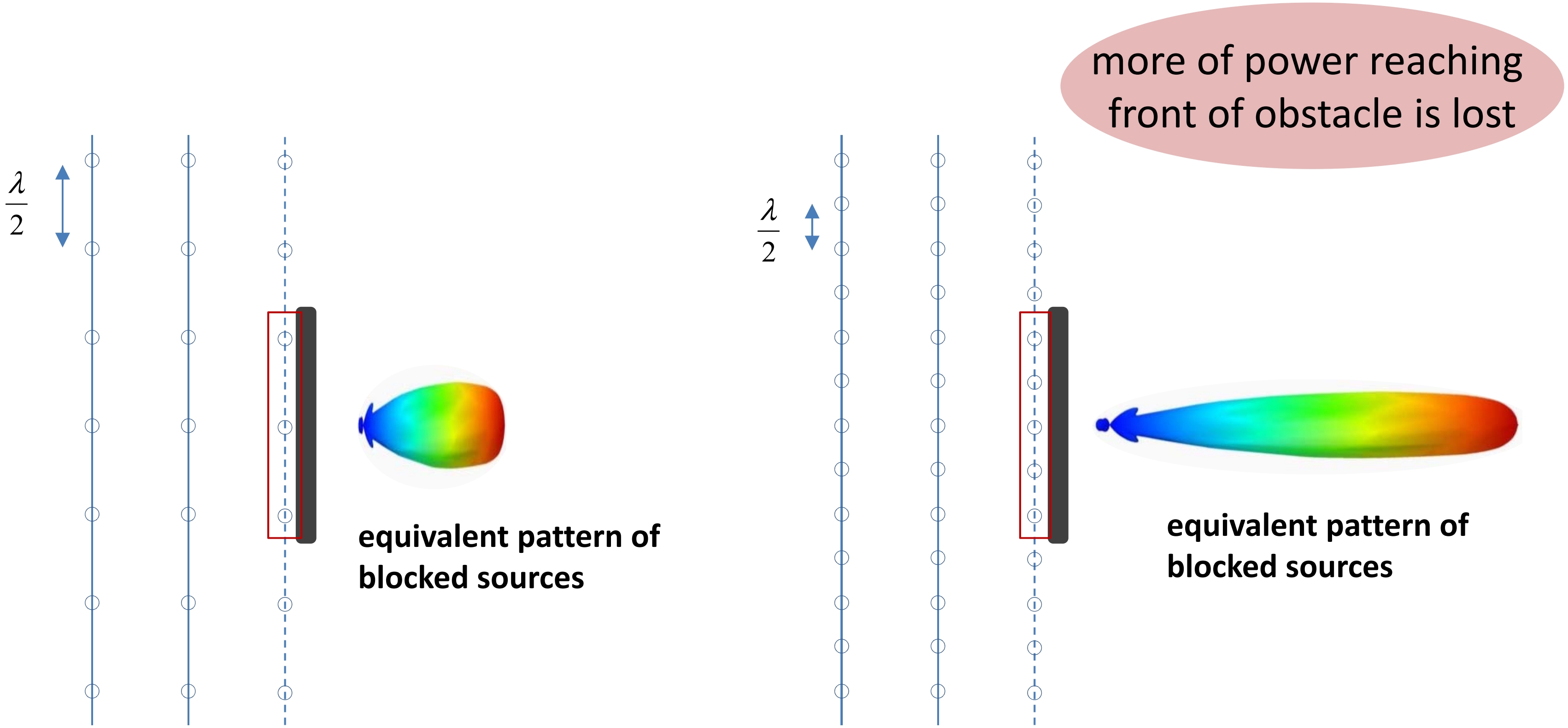
- If part of wavefront is blocked contribution of that portion is lost





Huygens' principle

- If part of wavefront is blocked contribution of that portion is lost



Blockage

- When path between TX and RX is blocked by an obstacle, link is diminished – and this can happen a lot
- Require beam steering to maintain link through alternate paths – reflections from environment surfaces

The overall mmwave channel

- Mainly free space (unobscured) line of sight and reflection
  - What does this mean for mmwave links..?

- Indoor and mobile scenarios:
  need for beam steering to deal with blockage

- Outdoor point-to-point links:
  sparse channel consisting of LOS and reflected paths
  channel mostly predictable from geometry

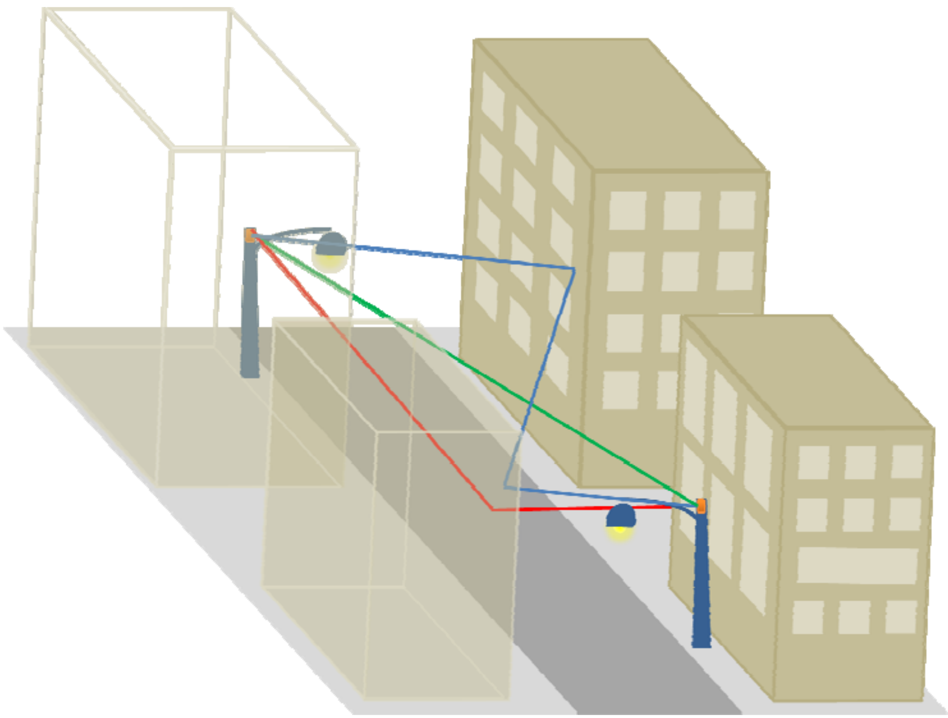
Outdoor point-to-point links

- Consequences of channel sparsity
  - CSI recoverable using faster sparse sensing techniques
  - More deterministic channel model in known environment geometry

$\Rightarrow$ Design guidelines for spatial and frequency diversity

The sparse multipath channel

- Lamp post to lamp post link inside street canyon
- Channel comprised of LOS and reflections from walls and ground – if they fall within antenna beams
- Fading can happen



small

$$h(t) = \sum_{i=1}^{M} G_i^{(t)} G_i^{(r)} L_i \left(\frac{\lambda}{4\pi D_i} \right)^2 \delta(t - D_i / c)$$

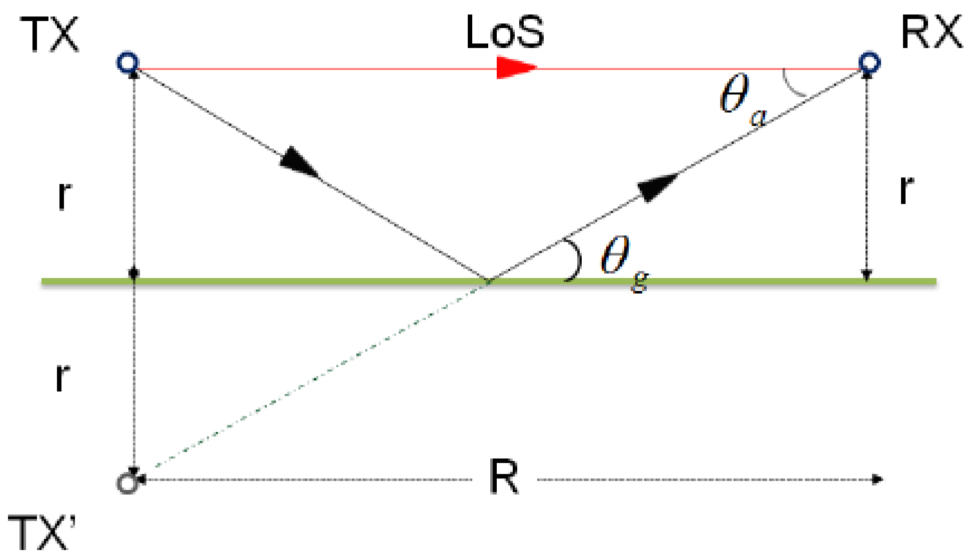
$$H(f) = \sum_{i=1}^{M} \alpha_i e^{j2\pi f \tau_i}$$

Antenna beamwidth can include single or double bounce reflections

Fading in a sparse multipath channel

Most basic case: **2 path channel**

- LOS
- One reflection

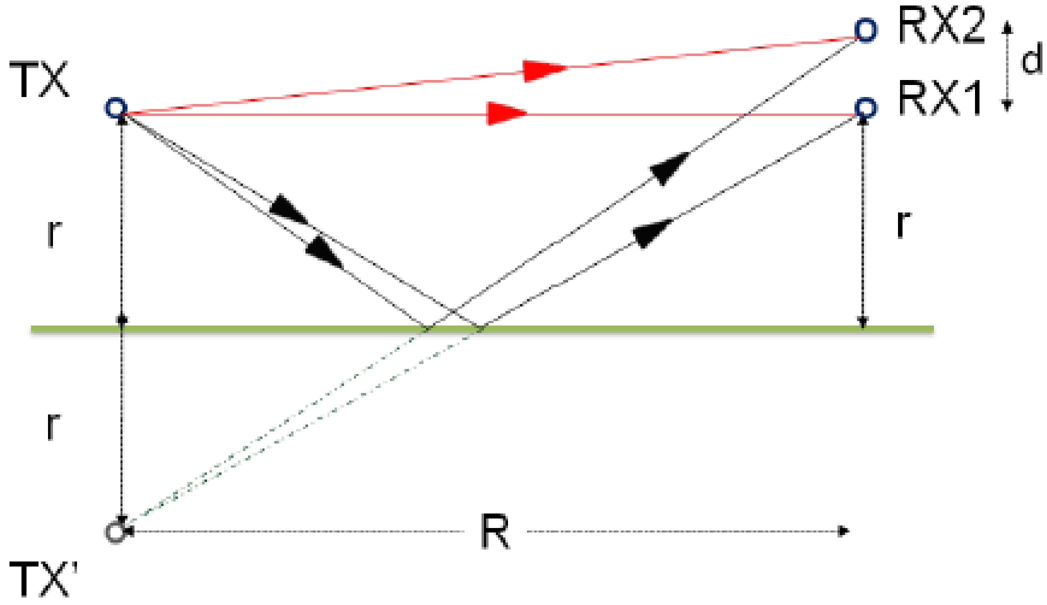


Fading in a sparse multipath channel

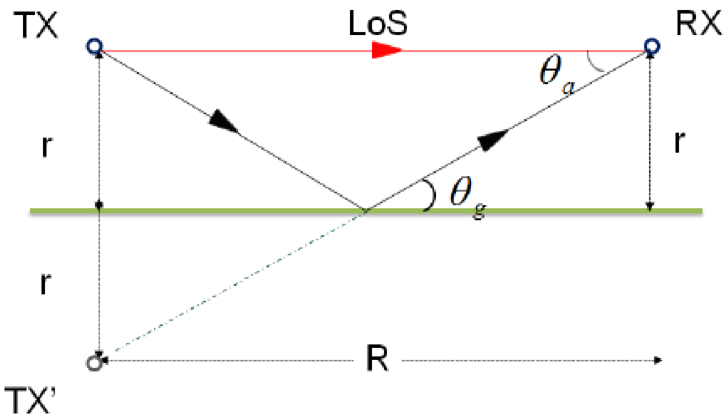
Spatial diversity using 2 receivers

Selection $\eta = \max(|h_1|^2, |h_2|^2)$

Maximum Ratio Combining $\eta = |h_1|^2 + |h_2|^2$



Basic two-ray channel

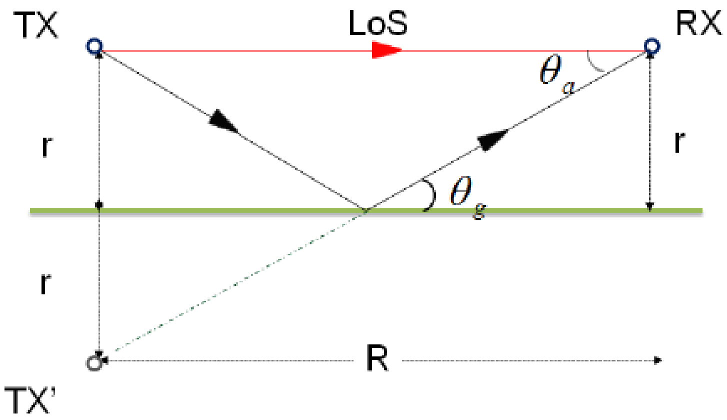


In SISO system, the outage probability relative to **the LoS link** is:

$$\boxed{P(|h| < \beta)}$$

$\beta \in (0,1]$ $1/\beta$: link margin, $h = 1 - \alpha e^{-j\phi}$

Basic two-ray channel



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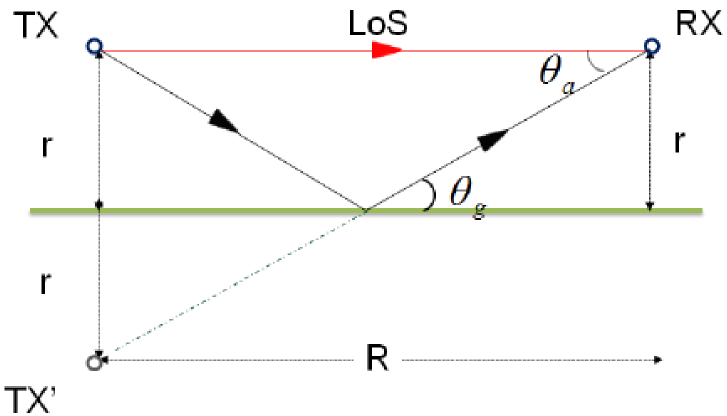
When does an outage occur?

Basic two-ray channel



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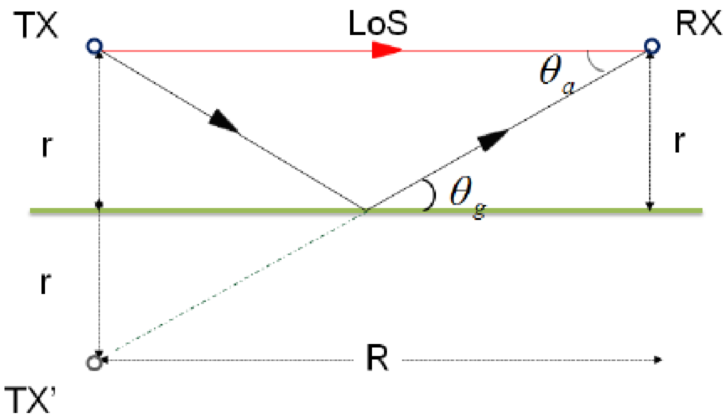
When does an outage occur?

LoS

Basic two-ray channel



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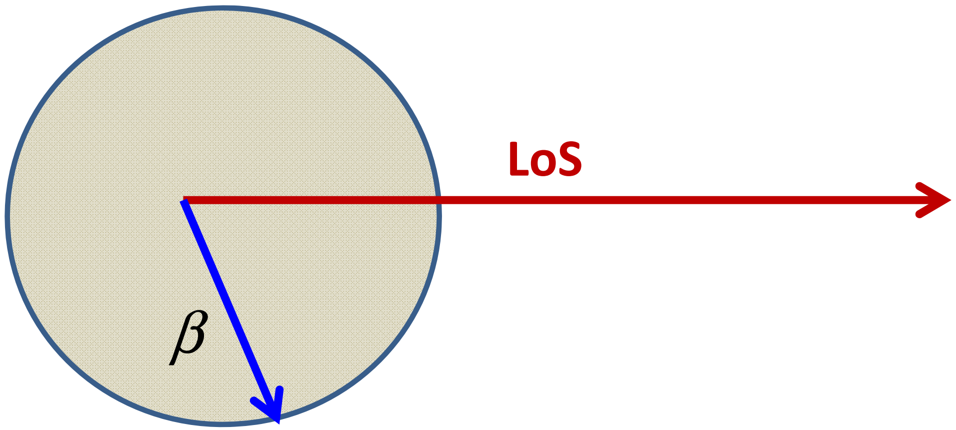
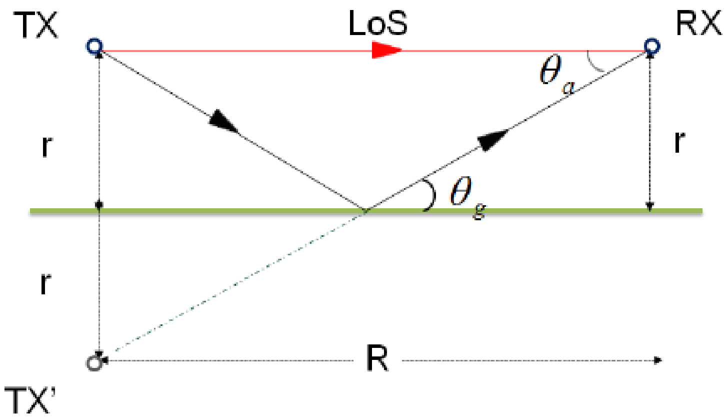


Basic two-ray channel



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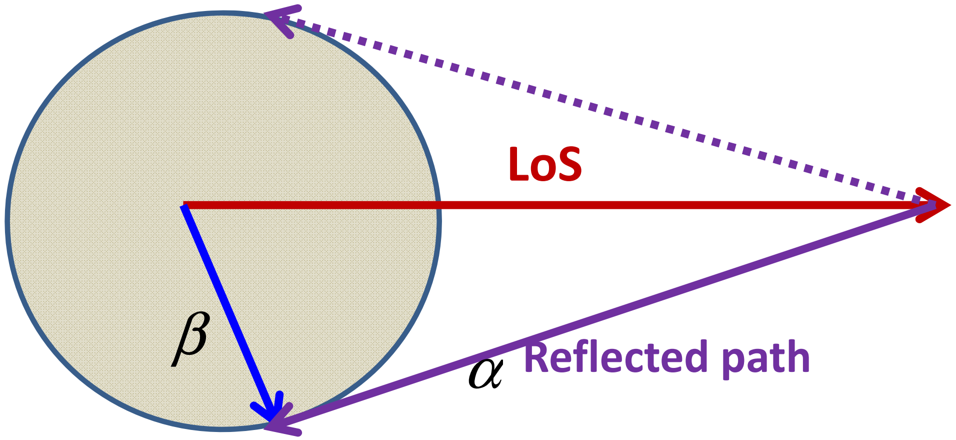
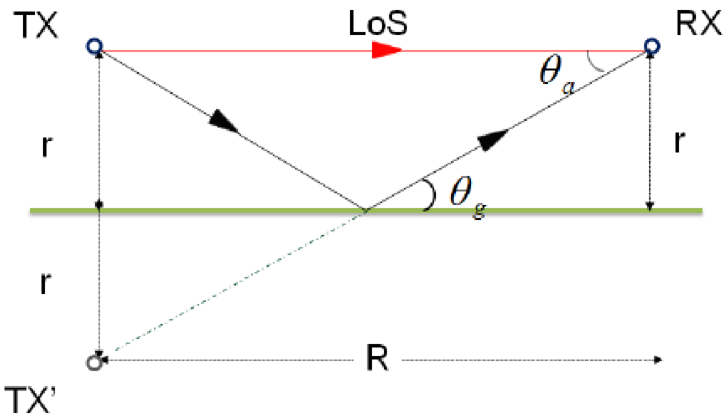


Basic two-ray channel



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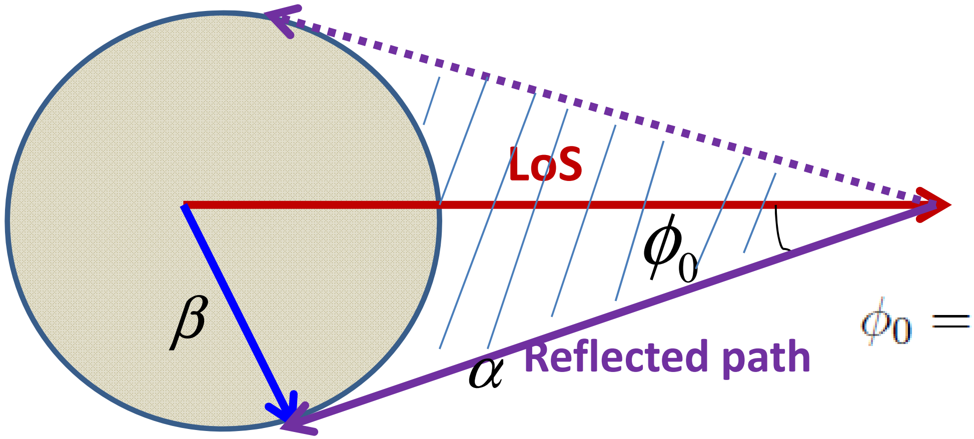
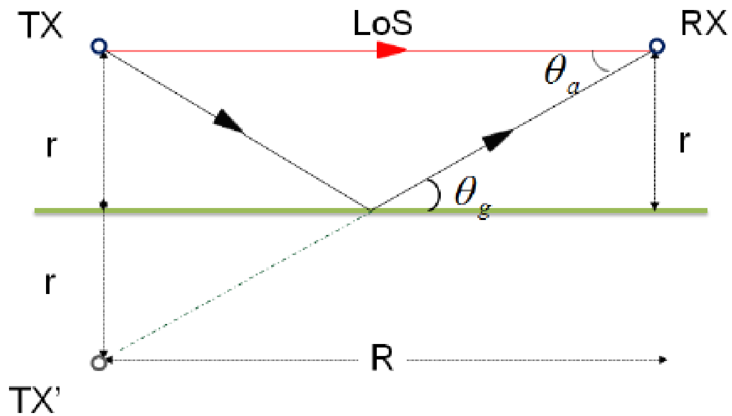


$$\phi_0 = \cos^{-1}\left(\frac{1+\alpha^2-\beta^2}{2\alpha}\right)$$

Basic two-ray channel

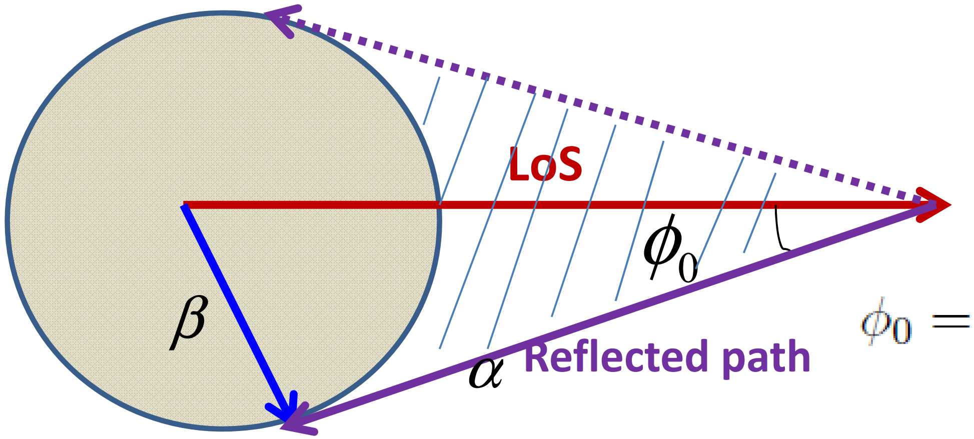


In SISO system, the outage probability relative to **the LoS link** is:

$$\boxed{P(|h|<\beta)}$$

$\beta \in (0,1]$ $1/\beta$: link margin, $h = 1 - \alpha e^{-j\phi}$

When does an outage occur?

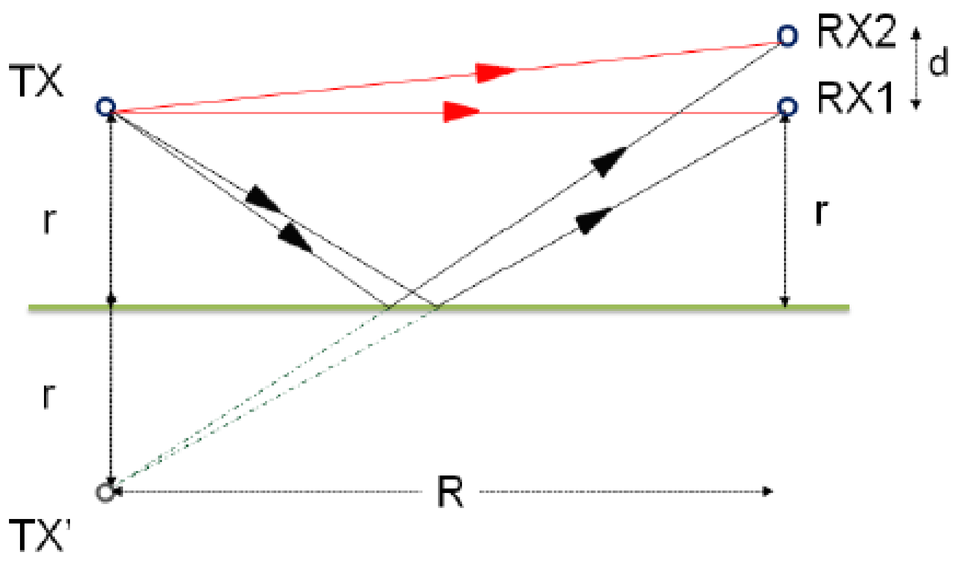


$$\phi_0 = \cos^{-1}\left(\frac{1+\alpha^2-\beta^2}{2\alpha}\right)$$

In SISO, the outage probability is: $\boxed{P_{out}(SISO) = \phi_0/\pi}$

For α = 1 and a link margin of **5 dB**, $P_{out}(SISO) = 0.56/\pi = 18\%$

Basic two-ray channel



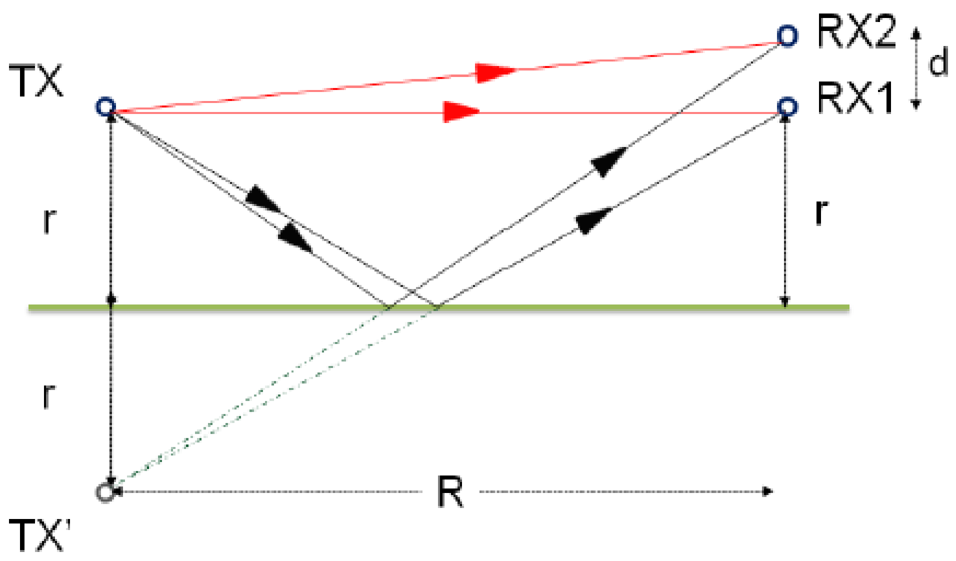
In SIMO system, the relative channel gain:

$$h_1 = 1 - \alpha e^{-j\phi}\ , \quad h_2 = 1 - \alpha e^{-j(\phi + \boxed{\gamma})}$$

The phase difference is given by: $\boxed{\gamma = \frac{2\pi}{\lambda}\Delta L_{reflected} \approx \frac{4\pi r d}{R\lambda}}$

$$\Delta L_{reflected} = \sqrt{R^2 + (2r+d)^2} - \sqrt{R^2 + (2r)^2} \approx \frac{2rd}{R}$$

Basic two-ray channel



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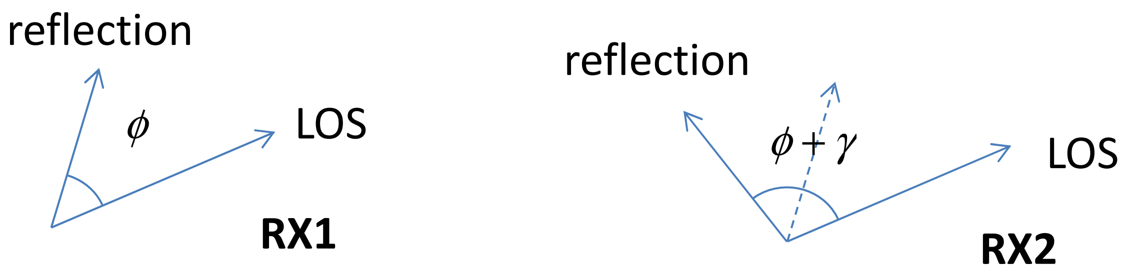


$\gamma = \pi \Rightarrow$ guaranteed constructive combination in at least one path

Basic two-ray channel

- Robustness

Range of antenna spacing: $$\boxed{\frac{\phi_0}{\pi}\frac{R}{2r} \le \frac{d}{\lambda} \le \left(1 - \frac{\phi_0}{\pi}\right)\frac{R}{2r}}$$

For the selective diversity scheme,
If one receiver sees a destructive fade,
the other one is **guaranteed** to see a constructive fade.

Examples of antenna spacing: $3.71 \le \frac{d}{\lambda} \le 16.29$ **5 dB link margin**

- Displacement d guarantees β for $R/r \in \left[\frac{2\pi d}{\lambda(\pi-\phi_0)}, \frac{2\pi d}{\lambda\phi_0}\right]$
- With R/r = fixed, $d \in \left[\frac{\phi_0\lambda}{2\pi}(R/r), \frac{(\pi-\phi_0)\lambda}{2\pi}(R/r)\right]$ guarantees β

Street canyon channel

- Including single reflection rays from walls and ground
- Horizontal and vertical diversity are **independent**

$$\gamma = \frac{2\pi}{\lambda}\left(\sqrt{R^2 + (2r_w + d_h)^2 + (2r_g + d_v)^2} - \sqrt{R^2 + (2r_w)^2 + (2r_g)^2}\right)$$

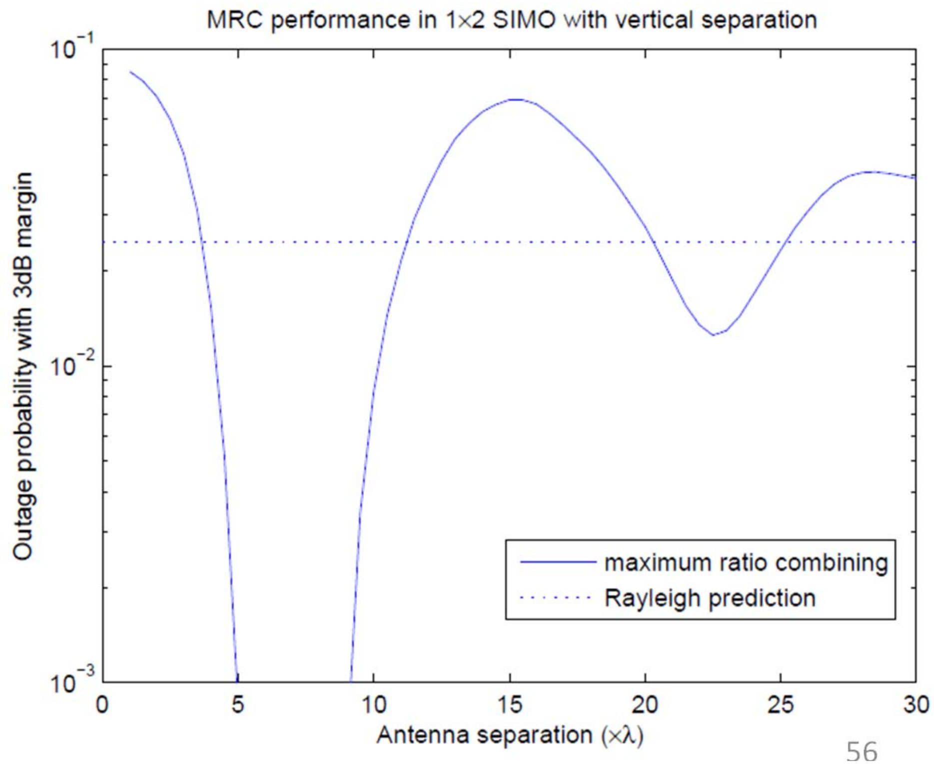
$$\approx \frac{4\pi r_w d_h}{\lambda D} + \frac{4\pi r_g d_v}{\lambda D}$$

$\Rightarrow$ Effective 2-path channel for <u>vertically</u> displaced antennas

Randomized geometry

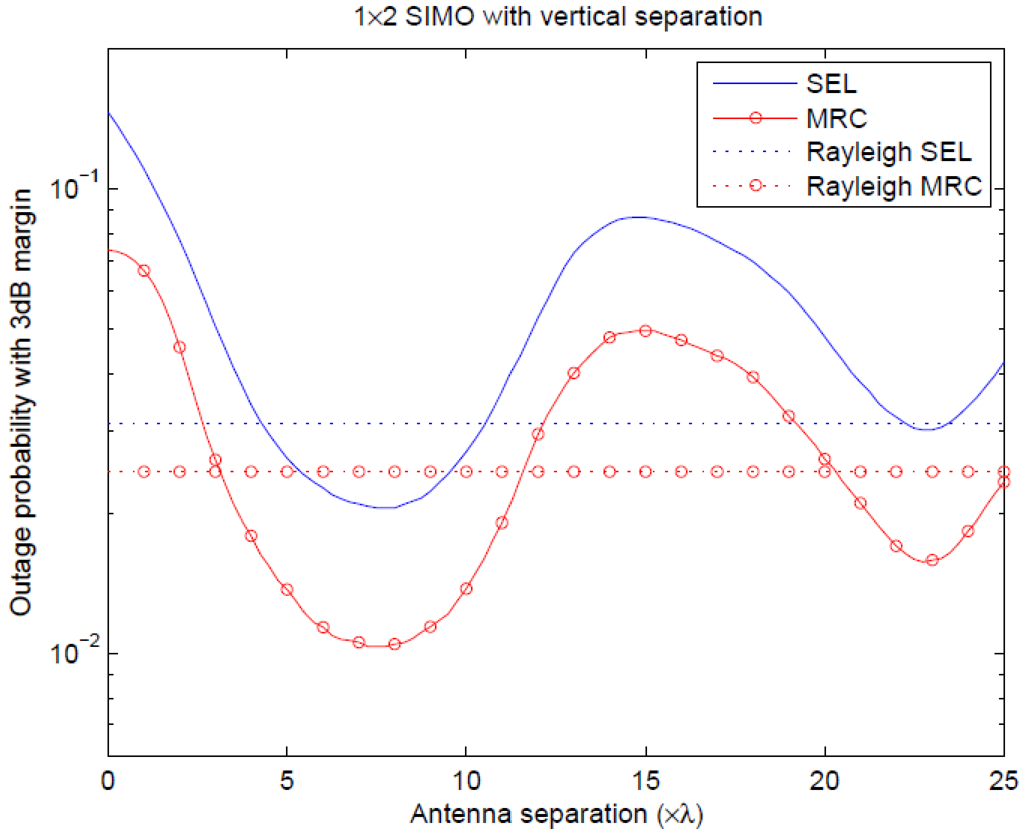
$r_w \in [4,20]$

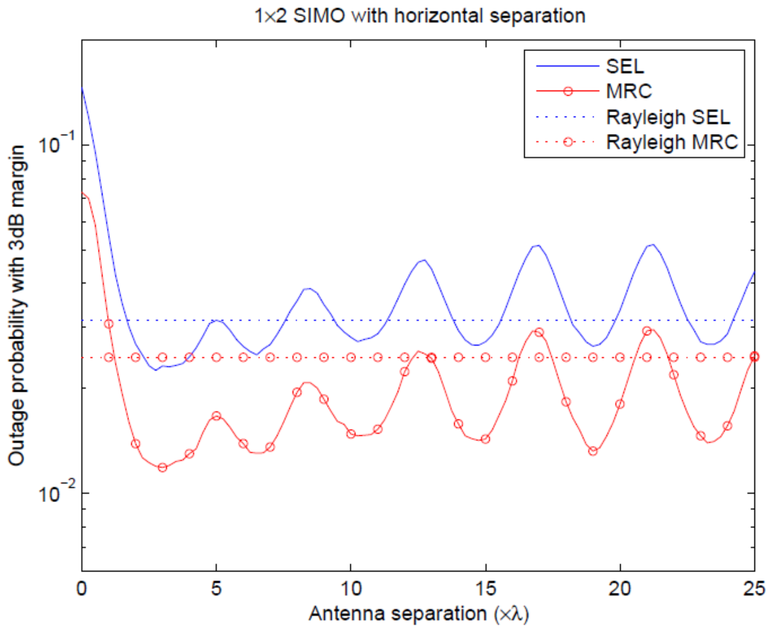
$r_g \in [4,8]$



Street canyon channel

- Including double reflection rays
- Less deterministic – but 2-ray guidelines apply to achieve maximum diversity gain

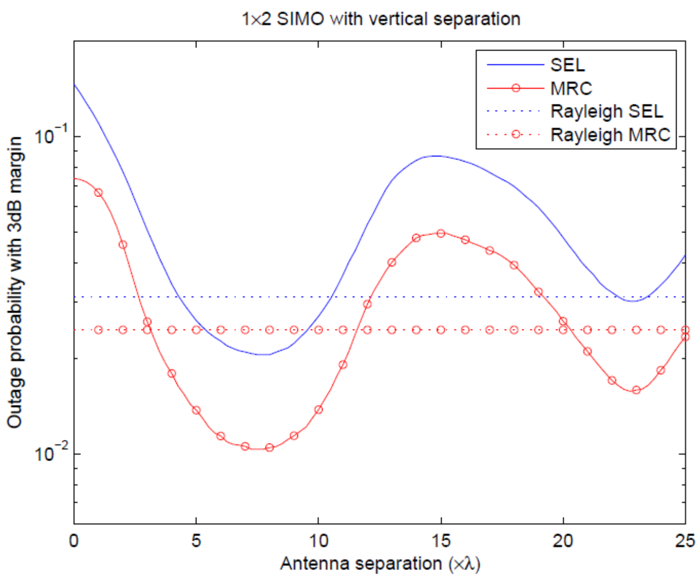


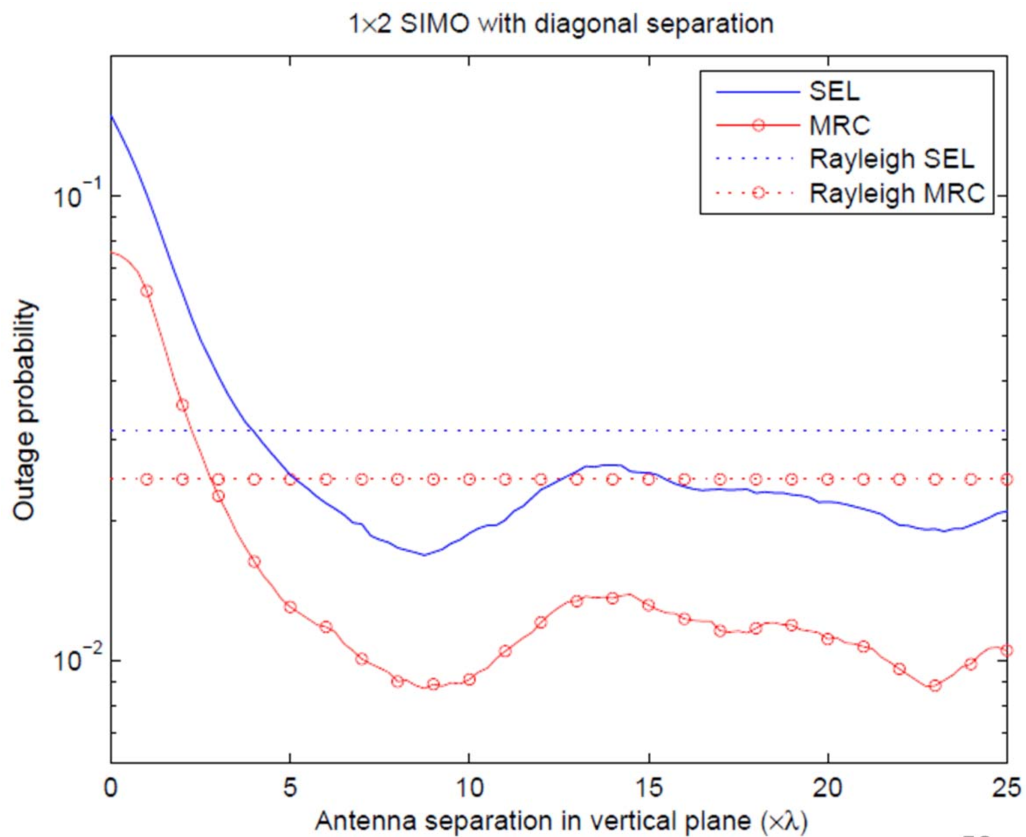


Street canyon channel

- Including double reflection rays
- Less deterministic – but 2-ray guidelines apply to achieve maximum diversity gain

Using diagonal spacing increases diversity





Frequency diversity

- Frequency diversity gain of bandwidth B

$$G = \frac{1}{f_{\max} - f_{\min}} \int_{f_{\min}}^{f_{\max}} |\, H(f)\,|^2 \; df$$

where

$$H(f) = \sum_{k=1}^{M} \alpha_k \exp(j2\pi\tau_k f)$$

$\alpha_1 = 1$ (LOS)

Frequency diversity

$$H(f) = \sum_{k=1}^{M} \alpha_k \exp(j2\pi\tau_k f) \qquad G = \frac{1}{f_{\max} - f_{\min}} \int_{f_{\min}}^{f_{\max}} |H(f)|^2 \, df$$

$$|H(f)|^2 = H(f).H(f)^*$$
$$= \sum_{k=1}^{M} |\alpha_k|^2 + \sum_{k=1}^{M} \sum_{l=k+1}^{M} 2|\alpha_k \alpha_l| \cos(2\pi(\tau_l - \tau_k) f + \angle\alpha_l - \angle\alpha_k)$$

$$G_B = \overbrace{\sum_{k=1}^{M} |\alpha_k|^2}^{G_{sat}}$$
$$+ \sum_{k=1}^{M} \sum_{l=k+1}^{M} \frac{2|\alpha_k \alpha_l|}{2\pi(\tau_l - \tau_k)B} \left[\sin(2\pi(\tau_l - \tau_k)(f_c + \frac{B}{2}) + \angle\alpha_l - \angle\alpha_k) - \sin(2\pi(\tau_l - \tau_k)(f_c - \frac{B}{2}) + \angle\alpha_l - \angle\alpha_k) \right]$$

Frequency diversity

$$G_B = G_{sat} + \sum_{k=1}^{M} \sum_{l=k+1}^{M} \frac{4|\alpha_k \alpha_l|}{2\pi(\tau_l - \tau_k)B} \left[\cos(2\pi(\tau_l - \tau_k) f_c + \angle\alpha_l - \angle\alpha_k) \sin(2\pi(\tau_l - \tau_k)\frac{B}{2}) \right]$$

$$f_c >> B, \frac{1}{\Delta\tau} \Rightarrow \quad E[G_B] = G_{sat}$$

$$E\left[|G_B - G_{sat}|^2\right] = \sum_{\substack{(k,l) \\ pairs}} E\left[\frac{8|\alpha_k \alpha_l|^2}{(2\pi(\tau_l - \tau_k)B)^2} \sin^2(2\pi(\tau_l - \tau_k)\frac{B}{2}) \right]$$

Diversity gain

$$E[G_B] = G_{sat} = 1 + E\left[\sum_{k=1}^{7} |\alpha_k|^2\right]$$

$$E\left[\| G_B - G_{sat} |^2\right] = E\left[\sum_{(k,l)\ pairs} 2|\alpha_k \alpha_l|^2 \left(\frac{\sin(2\pi(\tau_l - \tau_k)\frac{B}{2})}{2\pi(\tau_l - \tau_k)\frac{B}{2}}\right)^2\right]$$

$$\leq 2n\left(\frac{\sin(2\pi \min(\Delta\tau)\frac{B}{2})}{2\pi \min(\Delta\tau)\frac{B}{2}}\right)^2 \qquad \text{or} \approx 2\left(\frac{\sin(2\pi \min(\Delta\tau)\frac{B}{2})}{2\pi \min(\Delta\tau)\frac{B}{2}}\right)^2 \qquad \text{(dominant term)}$$

Diversity gain

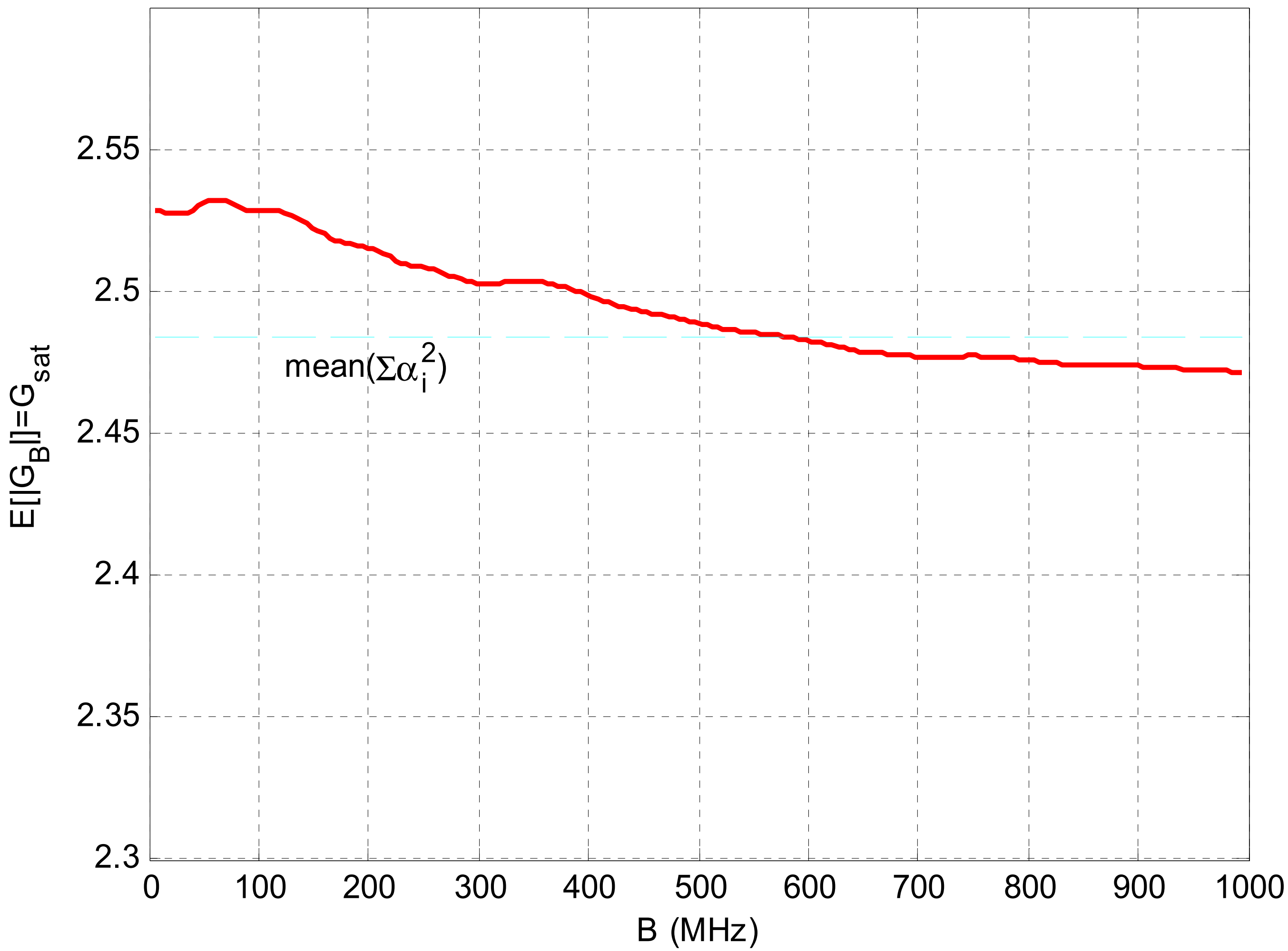
- When B is small => all components add up
- When B is large => dominant term

$$V \approx 2E\left[\left|\alpha_k \alpha_l\right|^2\right] \qquad B \to 0$$

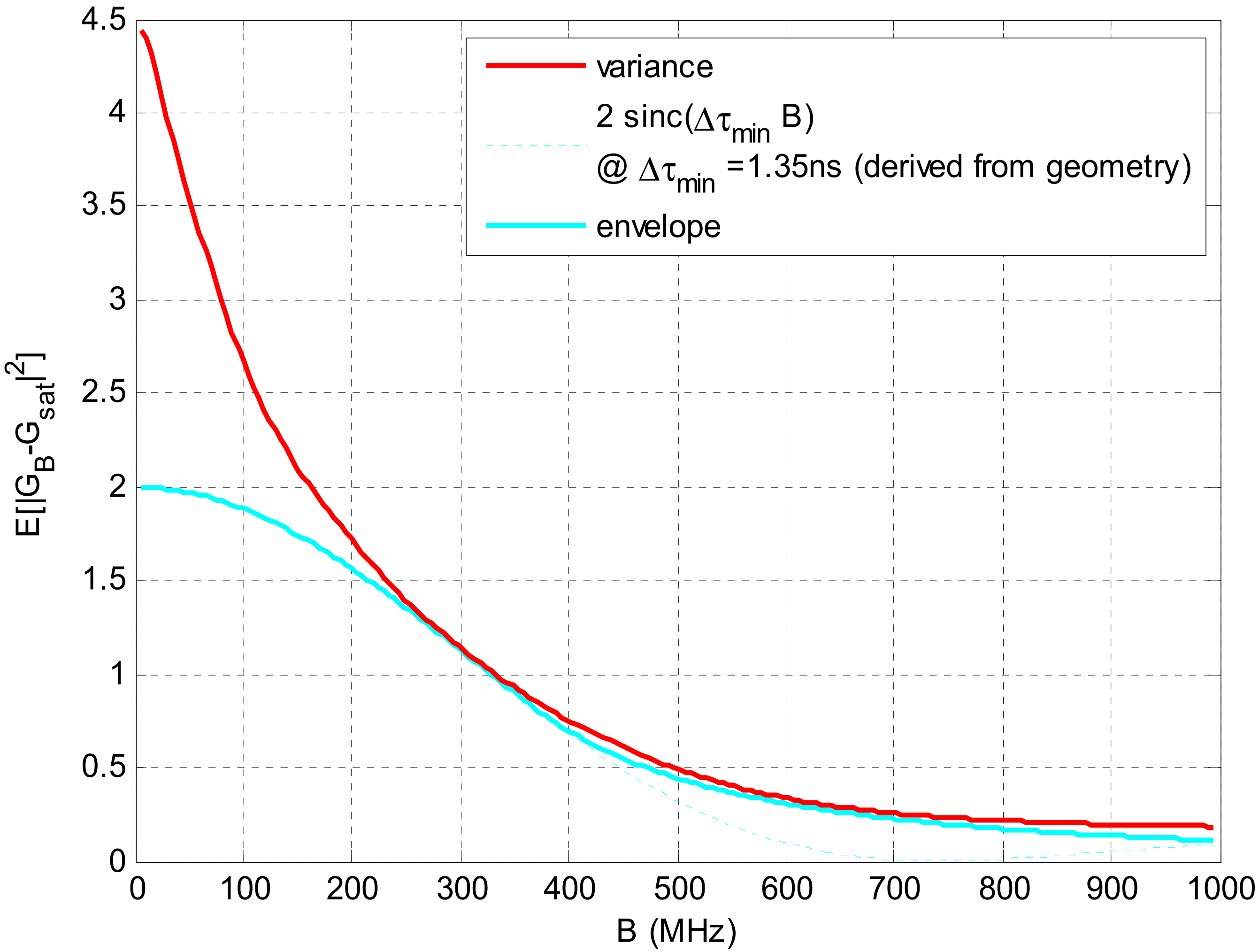
$$V \approx 2\,\mathrm{sinc}^2\left(\min(\Delta\tau)\times B\right) \quad \text{large } B$$

$\Rightarrow \mathrm{sinc}^2\left(\min(\Delta\tau)\times B\right)$ is representative of frequency diversity gain reliability

Mean of frequency diversity gain



Variance of frequency diversity gain



Frequency diversity

- In the lamp post to lamp post link with known geometry

$$\tau_i = \frac{d_i}{c}$$

$$d_i = \sqrt{L^2 + (mW)^2 + b(h_t + h_r)^2} \approx L + \frac{(mW)^2}{2L} + \frac{b(h_t + h_r)^2}{2L}$$

(m reflections from walls and b reflections from ground (b=0,1))

- Minimum delay is between LOS and single reflection from closest surface (near wall or ground)
- Guideline for diversity gain achieved from bandwidth B

Extending to SIMO

- Reminder: Frequency diversity gain:

$$G = \frac{1}{f_{\max} - f_{\min}} \int_{f_{\min}}^{f_{\max}} | H(f) |^2 \, df$$

- Selection and Maximum Ratio Combining

$$H_{Sel} = \max(H_1, H_2)$$

$$H_{MRC} = \sqrt{| H_1 |^2 + | H_2 |^2}$$

With high bandwidth, choice of displacement has no effect on mean or expected MRC gain, but affects its statistical variation

Extending to SIMO

- 1x2 SIMO with maximum ratio combining
- Expected diversity gain is $2G_{SISO}$
- Variance of diversity gain G_{SIMO}

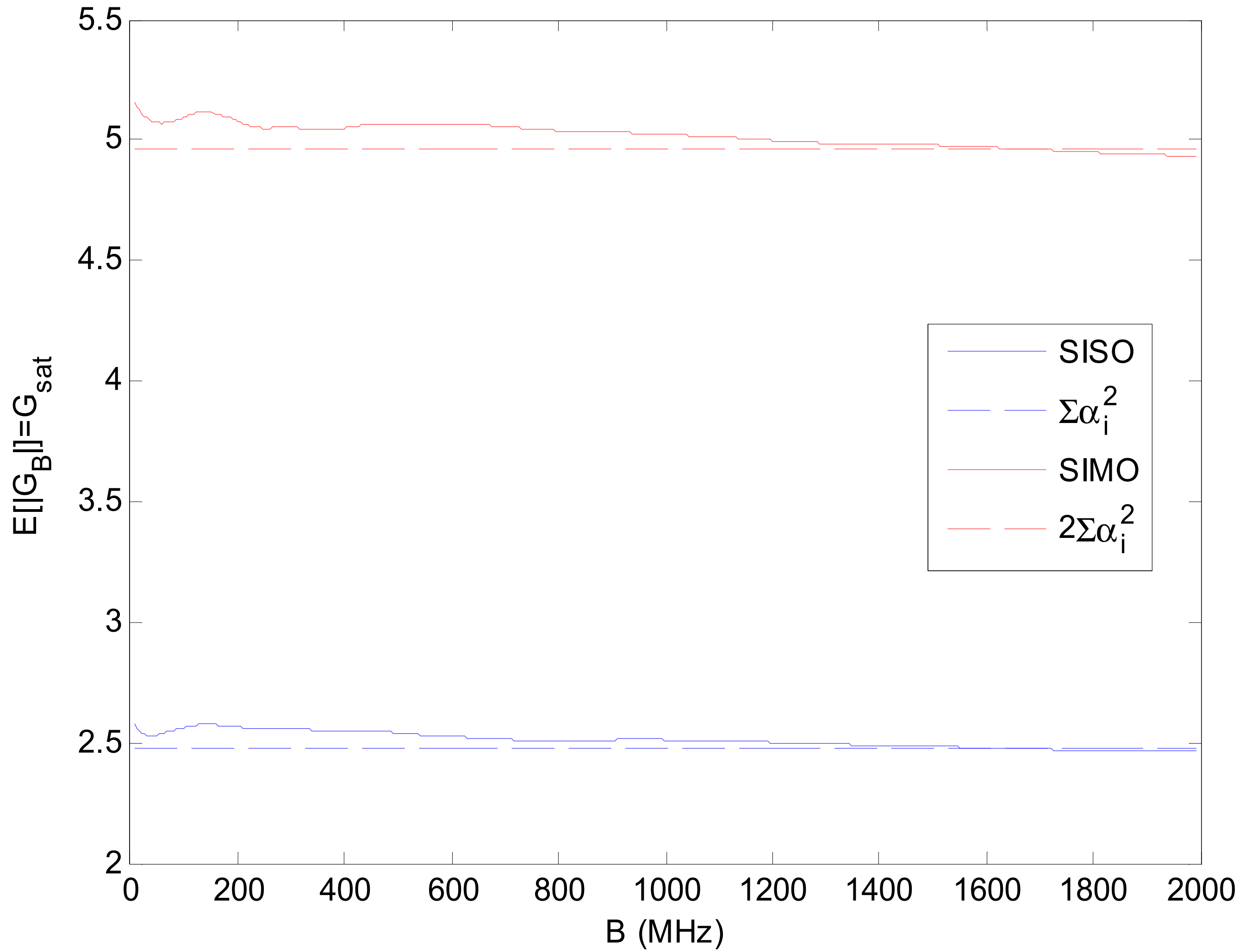
$= 2G_{SISO}$ for uncorrelated responses

$> 2G_{SISO}$ for positive correlation

$< 2G_{SISO}$ for negative correlation

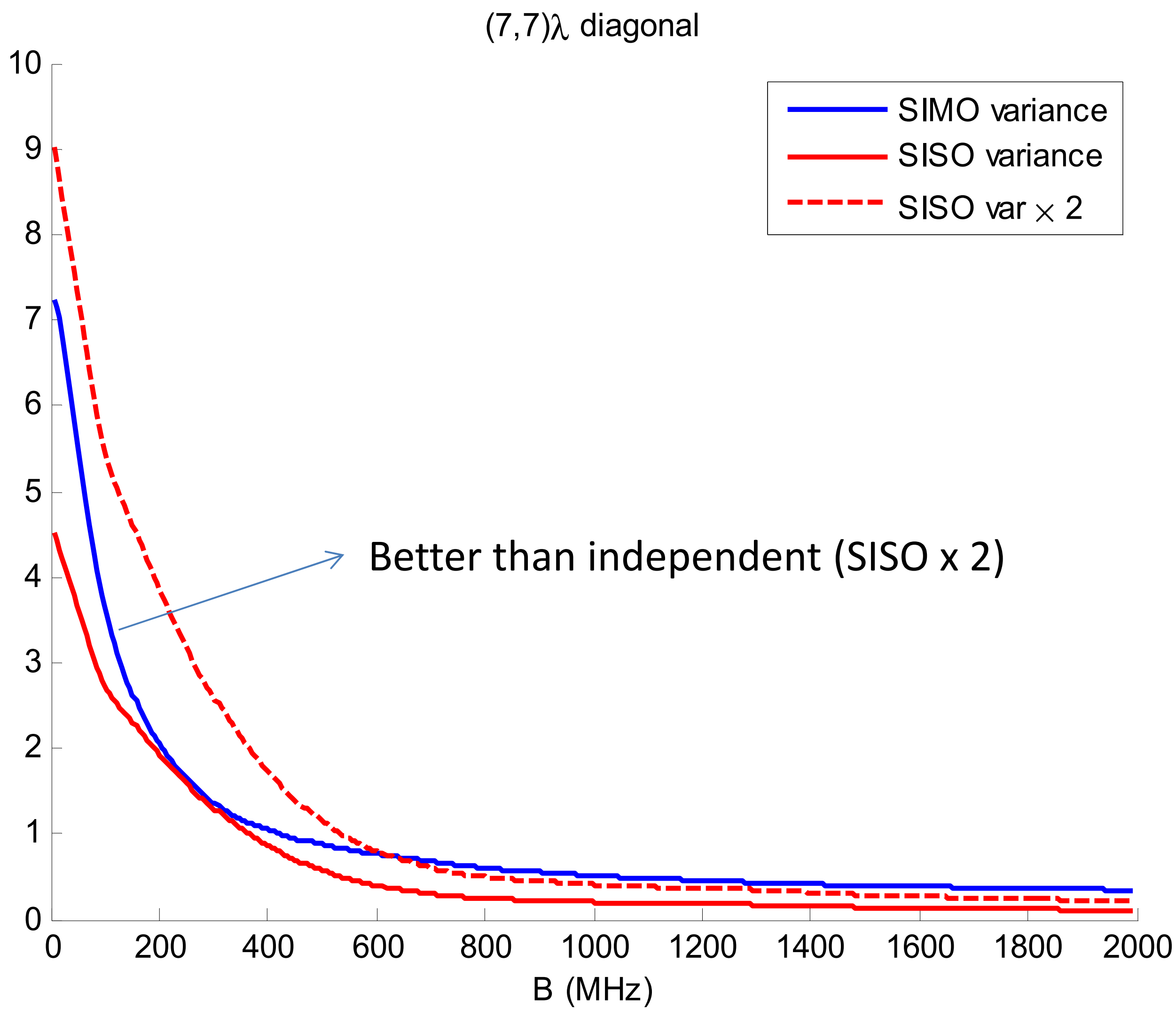
Good spacing – diagonal (7,7)λ

(mean)



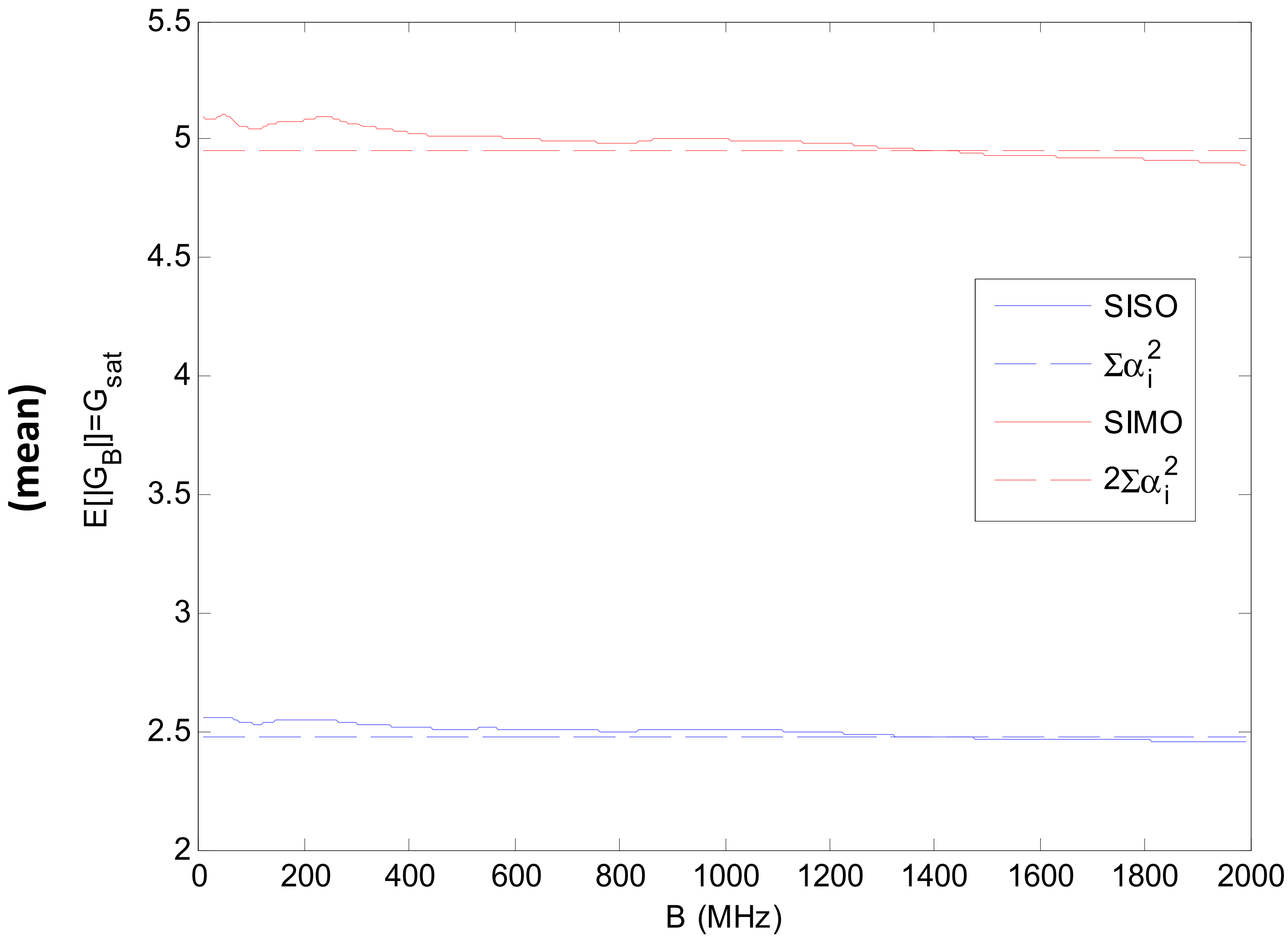
Good spacing – diagonal (7,7)λ

(variance)

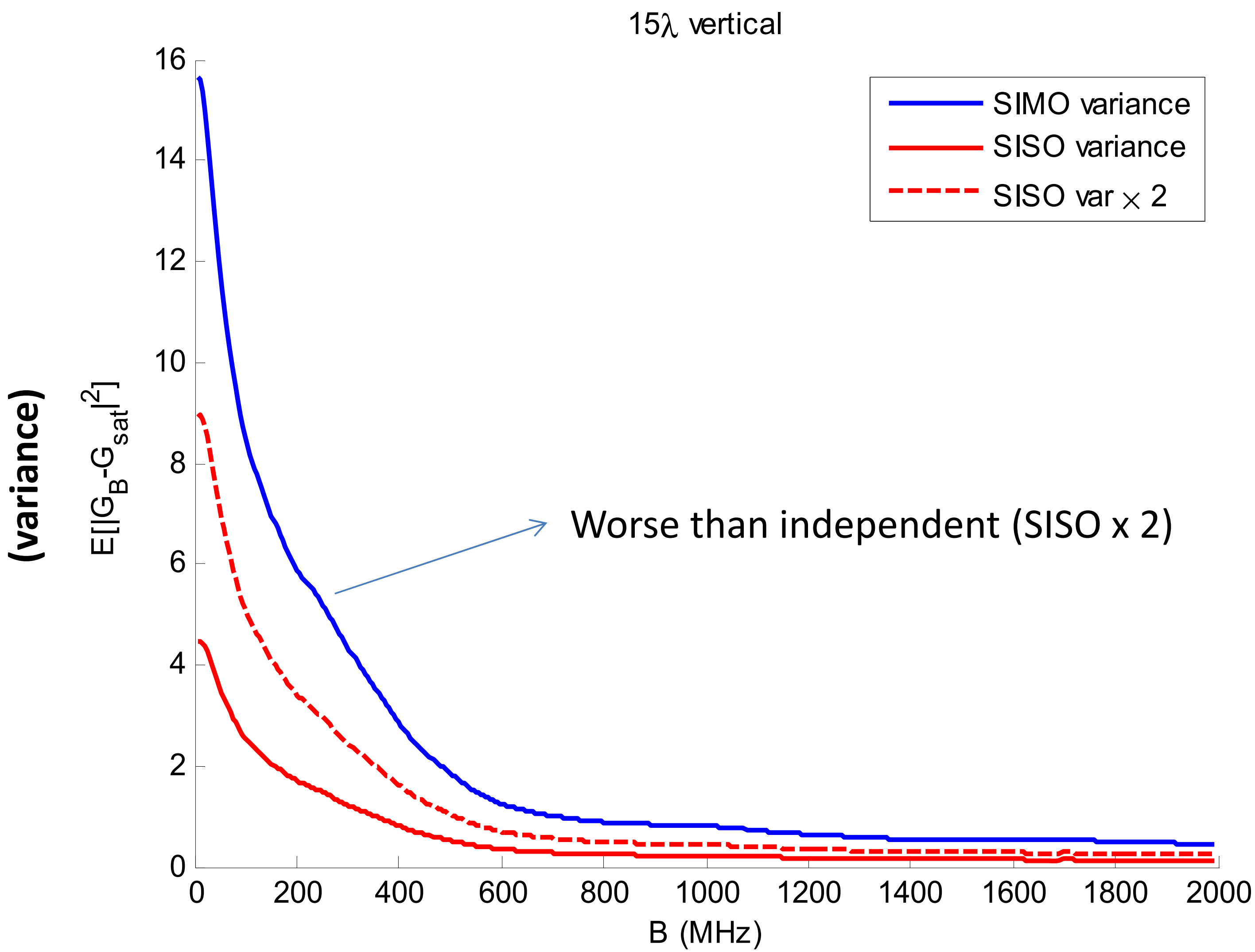


Better than independent (SISO x 2)

Bad spacing – vertical 15λ



Bad spacing – vertical 15λ



Take aways

- Spatial diversity is different for sparse multipath
- But it is not that interesting for the bandwidths of interest, since frequency diversity will do the trick
- Beamforming and spatial multiplexing are more interesting issues than diversity

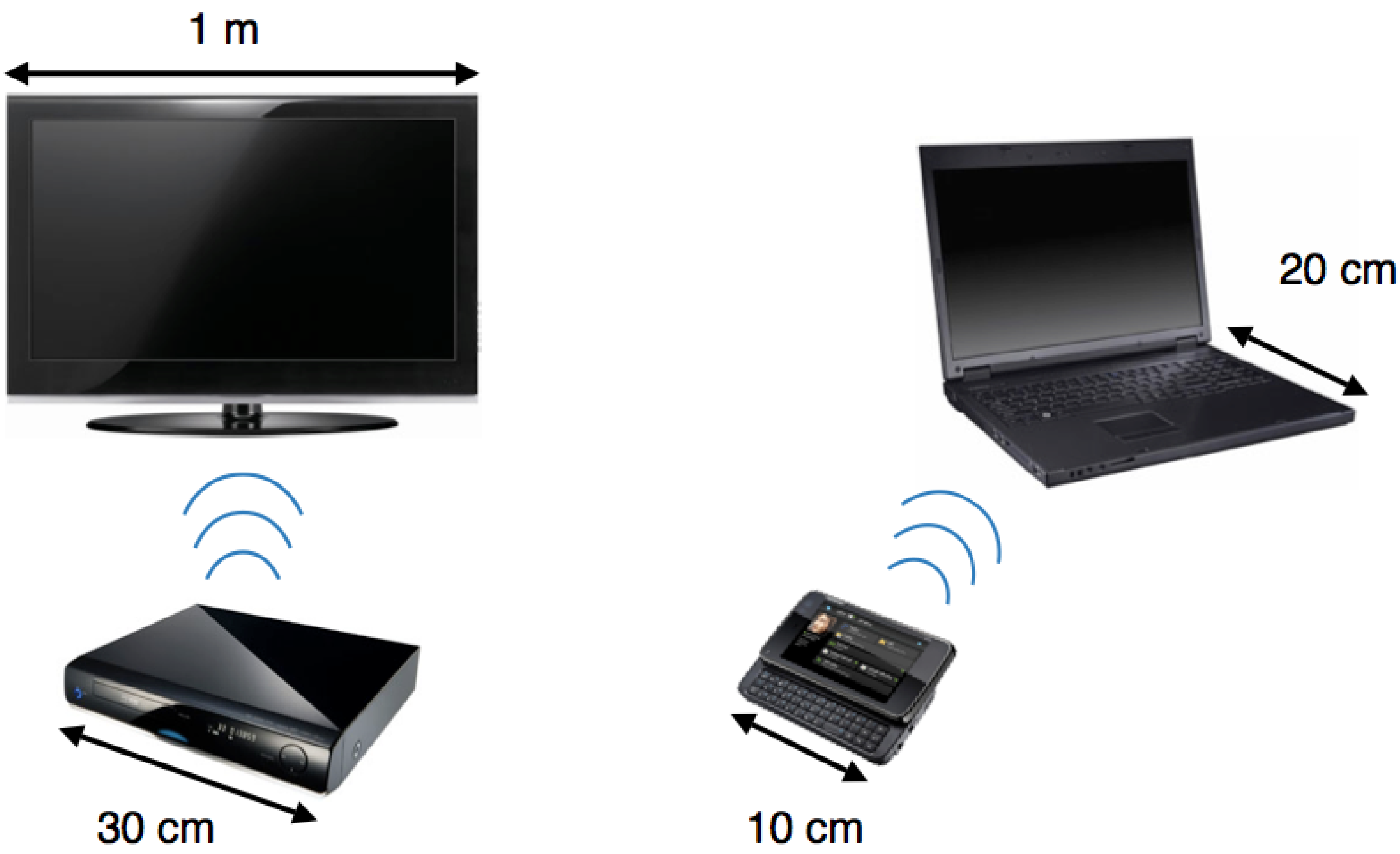
LoS MIMO, part 1

Fundamental limits
Array of subarrays
Location-dependent capacity
2010 Prototype

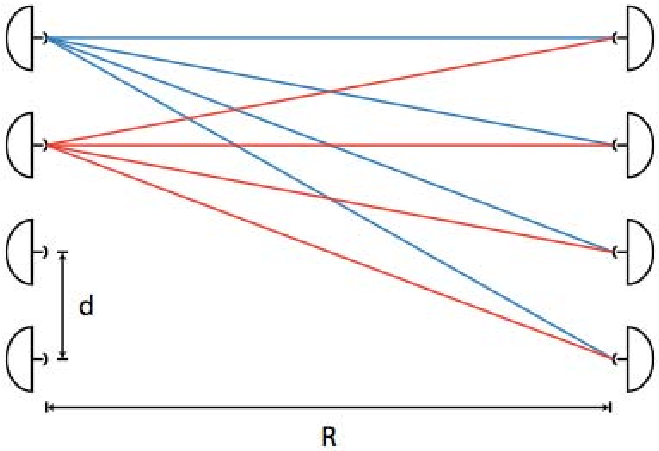
Collaborators
Dr. Colin Sheldon, Dr. Eric Torkildson,Dr. Munkyo Seo
Prof. Mark Rodwell

2016 Summer School, IISc Bangalore

How many degrees of freedom are available?



The Geometry of LoS MIMO: Rayleigh criterion



$$\mathbf{h}_1 = (1, e^{j\phi}, e^{j2^2\phi}, \ldots, e^{j(N-1)^2\phi})^T$$

$$\mathbf{h}_2 = (e^{j\phi}, 1, e^{j\phi}, \ldots, e^{j(N-2)^2\phi})^T$$

$$|\langle \mathbf{h}_1, \mathbf{h}_2 \rangle| = \left| \frac{\sin(N\phi)}{\sin\phi} \right|$$

Vectors are orthogonal when $N\phi = N\dfrac{\pi d^2}{\lambda R} = \pi$

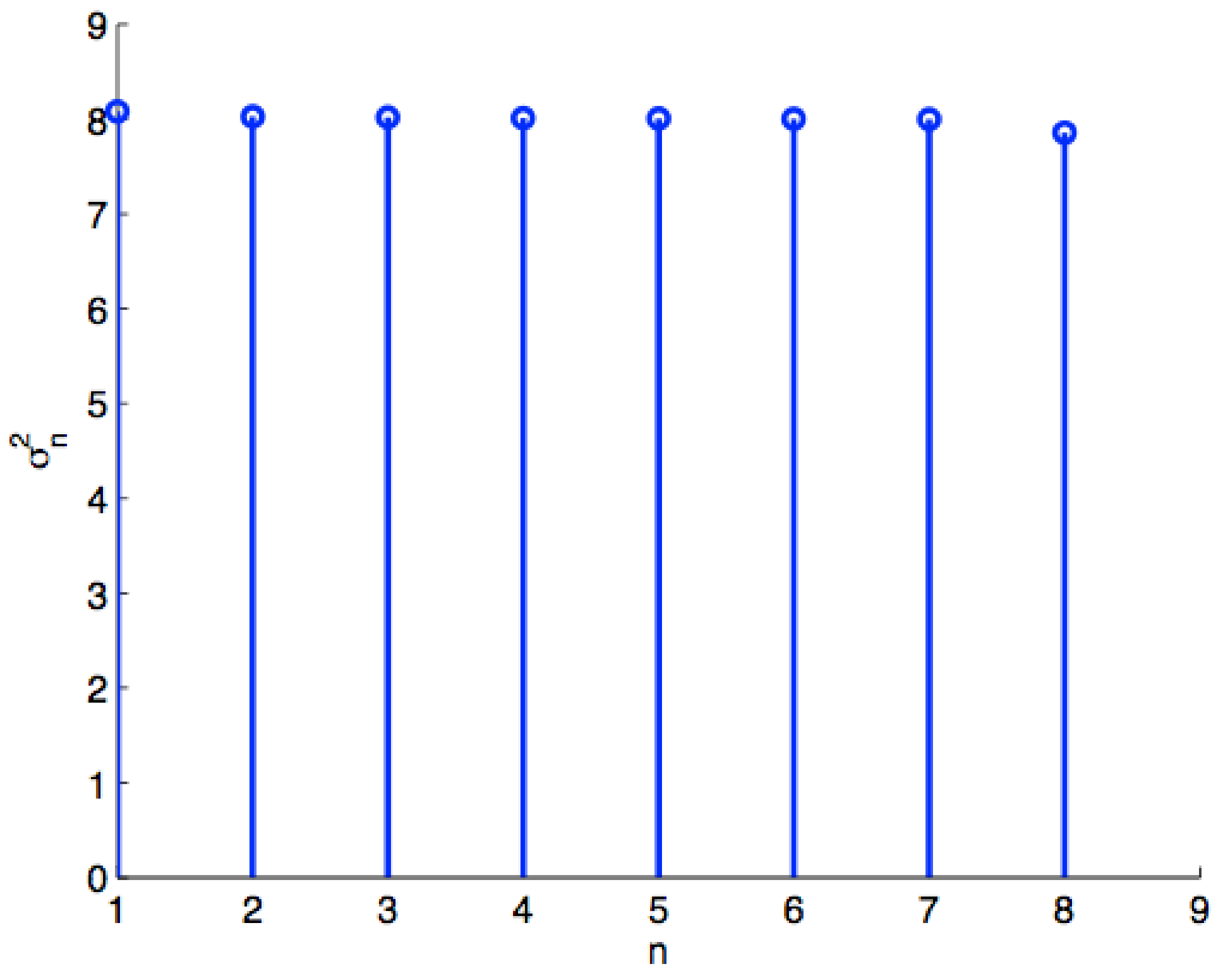
$$\boxed{d = \sqrt{\frac{\lambda R}{N}}}$$

Example

R=10 m, λ=5 mm, N=4

d =11 cm

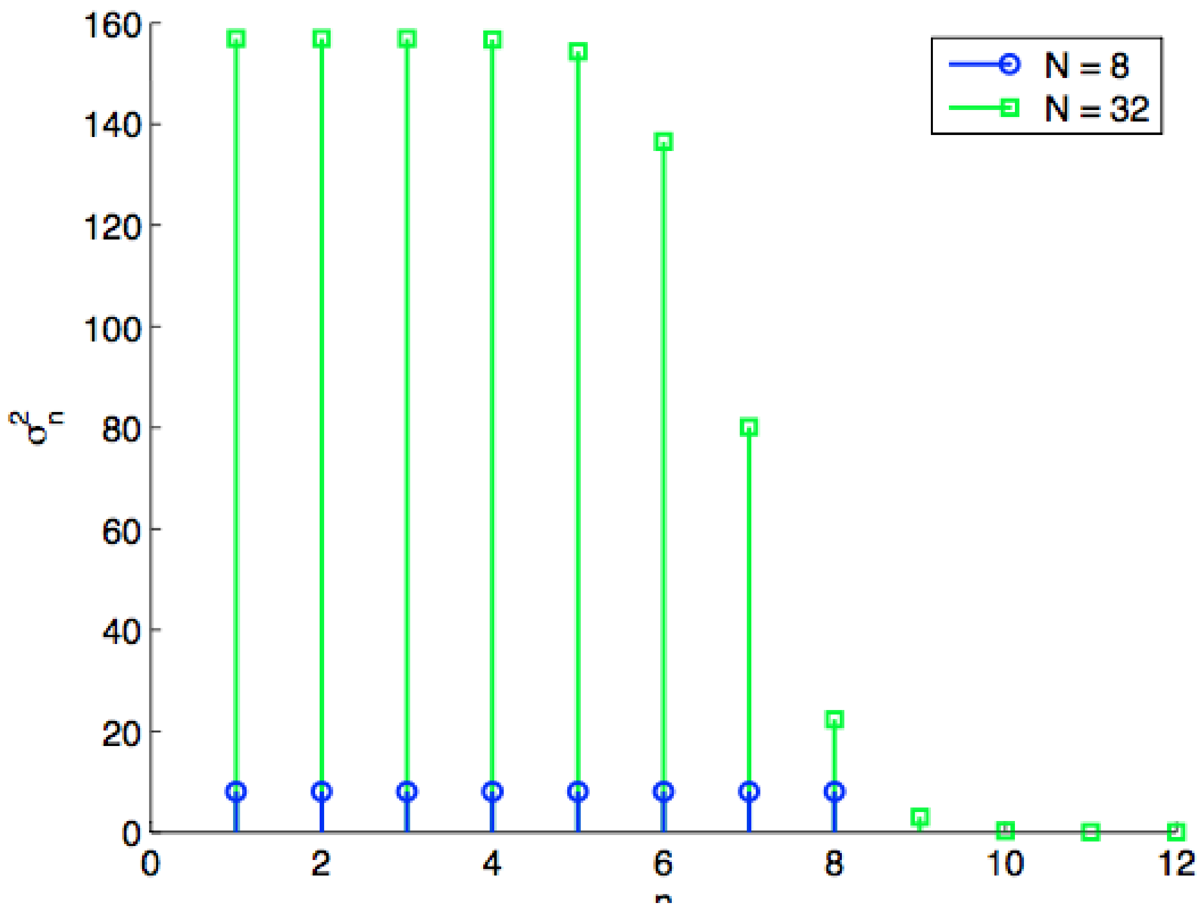
Spatial degrees of freedom



N element Rayleigh spaced array gives N degrees of freedom.
But plenty of room for more antenna elements…can we do better?

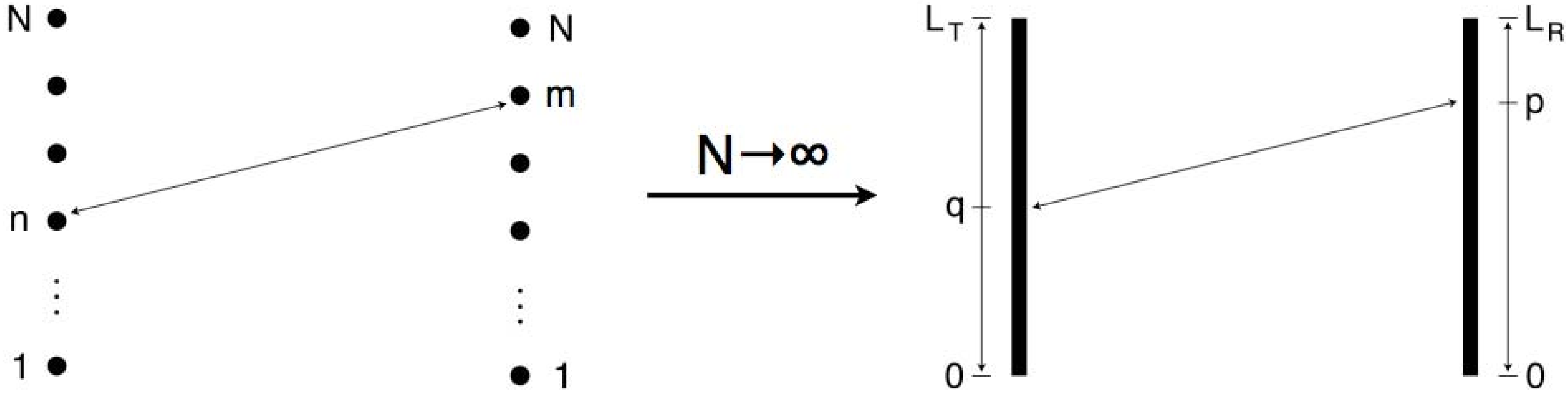
Increasing antenna count with fixed form factor

What happens with a four-fold increase to N=32?



What happens as N gets very large?

The continuous linear array limit



$$y_m = \frac{1}{N} \sum_{n=1}^{N} \underbrace{\exp\left(-i\frac{\pi}{\lambda R}\left(n d_T - m d_R\right)^2\right)}_{h_{m,n}} x_n \qquad y(p) = \frac{1}{L_T} \int_{-L_T/2}^{L_T/2} \underbrace{\exp\left(-i\frac{\pi}{\lambda R}(q-p)^2\right)}_{h(q,p)} x(q)\mathrm{d}q$$

Spatial prolate spheroidal waveforms

Directly analogous to classical prolate spheroidal analysis of bandlimited channel

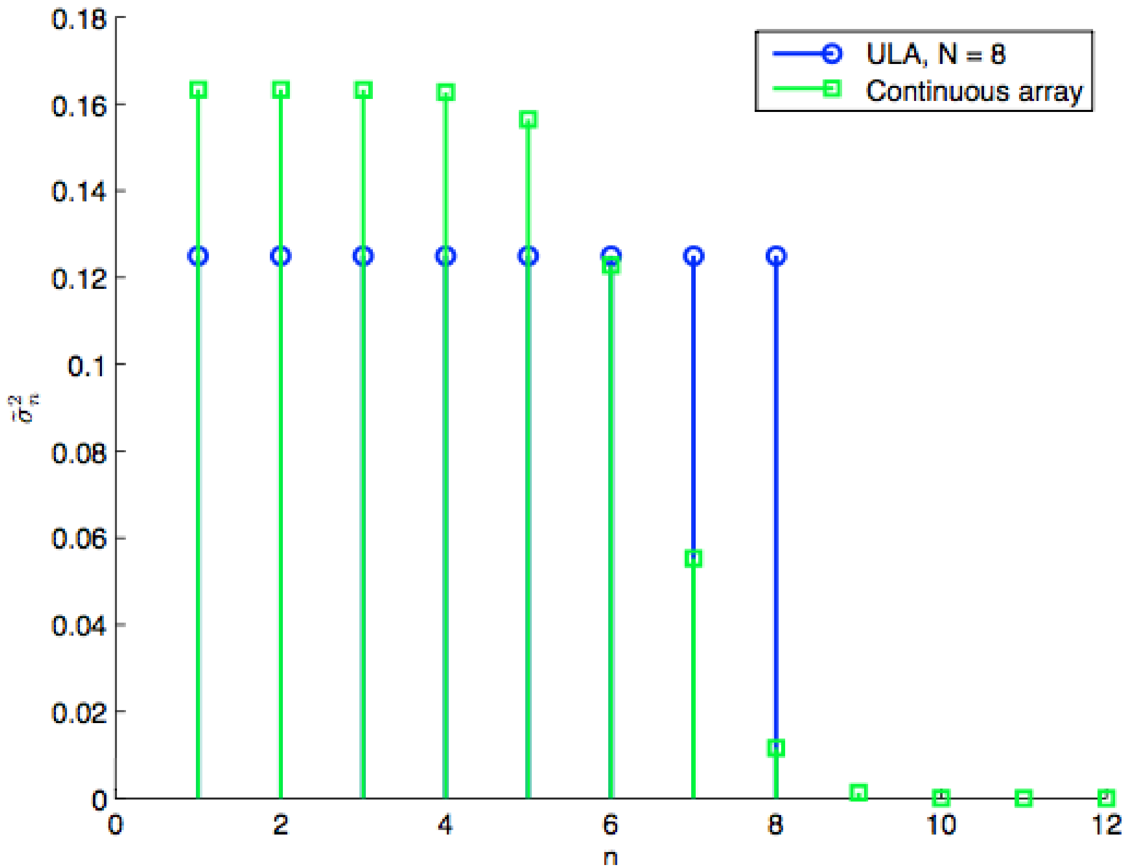
$$\lambda_n \phi_n(t) = \int_{-T/2}^{T/2} \frac{\sin 2\pi W(t-s)}{\pi(t-s)} \phi_n(s) ds$$

$$\frac{L_T^2}{\lambda R} |g_n|^2 \alpha_n(q) = \int_{-L_T/2}^{L_T/2} \frac{\sin 2\pi \frac{L_r}{2\lambda R}(q-q')}{\pi(q-q')} \alpha_n(q') dq'$$

Strength of LoS MIMO modes given by scaled prolate spheroidal e-values

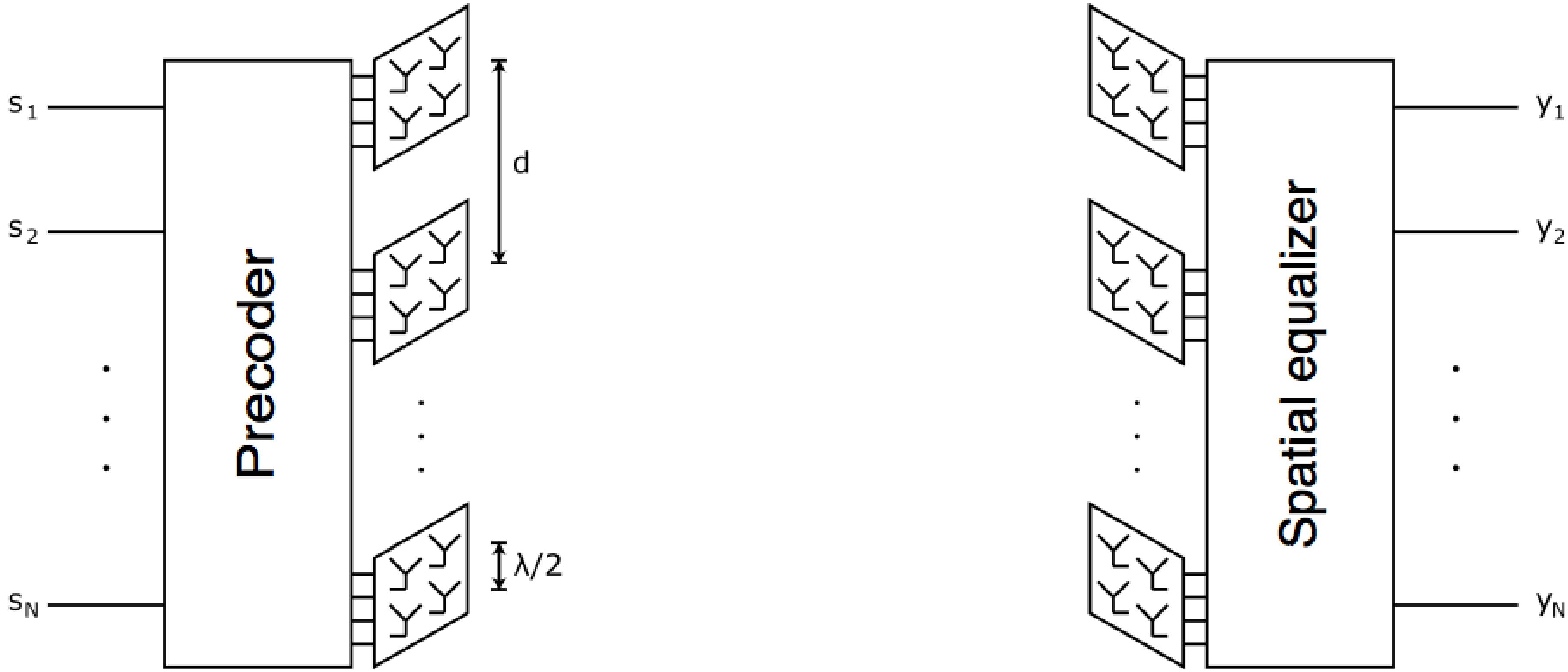
$$|g_n|^2 \approx 0 \text{ for } n > \frac{L_T L_R}{\lambda R}(1+\epsilon)$$

The optimality of Rayleigh spacing



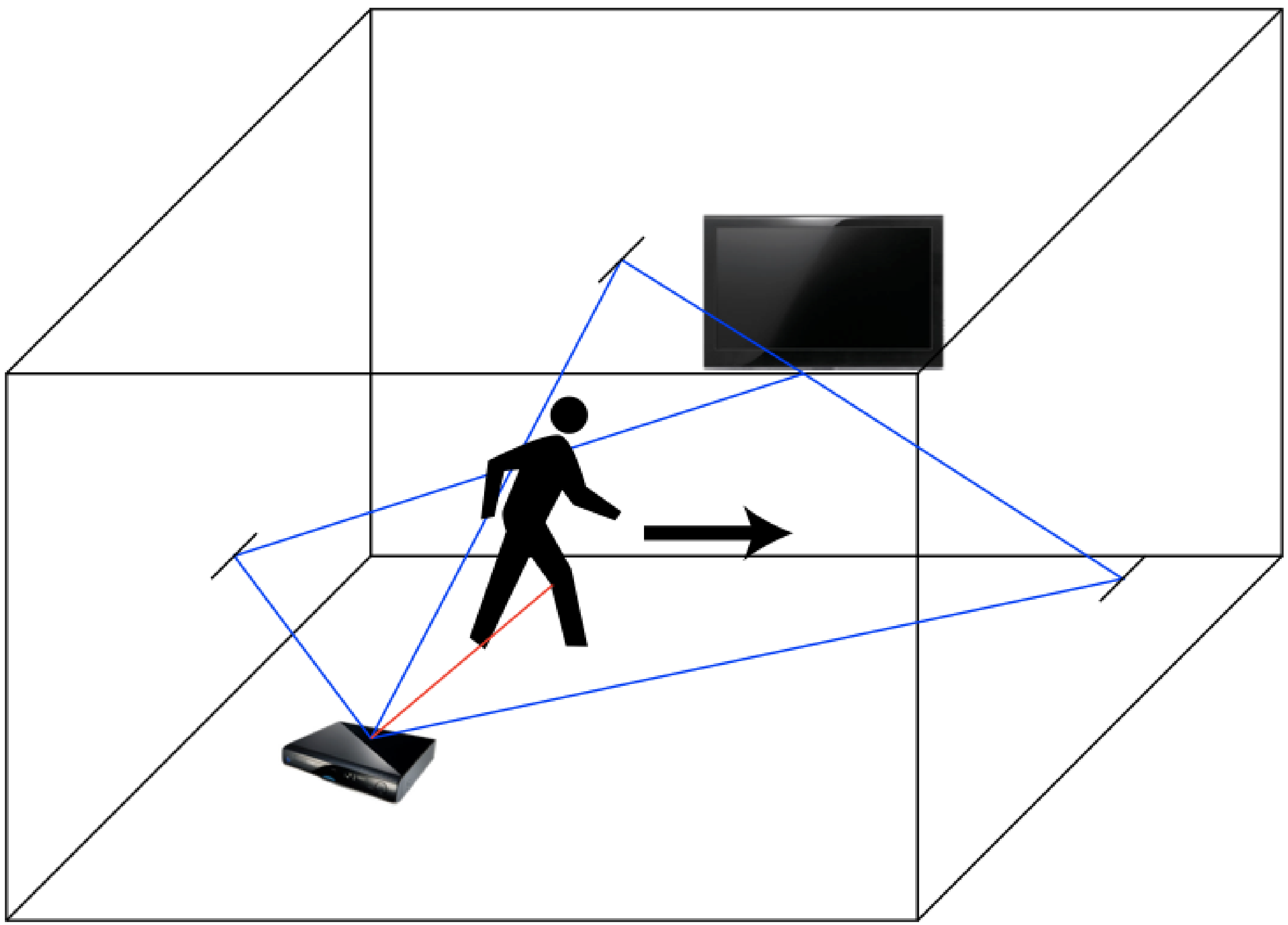
Rayleigh spacing essentially optimal in terms of degrees of freedom
But additional elements provide beamforming gain

Array of subarrays architecture

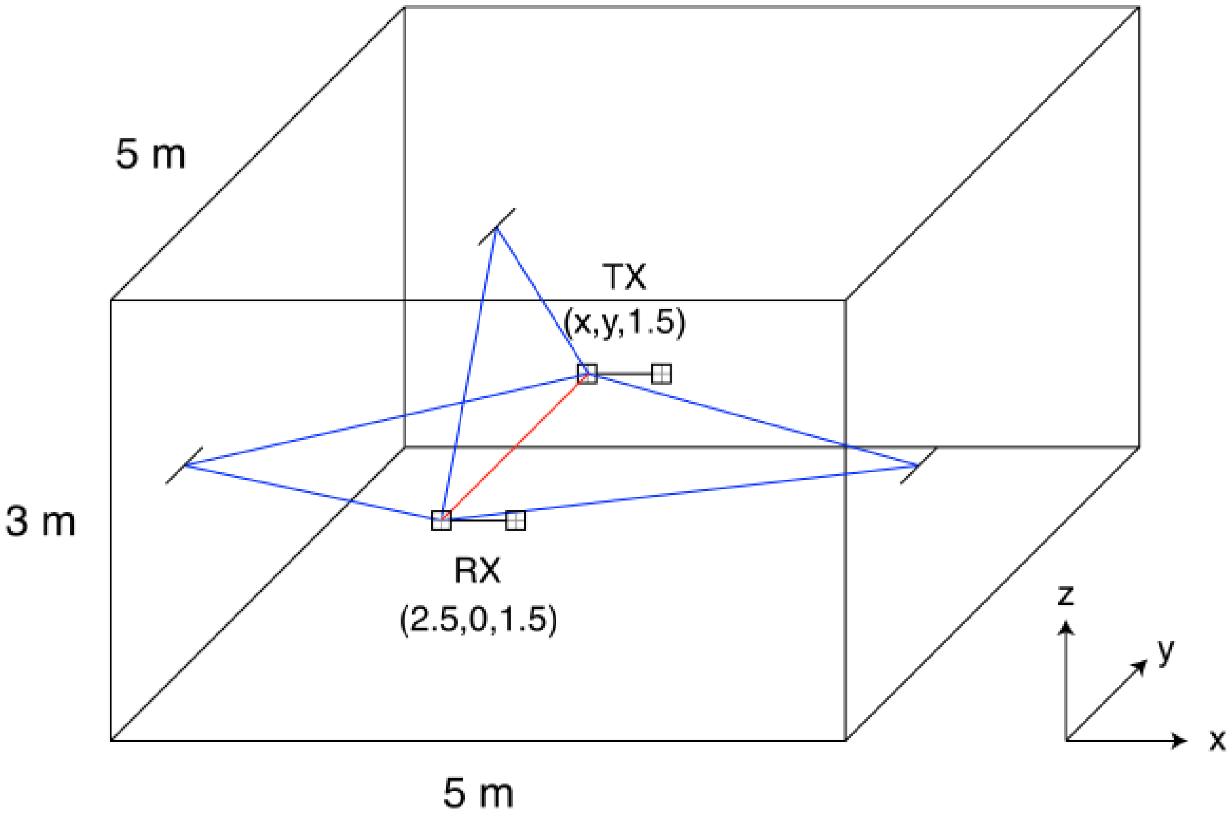


Rayleigh-spaced arrays: spatial multiplexing
Each array is a sub-wavelength spaced subarray: beamforming

Is this robust to reflections and blockage?



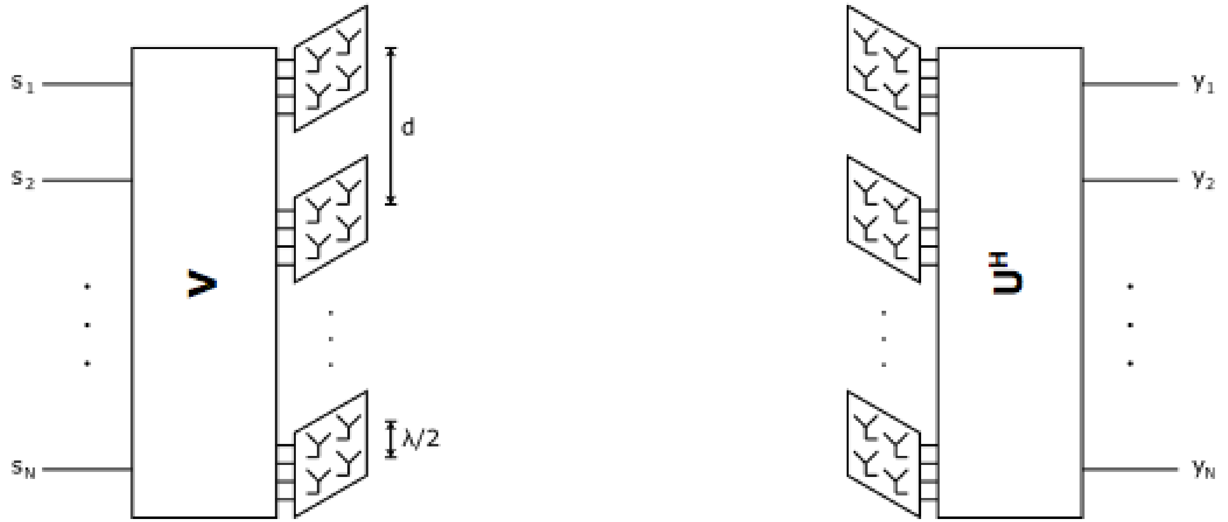
Modeling the Indoor Environment



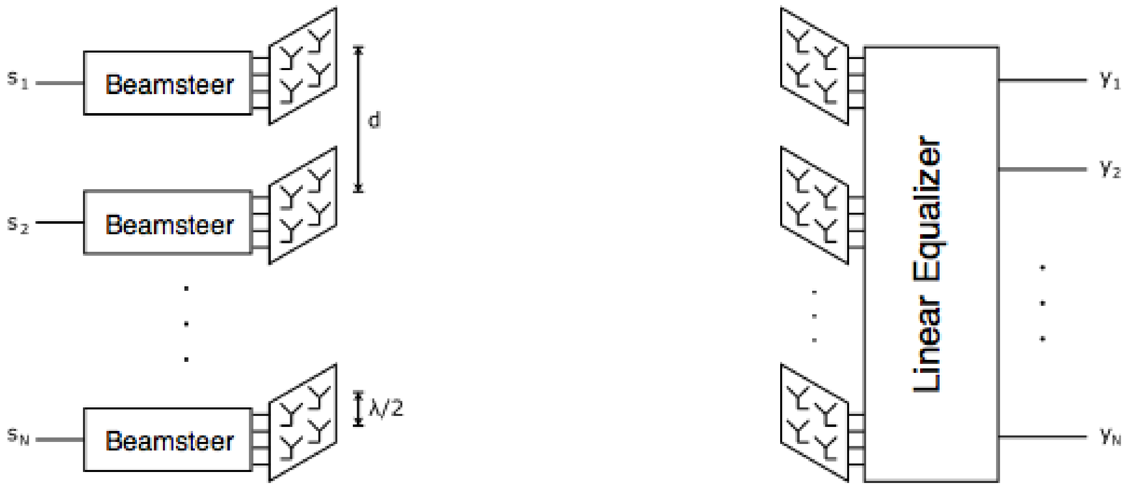
<u>Simulation parameters</u>

- Plasterboard walls $(\epsilon_r = 2.8,\ \sigma = 0.221)$
- Two 4x4 subarrays per node;
- d = 8 cm (optimized for R_o = 2.5 m)

Performance benchmarks

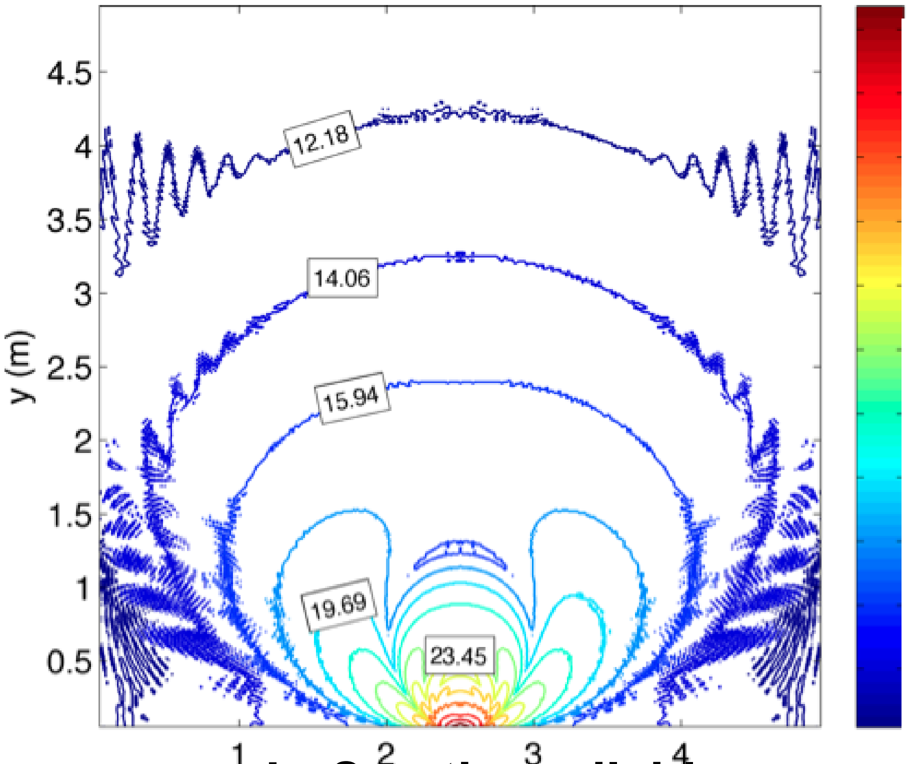


Linear precoding
Waterfilling

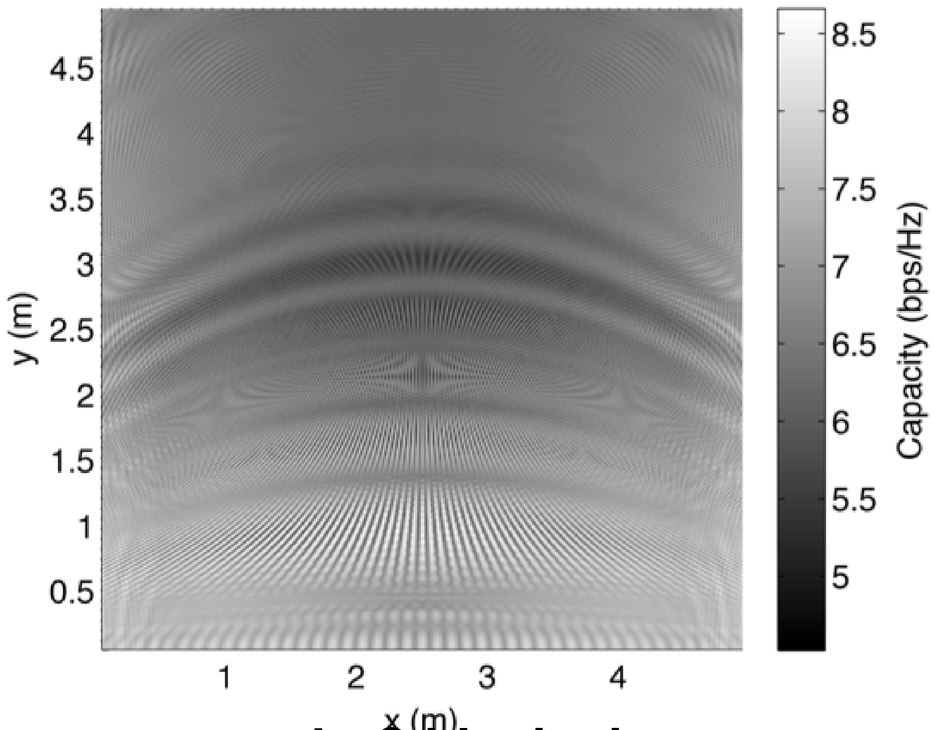


Transmit beamforming
Linear MMSE receiver

Capacity vs TX location (waterfilling)



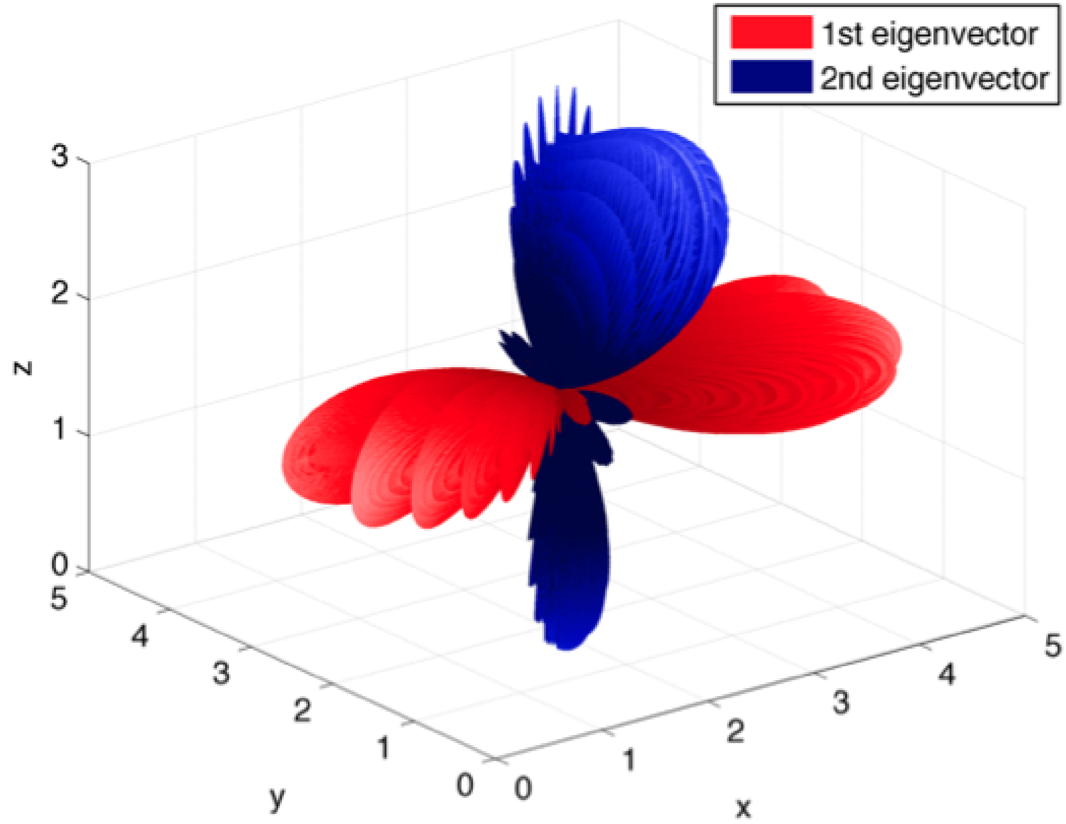
LoS path available



LoS blocked

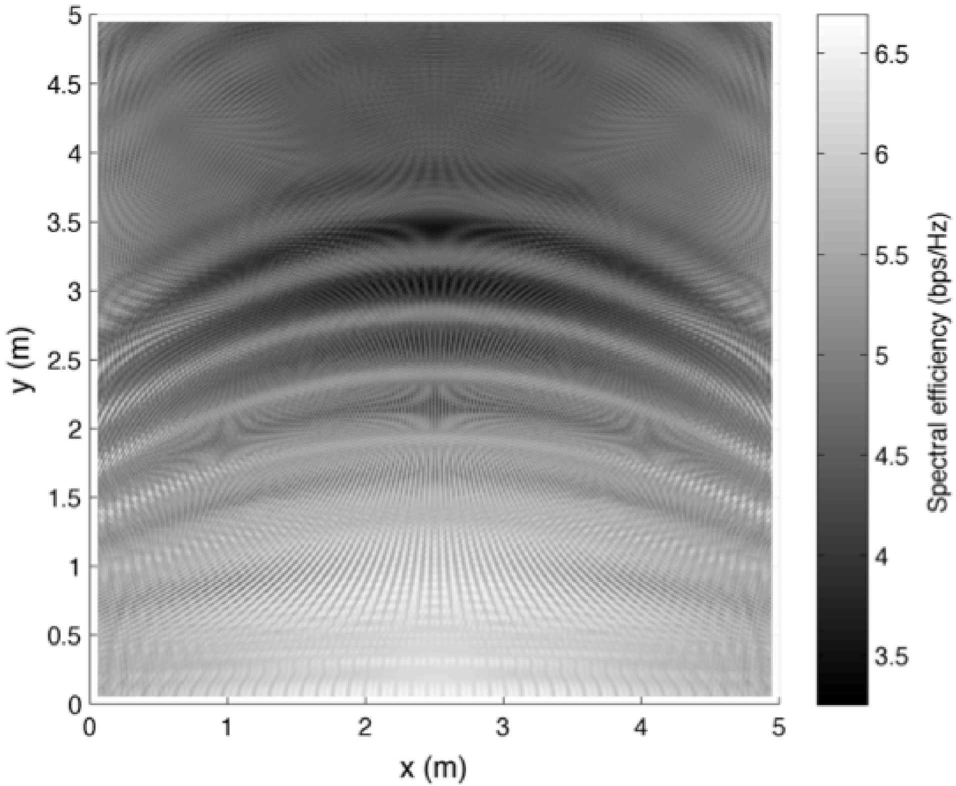
2x2 MIMO array of subarrays, Rayleigh spaced for TX at center
Each subarray is a 4x4 square array
TX power per element is -10 dBm

A typical blockage scenario

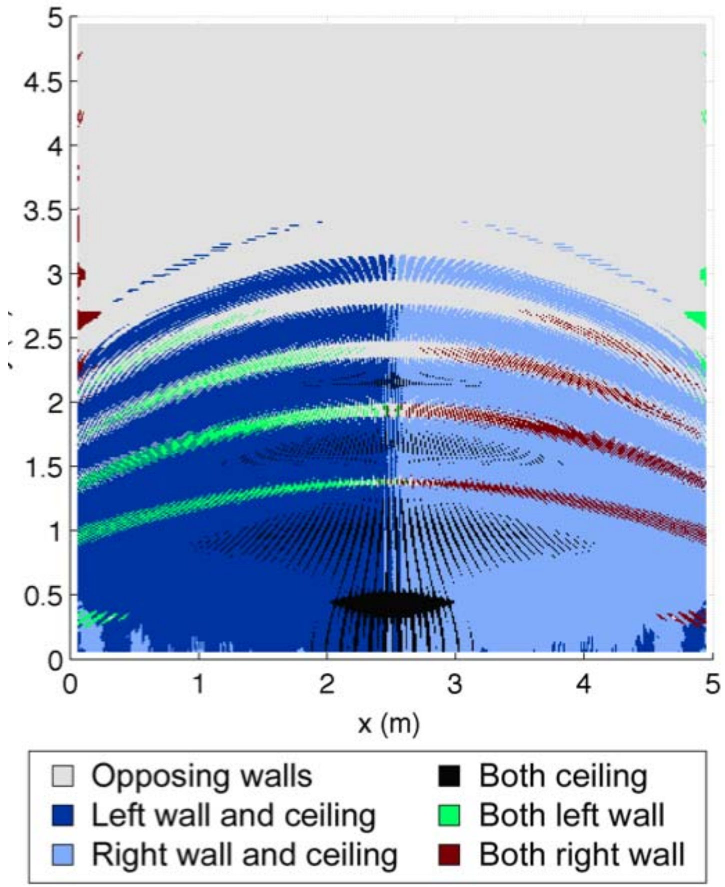


TX at center of room
First eigenmode uses wall, second eigenmode uses ceiling

Beamsteering/MMSE with LoS blockage



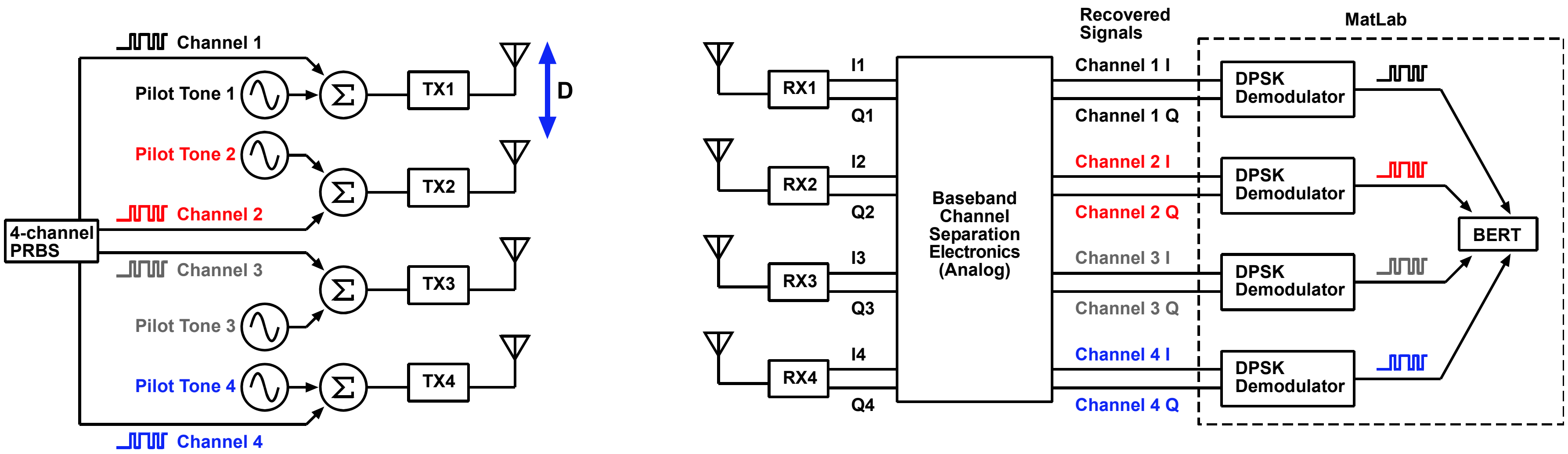
Capacity as a function of TX location



Optimal beams as a function of TX location

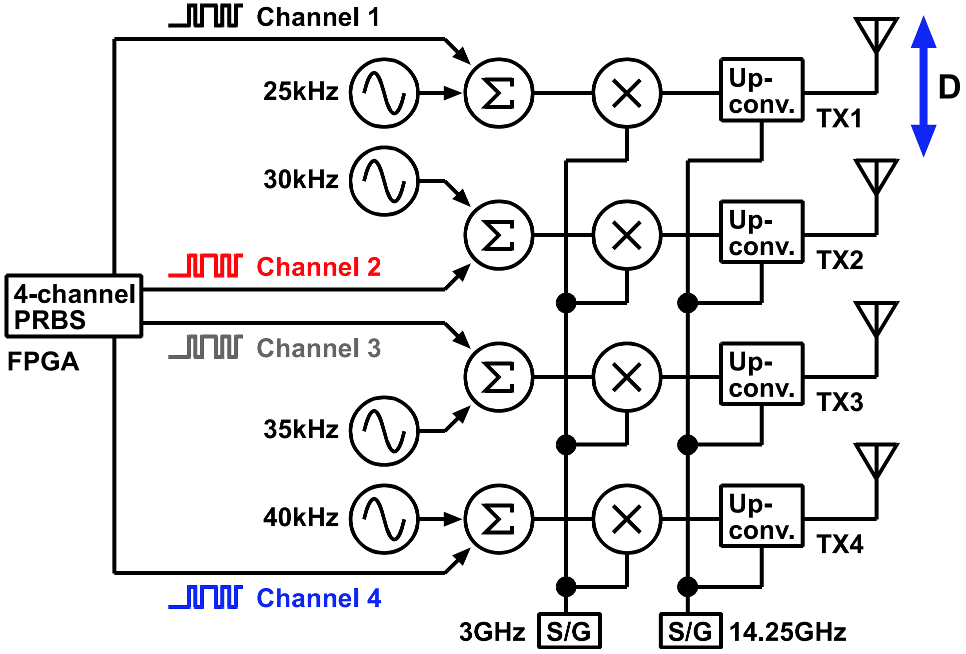
Acceptable performance even with LoS blockage with sufficient beam agility

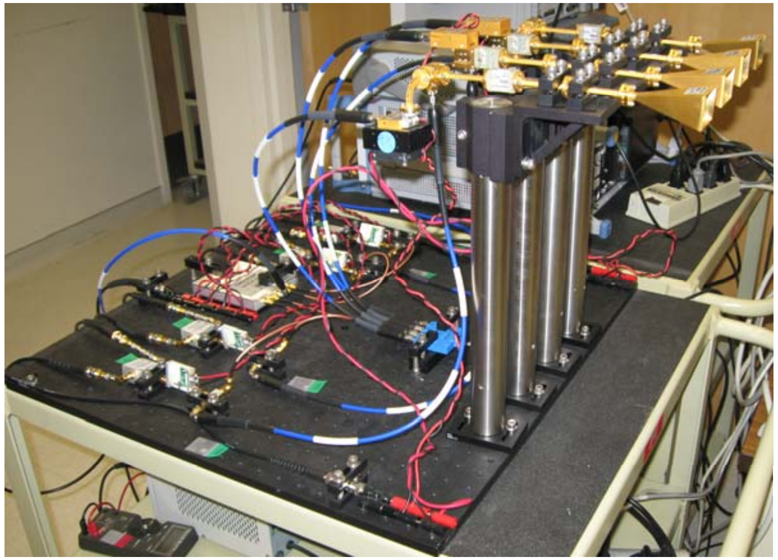
Demonstrating LoS MIMO: 4x4 Prototype



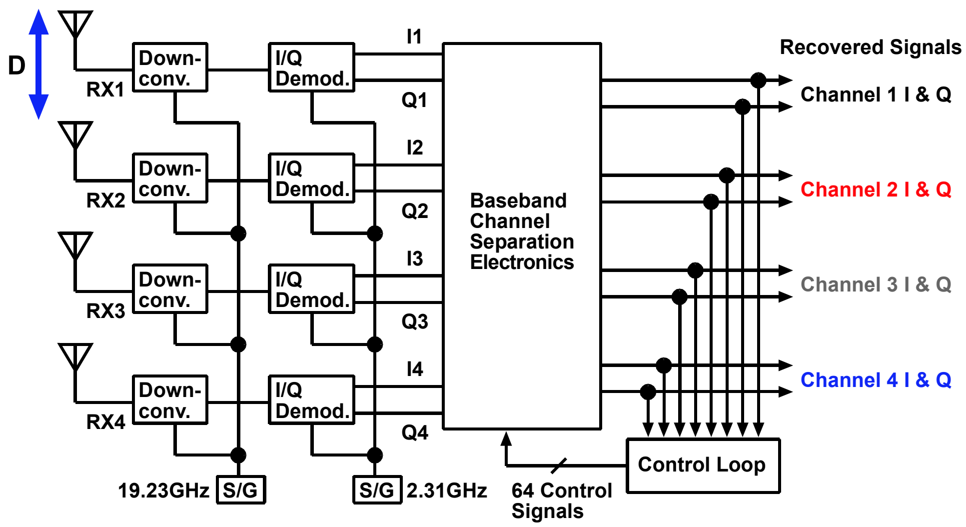
- Embedded pilot tones used to identify channels at the receiver
- Decouple receiver functions: channel separation and data demodulation
- Channel separation network implemented with baseband analog circuits

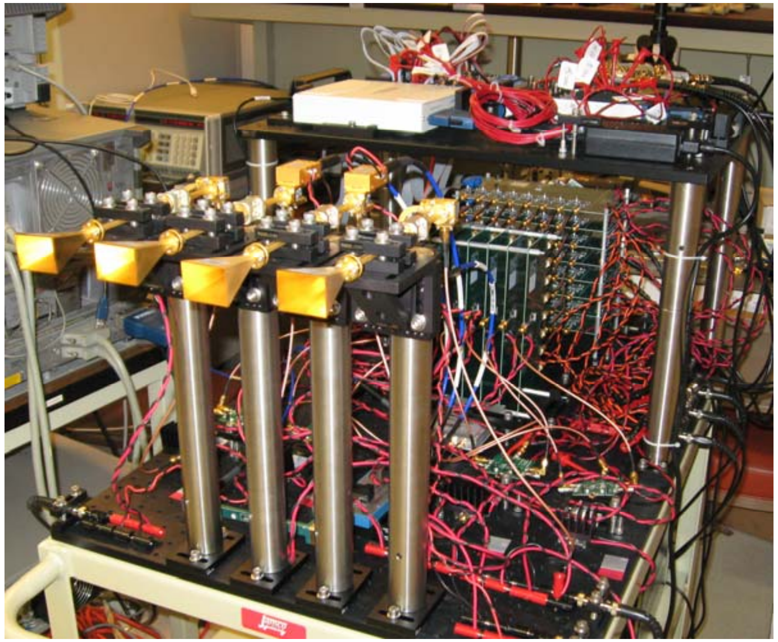
Transmitter Hardware Prototype



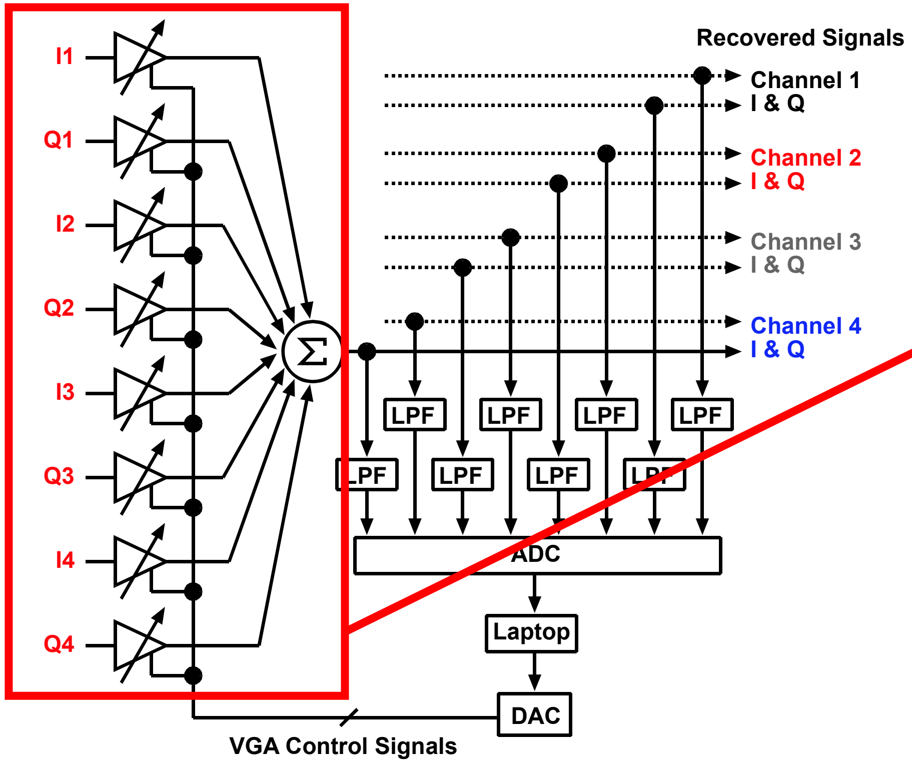


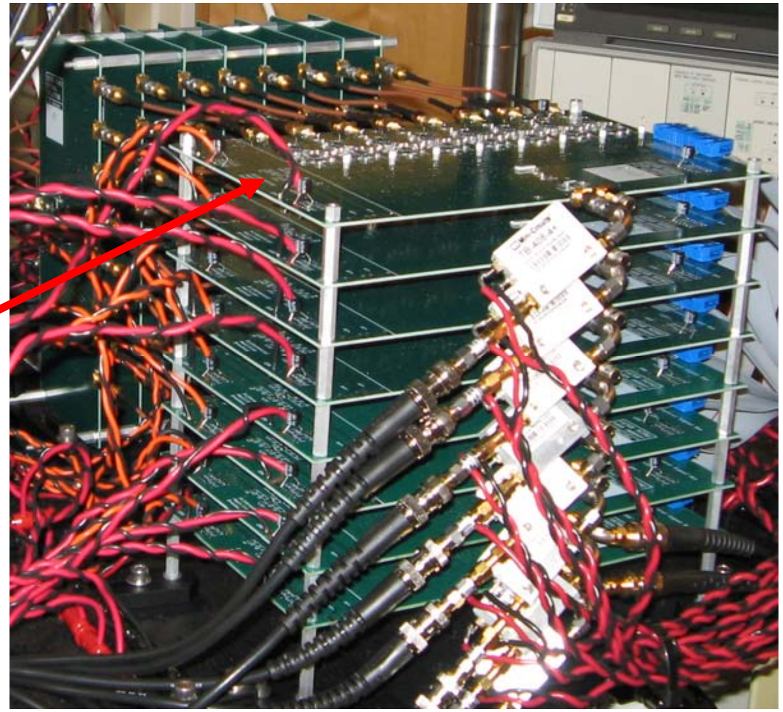
Receiver Hardware Prototype





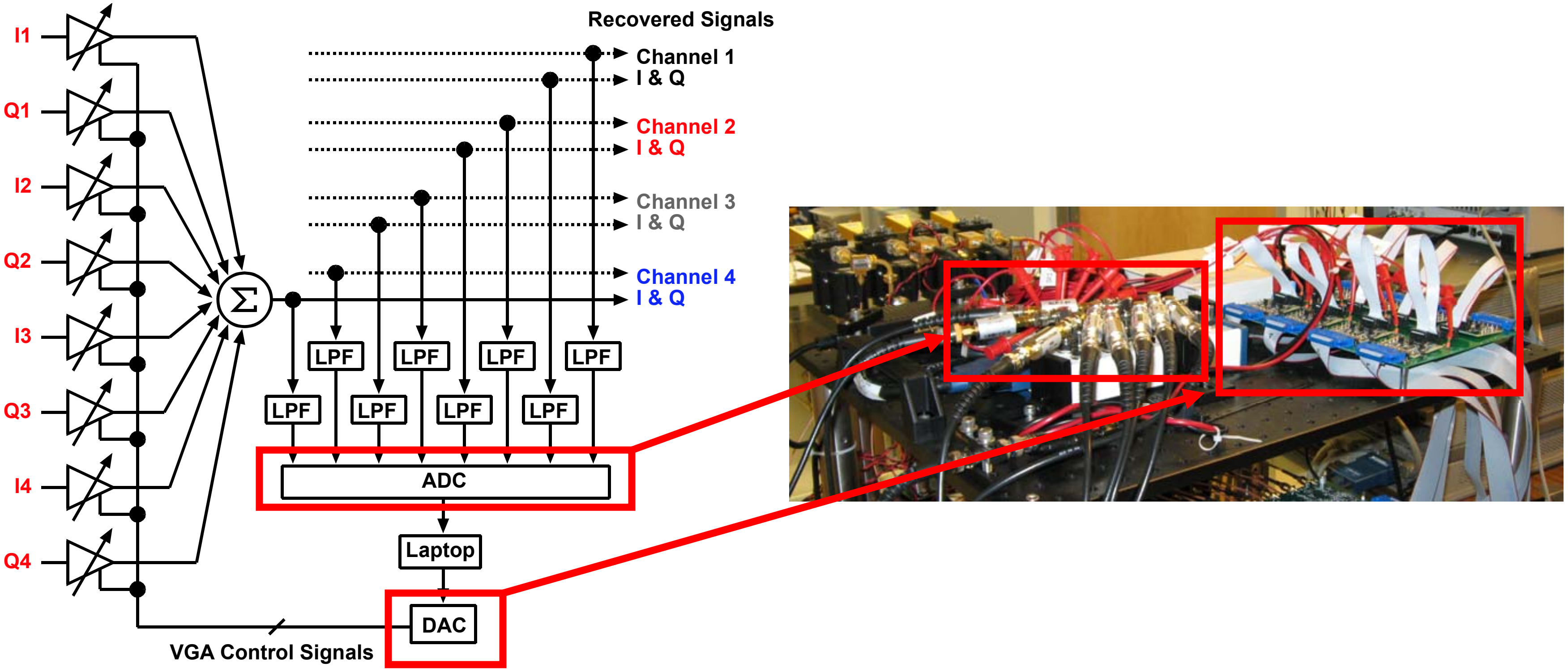
Channel Separation Prototype





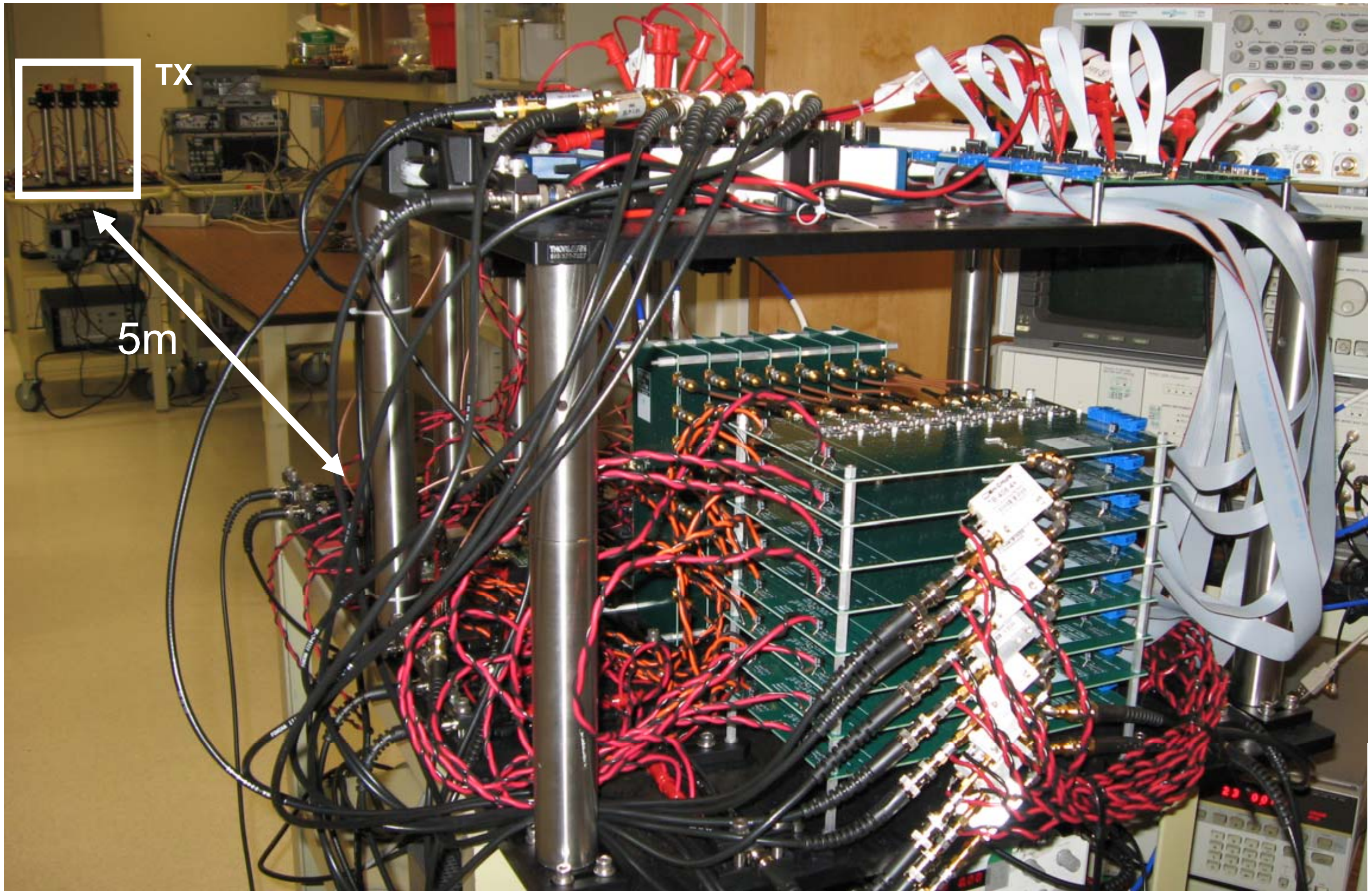
- VGAs are implemented as 4 quadrant analog multipliers using transistor array ICs
- Summation circuit consists of a resistor power combiner

Channel Identification and Control Loop



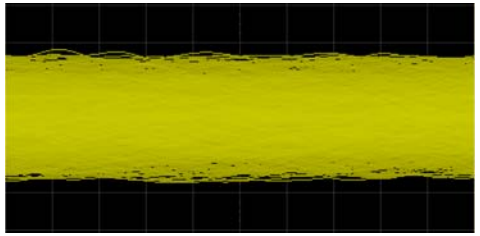
- Unique low frequency (25-40kHz) pilot tones added to each transmitter signal
- Control loop sets VGA control signals by maximizing desired pilot tone power
- Pilot tone signals from interfering transmitters are minimized

Indoor Radio Link Experiment



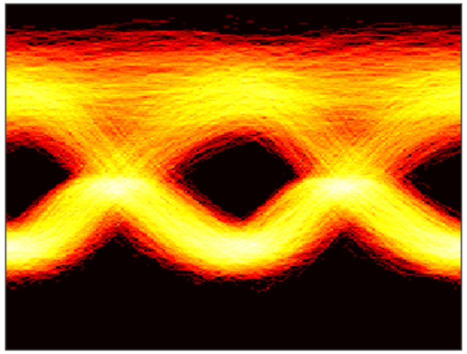
Time Domain Results

Before Channel Separation

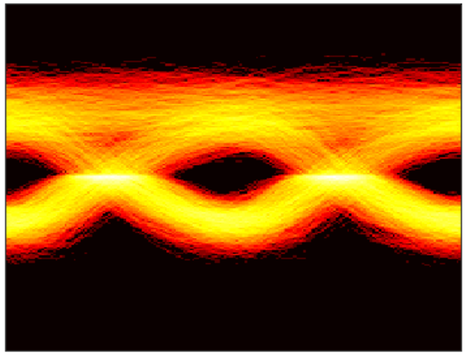


After Channel Separation

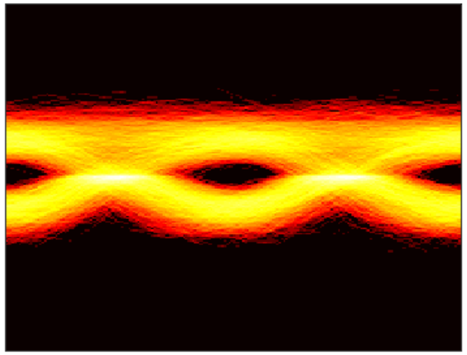
Channel 1



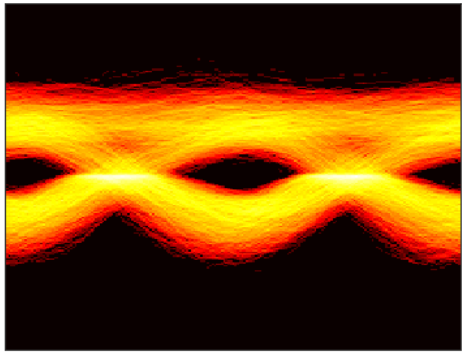
Channel 2



Channel 3

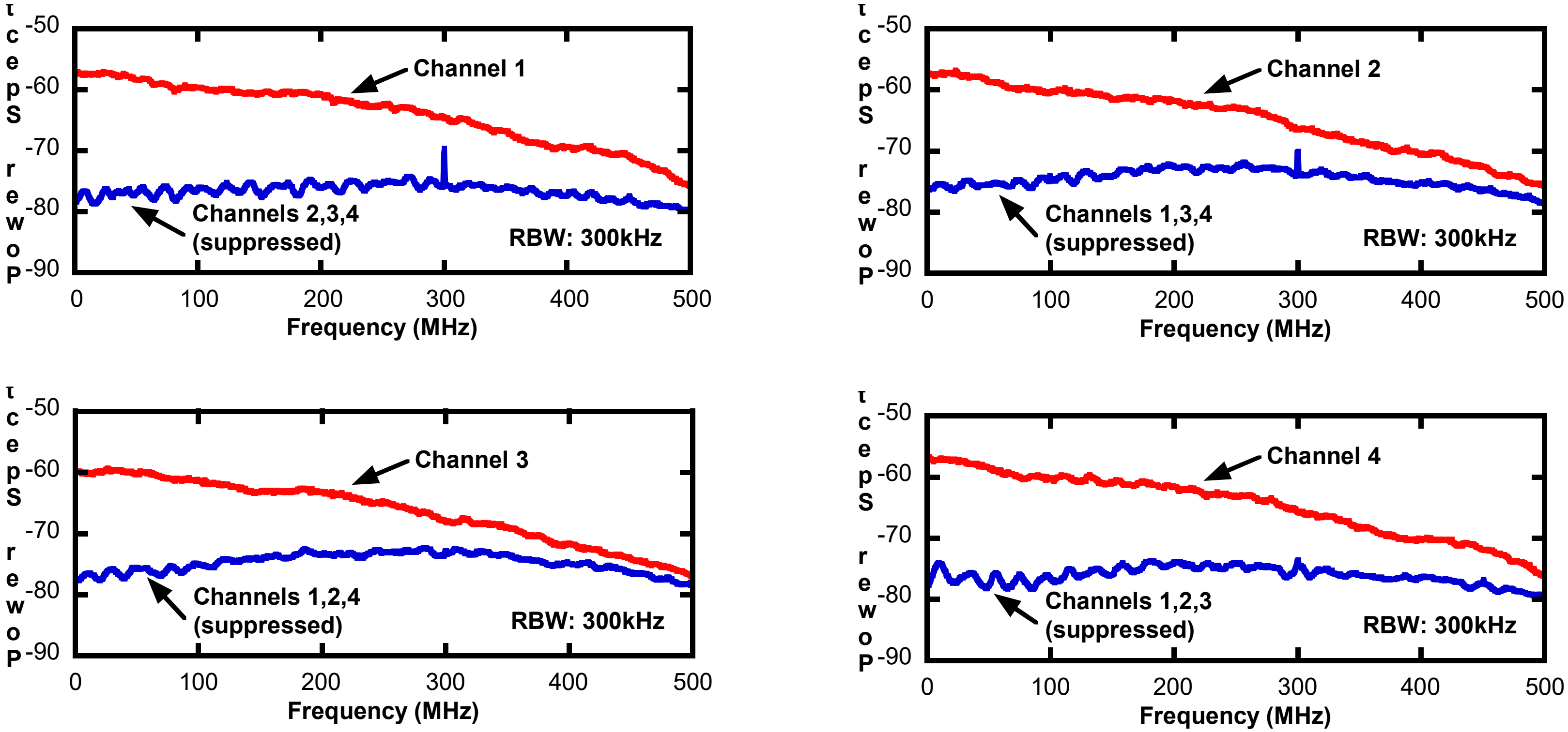


Channel 4



Differential data demodulation performed offline in software

Frequency Domain Results



Recovered Channel	BER	Signal-to-Interference Ratio (dB)
1	$<10^{-6}$	15
2	$<10^{-6}$	12
3	1.2×10^{-5}	10
4	$<10^{-6}$	14

Summary (from ~5 years ago)

- Information-theoretic analysis points to array of subarrays architecture
- Performance varies with TX location, but high spectral efficiencies despite LoS blockage
- Successful brassboarding verifies LOS MIMO geometry
  - Potentially applies to both indoor and outdoor systems
- Many interesting design challenges different from conventional MIMO
  - MIMO Processing for multiGigabit systems: natural hierarchy
  - Robustness to range mismatch
  - MultiGigabit baseband signal processing
  - Diversity/multiplexing tradeoffs for a new class of channels

Distributed LoS MIMO

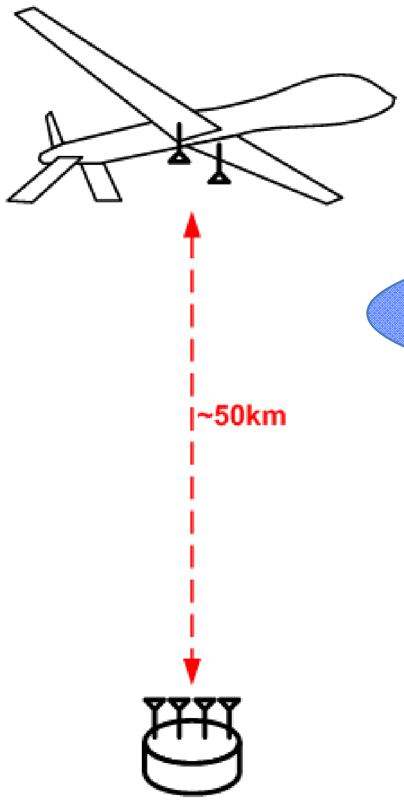
Motivated by DARPA 100G program

Sidestepping the Rayleigh limit for LoS spatial multiplexing: a distributed architecture for long-range wireless fiber

Andrew Irish, François Quitin, **Upamanyu Madhow** and Mark Rodwell
University of California, Santa Barbara

ICB INSTITUTE FOR COLLABORATIVE BIOTECHNOLOGIES

How to get 100 Gbps wireless over 50 km?



Must throw everything we know at it

Bandwidth ➔ mm wave band or higher

Power ➔ not THz or optics

Directivity ➔ mm wave band or higher

Spatial multiplexing ➔ geometry must support full rank MIMO matrix

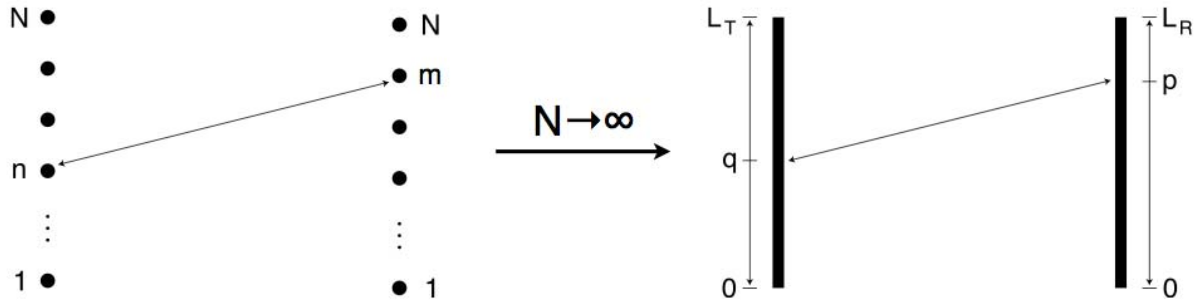
Focus today

Polarimetric multiplexing ➔ no conceptual hurdles, modulo hardware/signal processing design

LoS spatial muxing: a primer

Torklidson, Madhow, Rodwell, IEEE Trans. Wireless Comm., Dec 2011

The continuous Shannon limit



$$\underbrace{y_m = \frac{1}{N}\sum_{n=1}^{N} \exp\left(-i\frac{\pi}{\lambda R}\left(nd_T - md_R\right)^2\right) x_n}_{h_{m,n}} \qquad \underbrace{y(p) = \frac{1}{L_T}\int_{-L_T/2}^{L_T/2} \exp\left(-i\frac{\pi}{\lambda R}(q-p)^2\right) x(q)\mathrm{d}q}_{h(q,p)}$$

$$\lambda_n\phi_n(t) = \int_{-T/2}^{T/2} \frac{\sin 2\pi W(t-s)}{\pi(t-s)}\phi_n(s)ds$$

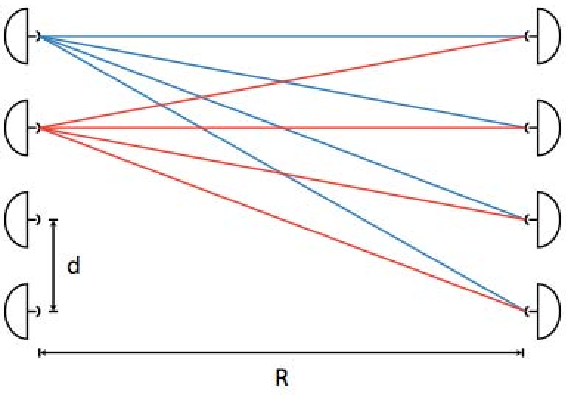
Spatial prolate spheroidal waveforms

$$\frac{L_T^2}{\lambda R}|g_n|^2\alpha_n(q) = \int_{-L_T/2}^{L_T/2} \frac{\sin 2\pi \frac{L_r}{2\lambda R}(q-q')}{\pi(q-q')}\alpha_n(q')dq'$$

$$|g_n|^2 \approx 0 \text{ for } n > \frac{L_T L_R}{\lambda R}(1+\epsilon)$$

Spatial “bandwidth” determined by form factor

The discrete Rayleigh limit



$$\mathbf{h}_1 = (1, e^{j\phi}, e^{j2^2\phi}, \ldots, e^{j(N-1)^2\phi})^T$$

$$\mathbf{h}_2 = (e^{j\phi}, 1, e^{j\phi}, \ldots, e^{j(N-2)^2\phi})^T$$

$$|\langle \mathbf{h}_1, \mathbf{h}_2 \rangle| = \left| \frac{\sin(N\phi)}{\sin\phi} \right|$$

Vectors are orthogonal when $N\phi = N\dfrac{\pi d^2}{\lambda R} = \pi$

$$\boxed{d = \sqrt{\frac{\lambda R}{N}}}$$

Example
R=10 m, λ=5 mm, N=4
d =11 cm

Perfect for short-range indoor 60 GHz comms

Generalizes to different spacing at TX and RX $d_T d_R = \dfrac{\lambda R}{N}$

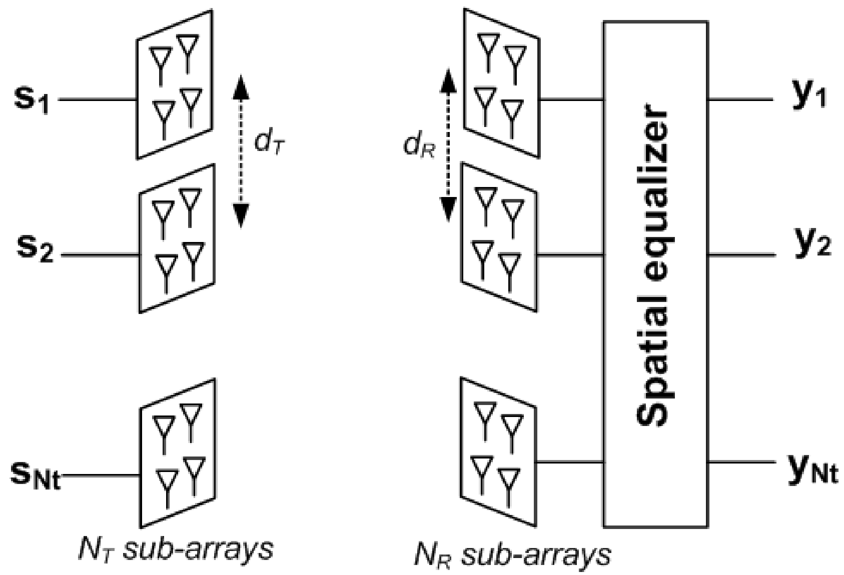
Achieves the spatial degrees of freedom promised by continuous Shannon limit

Array of subarrays architecture

Discrete array suffices to attain Shannon limit on degrees of freedom

Each element in the array can be a subarray providing beamforming gain

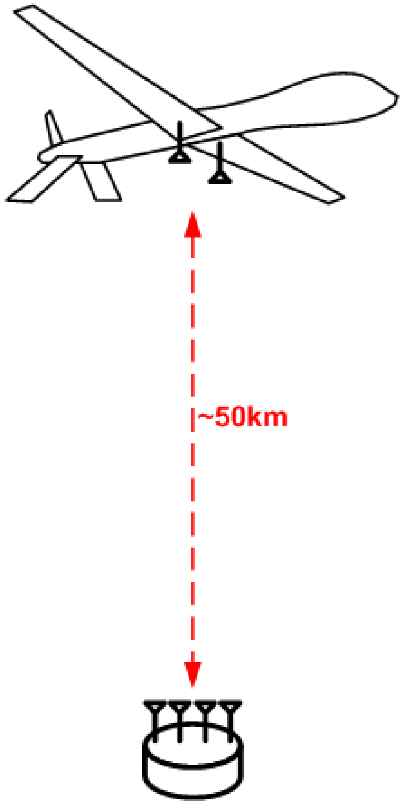
➔ **Array of subarrays architecture providing spatial multiplexing + beamforming**



$$d_T d_R = \frac{\lambda R}{N}$$

Back to 100 Gbps long-range link

We have a problem



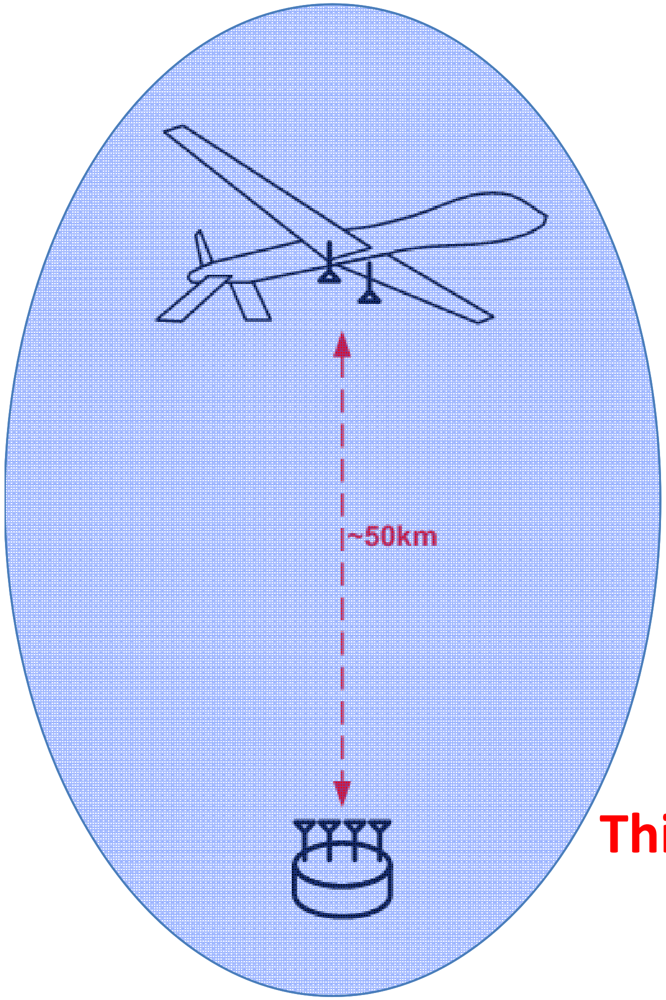
Example

75 GHz carrier frequency, 50 km range

Two-fold spatial multiplexing

$$d_T d_R = 100\ m^2$$

A dealbreaker?



Example

75 GHz carrier frequency, 50 km range
Two-fold spatial multiplexing

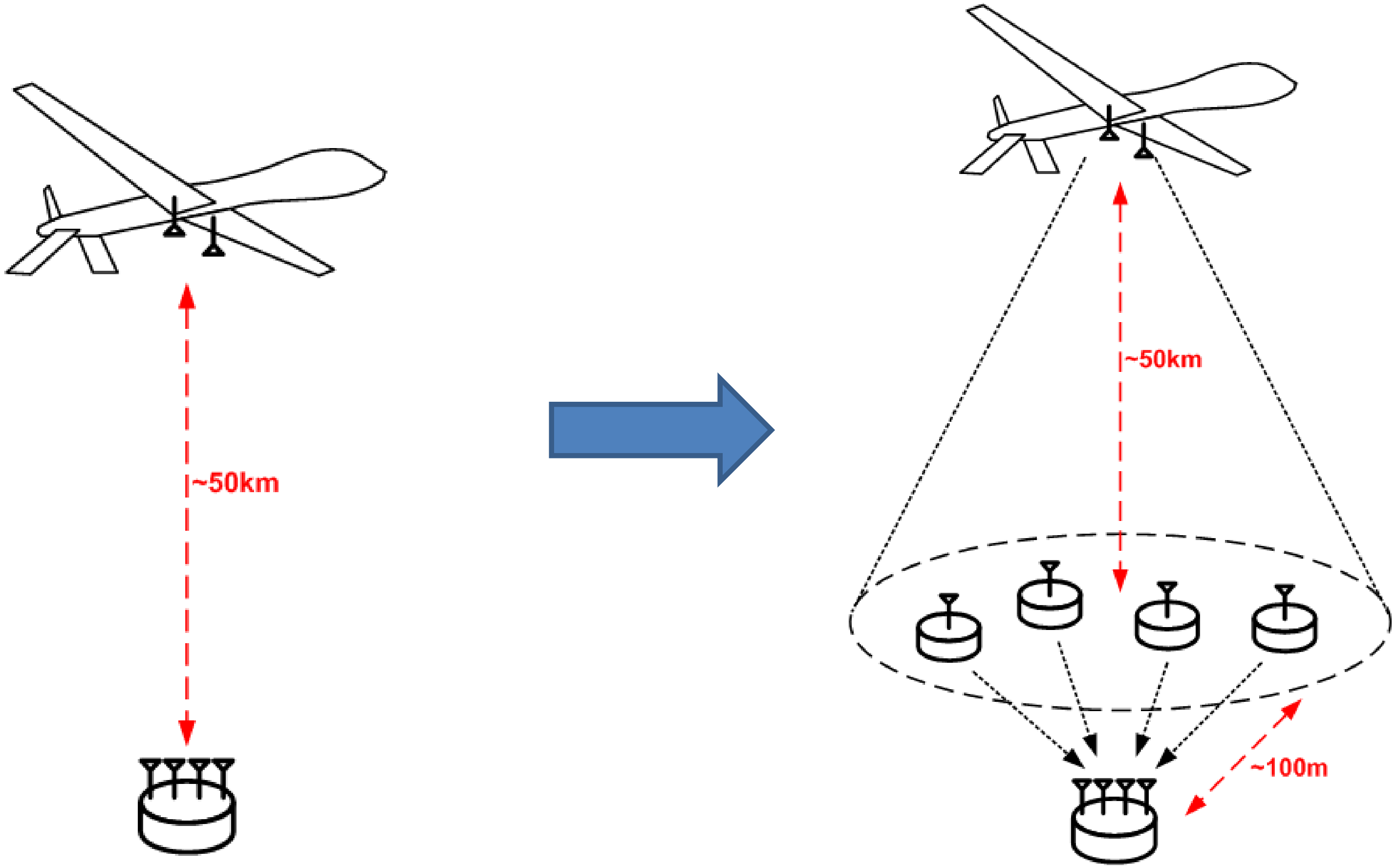
$$d_T d_R = 100\ m^2$$

Subarrays 1 m apart on aircraft
➔ Subarrays 100 m apart on the ground!

This picture does not work!

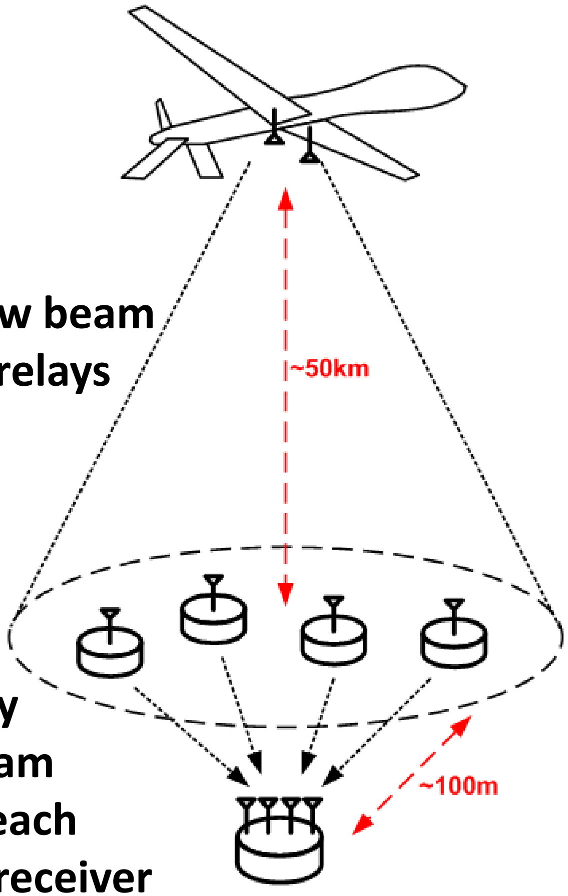
Distributed MIMO to the rescue

Synthesize full rank channel by spreading the receiver out



Anatomy of full rank DMIMO

Very narrow beam covers all relays



Moderately narrow beam between each relay and receiver

$\mathbf{H_1}$ full-rank thanks to spatial spread of relays

$\mathbf{H_2}$ diagonal => full-rank

Composite channel full-rank

Many design questions…

Level 1

How many relays?

How spread out?

Statistical rather than deterministic characterization

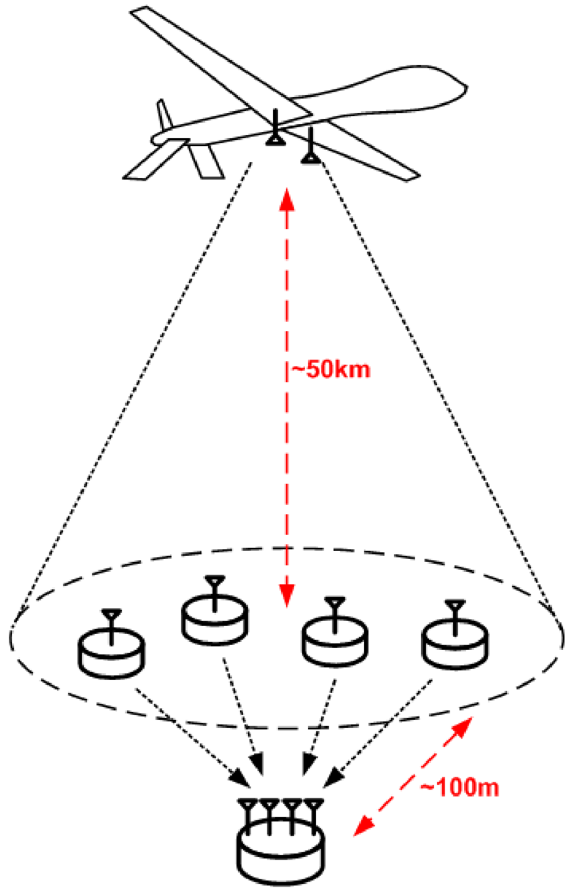
Level 2

How to attain the link budget?

How to design relaying hardware?

Level 3

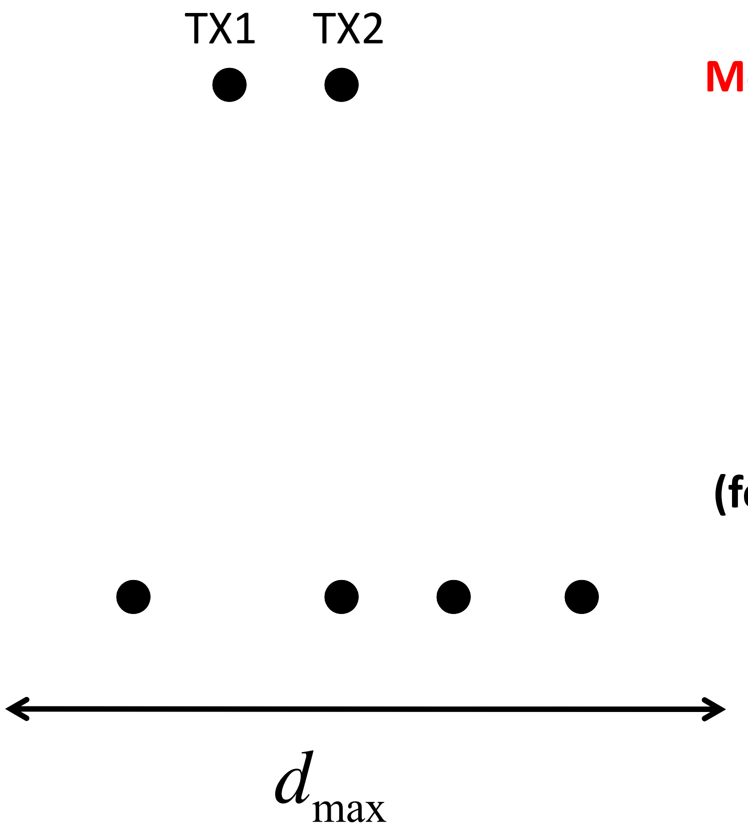
Signal processing architectures and algorithms



Level 1

Getting enough spatial degrees of freedom

Modeling relay geometry



Model for response of transmitters at relays

$$\mathbf{h}_1 = (e^{j\theta_{11}}, e^{j\theta_{12}}, e^{j\theta_{13}}, e^{j\theta_{14}})^T$$

$$\mathbf{h}_2 = (e^{j\theta_{21}}, e^{j\theta_{22}}, e^{j\theta_{23}}, e^{j\theta_{24}})^T$$

$$\theta_{ij}\ i.i.d.,\ Unif\,[0,2\pi]$$

(for "large enough" dispersal area)

Randomly dispersed relays

Zero-forcing performance

SNR degradation relative to spatial matched filter

$$1-|\rho|^2$$

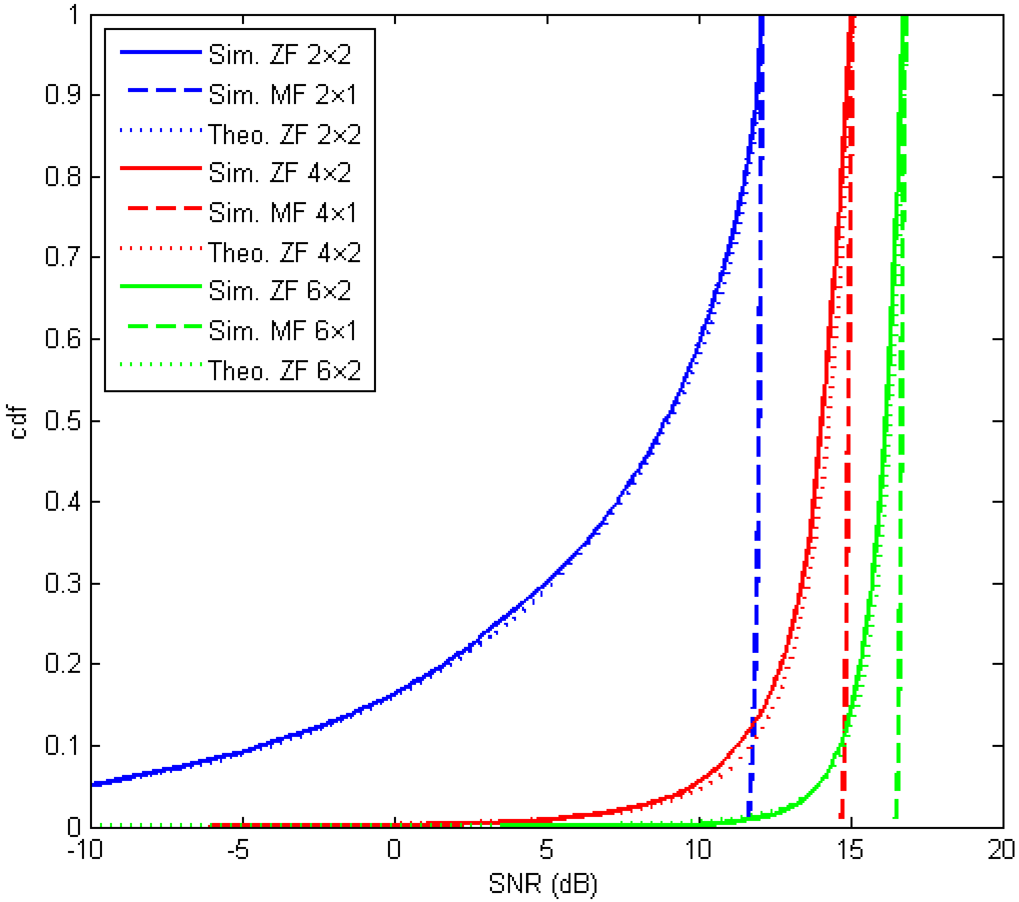
$$\rho = \frac{\langle \mathbf{h}_1, \mathbf{h}_2 \rangle}{\|\mathbf{h}_1\| \|\mathbf{h}_2\|}$$

Normalized cross-correlation

$$= \frac{1}{N_r} \sum_{k=1}^{N_r} e^{j(\theta_{1k} - \theta_{2k})}$$

➔apply CLT for moderately large number (4,6) of relays (and exact analysis for 2 relays)
➔for "large" number of relays, $|\rho|^2$ exponential with mean $1/N_r$

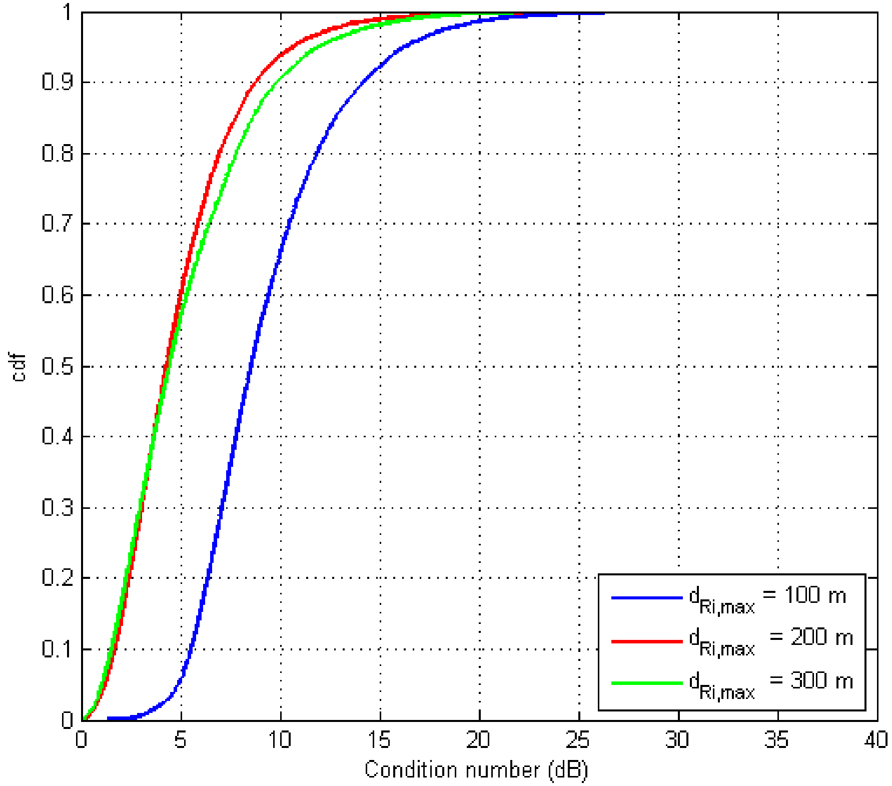
2 streams, 2-6 relays spread over 200 m



Noise enhancement from ZF demuxing not too bad for 4-6 relays
Performance with 2 relays is too variable
Analytical approximation closely matches simulations

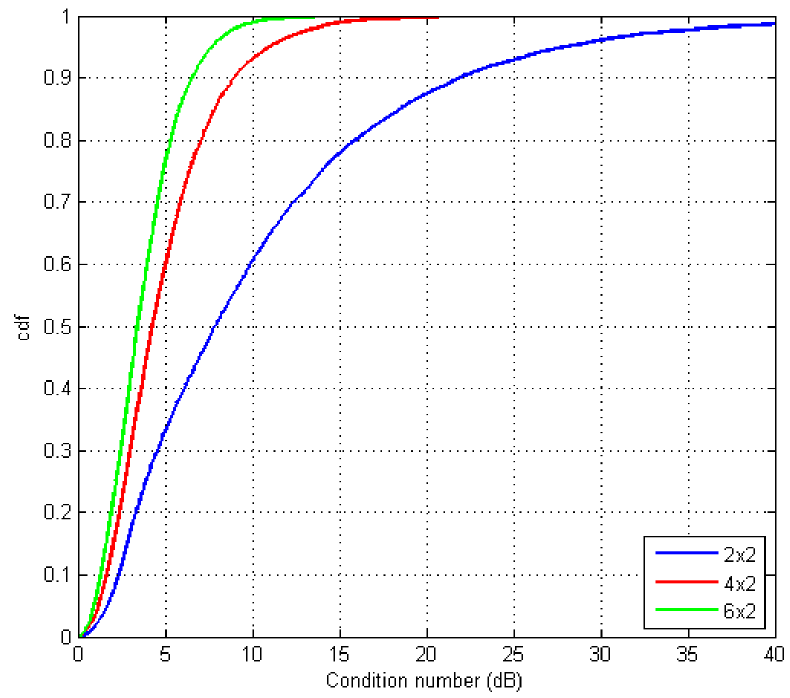
The effect of relay spread on channel rank

Spreading the relays over twice the Rayleigh limit achieves best rank



The effect of relay number on channel rank

Increasing the number of relays/Rx improves the matrix rank



Level 2

Getting enough power

50 km is difficult to achieve

Even with optimistic propagation assumptions…

$$P_r = P_t + G_t + G_r - PL - AL - L$$

52 dBi 164 dB 25 dB 10 dB

For 8.9 dB receive SNR (sufficient for high-rate coded QPSK):

TX power of 34 dBm even with 52 dBi beamformers on each side

How are we going to get 1-2 Watts of transmit power in E-band?

How are we going to get 52 dB of beamforming gain?

But not impossible...

$$P_r = P_t + \underbrace{G_t + G_r}_{52\text{ dBi}} - \underbrace{PL}_{164\text{ dB}} - \underbrace{AL}_{25\text{ dB}} - \underbrace{L}_{10\text{ dB}}$$

Where are we going to get 1-2 Watts of transmit power in E-band?

How are we going to get 52 dB of beamforming gain?

Possible approaches:

1) Mechanically steerable dish antenna and III-V power amplifier
2) 32x32 electronically steerable array with 22 dBi per-element gain and 12 dBm per-element power (**achievable in silicon**)

Much cooler!

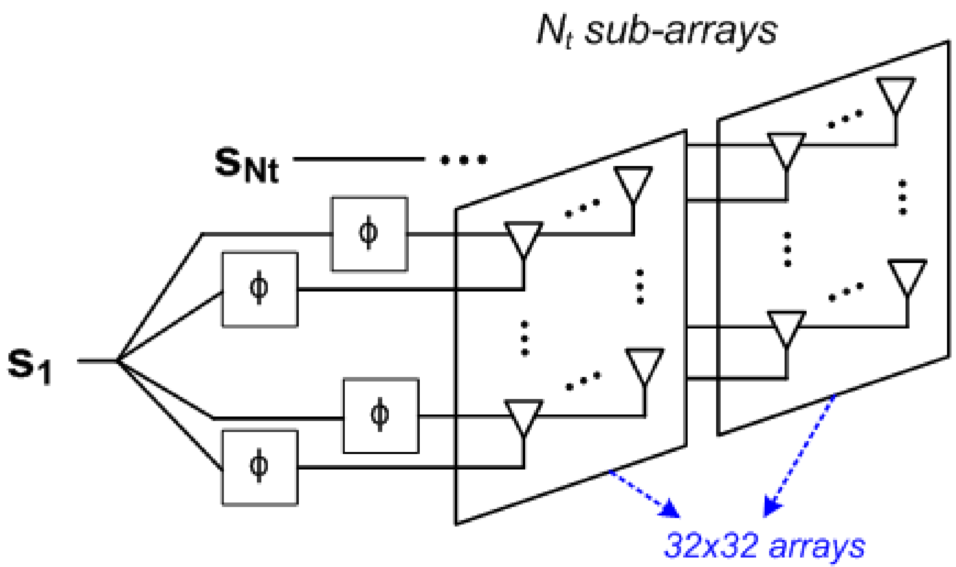
Level 3

We're finally ready to talk about transceiver design…

Transmitter

Array of 1000-element subarrays

- Subarrays provide beamforming gain
- N_t data streams, one on each subarray

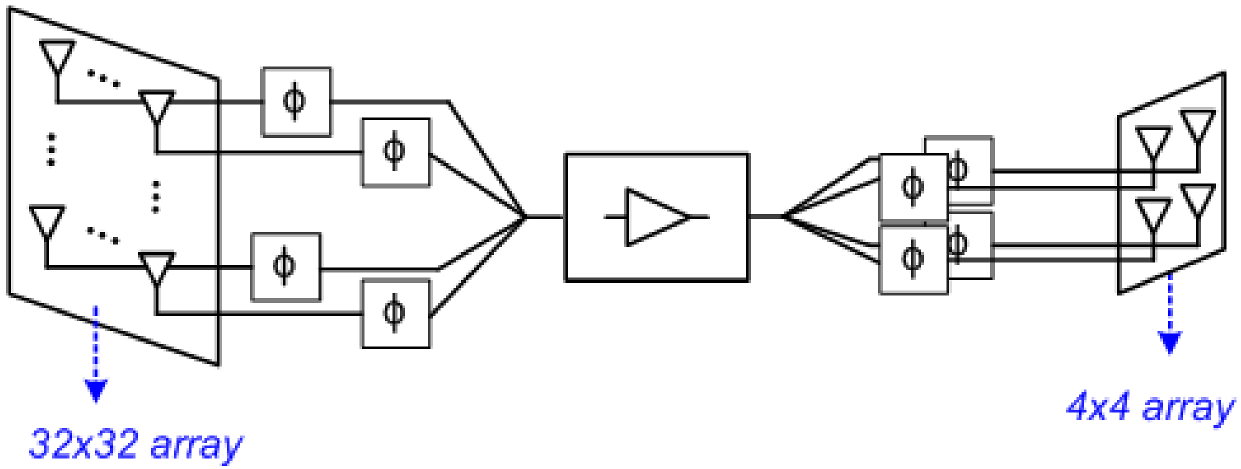


0.5 degree beam at 50 km covers 400 m ➔ can cover all relays with one beam
We know how to do RF beamforming with 1000-element arrays (ITA 2012)

Relay

1000-element sky-facing subarray: receive beamforming over long link

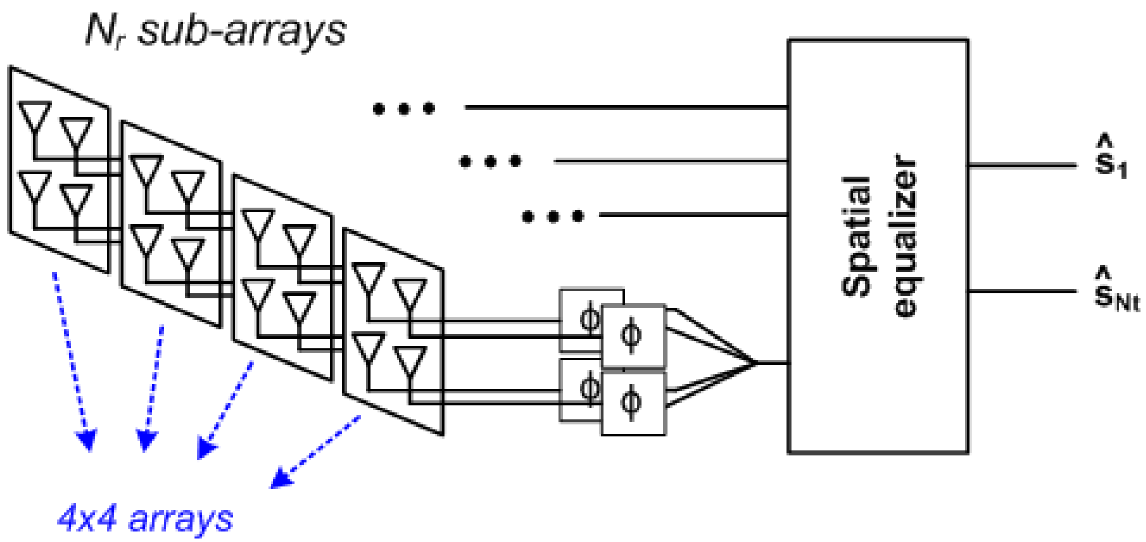
Smaller subarray: transmit beamforming over short link to receiver



Receiver

N_r subarrays, one for each relay

Receive beamforming from subarray to relay creates diagonal relay-RX channel



Wrapping up

The good news

- Long-range ``wireless fiber'' is more attainable that we think
  - 5 GHz x dual polarization x 4-fold spatial multiplexing x 2.5 bps/Hz = 100 Gbps
- Distributed architecture can get around form factor constraints to provide spatial muxing

Many challenges remain

- Building very large subarrays
  - 1000-element arrays to get desired directivity and reasonable power per element
- Relay design
  - FDD first, then full duplex?
- High spectral efficiency at high bandwidths ➔ high dynamic range required
  - How best to handle the ADC bottleneck?
- Adaptive signal processing at multiple layers

Achieving multiple degrees of freedom in long-range mm-wave MIMO channels using randomly distributed relays

Andrew T. Irish, Francois Quitin, Upamanyu Madhow, and Mark Rodwell

UC Santa Barbara

Asilomar SSC. November 5, 2013

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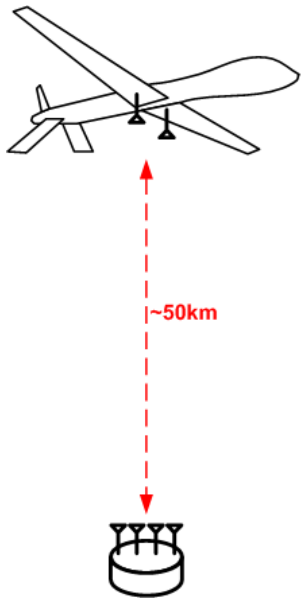
How to do long-range “wireless fiber”?



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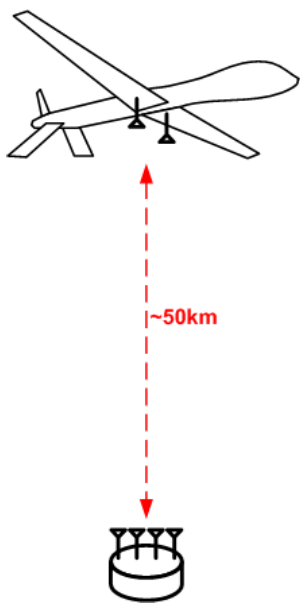

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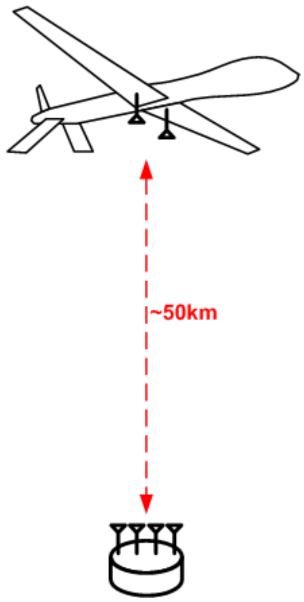

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Millimeter-wave carrier frequencies

- 5 GHz + bandwidth
- Directivity

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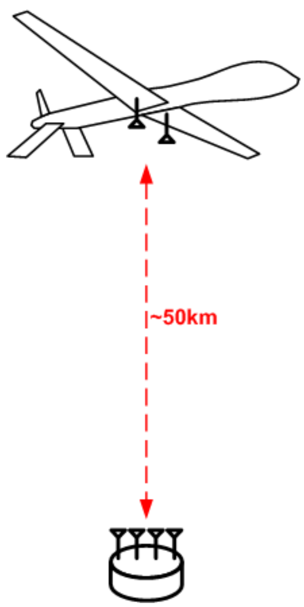

- 10 – 50 km range
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Millimeter-wave carrier frequencies

- 5 GHz + bandwidth
- Directivity
- But...
  1. Can we overcome the huge pathloss?
  2. Can we use spatial multiplexing (MIMO)?

Is 50 km too far?

[1] A. Irish et al. “Sidestepping the Rayleigh limit for LoS spatial multiplexing: A distributed architecture for long-range wireless fiber”. In: *2013 Information Theory and Application Workshop (ITA)*. 2013.

Is 50 km too far?

- Examined link budget for $\sim$ 10 Gbps SISO link at $f_c = 73.5$ GHz with $BW = 5$ GHz[1]

$$P_R = \underbrace{P_T}_{34\text{ dBm}} + \underbrace{G_T + G_R}_{52\text{ dBi (each)}} - \underbrace{PL}_{164\text{ dB}} - \underbrace{AL}_{25\text{ dB}} - \underbrace{L}_{10\text{ dB}}$$

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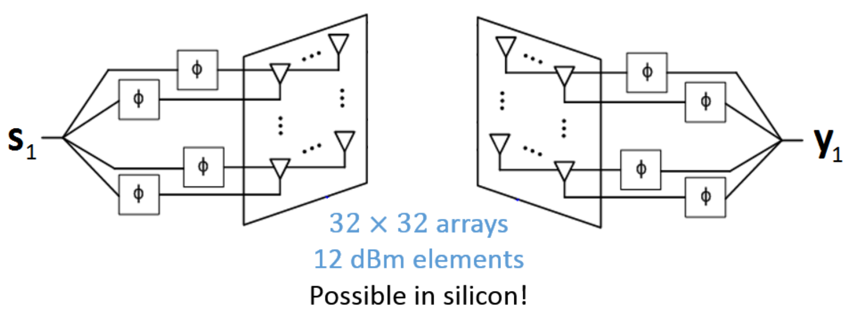

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$$\boxed{\text{As system designers, how do we deal with this?}}$$

Some background on LOS MIMO

[2] E. Torkildson, U. Madhow, and M. Rodwell. "Indoor Millimeter Wave MIMO: Feasibility and Performance". In: *Wireless Communications, IEEE Transactions on* 10.12 (2011), pp. 4150 –4160. ISSN: 1536-1276. DOI: 10.1109/TWC.2011.092911.101843.

Some background on LOS MIMO

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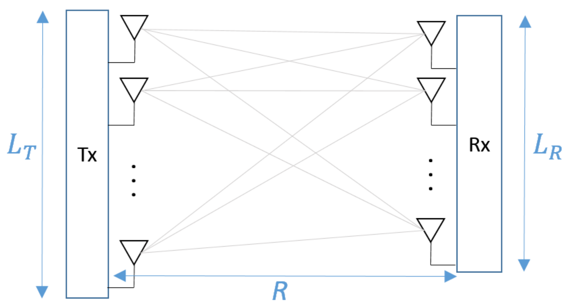

Linear arrays: $DOF \approx \dfrac{L_T L_R}{R\lambda} + 1$

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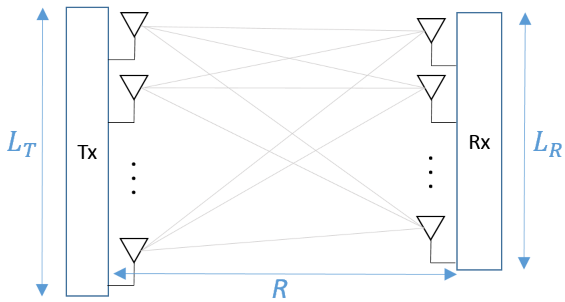

$$\underline{\text{Linear arrays:}}\ \ DOF \approx \frac{L_T L_R}{R\lambda} + 1$$

- Easily generalizes to square arrays of side lengths L_T, L_R.

$$DOF \approx \left(\frac{L_T L_R}{R\lambda}\right)^2 + 1$$

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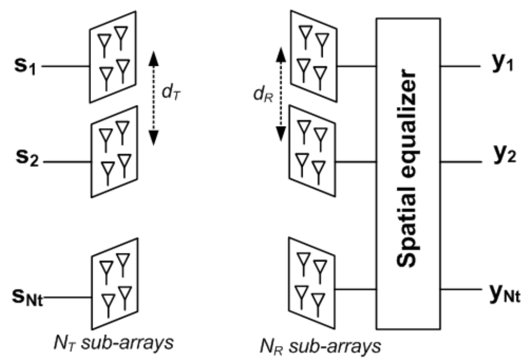
- array of sub-arrays architecture:
  - sub-arrays provide beamforming gain
  - arrays of sub-arrays provide spatial muxing



$$L_T = (N_T - 1)d_T, \quad L_R = (N_R - 1)d_R$$

Now back to our scenario

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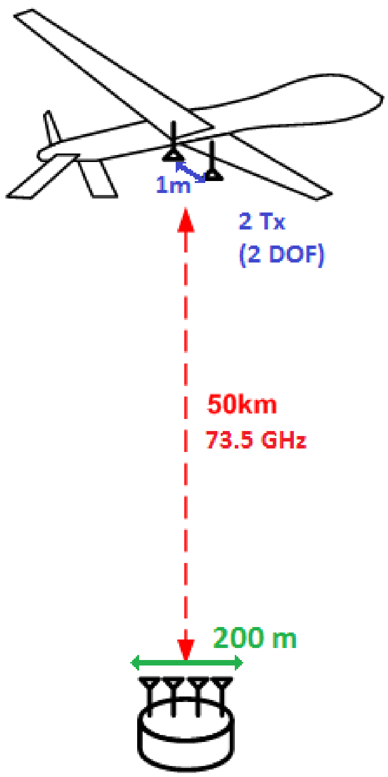
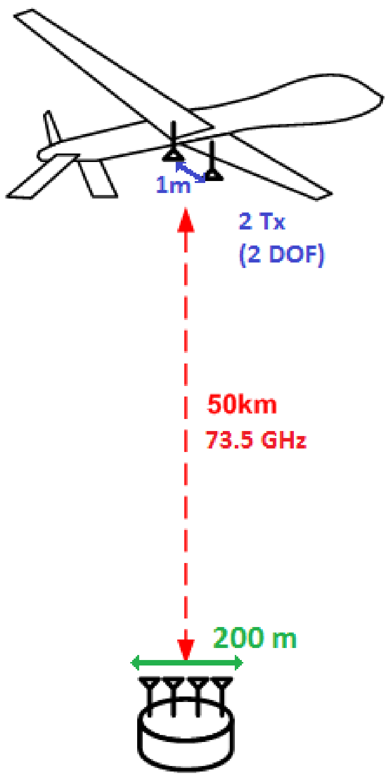
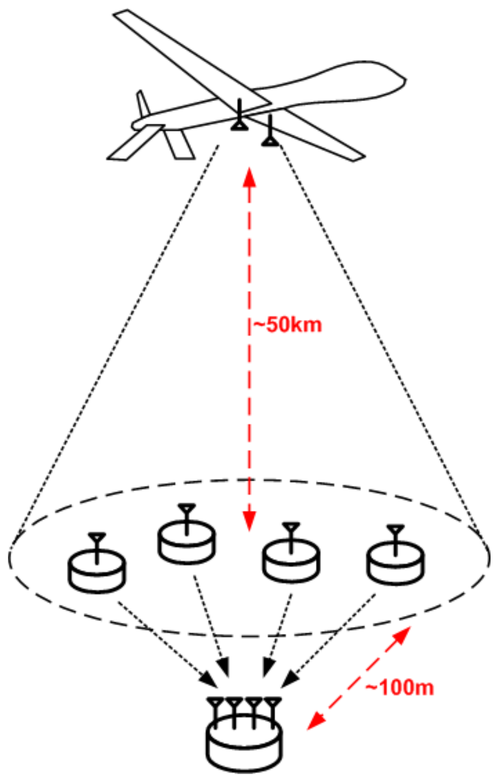


Problem: Rayleigh criterion
$\Rightarrow$ receive array is 200 m long

Now back to our scenario

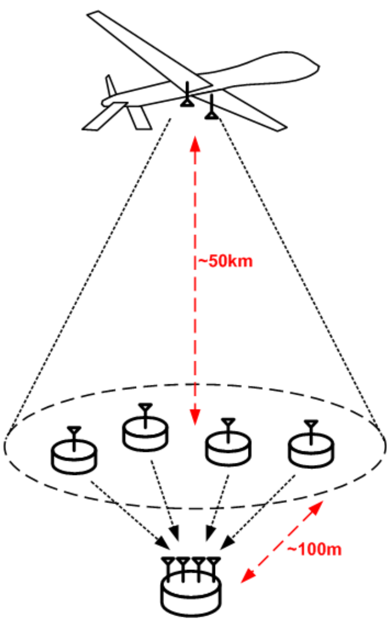




Problem: Rayleigh criterion
$\Rightarrow$ receive array is 200 m long

Solution: Introduce amplify and forward relays

How does the relay-based approach work?



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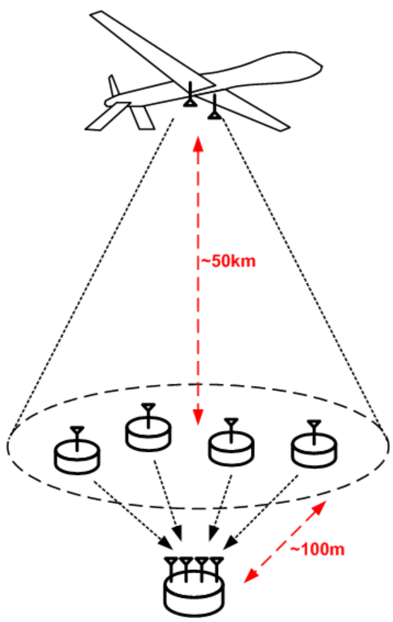

- AF relays $=$ **virtual** Rx array
  - **Random** placement $\rightarrow$ *Scalable*

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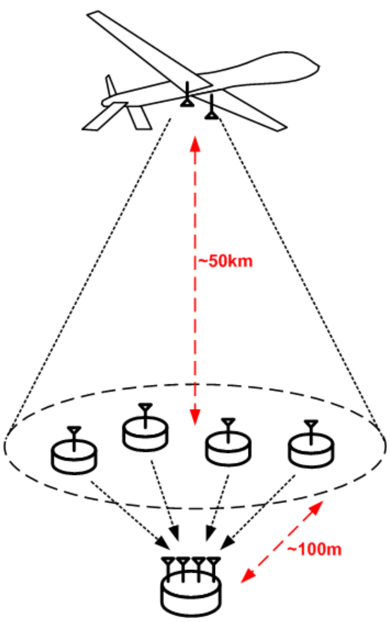

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  - ZF spatial equalization $\rightarrow$ low system complexity

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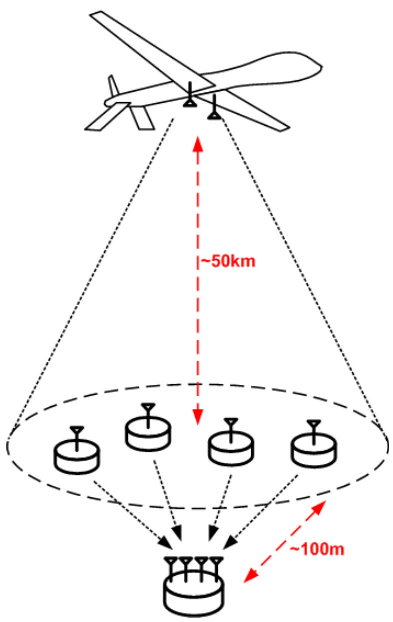

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- Abstraction: ignore short link!
  - System performance depends on long-link channel $\mathbf{H}$

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How large of an area do we need for the relays?

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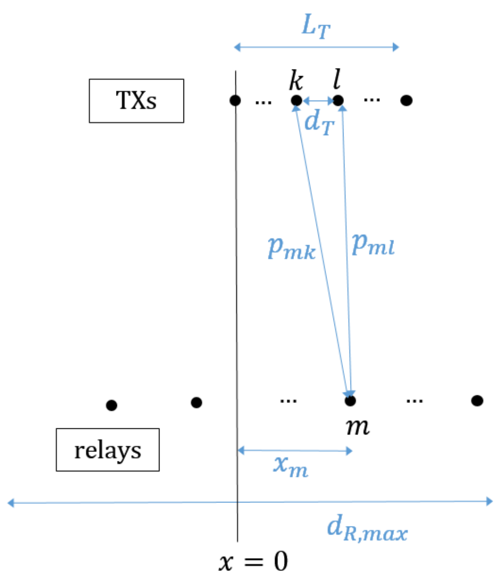

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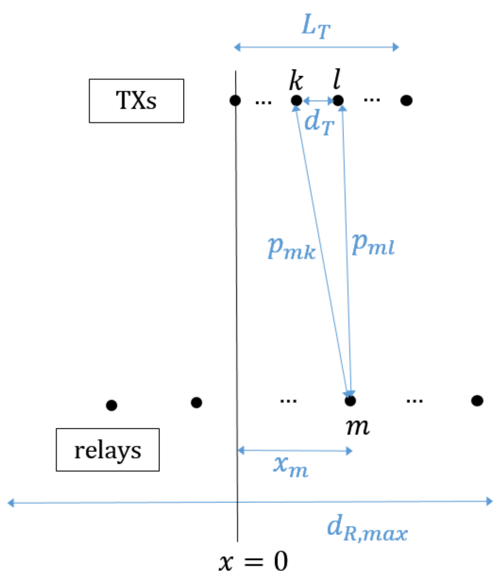

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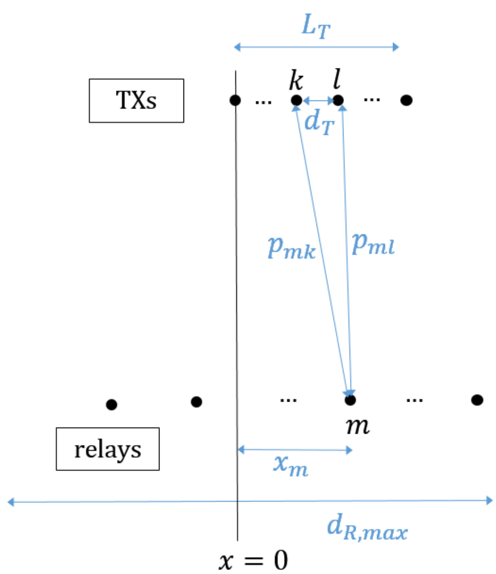

<u>Want zero inter-TX correlation</u>

Since $p_{mk} \gg x_m$ we get

$$\langle \mathbf{h}_k \mathbf{h}_l \rangle \propto \sum_{m=0}^{N_R-1} e^{j\frac{2\pi}{\lambda}(p_{mk}-p_{ml})}$$

$$\approx \sum_{m=0}^{N_R-1} e^{j\frac{2\pi}{\lambda R}(l-k)d_T x_m}$$

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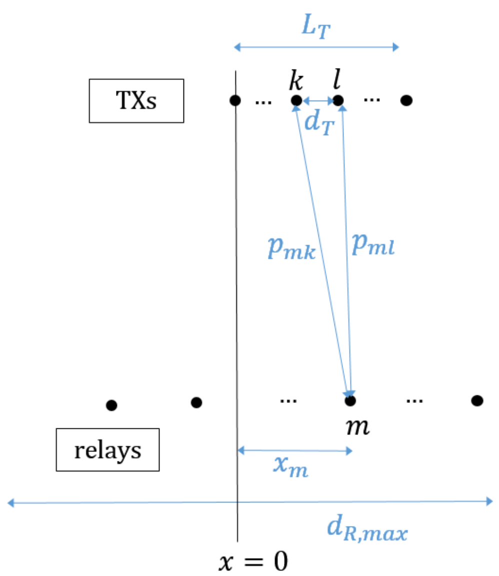

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Since $p_{mk} \gg x_m$ we get

$$\langle \mathbf{h}_k \mathbf{h}_l \rangle \propto \sum_{m=0}^{N_R-1} e^{j\frac{2\pi}{\lambda}(p_{mk}-p_{ml})}$$

$$\approx \sum_{m=0}^{N_R-1} e^{j\frac{2\pi}{\lambda R}(l-k)d_T x_m}$$

Uniform phases yield 0 corr, so:

$$x_m \sim U\left(-\frac{\lambda R}{2(l-k)d_T}, \frac{\lambda R}{2(l-k)d_T}\right)$$

How large of an area do we need for the relays? (cont.)

- Required region is largest when $l - k = 1$. Result (1D):

$$\boxed{x_m \sim U\left(-\frac{\lambda R}{2d_T}, \frac{\lambda R}{2d_T}\right) \Rightarrow \textit{on avg.}\ \text{spatial responses uncorrelated}}$$

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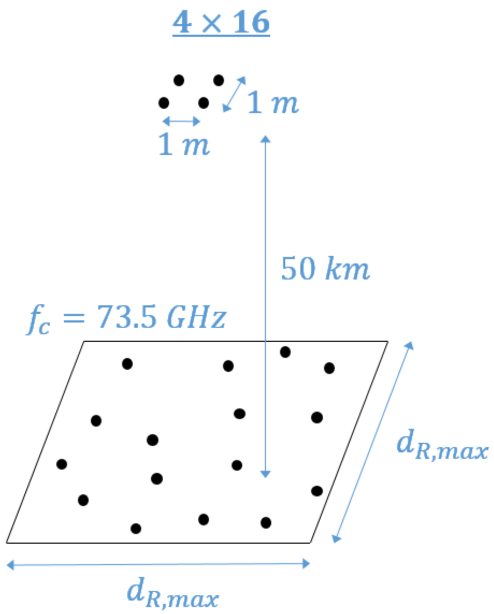
- **Generalization to 2D?**
  - Assume alignment of relay region with array

Theorem

Consider a regular square Tx array with minimum inter-element spacing of d_T. If relays are placed uniformly in a square of side $\frac{\lambda R}{d_T}$ then spatial responses are uncorrelated on average.

Verified using Monte Carlo simulations



$\Rightarrow$ "best" $d_{R,max} = \frac{\lambda R}{d_T} = 200$ m

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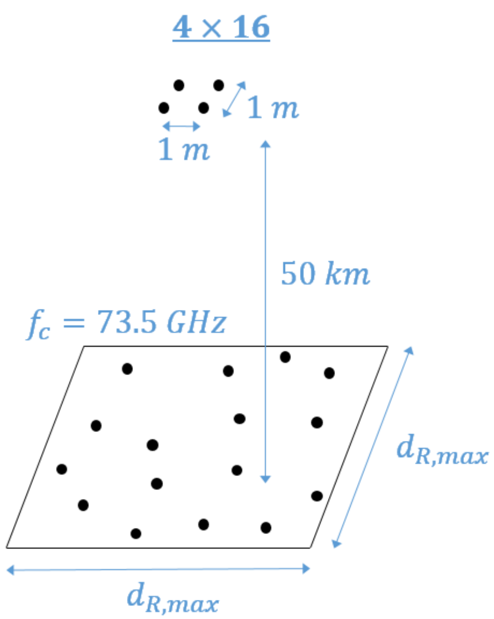

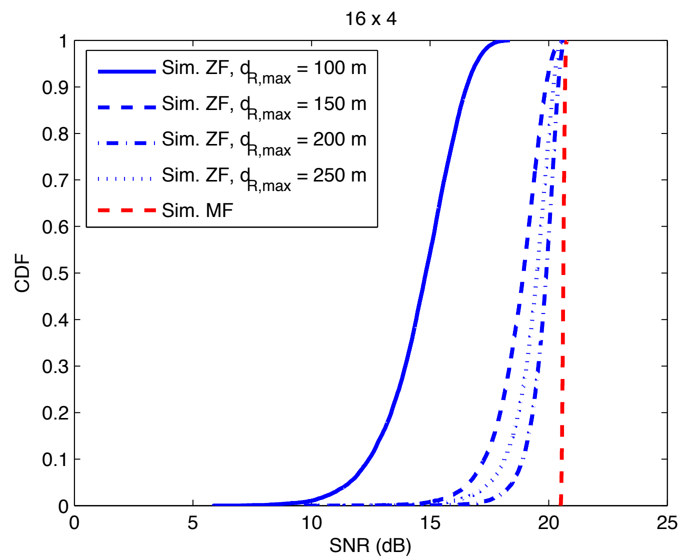

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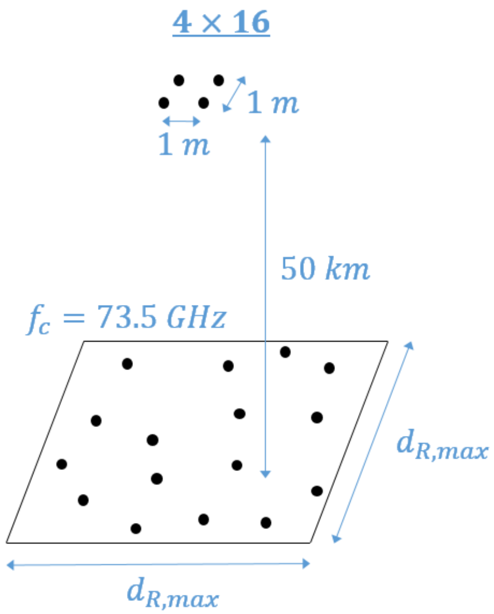

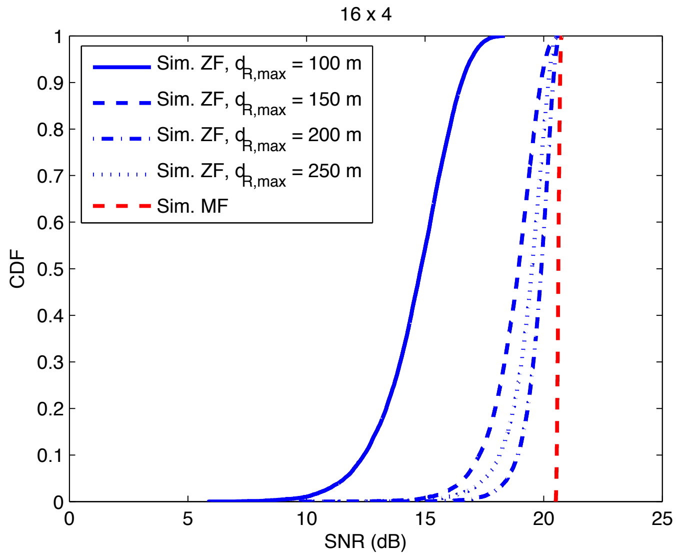

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- Validates predicted best $d_{R,max} = 200$ m

OK, how then to predict system performance?

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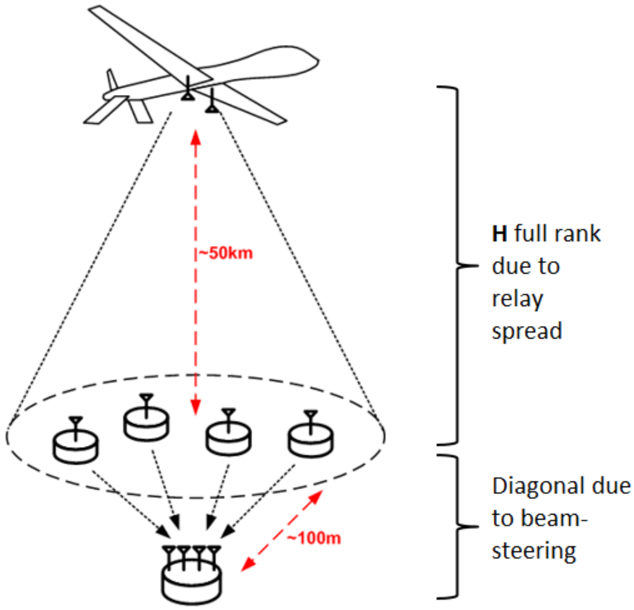

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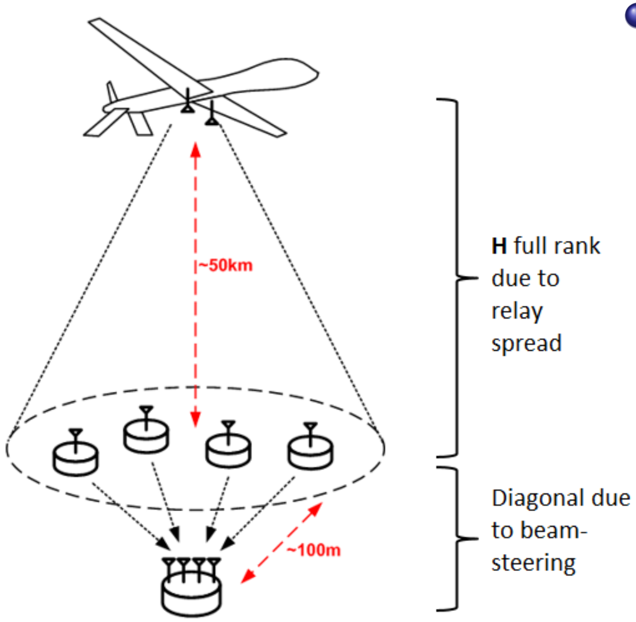

- Relay spread “large enough” $\Rightarrow$ **H** well-modeled as finite dimensional i.i.d. random phasor matrix

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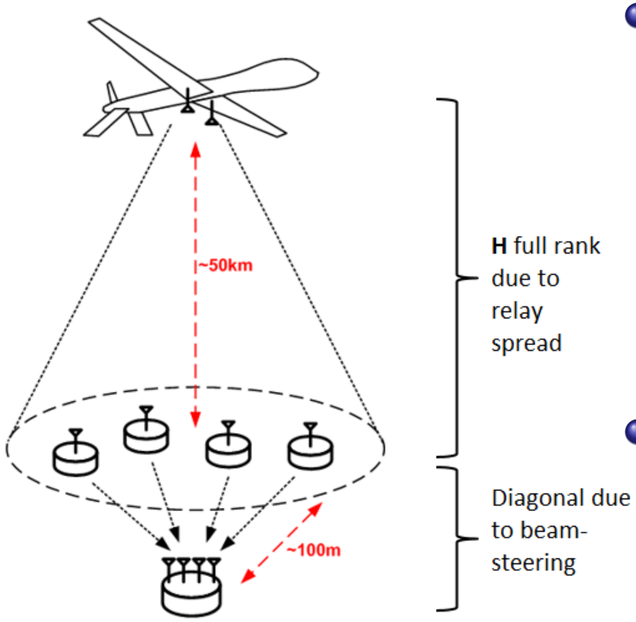

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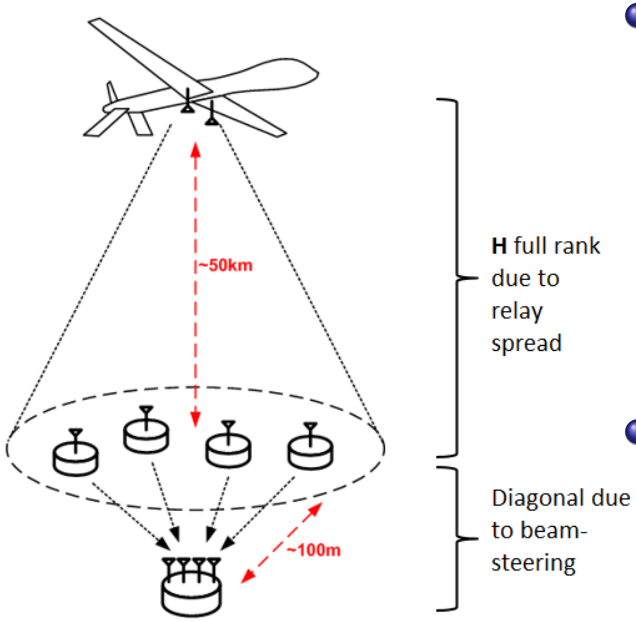

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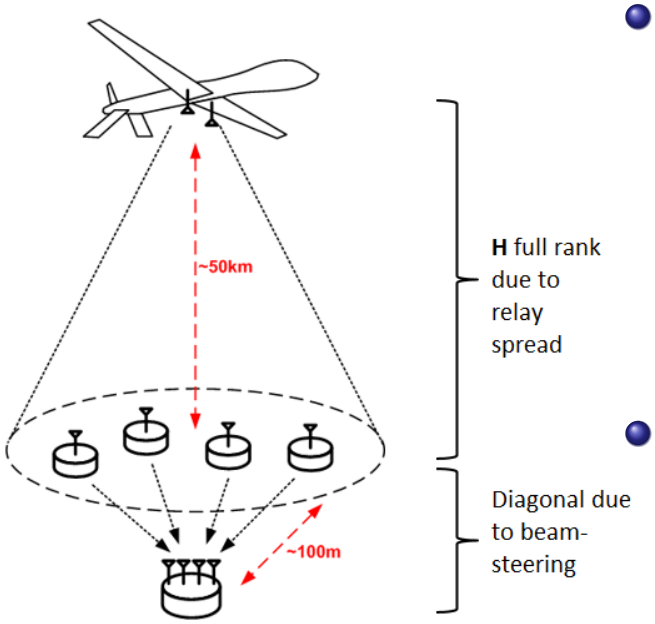

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- Most methods of random matrix analysis for Gaussian / binary / asymptotic cases

- **Question:** How do system dimensions N_R (relays) and N_T (TX) relate to ZF noise enhancement?
- **Answers:**
  - Approach # 1: *bound* the ZF SNR loss
  - Approach # 2: *approximate* ZF SNR loss

Chebyshev bound on ZF SNR loss

Theorem

Let $\rho_i = \frac{1}{N_R} \langle \mathbf{h}_1, \mathbf{h}_i \rangle$ *and* $X = \sum_{i=2}^{N_T} |\rho_i|^2$*. For* $0 < \delta < \frac{N_R - N_T + 1}{N_R}$*, the ZF gain* $\Delta \in [0, 1]$ *is upper-bounded as follows*

$$\Pr[\Delta < \delta] \leq \frac{\mathcal{C}(X; k)}{\left(\frac{N_R - N_T + 1}{N_R} - \delta\right)^k}$$

where $\mathcal{C}(X; k)$ *is the kth central moment of* X*, computed analytically.*

[3] Irish et al., "Sidestepping the Rayleigh limit for LoS spatial multiplexing: A distributed architecture for long-range wireless fiber".

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- **Note:** For the 2×2 system, an exact form is known[3]

$$\Pr[\Delta < \delta] = \frac{2}{\pi} \arccos \sqrt{1 - \delta}$$

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$$\Rightarrow \Delta \geq 1 - \sum_{i=2}^{N_T} |\rho_i|^2, \qquad \rho_i = \tfrac{1}{N_R} \langle \mathbf{h}_1, \mathbf{h}_i \rangle = \tfrac{1}{N_R} \textstyle\sum_{k=1}^{N_R} e^{j\theta_{ki}}$$

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  - Apply bi/multinomial theorems $\rightarrow$ Central moments of $X = \sum_{i=2}^{N_T} |\rho_i|^2$
  - Apply Markov inequality on $(X - \mathbb{E}(X))^k$ $\rightarrow$ Chebyshev bound on $1 - \Delta$

Beta approximation to ZF SNR loss

Theorem

For $N_R \geq N_T > 2$, the ZF gain $\Delta \in [0, 1]$ approximately follows a beta distribution:

$$\Delta \sim Beta\,(N_R - N_T + 1, N_T - 1)$$

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- **Note:** For $N_T = 2$, a tighter approximation is known[4]

$$\Delta = 1 - |\rho|^2 \text{ where } |\rho|^2 \sim \text{Exp}\,(N_R)$$

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$$\frac{1}{N_T - 1} \mathbf{H}_{-1} \mathbf{H}_{-1}^H \approx \mathbf{I} + \sqrt{\frac{2}{N_T - 1}} \underbrace{\mathbf{W}}_{\text{GUE Wigner Matrix}}$$

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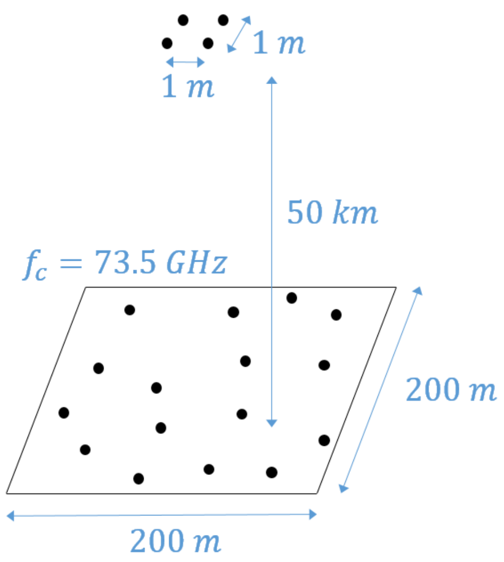
    $\Rightarrow$ Interf. subspace is randomly oriented w.r.t. $\mathbf{h}_1$
  - *Flip view:* $\mathbf{h}_1$ is randomly oriented w.r.t. fixed interf. subspace

    $\Rightarrow$ Entries of $\mathbf{h}_1$ can be thought as i.i.d. circular Gaussian RVs

    $\Rightarrow$ Signal/interf. energy are independent Chi-squared RVs

    $\Delta = \frac{P_S}{P_S + P_I} \Rightarrow$ ZF SNR gain is a Beta RV

Comparison with MC Simulations: Beta approx. on ZF SNR loss



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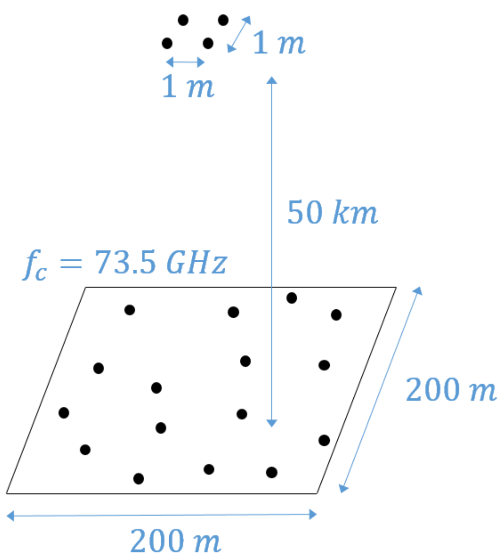

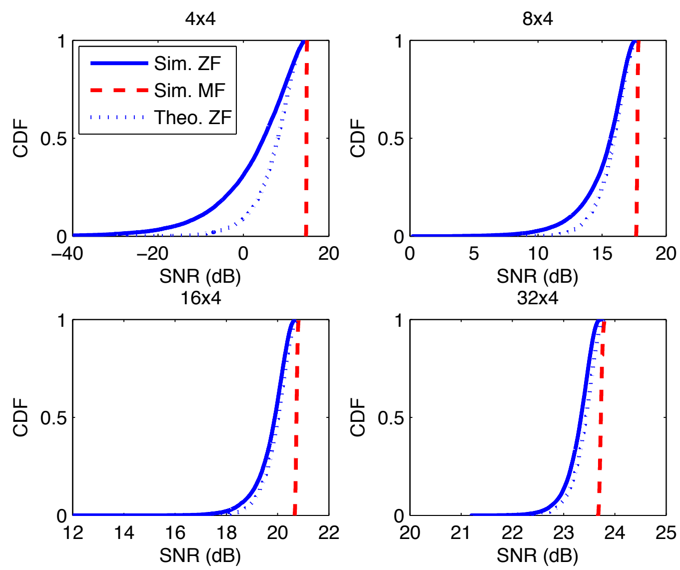

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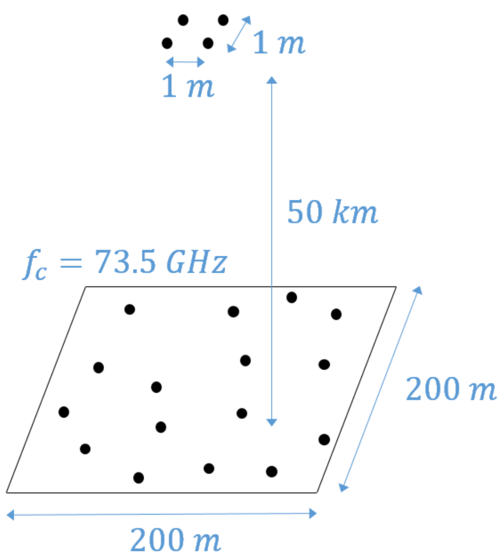

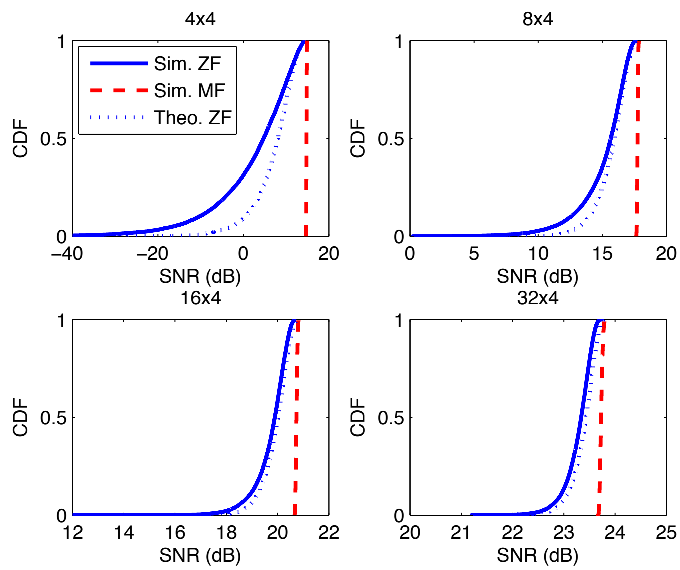

- ZF SNR improved by increasing the number of relays

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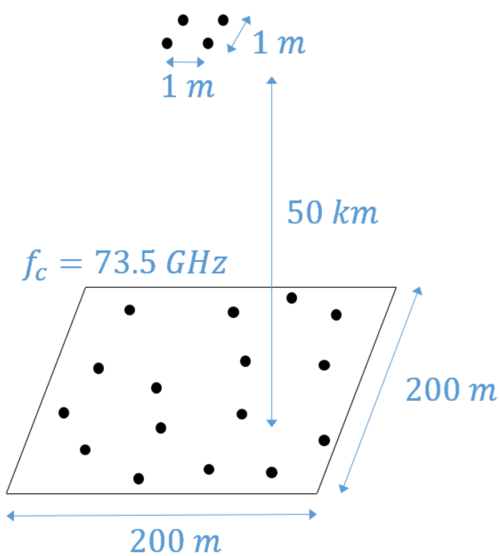
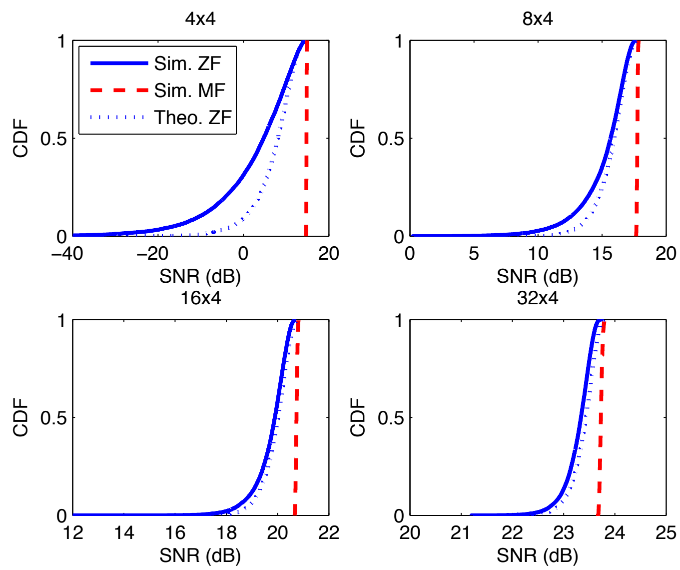
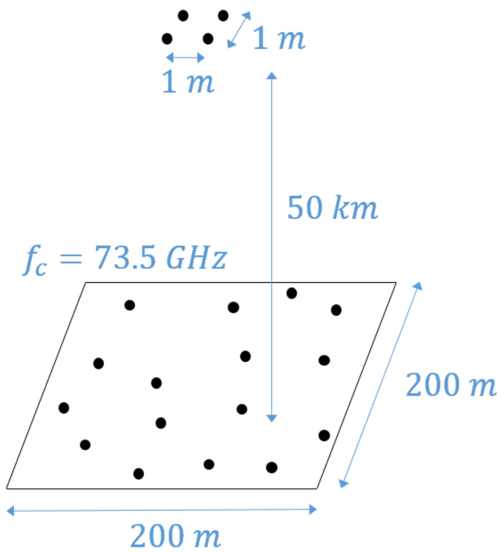




- ZF SNR improved by increasing the number of relays
- Theoretical (Beta) approx.
  - Slightly off for the 4×4 system
  - For ≥ 8 relays is within 2 dB of the simulations.

Comparison with MC Simulations: Chebyshev bound on ZF SNR loss



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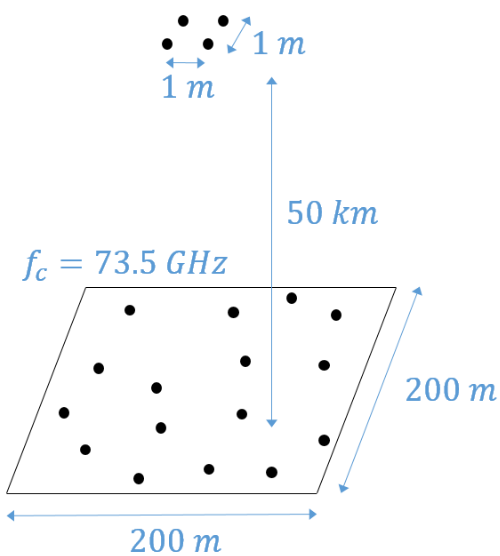

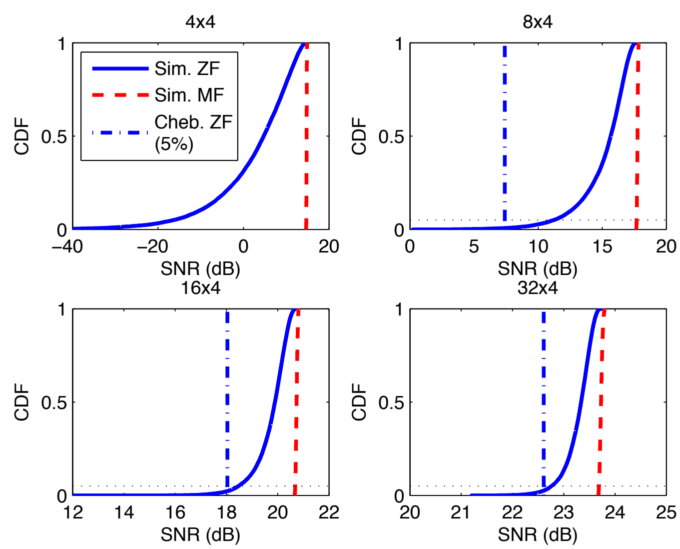

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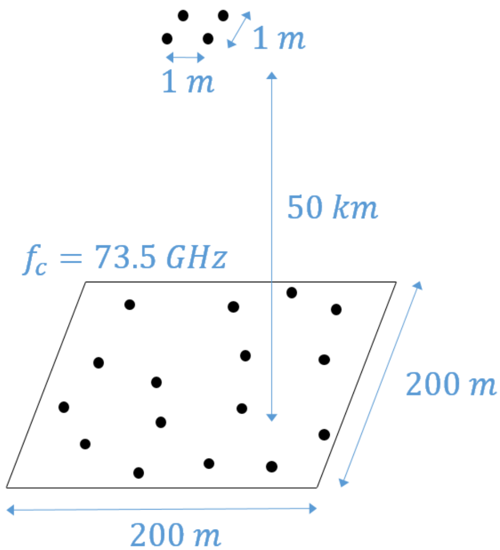

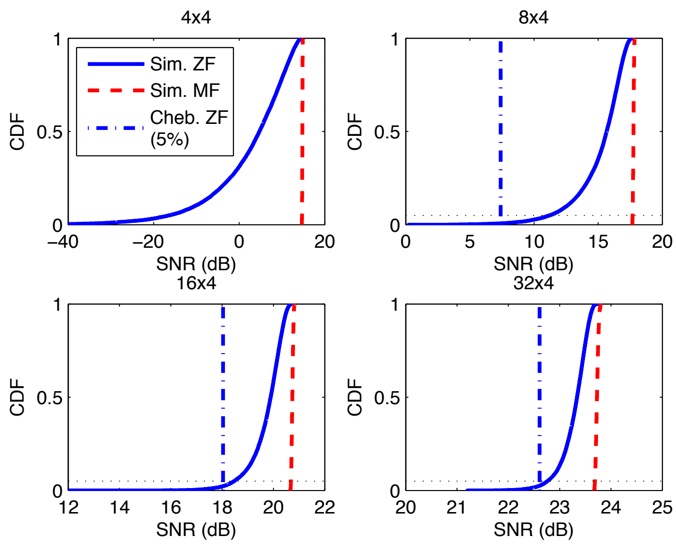

- 5 % Chebyshev bound is tight: optimized over $2 \leq k \leq 30$
  - $\{3.81, 0.43, 0.15\}$ dB gap for $N_R \in \{8, 16, 32\}$
  - For 4×4 system no feasible 5 % Chebyshev bound found

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- Presented bound/approximation on ZF noise enhancement, depend on N_R, N_T only
    - Very close to Monte-Carlo simulations as the number of relays increases
    - For 4 Tx, need $\geq$ 8 relays to achieve reasonable outage probability; excellent performance with 16 relays

LoS MIMO
Towards 100 Gbps at 100m

Hardware-Constrained Signal Processing for mm-wave LoS MIMO Links

Nov. 11th , 2015

B. Mamandipoor[1], M. Sawaby[2], A. Arbabian[2], U. Madhow[1]

(1) ECE Dept., University of California, Santa Barbara

(2) EE Dept., Stanford University

Outline

- **Introduction**
- **Channel Model**
- **Optimized MIMO Processing**
- **Conclusion**

PART I

INTRODUCTION

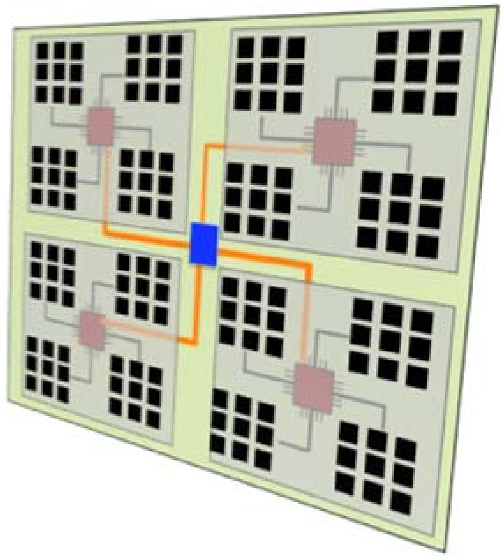
LoS MIMO: Spatial Multiplexing

- Spatial Multiplexing over pure Line-of-Sight channel at mm-wave

- Degrees of freedom of a 2-dimensional LoS MIMO channel

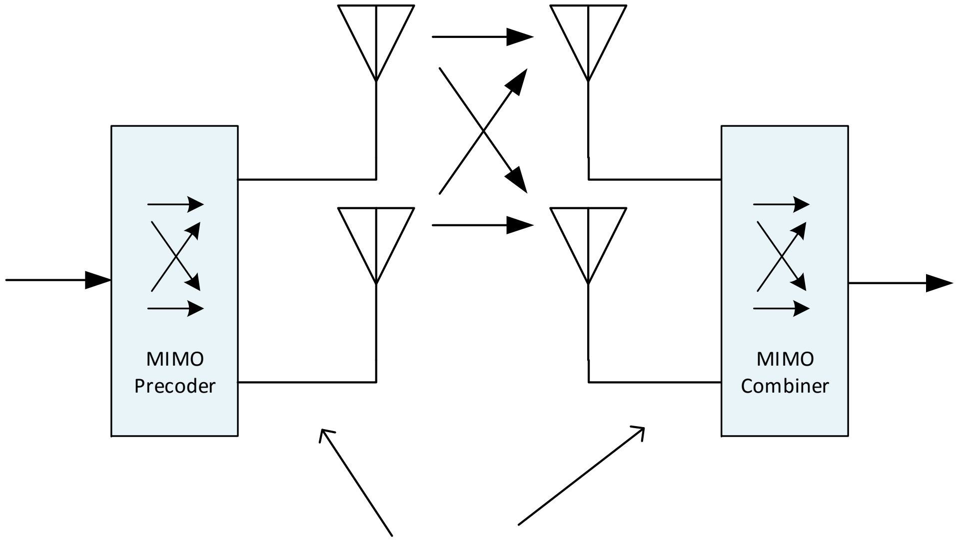
$$\mathrm{DoF} \approx \left(\frac{L_T L_R}{R\lambda}\right)^2$$

- Leads to an array of sub-arrays
  - ✓ Sub-arrays provide beamforming gain
  - ✓ Array of sub-arrays provides spatial multiplexing gain

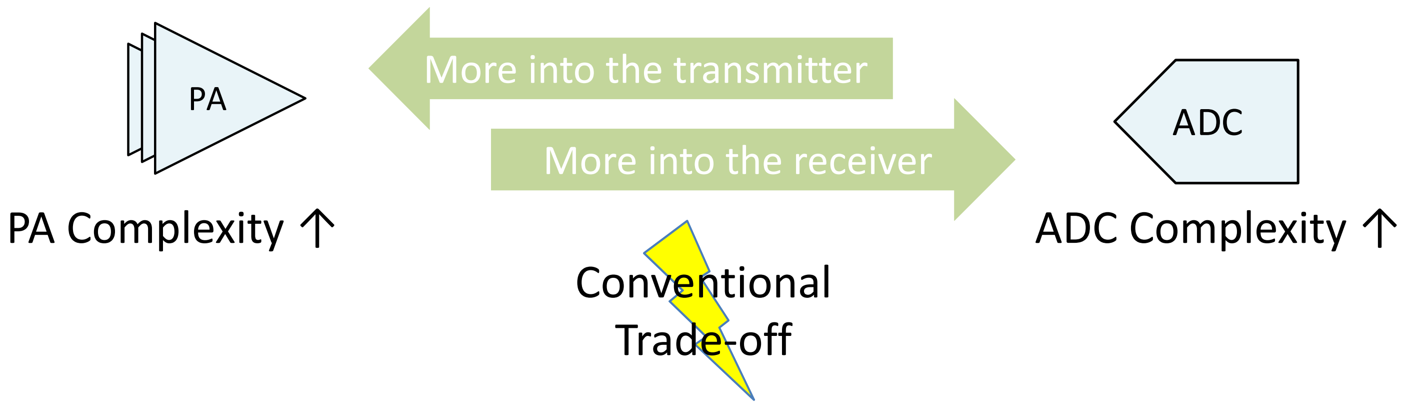


E. Torkildson, U. Madhow, M. Rodwell, "Indoor Millimeter Wave MIMO: Feasibility and Performance, " IEEE Transactions on Wireless Communications, vol. 10, no. 12, pp. 4150-4160, December 2011.

HW Complexity



High fan in/ fan out
Networks Running at high
speeds

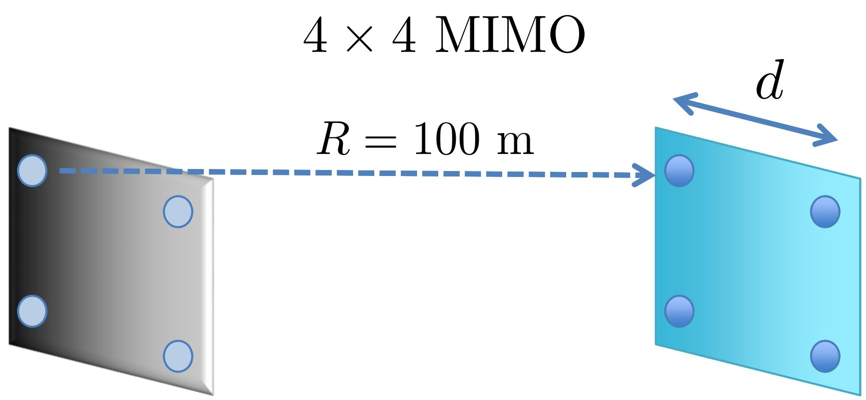


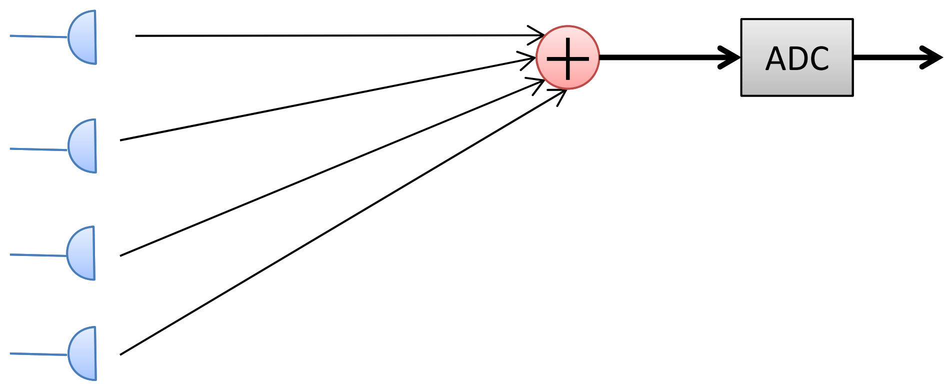
ADC Bottleneck at High Rates

Frequency: 130 GHz

Data rate : 20 Gbps/real dim

Overall data rate: 160 Gbps

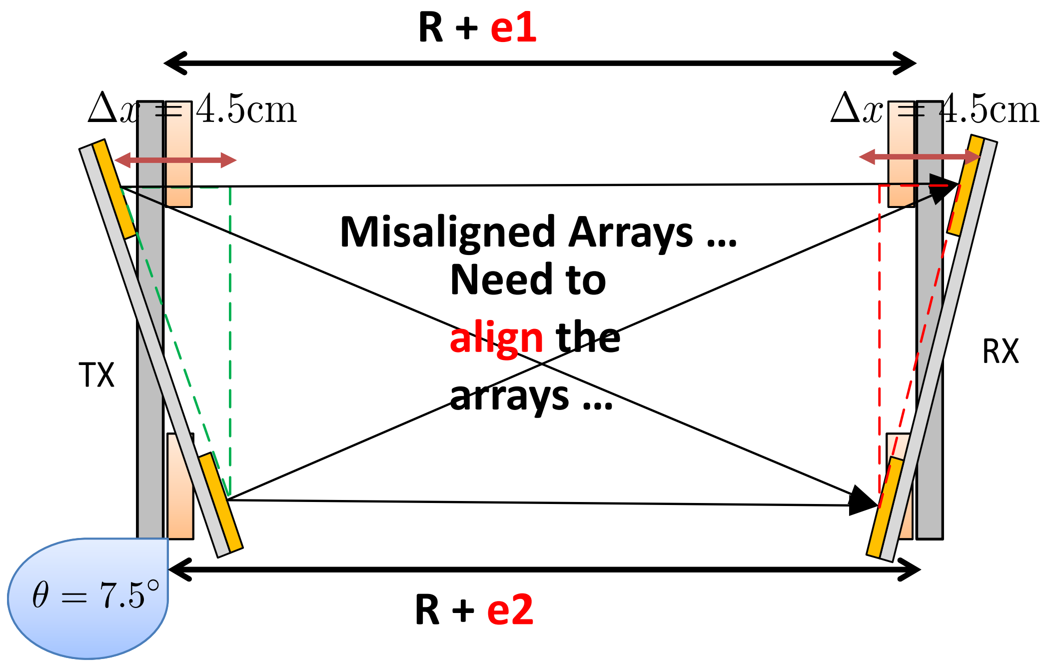




Large dynamic range $\rightarrow$ higher resolution
High data rate $\rightarrow$ higher sampling rate

hard to realize!

Misaligned Array



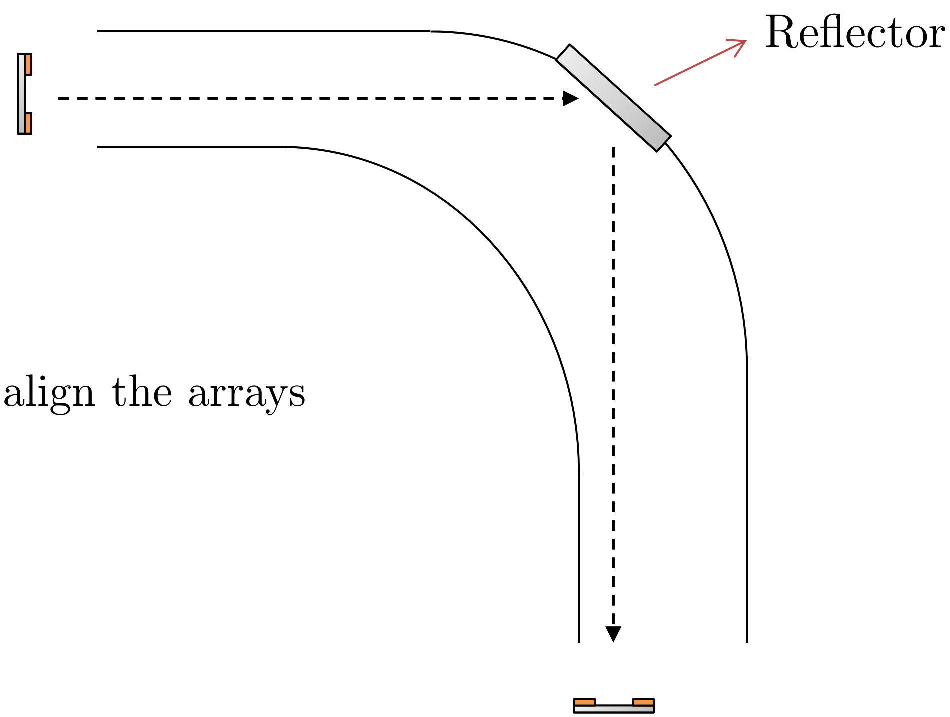
$f = 130$ GHz $\longrightarrow$ $\lambda = c/f = 2.3$ mm

$B = 20$ GHz $\longrightarrow$ $T \approx 1/B = 50$ psec

One Symbol: $\begin{cases} T = 50\text{psec} \\ \text{spatial length} = 1.5\text{cm} \end{cases}$

Reflectors

- Using reflectors for sidestepping LoS blockage

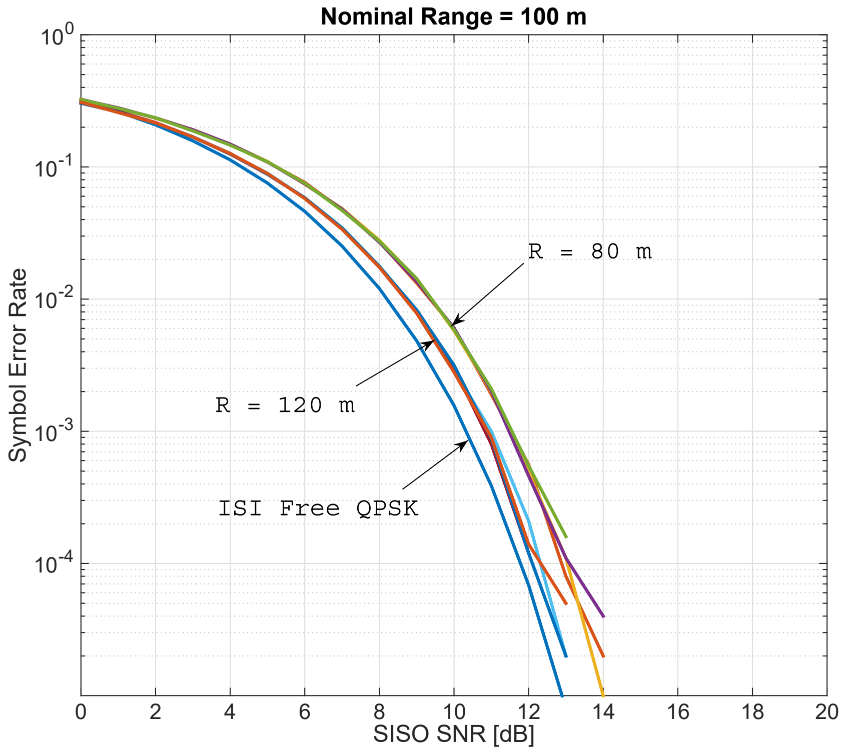


$\rightarrow$ More difficult to align the arrays

Range Deviation is Less Degrading!

Range deviation does not introduce memory in the channel.

$$\text{Channel matrix} \rightarrow \begin{cases} \text{Non-unitary} \\ \text{Memory-less} \end{cases}$$



E. Torkildson, C. Sheldon, U. Madhow, M. Rodwell, "Nonuniform Array Design for Robust Millimeter-Wave MIMO Links," in Proc. IEEE Global Communications Conference 2009 (Globecom), Honolulu, Hawaii, November 2009.

PART II

CHANNEL MODEL

Ideal Channel (aligned)

$R:$ range [m]

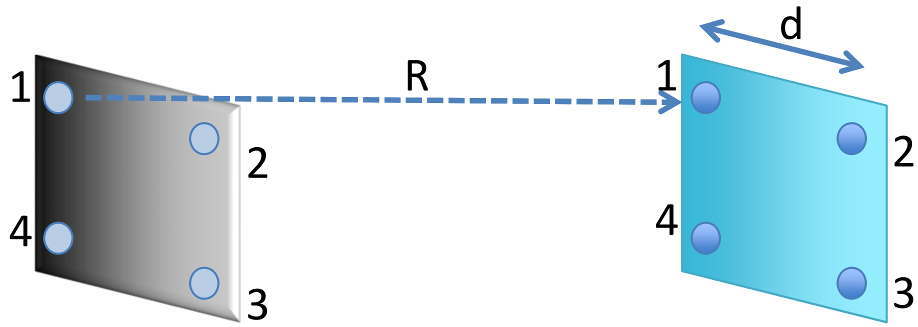
$d:$ inter-element spacing [m]

$f:$ carrier frequency [Hz]

$\lambda:$ carrier wavelength [m]

$T:$ symbol duration [sec]

$B:$ bandwidth [Hz]



$$H = \begin{bmatrix} 1 & e^{j\phi} & e^{j2\phi} & e^{j\phi} \\ e^{j\phi} & 1 & e^{j\phi} & e^{j2\phi} \\ e^{j2\phi} & e^{j\phi} & 1 & e^{j\phi} \\ e^{j\phi} & e^{j2\phi} & e^{j\phi} & 1 \end{bmatrix}$$

$$\boxed{\phi \approx \frac{2\pi}{\lambda} \frac{d^2}{2R}}$$

E. Torkildson, B. Ananthasubramaniam, U. Madhow and M. Rodwell, Millimeter-wave MIMO: Wireless Links at Optical Speeds (Invited Paper) Proc. of 44th Allerton Conference on Communication, Control and Computing, Monticello, Illinois, Sept 2006.

Rayleigh Criterion

Condition for columns of H to be orthogonal

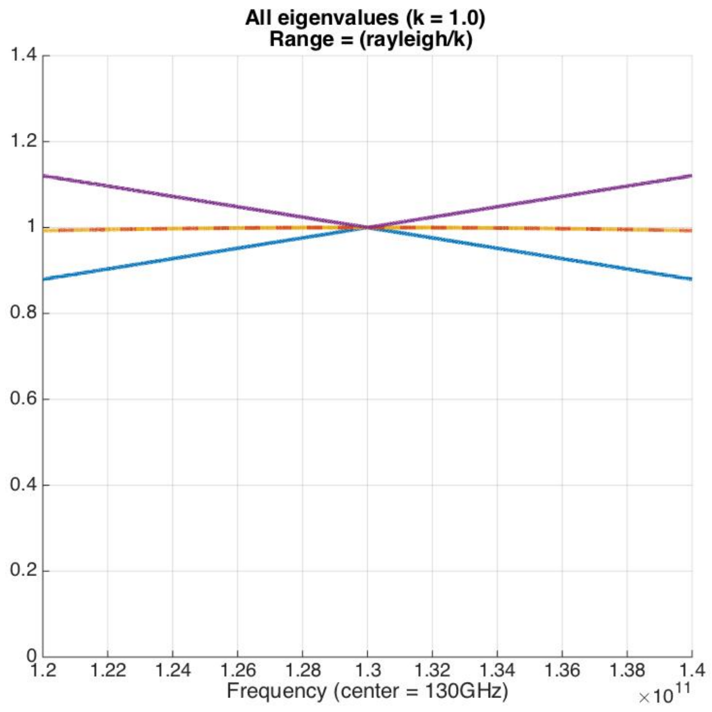
$$\phi = k\frac{\pi}{2} \text{ where } k \in \{1, 3, 5, \ldots\}$$

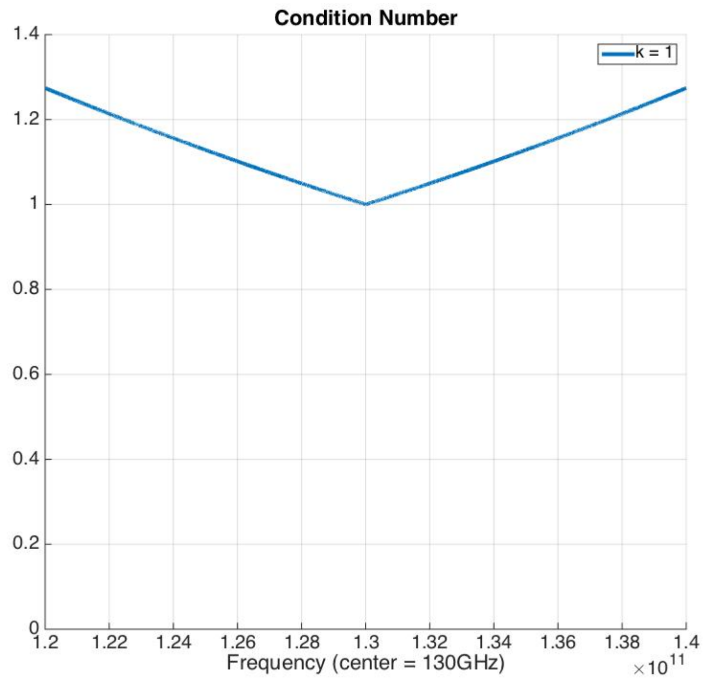
$$\phi \approx \frac{2\pi}{\lambda}\frac{d^2}{2R} = k\frac{\pi}{2} \quad \Rightarrow \quad \boxed{R = \frac{2d^2}{k\lambda} \quad \text{where } k \in \{1, 3, 5, \ldots\}}$$

Therefore, $\quad d_{\min} = \sqrt{\frac{R\lambda}{2}}$

if $\quad R = 100\text{m} \rightarrow d_{\min} = \sqrt{\frac{100 \times 2.3 \times 10^{-3}}{2}} \approx 34\text{cm}$

Frequency Independence



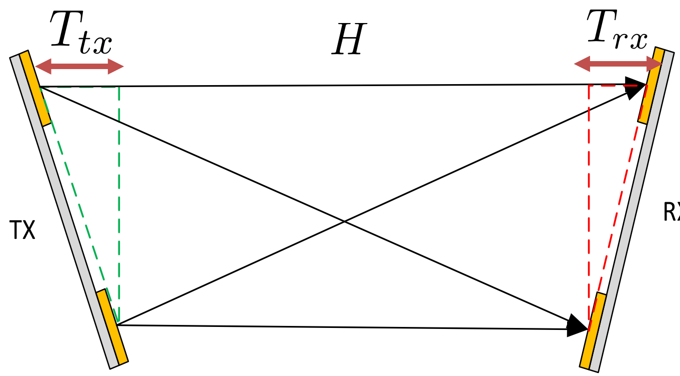


Ideal (aligned) channel is approximately frequency independent in 20 GHz bandwidth.

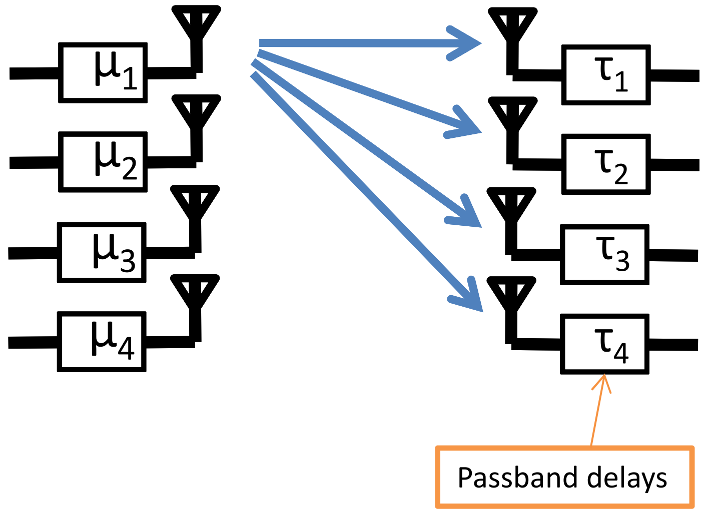
$$H_{\text{ideal}} = \begin{bmatrix} 1 & j & j & -1 \\ j & 1 & -1 & j \\ j & -1 & 1 & j \\ -1 & j & j & 1 \end{bmatrix}$$

Misaligned Channel

- Decomposition of the misaligned channel



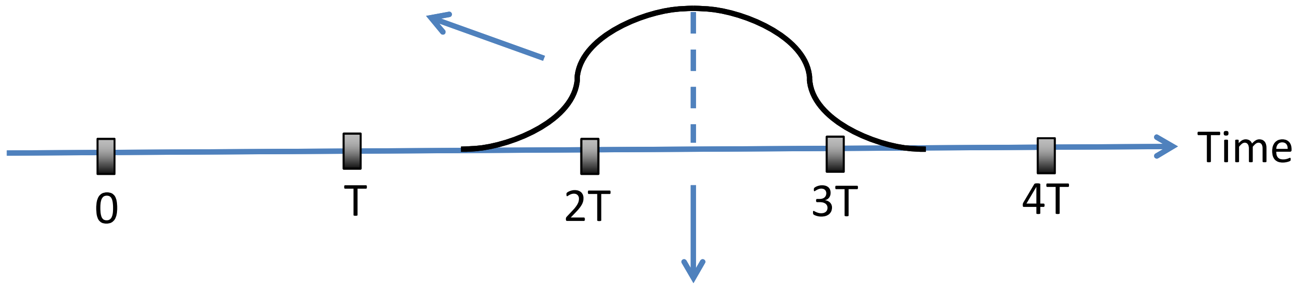
$$H_{new} \approx T_{rx} H T_{tx}$$



$$H_{new} \approx \underbrace{\begin{bmatrix} z^{-\tau_1} & 0 & 0 & 0 \\ 0 & z^{-\tau_2} & 0 & 0 \\ 0 & 0 & z^{-\tau_3} & 0 \\ 0 & 0 & 0 & z^{-\tau_4} \end{bmatrix}}_{\text{Rx misalignment}} \underbrace{\begin{bmatrix} 1 & j & j & -1 \\ j & 1 & -1 & j \\ j & -1 & 1 & j \\ -1 & j & j & 1 \end{bmatrix}}_{\text{Ideal channel}} \underbrace{\begin{bmatrix} z^{-\mu_1} & 0 & 0 & 0 \\ 0 & z^{-\mu_2} & 0 & 0 \\ 0 & 0 & z^{-\mu_3} & 0 \\ 0 & 0 & 0 & z^{-\mu_4} \end{bmatrix}}_{\text{Tx misalignment}}$$

Channel from i^{th} Tx to j^{th} Rx element

$$\alpha_{ij} P(t - \mu_i - \tau_j)$$



$$\mu_i + \tau_j$$

$$\alpha_{ij} \triangleq h_{ij} e^{-j2\pi f_c(\mu_i + \tau_j)}$$

$$H_{ij}[n] = \alpha_{ij} P(nT - \mu_i - \tau_j) \qquad \leftarrow \text{sample at the symbol rate}$$

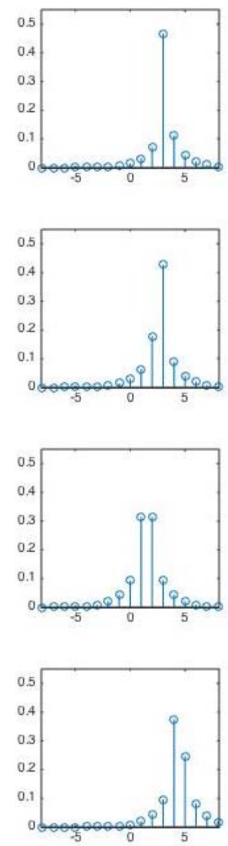
$$\boxed{H[n] = \sum_{i=0}^{L} H_i \delta(n - i) \qquad H_i \in \mathbb{C}^{M \times N}}$$

Part III

OPTIMIZED MIMO PROCESSING

Cannot Afford Oversampling

- All Rx elements sample at the symbol rate (ADC)
- Aligned channel → single-tap demultiplexer
- Misaligned channel → oversampling comes for free in the spatial domain!



Question: How does all-digital linear space-time equalizer perform in the misaligned channel?

Space-Time Equalizer

$$H[n] = \sum_{i=0}^{L} H_i \delta(n-i) \qquad\qquad H_i \in \mathbb{C}^{M \times N}$$

Looking at a time window of size W

$$\boxed{\mathbf{y} = \mathbf{U}\mathbf{s} + \mathbf{z}}$$

$$\mathbf{s} = \Big[\bar{s}[n], \bar{s}[n-1], \ldots, \bar{s}[n-L-W+1]\Big]^T$$

$$\mathbf{y} = \Big[\bar{y}[n], \bar{y}[n-1], \ldots, \bar{y}[n-W+1]\Big]^T$$

$$\bar{s}[n] = \Big[s_1[n], s_2[n], \ldots, s_N[n]\Big] \rightarrow \text{Transmitted symbols at time n}$$

$$\bar{y}[n] = \Big[y_1[n], y_2[n], \ldots, y_M[n]\Big] \rightarrow \text{Received signals at time n}$$

MMSE Equalizer

- Block Toeplitz matrix of channel coefficients:

$$\mathbf{U} = \begin{bmatrix} H_0 & H_1 & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & H_0 & H_1 & \dots & \mathbf{0} \\ \vdots & & & & \\ \mathbf{0} & \mathbf{0} & \dots & H_{L-1} & H_L \end{bmatrix}_{WM \times N(L+W-1)}$$

MMSE equalizer:

$$C_{\mathrm{MMSE}} = \left(\mathbf{U}\mathbf{U}^H + \frac{1}{\rho}\mathbb{I} \right)^{-1} \mathbf{U}\mathbf{e}$$

$$\text{SISO SNR} = \rho \triangleq \frac{\sigma_s^2}{2\sigma^2}$$

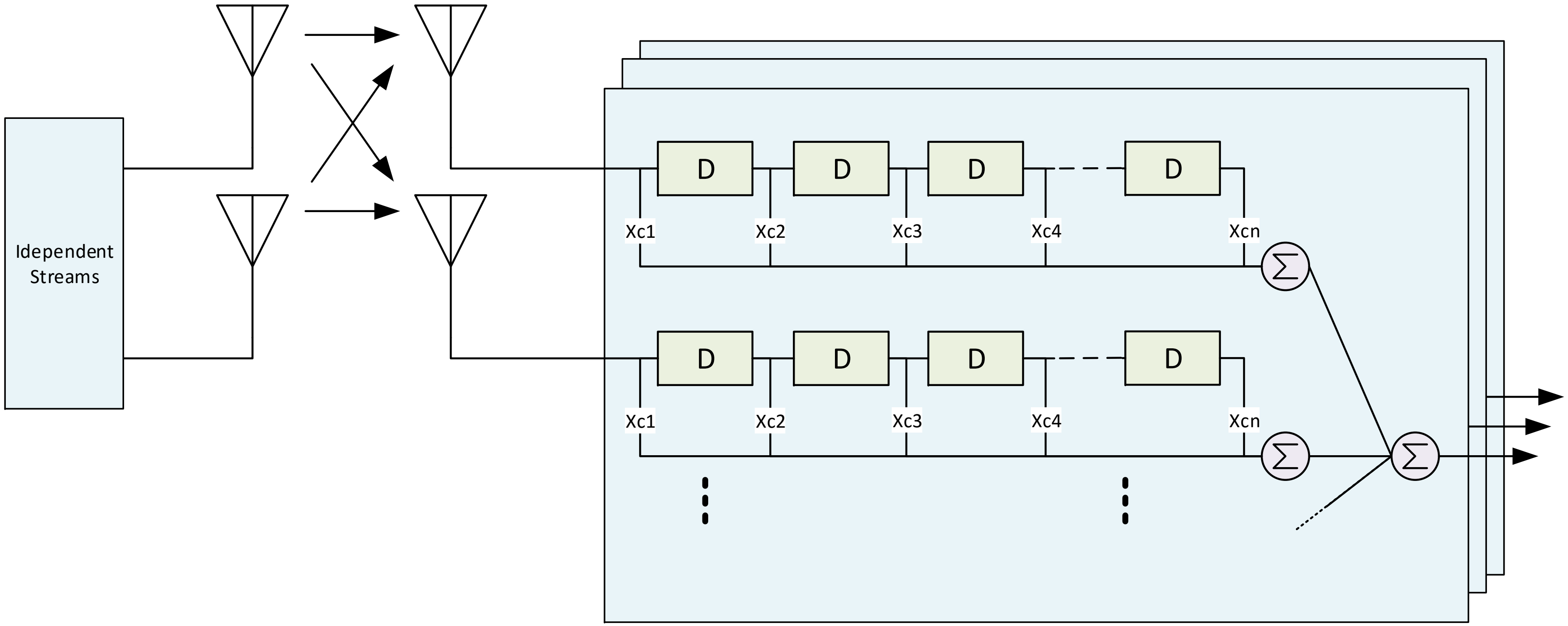
Choosing one symbol for each Tx

$$\mathbf{e} = \left[\bar{e}[n], \bar{e}[n-1], \ldots, \bar{e}[n-W+1]\right]^T$$

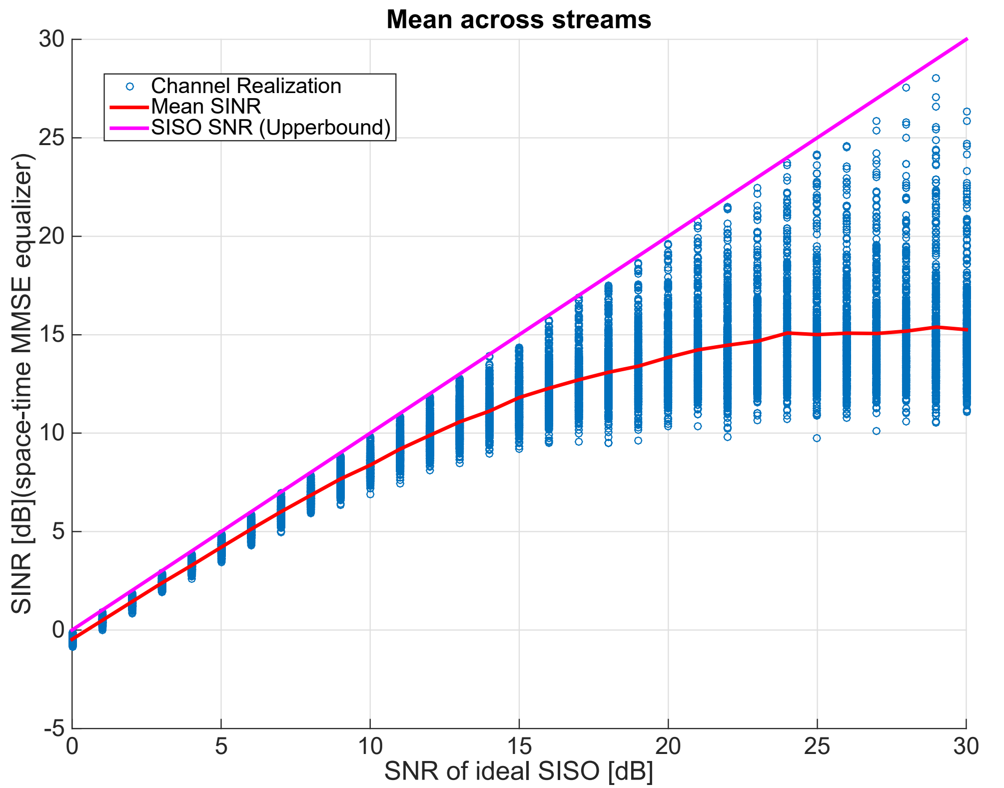
i^{th} element of each vector corresponds to Tx element #i

only one of the elements corresponding to Tx#i is equal to one

Equalizer Implementation



Output SINR

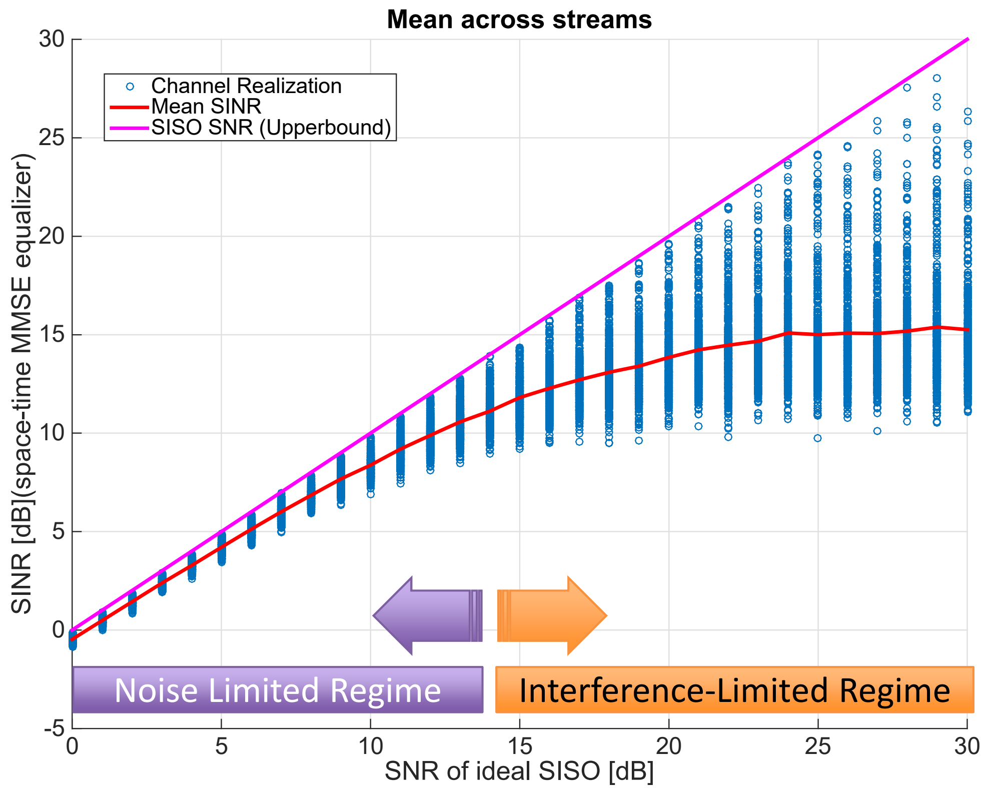


$$\begin{cases} \text{Raised cosine pulse with roll-off} = 0.25 \\ \tau_i, \mu_j \sim \ \text{Uniform}[0, 150e-12] \\ \text{Window size } W = 5 \end{cases}$$

$$\text{SINR} = \frac{|\langle c, u_0 \rangle|^2}{\sum_{i \neq 0} |\langle c, u_i \rangle|^2 + \frac{1}{\rho}||c||^2}$$

$$\text{SISO SNR} = \rho \triangleq \frac{\sigma_s^2}{2\sigma^2}$$

Output SINR

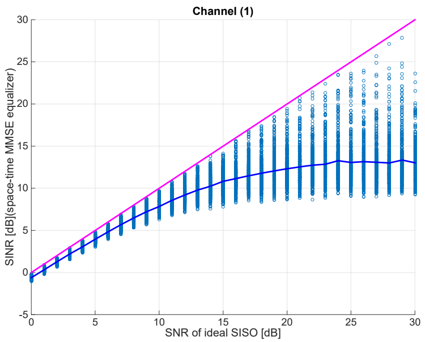


Noise-limited regime $\rightarrow$ SINR less sensitive to channel realization

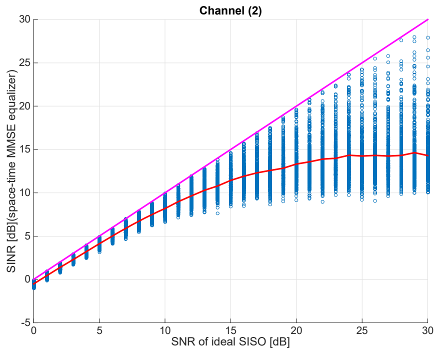
Interference-limited regime $\rightarrow$ SINR highly sensitive to channel realization

Question: For a given SNR at interference-limited regime, is it possible to increase the SINR by adding sub-symbol delays?

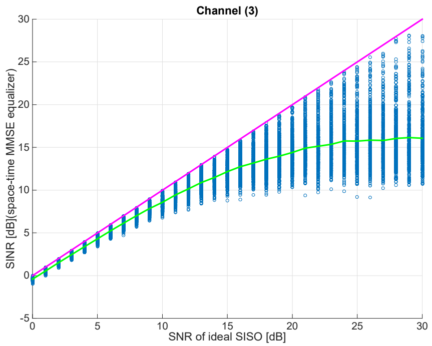
Output SINR



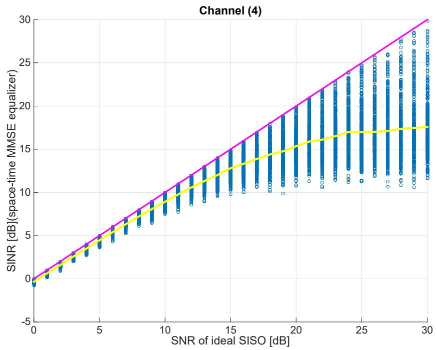
Channel (1)
SINR [dB](space-time MMSE equalizer)
SNR of ideal SISO [dB]



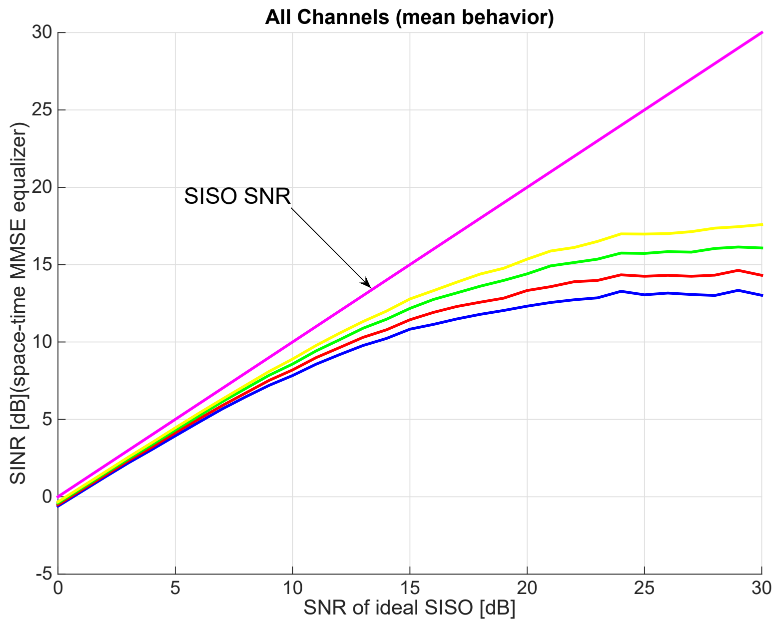
Channel (2)
SINR [dB](space-time MMSE equalizer)
SNR of ideal SISO [dB]



Channel (3)
SINR [dB](space-time MMSE equalizer)
SNR of ideal SISO [dB]

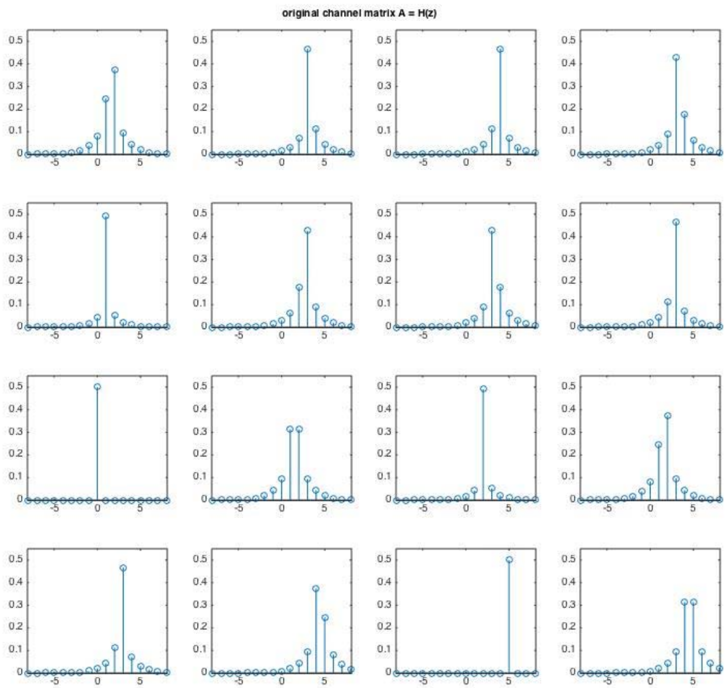


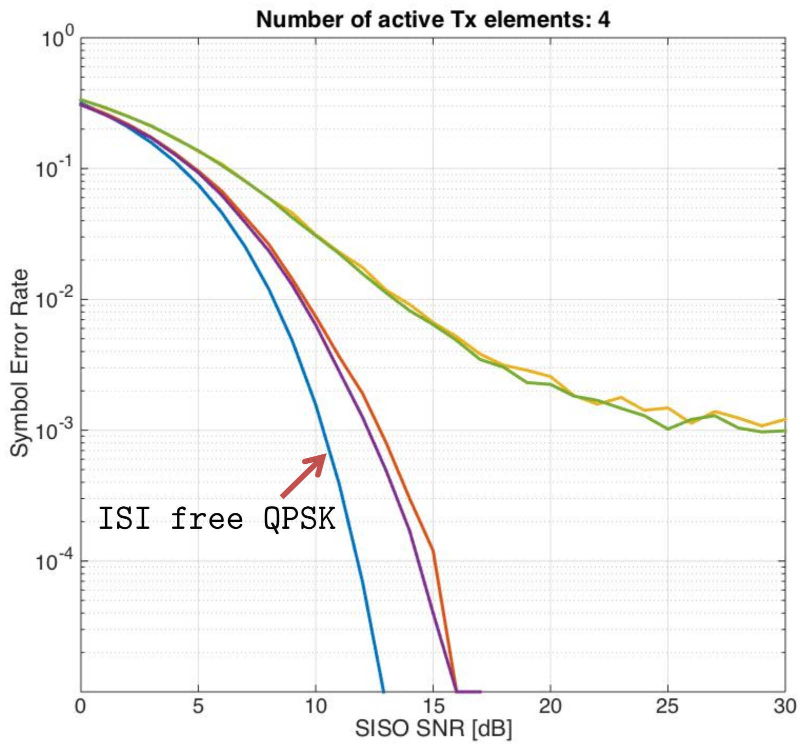
Channel (4)
SINR [dB](space-time MMSE equalizer)
SNR of ideal SISO [dB]



All Channels (mean behavior)
SISO SNR
SINR [dB](space-time MMSE equalizer)
SNR of ideal SISO [dB]

Channel Realization (tough)

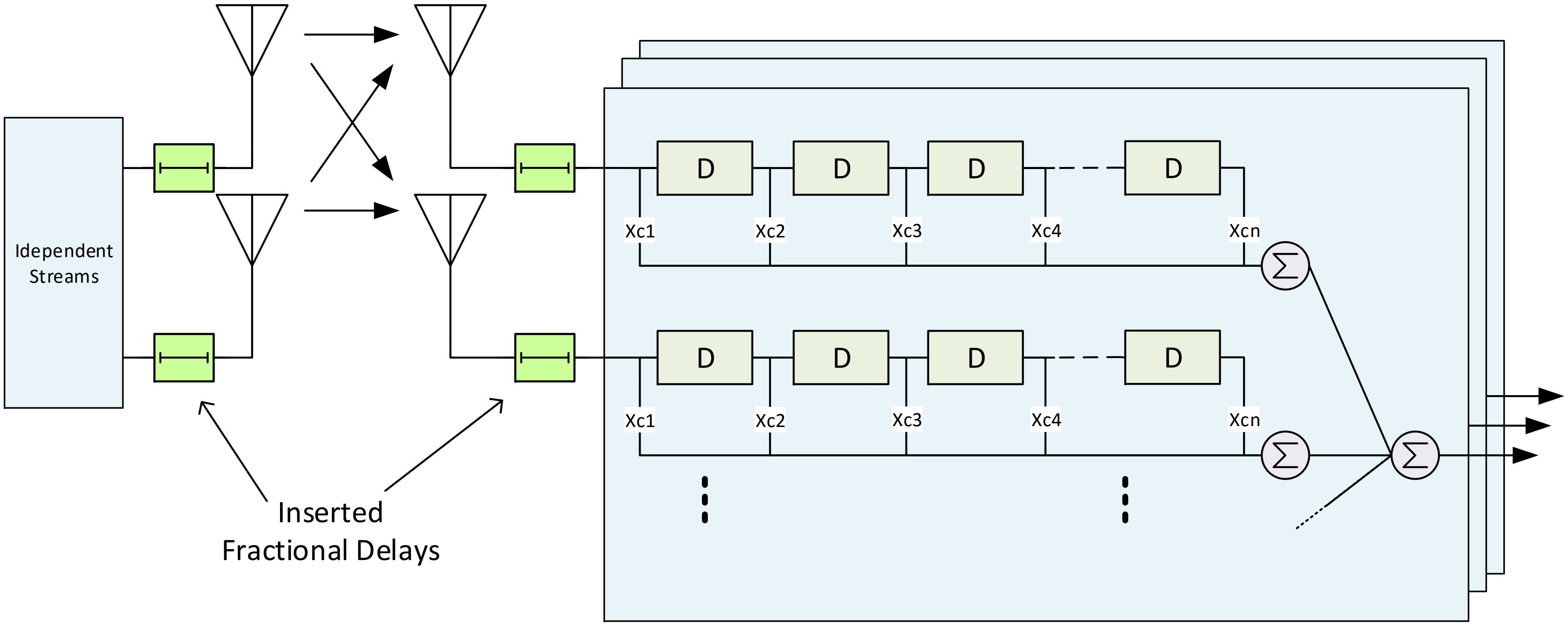




TAU = 1.0e-09 *[0.0847 0.0597 0 0.1445]

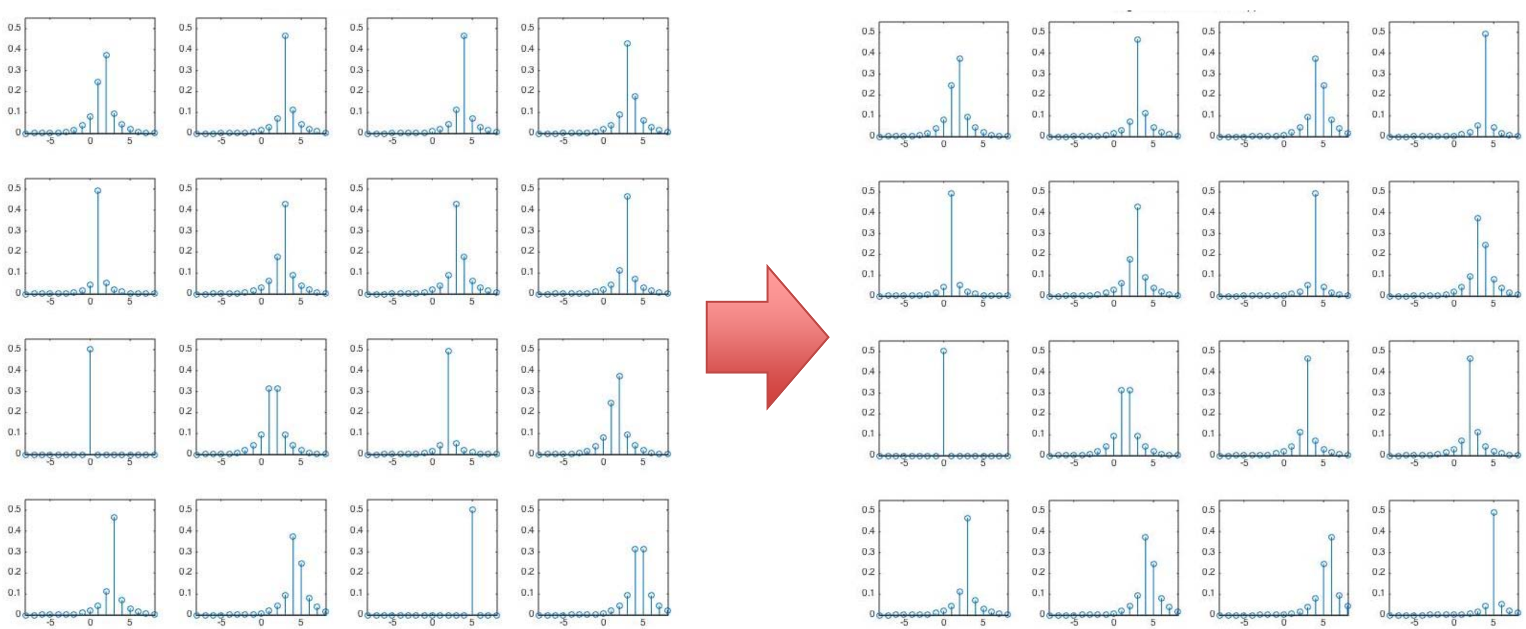
MU = 1.0e-09 *[0 0.0791 0.1100 0.0811]

Modified Equalizer Implementation



Sub-symbol delay at the Tx side

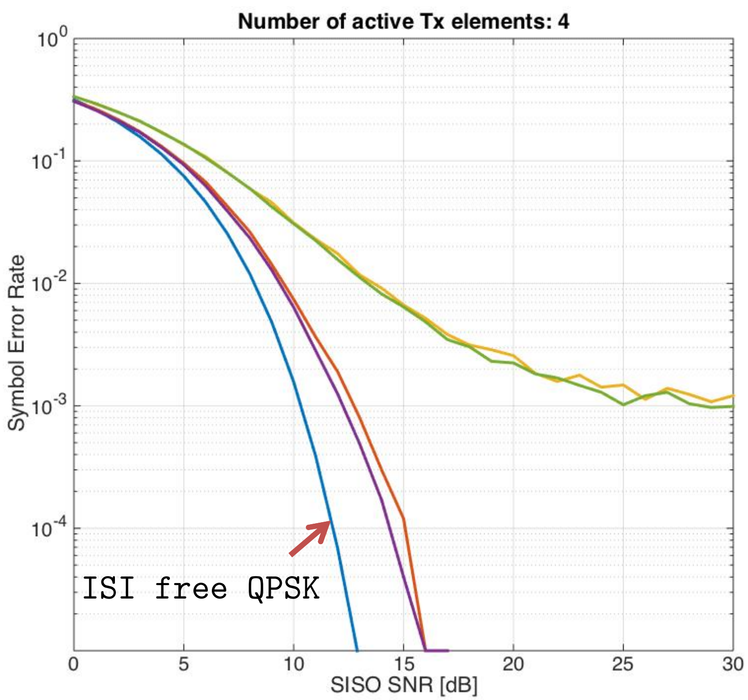
Delays added to these columns

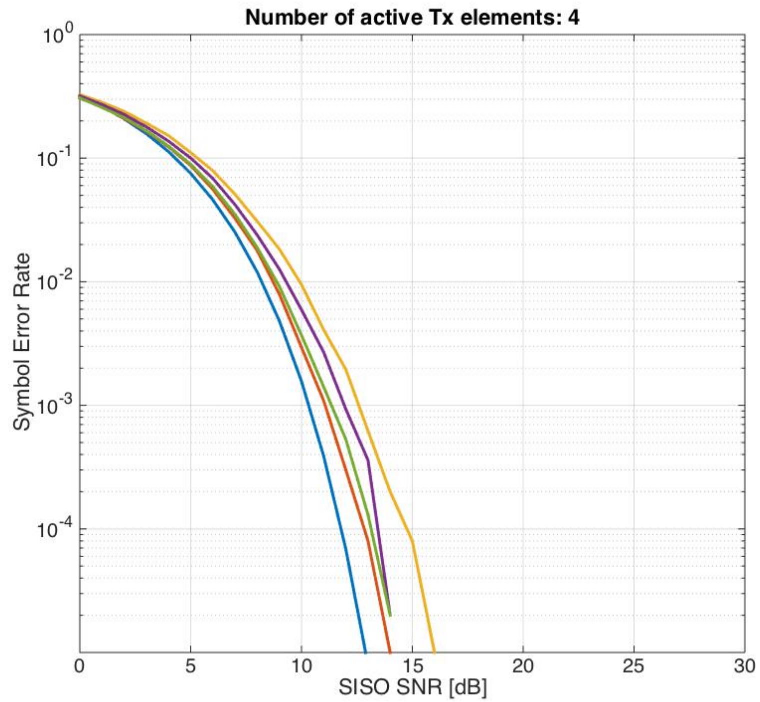


$[\text{Tx}\#3] \rightarrow +30$ psec

$[\text{Tx}\#4] \rightarrow +30$ psec

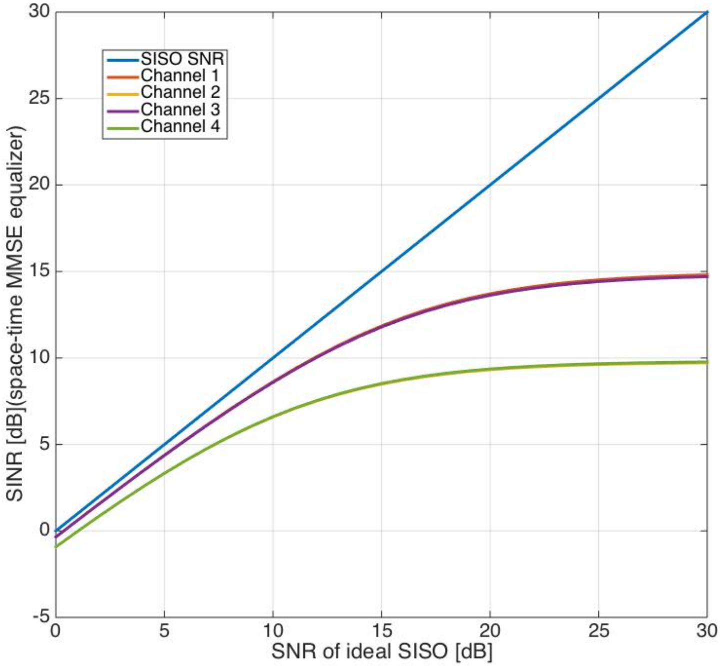
Sub-symbol delay at the Tx side

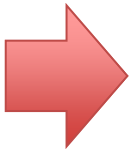


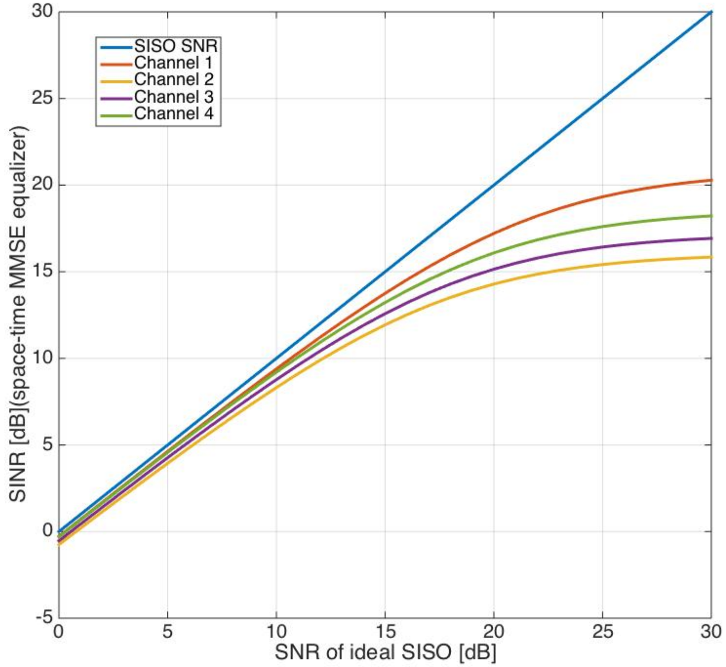


$[\mathrm{Tx}\#3] \rightarrow +30$ psec
$[\mathrm{Tx}\#4] \rightarrow +30$ psec

Sub-symbol delay at the Tx side

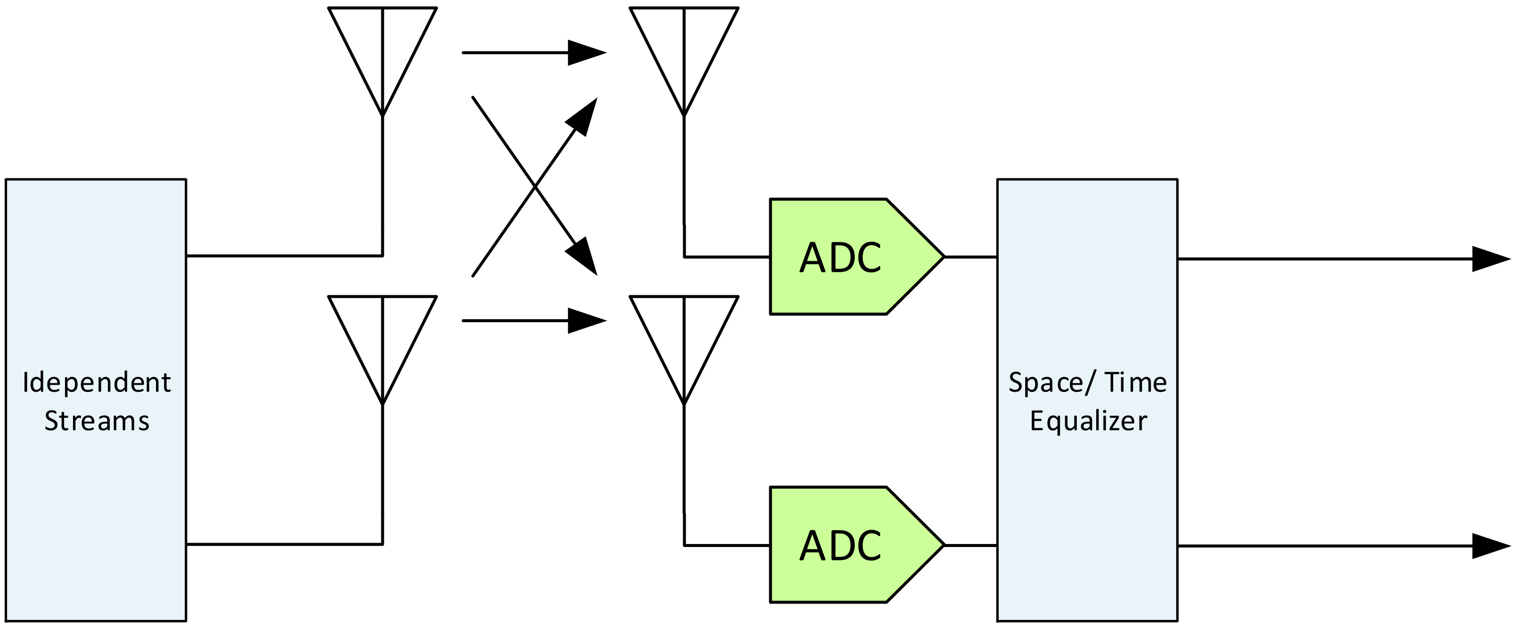






$[\mathrm{Tx}\#3] \rightarrow +30$ psec
$[\mathrm{Tx}\#4] \rightarrow +30$ psec

HW Insight: Optimum Sampling Position



- Relax ADC requirements
- Decomposition of the space/ time equalizer

Analog Processing

- Independent Tx streams
  → multiples of symbol delay at Tx side is irrelevant

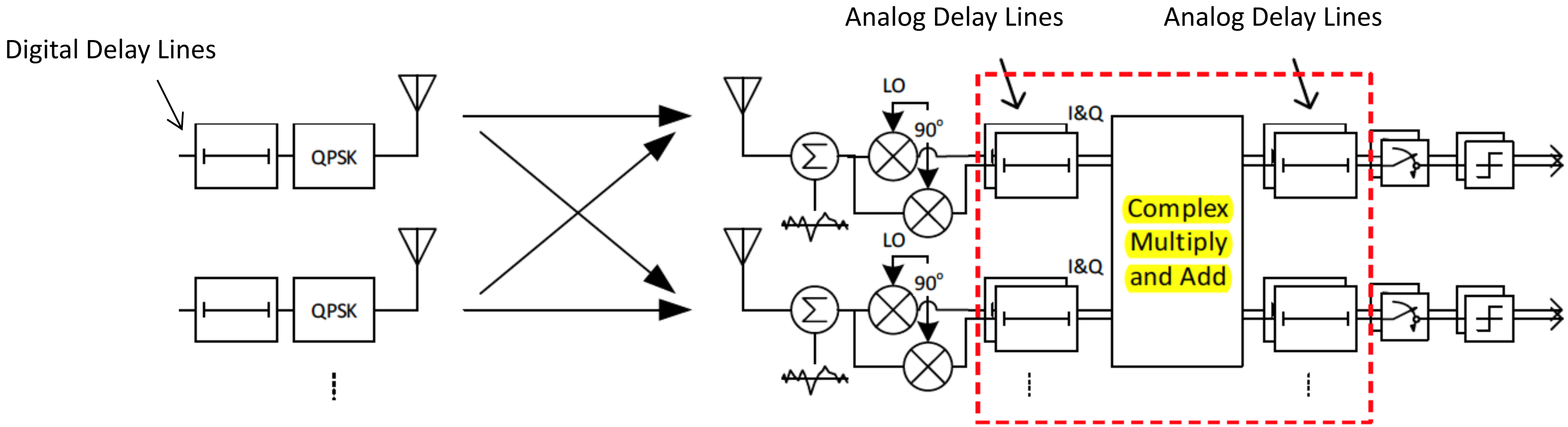
- Analog delay lines at the Rx side
  → Multiple-symbol length with sub-symbol precision
  → Can completely account for the Rx misalignment

$$Y = \underbrace{T_{tx,\epsilon}^{-1} H^{-1} T_{rx}^{-1}}_{\text{Analog Processing}} T_{rx} H T_{tx} T_{tx,n}^{-1} U$$

Analog CSN

Rx alignment and channel inversion in analog:

$$Y = {\color{blue}T_{tx,\epsilon}^{-1} H^{-1} T_{rx}^{-1}} \; T_{rx} H T_{tx} \; {\color{red}T_{tx,n}^{-1}} \; U$$



→ Analog delay lines

→ Analog matrix demultiplexer

Part IV

CONCLUSION

Takeaways & Open Issues

- Symbol-rate sampling followed by space-time equalizer
  - Need high resolution ADC
  - Performance is highly sensitive to channel realization
  - Possibility of recovering by sub-symbol delays at Tx/Rx side

- Optimization of sub-symbol delays
- Non-linear strategies

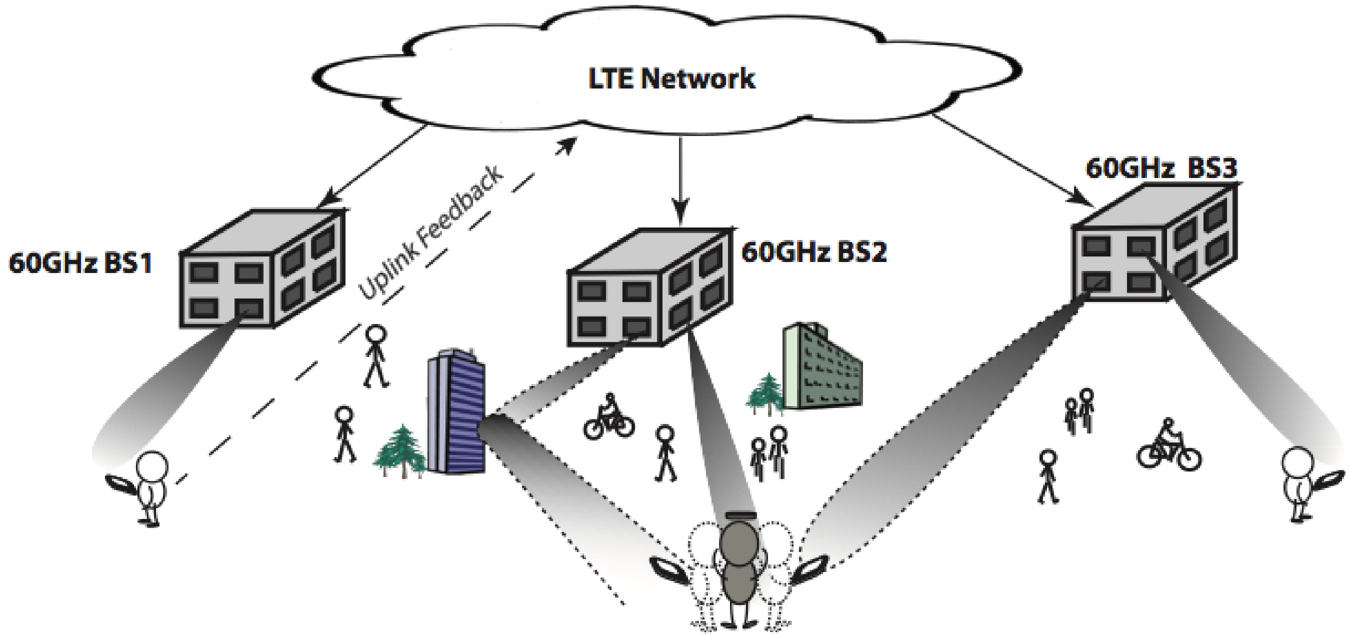
Takeaways & Open Issues

- Analog pre-processing before ADC
  - Analog delay line (align Rx array)
  - Analog matrix demultiplexer (eliminating ICI using a single-tap)
- Channel estimation and adaptation algorithms
- Estimates of power/hardware saving compared to fully digital architectures

Compressive picocellular architectures

mm wave for small cells

- Up the ante on spatial reuse
  - Highly directional mm wave (+LTE) to the mobile
  - 28 GHz (industry), 60 GHz (can leverage WiGig)

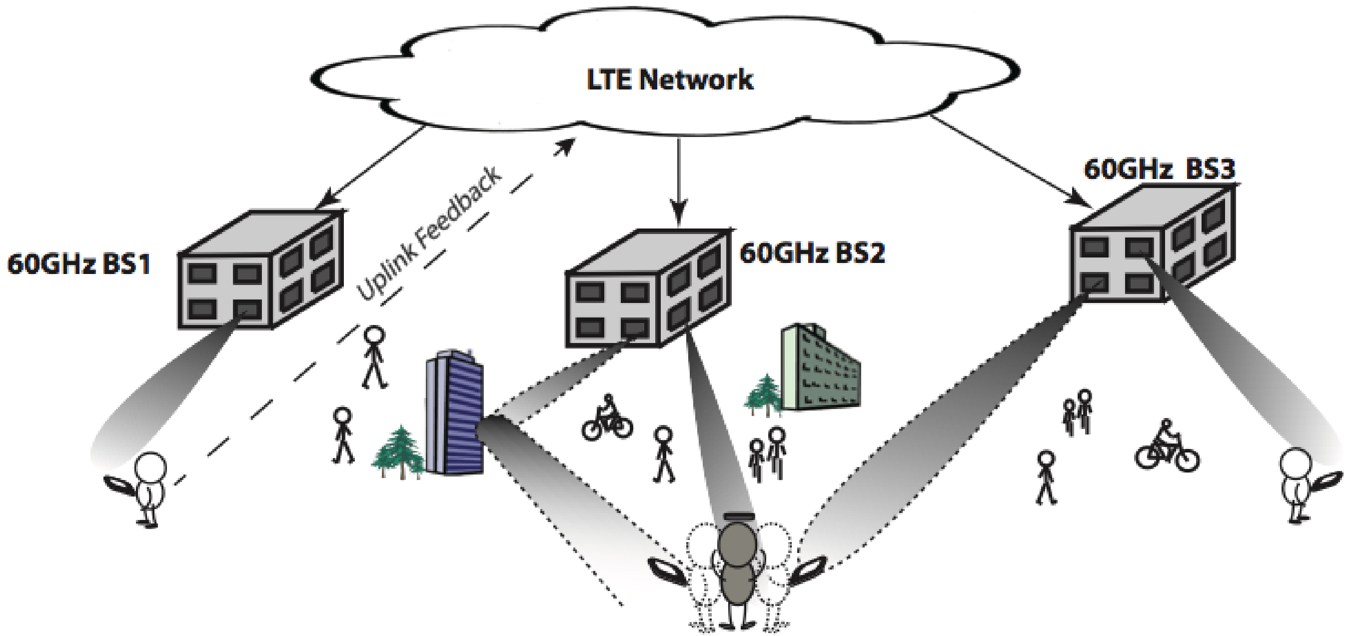


eive-

Need robustness to blockage by user's body and other obstacles

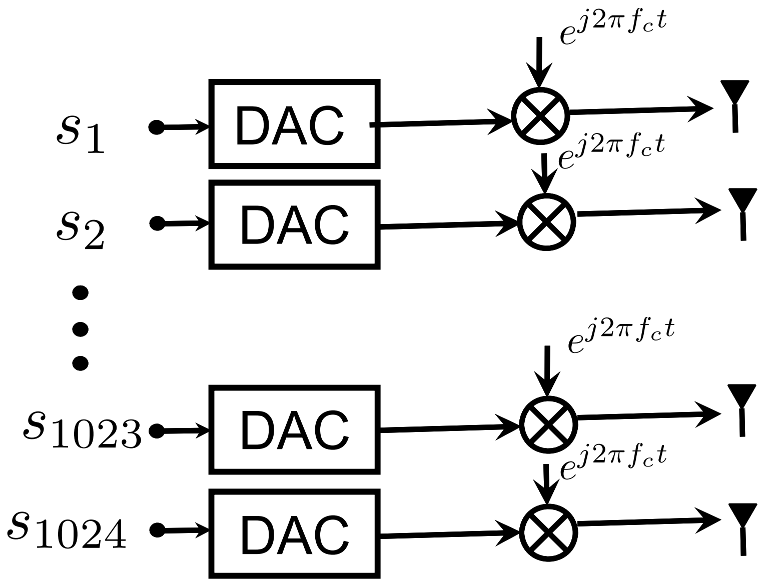
Beamsteering with very large arrays

(The key to "unlimited" spatial reuse)



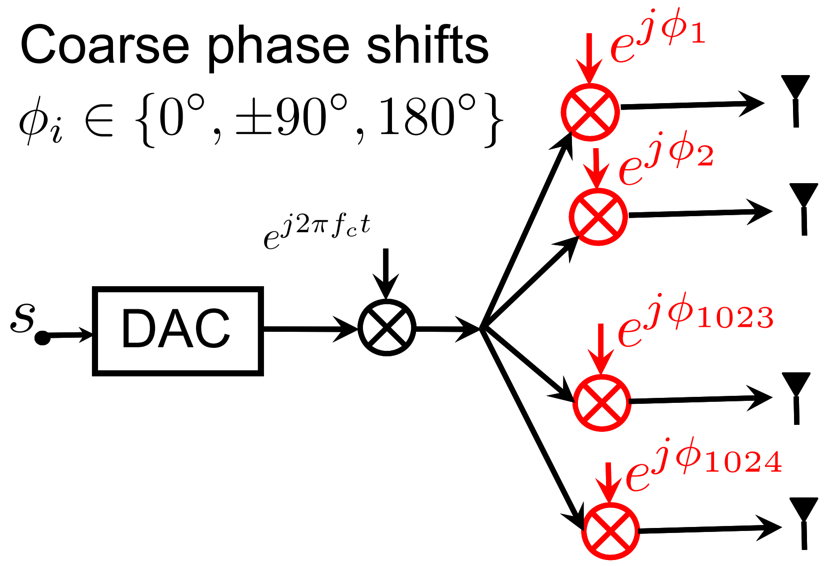
Beamforming today

DSP-centric, one RF chain per antenna element



Does not scale to 1000 elements!

RF Beamforming with hardware constraints

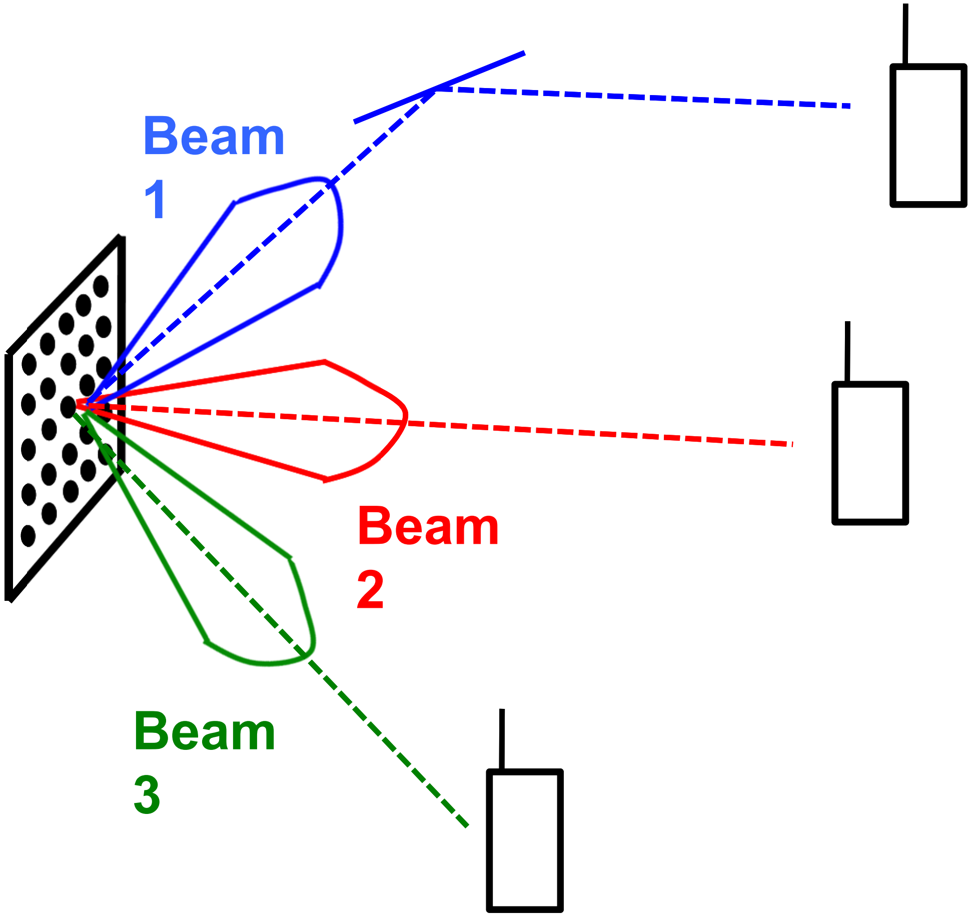


Much more feasible

But how do we adapt it?

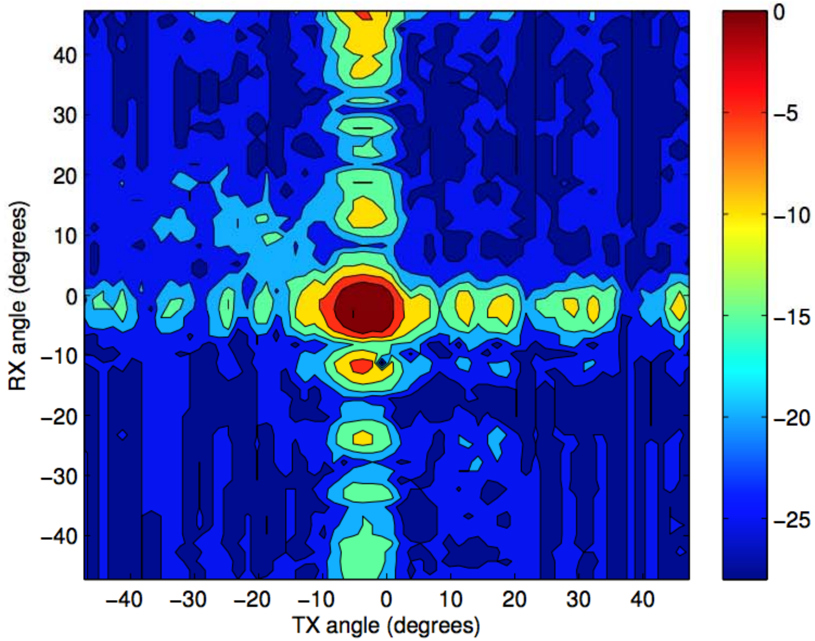
No access to individual elements ➔ least squares does not work

Beam scanning architecture unattractive

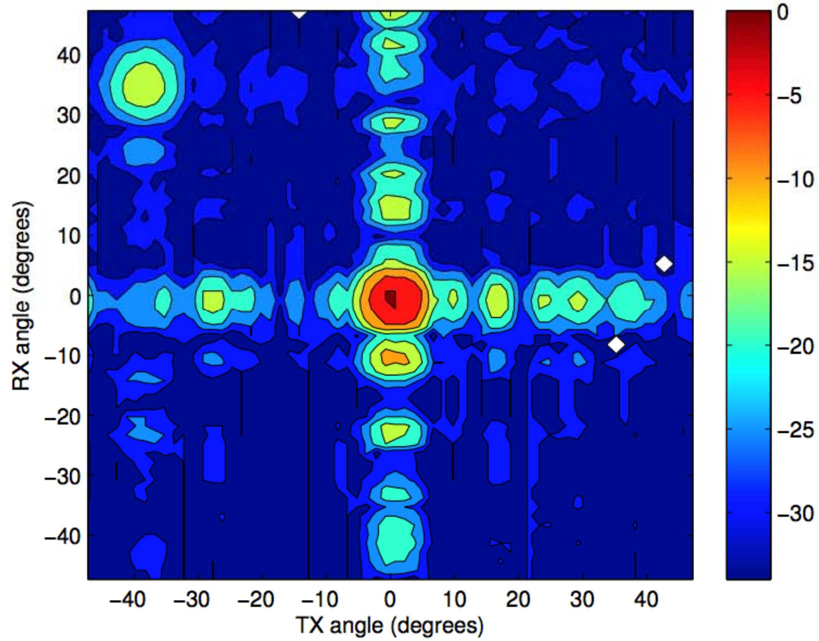


- Requires fine control of phases
- Slow adaptation

Mm wave channel is sparse



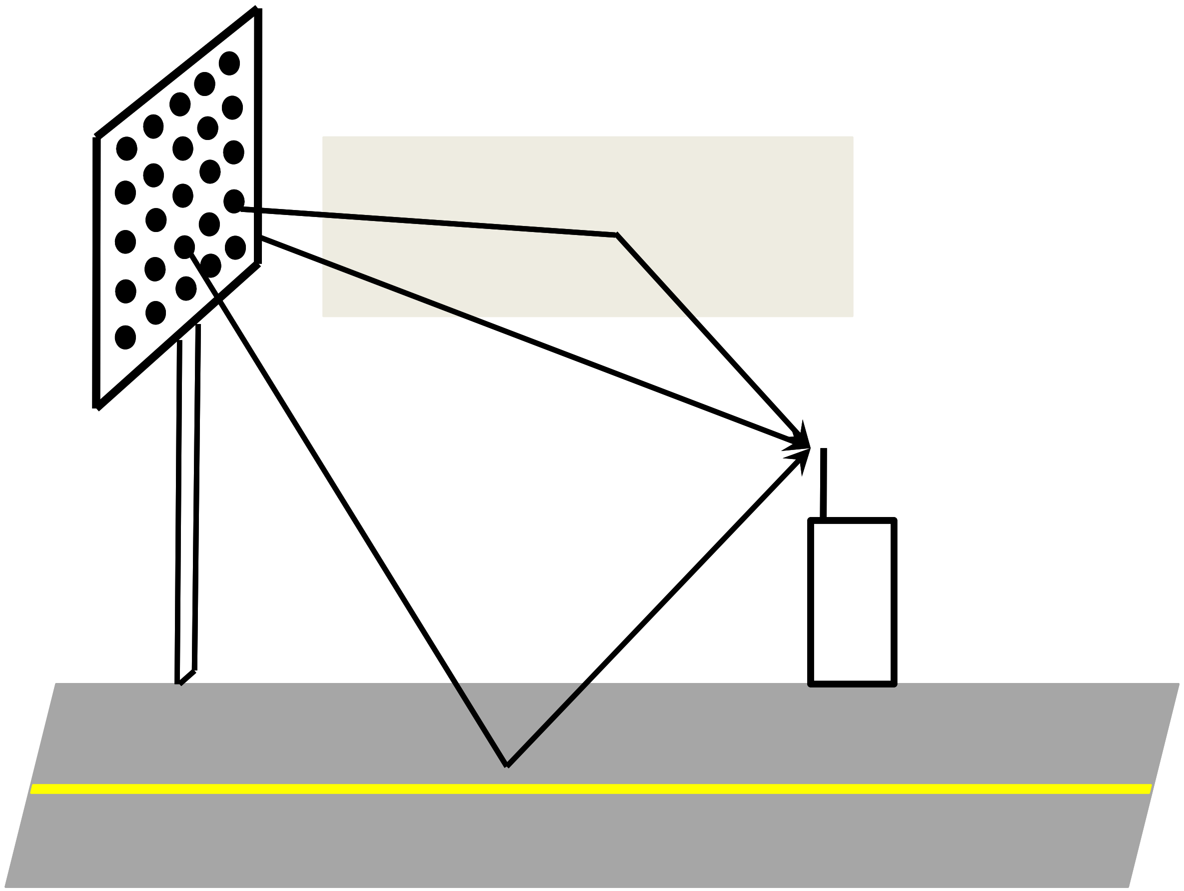
One path



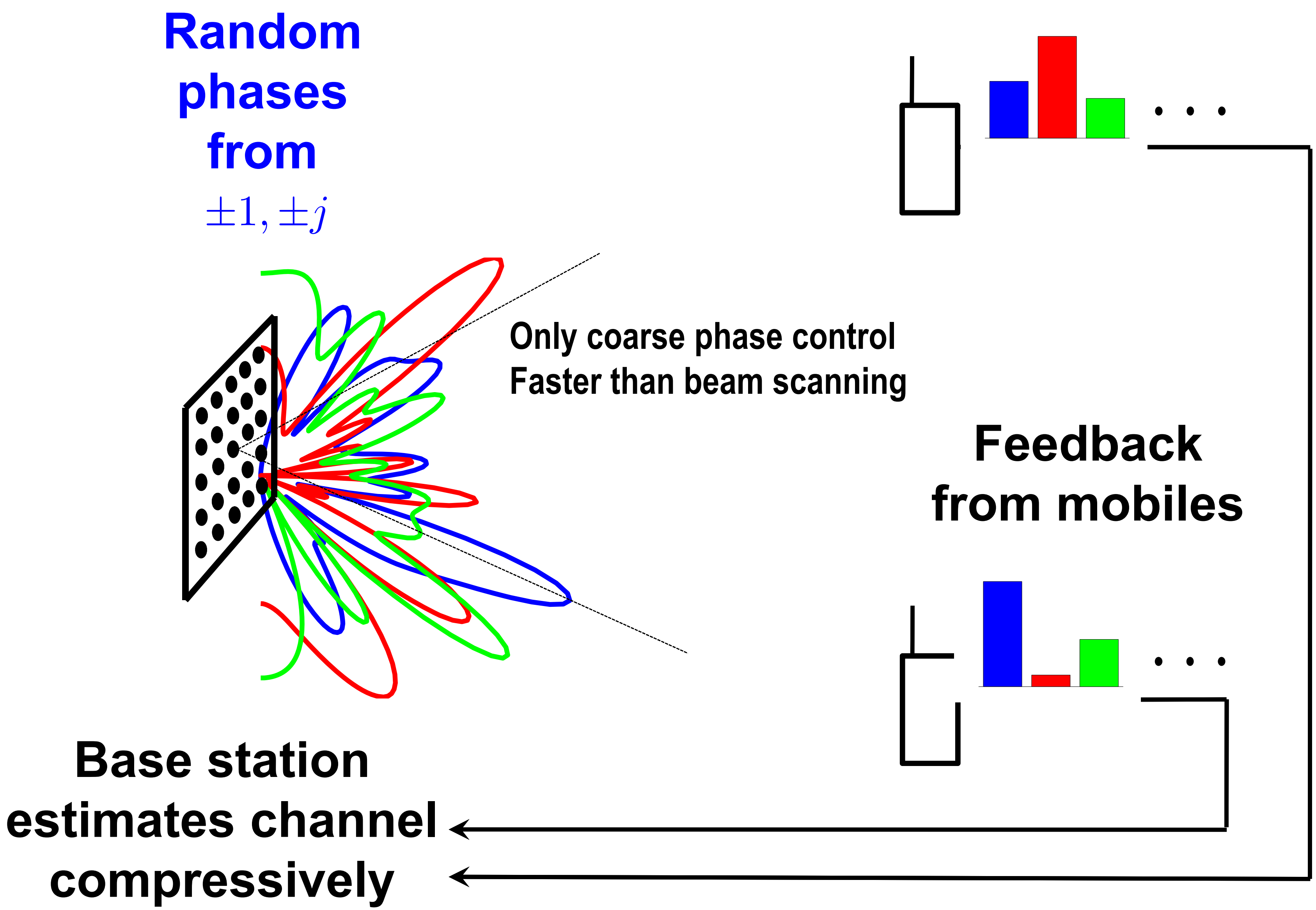
Two paths

Experiments on UCSB campus
using FB Terragraph nodes

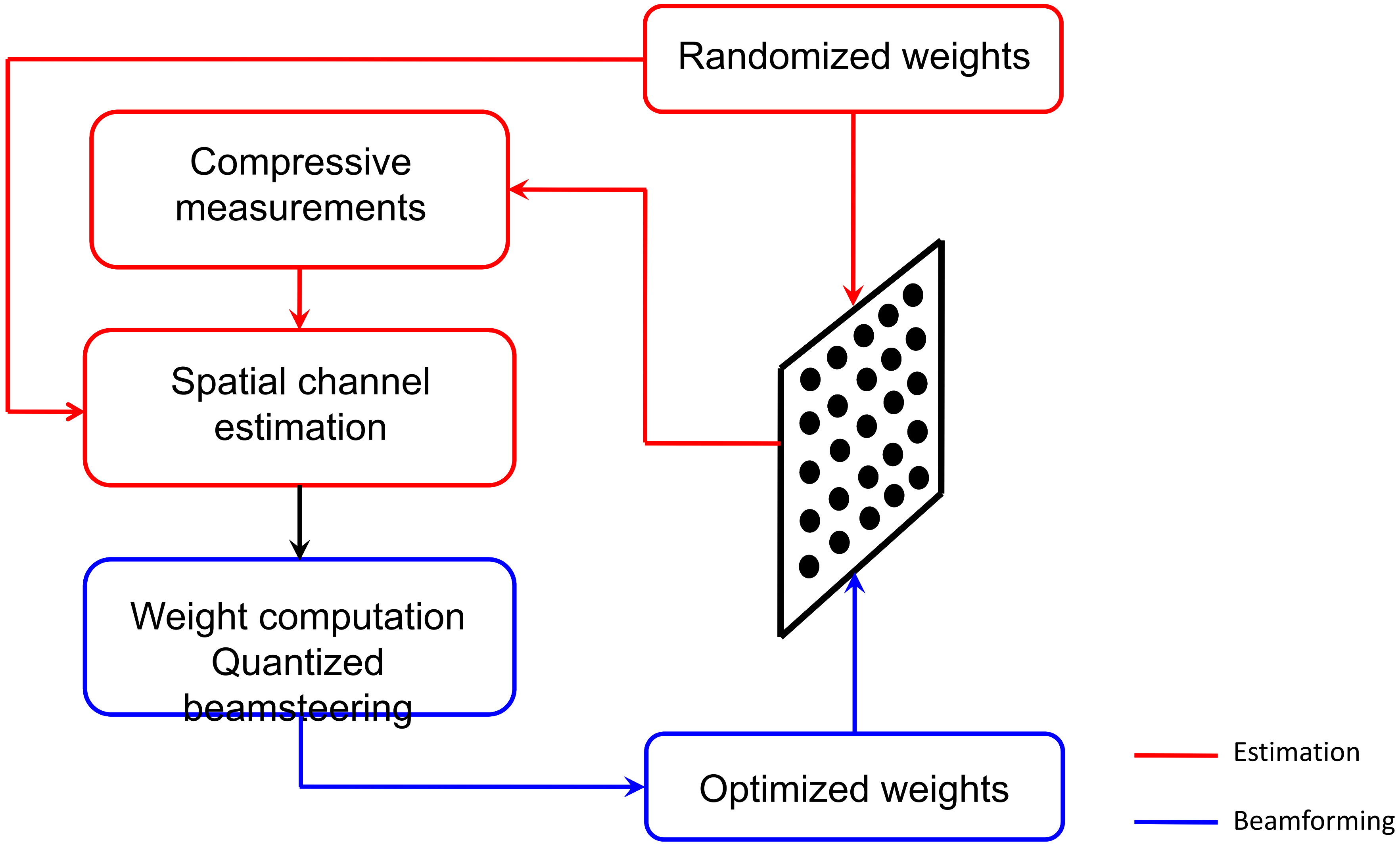
Can we exploit the sparsity of the mm wave channel?



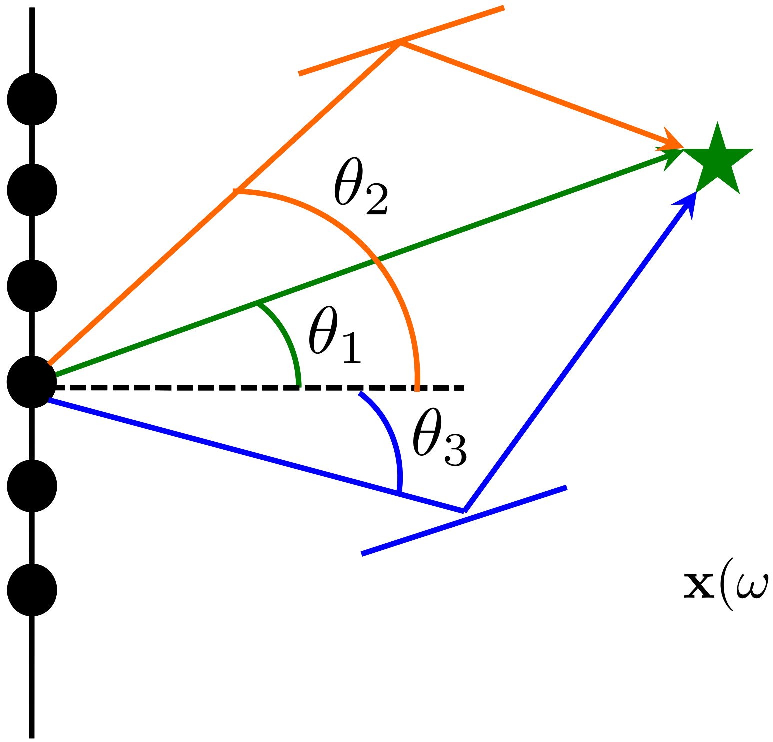
Compressive adaptation



Compressive Adaptation Architecture



Estimation problem



Channel is a sum of a few sinusoids

$$\mathbf{h} = g_1\mathbf{x}(\omega_1) + g_2\mathbf{x}(\omega_2) + g_3\mathbf{x}(\omega_3)$$

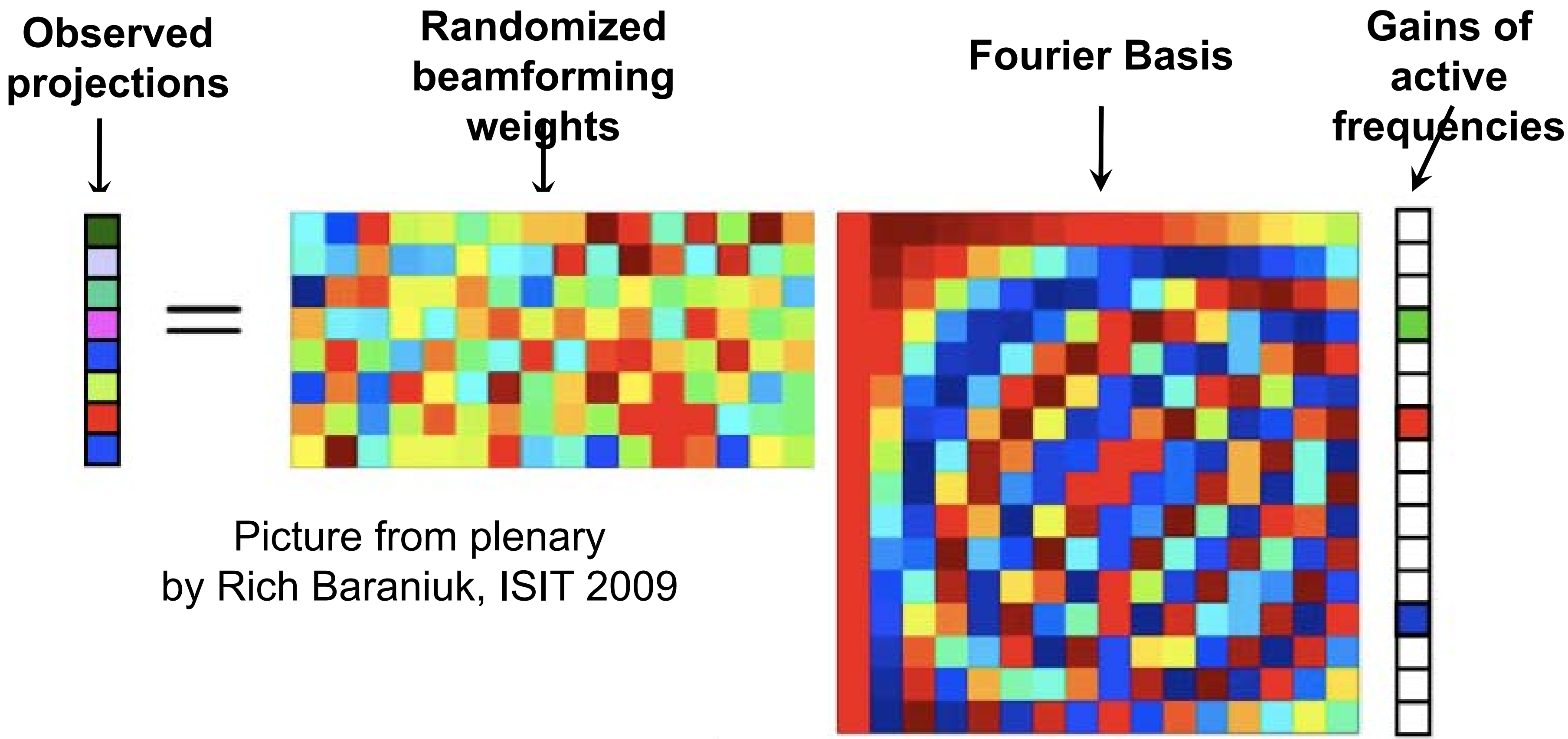
$$\mathbf{x}(\omega) = \begin{pmatrix} 1 \\ e^{j\omega} \\ e^{j2\omega} \\ \vdots \\ e^{j(N-1)\omega} \end{pmatrix} \quad \omega_i = \frac{2\pi d}{\lambda}\sin\theta_i$$

Mobile makes compressive measurements

$$y_i = \mathbf{a}_i^T\mathbf{h}, i = 1, 2, \ldots, M$$

Estimate gains and spatial frequencies from compressive measurements

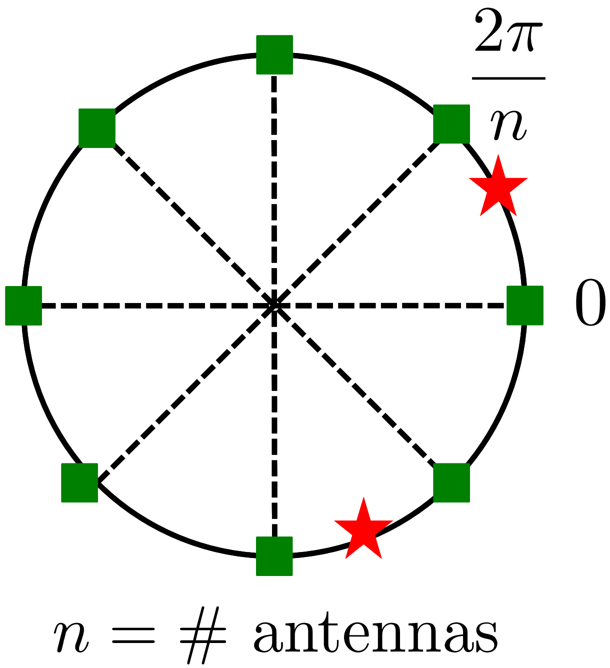
Can we use standard compressed sensing?



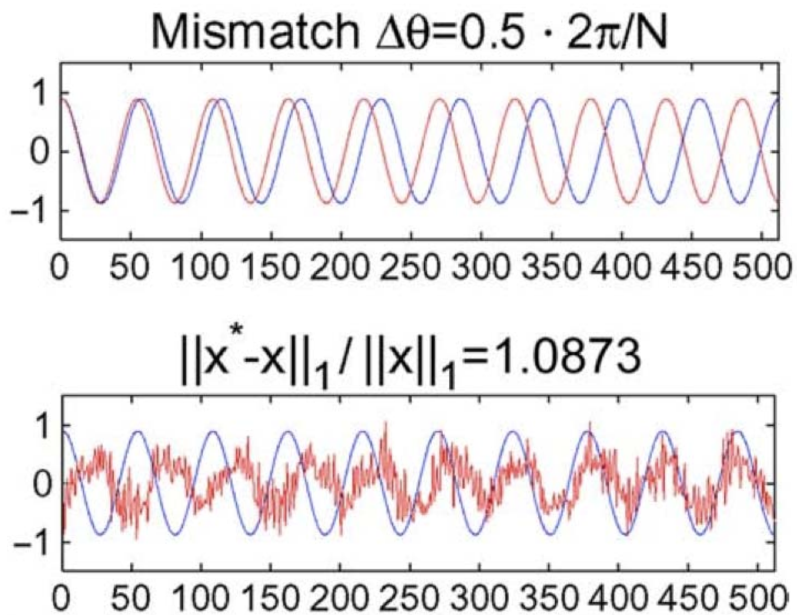
Picture from plenary
by Rich Baraniuk, ISIT 2009

$$\mathbf{y} = \sum_{k=1}^{L} g_k \mathbf{A}\mathbf{x}(\omega_k) + \mathbf{n}$$

Not quite: because of basis mismatch



Frequencies come from a continuum, not a grid



With standard CS, off-grid frequencies can have large estimation errors

Sensitivity to Basis Mismatch in Compressed Sensing,
Y. Chi, L. Scharf, A. Pezeshki, R. Calderbank

Need algorithms and theory for compressive estimation!

Key Results

- Compressive estimation is equivalent to regular estimation if certain isometries are preserved
- Equivalence characterized based on fundamental estimation-theoretic bounds
  - Ziv-Zakai bound, Cramer-Rao bound
- Super-resolution algorithms for regular estimation will work for compressive estimation as well
  - State of the art algorithm: NOMP
- Compressive estimation is a promising basis for a picocellular architecture

Plan

- Review of fundamental estimation-theoretic bounds
  - Ziv-Zakai bound in particular (because it deserves more publicity than it has gotten)
- Super-resolution algorithm for estimating a mixture of sinusoids
  - Newtonized Orthogonal Matching Pursuit (NOMP)
- Theory of compressive estimation
- Results for picocellular settings

Ziv-Zakai bound reviewed on the board

Discussion based on:
Bell, Steinberg, Ephraim, Van Trees, “Extended Ziv-Zakai Lower Bound for Vector Parameter Estimation,” *IEEE Trans. Information Theory,* March 1997.

Original paper on ZZB:
J. Ziv and M. Zakai, “Some lower bounds on signal parameter estimation,” *IEEE Trans. Information Theory,* May 1969.

Frequency Estimation for a Mixture of Sinusoids: A Near-Optimal Sequential Approach

“Newtonized Orthogonal Matching Pursuit”

B. Mamandipoor*, D. Ramasamy, U. Madhow

ECE Department, University of California, Santa Barbara

December 2015

References

B. Mamandipoor, D. Ramasamy, U. Madhow, “Frequency estimation for a mixture of sinusoids: a near-optimal sequential approach,” GlobalSIP 2015.

B. Mamandipoor, D. Ramasamy, U. Madhow, “Newtonized Orthogonal Matching Pursuit: Frequency Estimation over the Continuum,” to appear, IEEE Trans. Signal Processing.

Outline

- Introduction
- Proposed sequential algorithm
- Stopping criteria: CFAR
- Convergence
- Performance evaluation

Formulation

Unit norm sinusoid of frequency $\omega \in [0, 2\pi)$

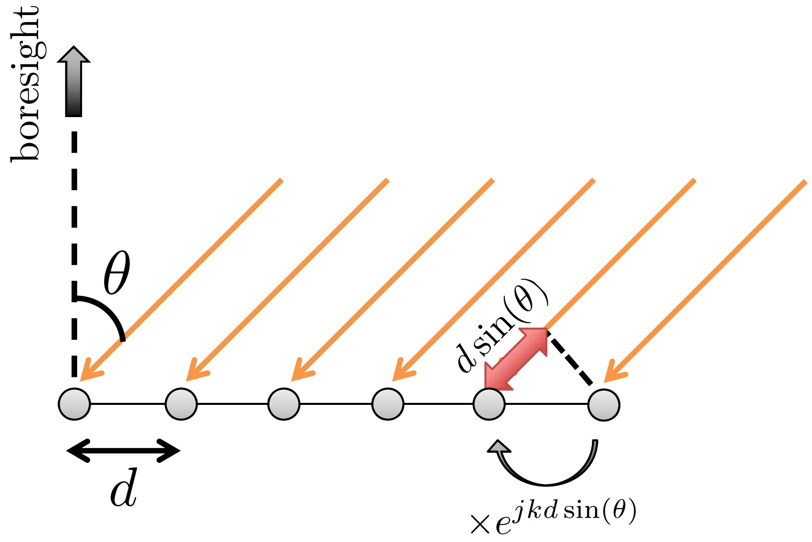
$$\mathbf{x}(\omega) = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & e^{j\omega} & \dots & e^{j(N-1)\omega} \end{bmatrix}^T$$

Mixture of sinusoids:

$$\underbrace{\mathbf{y}}_{\in \mathbb{C}^N} = \sum_{l=1}^{K} \underbrace{g_l}_{\in \mathbb{C}} \mathbf{x}(\omega_l) + \underbrace{\mathbf{z}}_{\sim CN(\mathbf{0}, \sigma^2 \mathbb{I})}$$

Goal: Estimate $\{(g_l, \omega_l), l = 1, 2, \dots, K\}$ and K

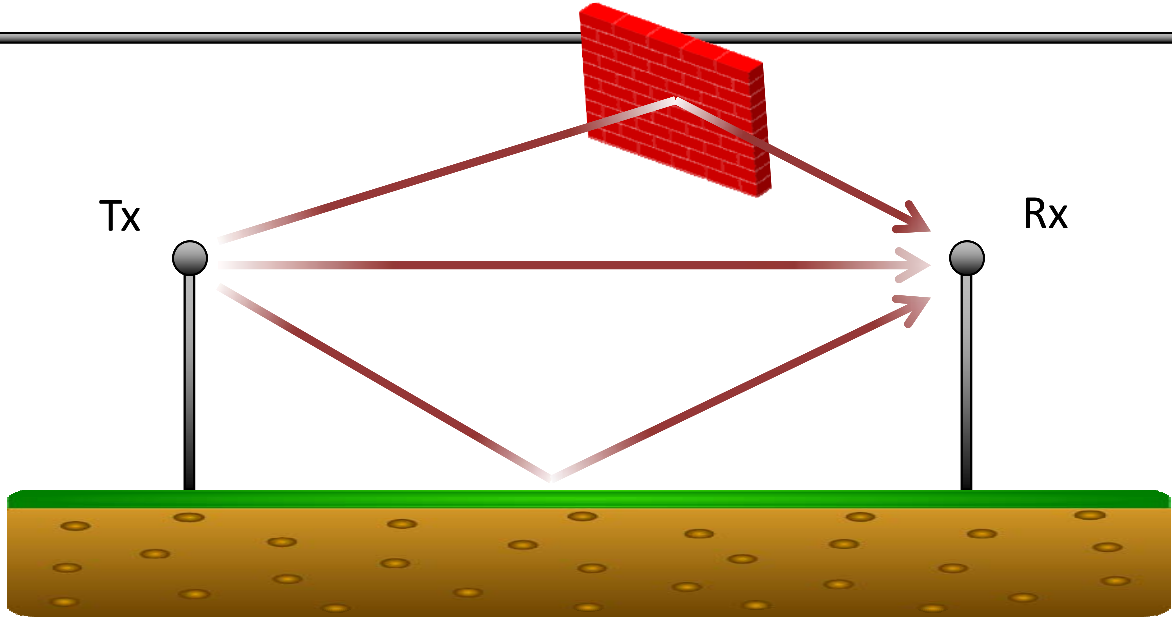
DoA Estimation



$$\omega = kd\sin(\theta) \rightarrow \mathbf{y} = g\mathbf{x}(\omega)$$

Direction of Arrival estimation $\rightarrow$ Frequency estimation problem

Multipath Channel Estimation



Channel impulse response: $h(t) = \sum_{l=1}^{K} g_l \delta(t - \tau_l)$

Channel transfer function: $H(f) = \sum_{l=1}^{K} g_l e^{-j2\pi f \tau_l}$

Sample Uniformly in Frequency Domain

$$H = \sum_{l=1}^{K} g_l \mathbf{x}(\omega_l) \qquad \omega_l = -2\pi \Delta f \tau_l$$

Single Frequency Estimation

Unknown parameters are frequency and complex gain.

Maximum Likelihood: $\max_{\{g,\omega\}} 2\Re\{\mathbf{y}^H g\mathbf{x}(\omega)\} - |g|^2||\mathbf{x}(\omega)||^2$

GLRT: first maximize over all possible complex gains, then maximize over frequencies.

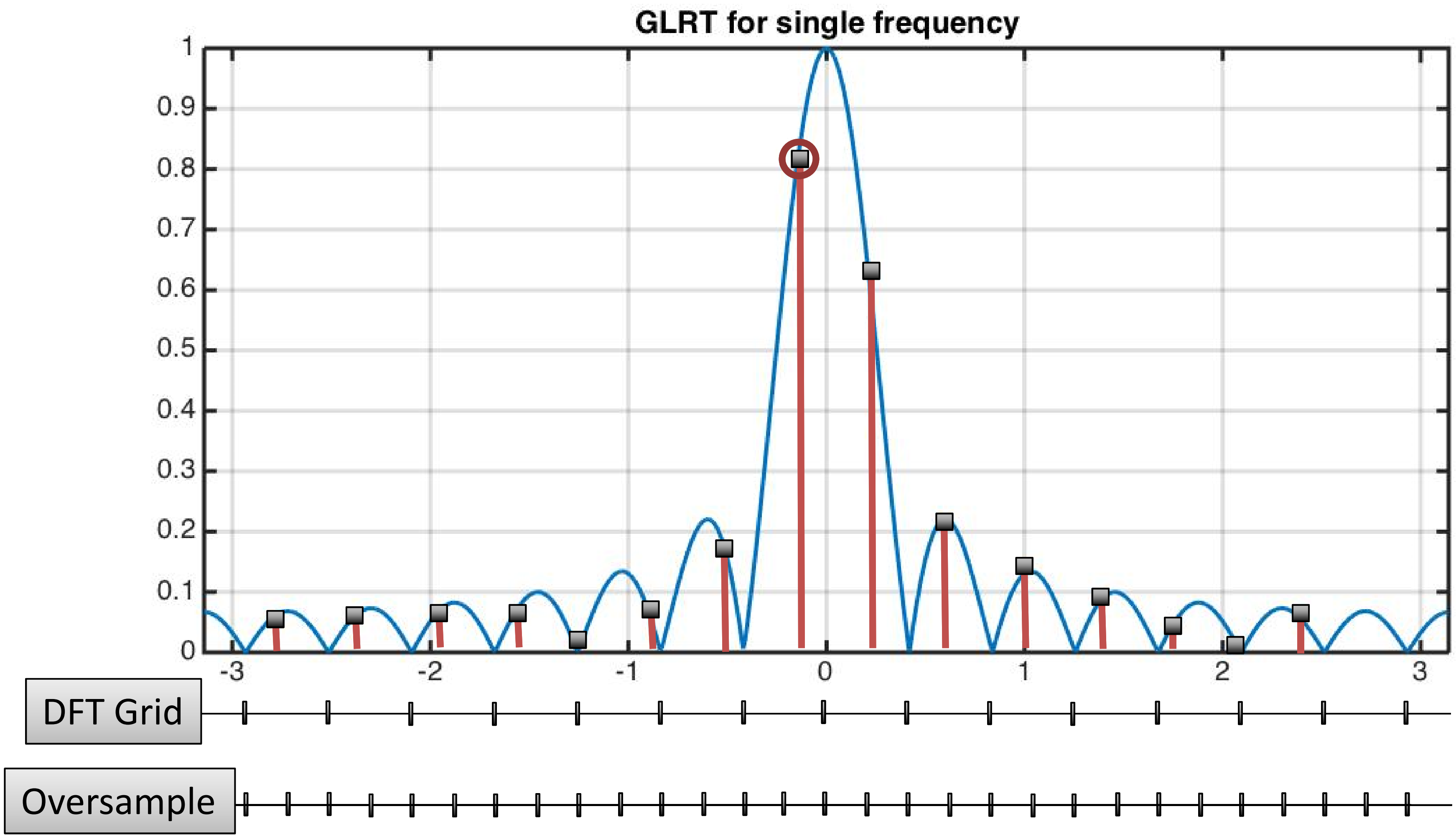
$$\max_{g} 2\Re\{\mathbf{y}^H g\mathbf{x}(\omega)\} - |g|^2||\mathbf{x}(\omega)||^2 \quad \longrightarrow \quad \hat{g} = \frac{\mathbf{x}(\omega)^H\mathbf{y}}{||\mathbf{x}(\omega)||^2}$$

Next, we maximize for the frequency: $\max_{\omega} G_{\mathbf{y}}(\omega)$

$$\text{Where,} \quad G_{\mathbf{y}}(\omega) = \frac{|\mathbf{x}(\omega)^H\mathbf{y}|^2}{||\mathbf{x}(\omega)||^2}$$

Grid & Refine

Grid & Refine


GLRT for single frequency
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-3
-2
-1
0
1
2
3
DFT Grid
Oversample

Newtonized OMP (NOMP)

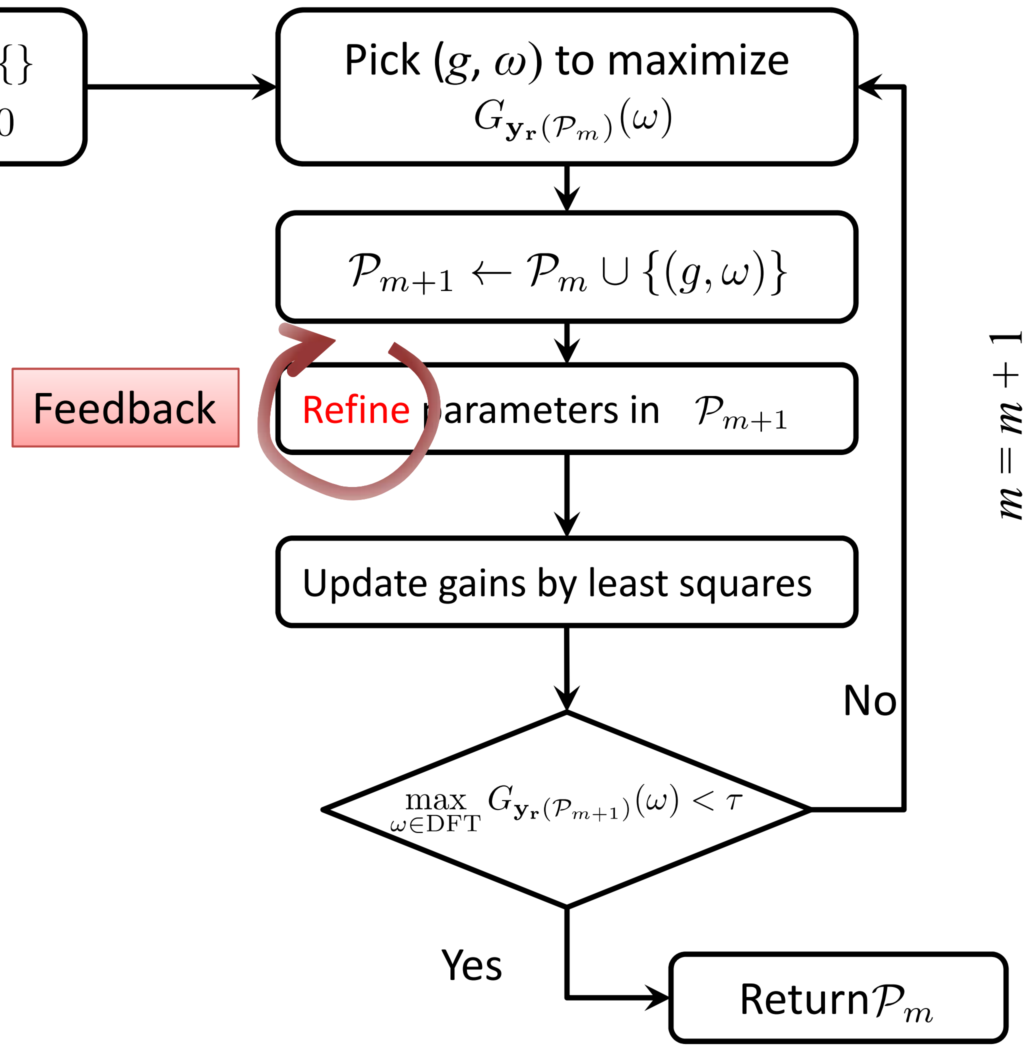
GLRT cost function

$$G_{\mathbf{y}}(\omega) = \frac{|\mathbf{x}(\omega)^H \mathbf{y}|^2}{||\mathbf{x}(\omega)||^2}$$

Residual response

$$\mathcal{P} = \{(g_l, \omega_l), l = 1, \ldots, k\}$$

$$\mathbf{y_r}(\mathcal{P}) = \mathbf{y} - \sum_{l=1}^{k} g_l \mathbf{x}(\omega_l)$$



[1] B. Mamandipoor, D. Ramasamy, U. Madhow, “Newtonized Orthogonal Matching Pursuit: Frequency Estimation over the Continuum,”arXiv preprint arXiv:1509.01942, 2015.

Stopping Criteria: CFAR

- Common strategy in detection problems → use noise model only
- Criterion: if noise can explain the observation, then assume no target
- We develop a similar criteria for the frequency estimation algorithm:

$$\max_{\omega \in \mathrm{DFT}} G_{\mathbf{y_r}(\mathcal{P}_{m+1})}(\omega) < \tau \qquad \text{Assuming } \mathbf{y_r}(\mathcal{P}_{m+1}) \text{ is pure noise!}$$

$$||\mathcal{F}\mathbf{y_r}(\mathcal{P}_{m+1})||_{\infty}^{2} < \tau$$

Probability of false alarm:

$$Pr\{||\mathcal{F}\mathbf{y_r}(\mathcal{P}_{m+1})||_{\infty}^{2} > \tau\} = P_{\mathrm{fa}}$$

Stopping Criteria: CFAR

P_{fa} is the nominal false alarm rate.

$$\tau = \sigma^2 \log(N) - \sigma^2 \log\log\left(\frac{1}{1-P_{\text{fa}}}\right)$$

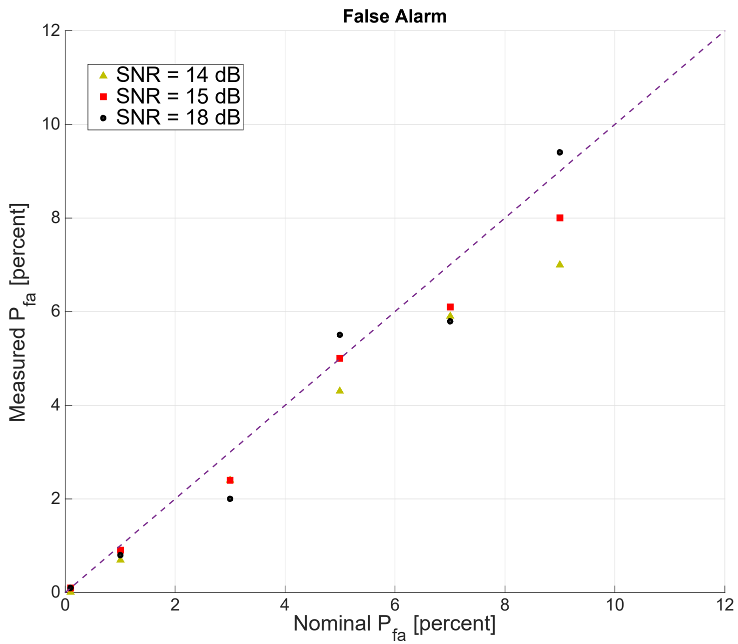
300 runs of NOMP

#sinusoids $K = 16$

#observations $N = 256$

fixed nominal SNR

$\Delta\omega_{\text{min}} = 2.5\Delta_{\text{dft}}$



Simulation result: measured false alarm rate is in agreement with the the nominal value.

Probability of Miss and ROC

Taking into account the effect of noise

Ignoring the "interference" from other sinusoids

$$P_{\text{miss}} \approx 1 - Q_1\left(0.88\sqrt{2SNR}, \sqrt{2\tau/\sigma^2}\right)$$

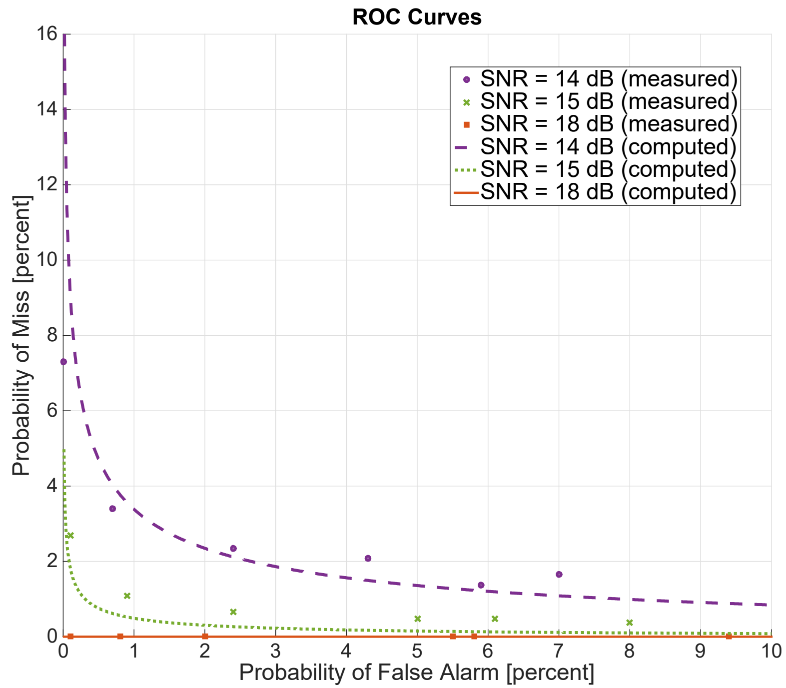
300 runs of NOMP

#sinusoids $K = 16$

#observations $N = 256$

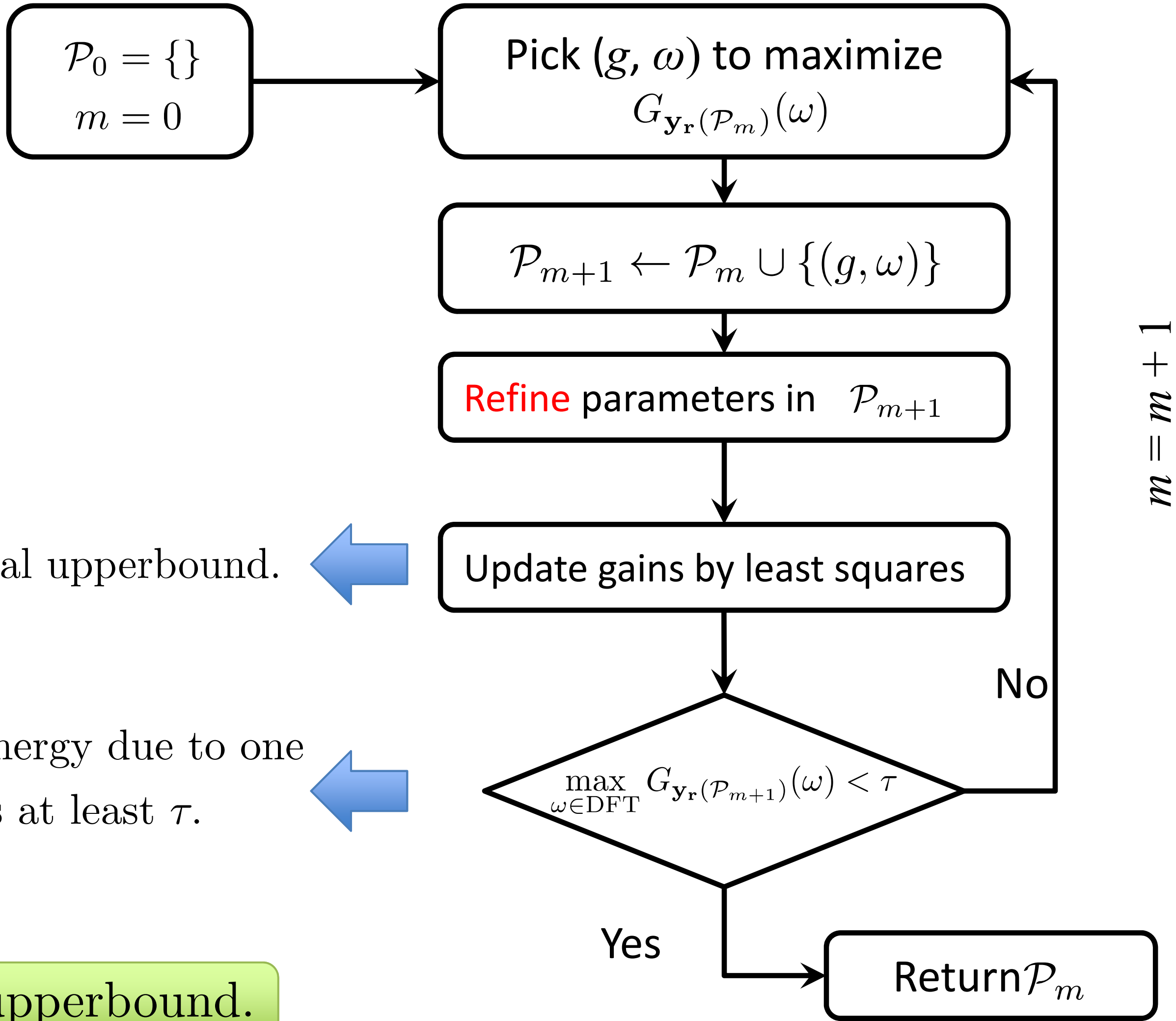
fixed nominal SNR

$\Delta\omega_{\min} = 2.5\Delta_{\text{dft}}$



The resulting ROC turns out to be in remarkable agreement with simulations.

Convergence: bounding # iterations



#observations (N) is a trivial upperbound.

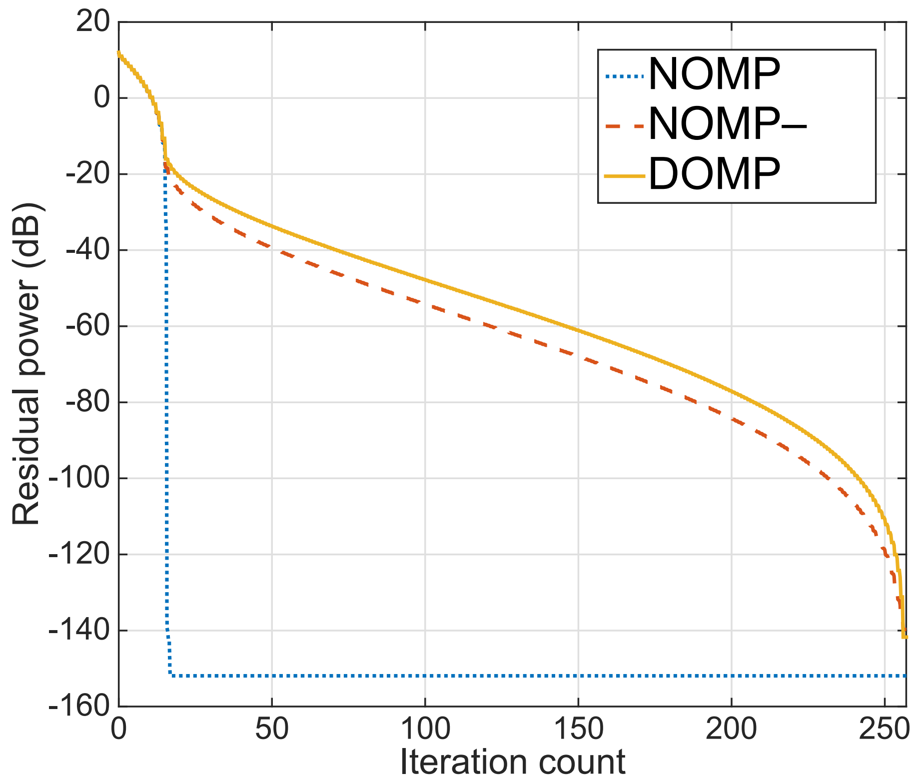
Reduction of the residual energy due to one iteration of the algorithm is at least τ.

$\min\{N, ||\mathbf{y}||^2/\tau\}$ is an upperbound.

Empirical Convergence Rate

NOMP – : just a single refinement step for the newly detected sinusoid
DOMP : discretize the parameter space and apply OMP

average over 1000 runs
#sinusoids $K = 16$
fixed nominal SNR
$\Delta\omega_{\min} = 2.5\Delta_{\text{dft}}$

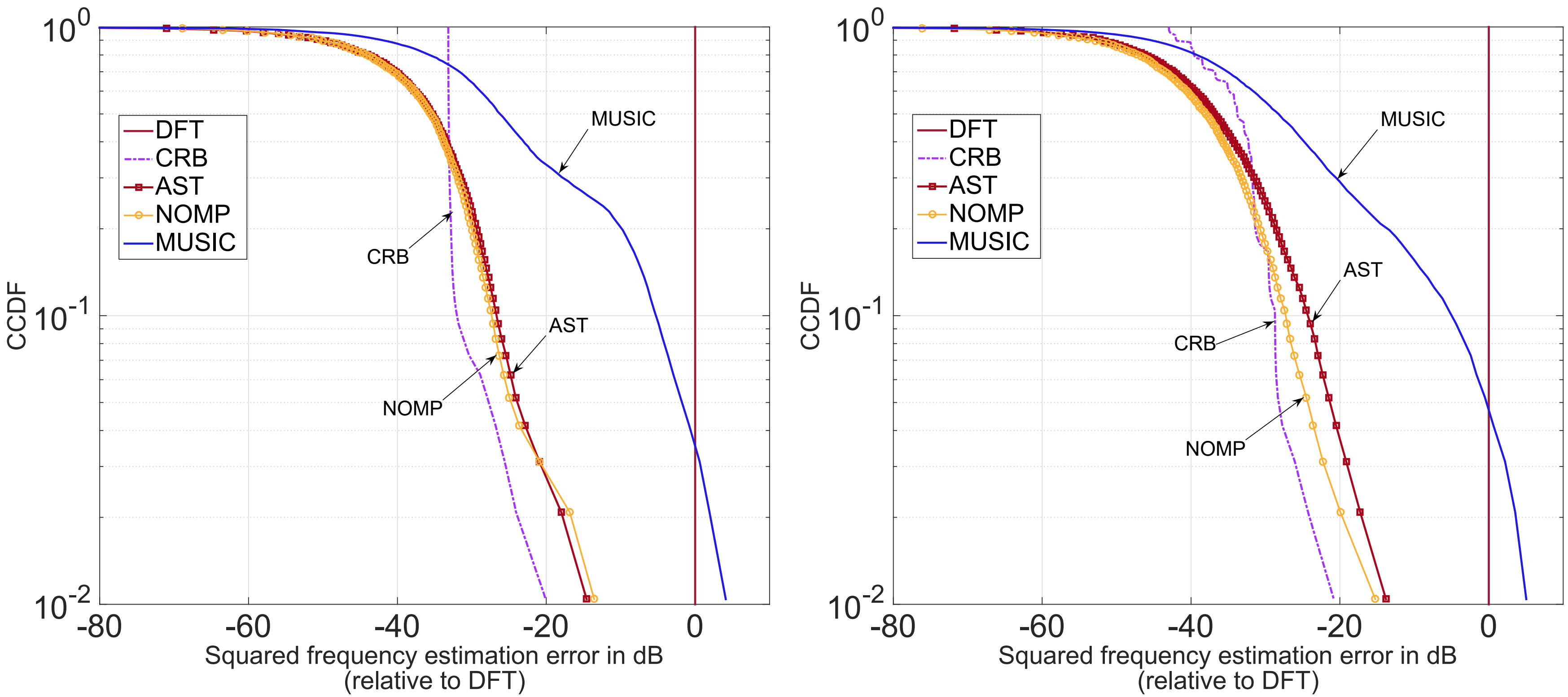


Cyclic refinements are critical for speeding up convergence

Performance – Accuracy

Fixed SNR: 25 dB

SNR ~ Uniform[15,35)

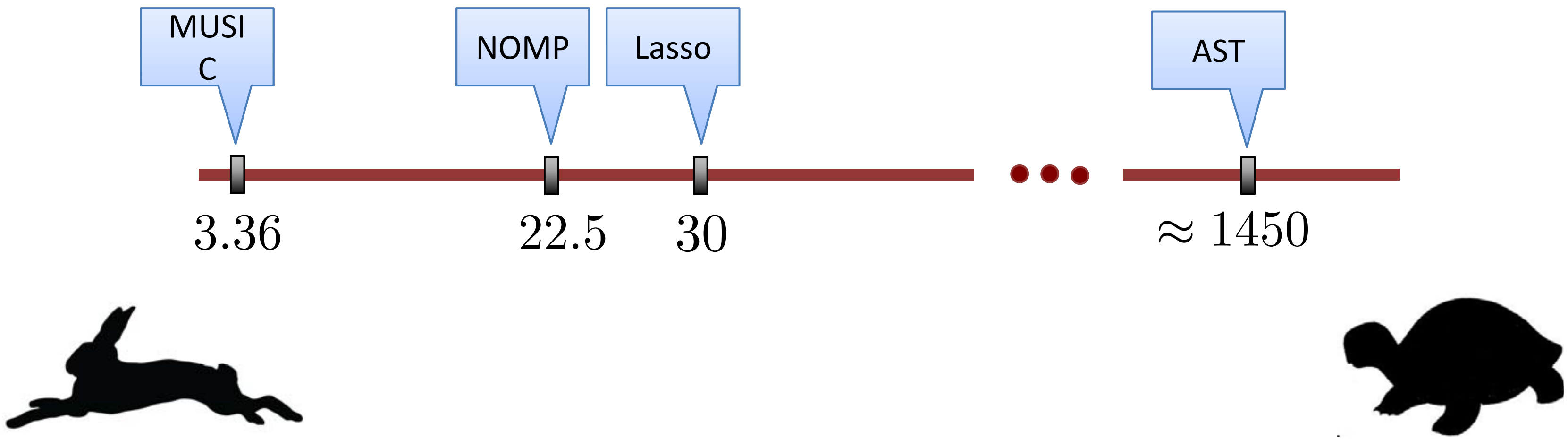


Comparison with state-of-the-art algorithm: Atomic norm Soft Thresholding (AST)

[2] B. N. Bhaskar, G. Tang, and B. Recht, "atomic norm denoising with applications to line spectral estimation," arXiv preprint arXiv:1204.0562, 2012

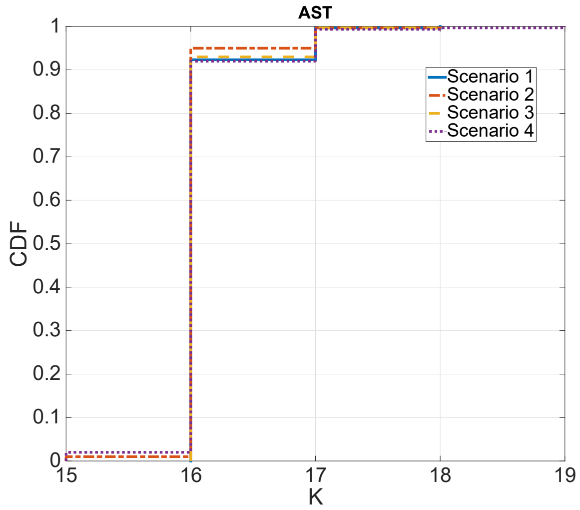
Performance – Speed

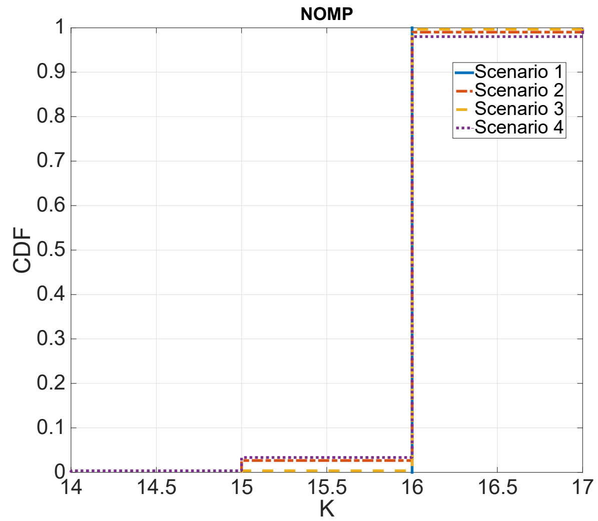
Run time of various algorithms over 300 simulation runs
(#sinusoids in the mixture =16)



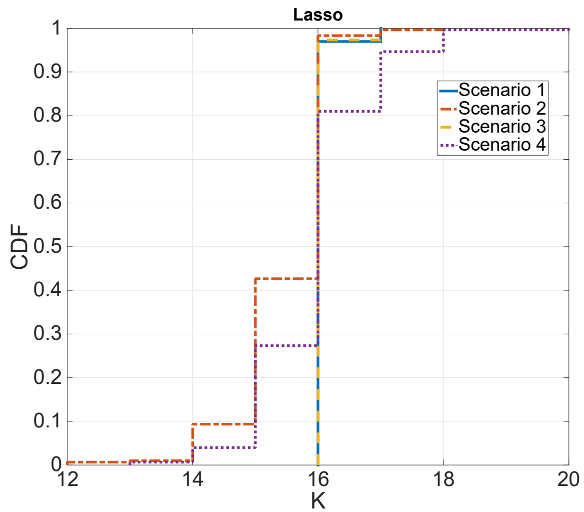
Model Order (K) Estimation

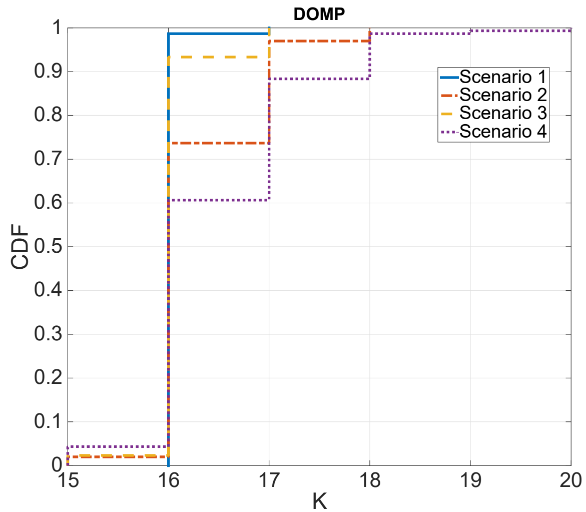
Scenarios	SNR (dB)	$\Delta\omega_{\text{min}}/\Delta_{\text{dft}}$
1	25	2.5
2	25	0.5
3	Uniform$[15, 35]$	2.5
4	Uniform$[15, 35]$	0.5





When minimum separation between frequencies is small, Lasso and DOMP make errors!





Takeaways

- Near-optimal sequential algorithm for the problem of estimating frequencies and gains in a noisy mixture of sinusoids.
- Superior estimation accuracy compared to state-of-the-art with significantly lower computational complexity.
- CFAR-based stopping criteria that leads to accurate model order estimation.
- Characterizing the effect of Newton refinements of the rate of convergence remains as an open issue.

You can download a MATLAB implementation of the algorithm here:
https://bitbucket.org/wcslspectralestimation/continuous-frequency-estimation/src/NOMP

Questions??

Sparse Approximation: Formulation

Find sparse $\mathbf{g}$ such that $\mathbf{y} = \mathbf{X}(\omega)\mathbf{g}$

fat matrix

$\#\text{rows} = N$

$\#\text{columns} = \infty$

Finding maximally sparse representation:

$$\min_{\mathbf{g}} ||\mathbf{g}||_0 \text{ such that } \mathbf{y} = \mathbf{X}(\omega)\mathbf{g}$$

known to be NP-hard!

Sparse Approximation: Methods

Convex Optimization

Atomic Norm Soft Thresholding
Basis Pursuit
Lasso
…

Greedy Methods

Matching Pursuit
Orthogonal Matching Pursuit (OMP)
…
Newtonized OMP (NOMP)

Sparse Approximation

Convex Optimization

Atomic Norm Soft Thresholding
Basis Pursuit
Lasso
…

Greedy Methods

Matching Pursuit
Orthogonal Matching Pursuit
…

	Convex Optimization	Greedy Methods
Convergence Guarantees:	Yes	Yes
Performance :	Consistently good	Surprisingly good
Computational Complexity:	High	Low

Cyclic Refinement

- Interpreted as a Feedback mechanism

$$\text{maximize}_{g,\omega} S(g,\omega)$$

$$S(g,\omega) = \; 2\Re\{\mathbf{y}^H g\mathbf{x}(\omega)\} - |g|^2||\mathbf{x}(\omega)||^2$$

$$\text{Newton step: } \hat{\omega} = \omega - \dot{S}(g,\omega)/\ddot{S}(g,\omega)$$

Refinement Acceptance Condition (RAC): we accept refinement step only if residual energy decreases.

Rate of convergence

Maximizing the GLRT cost function over [0,2π] is consistent with that over the oversampled grid Ω with oversampling factor γ[2].

$$\begin{aligned}\max_{\omega\in\Omega}\sqrt{G_{\mathbf{y}}(\omega)} &\leq \sup_{\omega\in[0,2\pi)}\sqrt{G_{\mathbf{y}}(\omega)} \\ &\leq \left(1-\frac{2\pi}{\gamma}\right)^{-1}\max_{\omega\in\Omega}\sqrt{G_{\mathbf{y}}(\omega)}.\end{aligned}$$

Atomic Norm Definition:

The atomic set of unit norm sinusoids: $\mathcal{A}=\{e^{j\phi}\mathbf{x}(\omega):\phi,\omega\in[0,2\pi)\}$

Atomic norm for $\mathbf{s}$: $||\mathbf{s}||_{\mathcal{A}} \triangleq \inf\{t>0:\mathbf{s}\in t\ \mathrm{conv}(\mathcal{A})\}$

$$||\mathbf{s}||_{\mathcal{A}}=\inf\left\{\sum_l |g_l| : \mathbf{s}=\sum_l g_l\mathbf{x}(\omega), \mathbf{x}(\omega)\in\mathcal{A}\right\}$$

[2] B. N. Bhaskar, G. Tang, and B. Recht, “atomic norm denoising with applications to line spectral estimation,” arXiv preprint arXiv:1204.0562, 2012

Rate of Convergence

Bound on the *rate of convergence*:

$$||\mathbf{y}_\mathbf{r}(\mathcal{P}_m)|| \leq (m+1)^{-1/2} \left(1 - \frac{2\pi}{\gamma}\right)^{-1} ||\mathbf{y}||_\mathcal{A}$$

Oversampling Factor
compared to DFT

Comparing with OMP over the continuum:

$$||\mathbf{y}_\mathbf{r}(\mathcal{P}_m)|| \leq (m+1)^{-1/2} ||\mathbf{y}||_\mathcal{A}$$

Discretizing the parameter space does not cost us much in terms of convergence rate.

Open problem: characterizing the effect of Newton refinements on increasing the convergence rate?

AST

Mixture of sinusoids:

$$\underbrace{\mathbf{y}}_{\in \mathbb{C}^N} = \sum_{l=1}^{K} \underbrace{g_l}_{\in \mathbb{C}} \mathbf{x}(\omega_l) + \mathbf{z}$$

The atomic set of unit norm sinusoids: $\mathcal{A} = \{e^{j\phi}\mathbf{x}(\omega) : \phi, \omega \in [0, 2\pi)\}$

Atomic norm for $\mathbf{s}$: $||\mathbf{s}||_{\mathcal{A}} \triangleq \inf\{t > 0 : \mathbf{s} \in t \text{ conv}(\mathcal{A})\}$

When centroid of the conv$(\mathcal{A})$ is at the origin, the atomic norm can be rewritten as

$$||\mathbf{s}||_{\mathcal{A}} = \inf \left\{ \sum_{l} |g_l| : \mathbf{s} = \sum_{l} g_l \mathbf{x}(\omega), \mathbf{x}(\omega) \in \mathcal{A} \right\}$$

Measurement model: $\mathbf{y} = \mathbf{s} + \mathbf{z}$

AST formulation: $\text{minimize}_{\mathbf{s}} ||\mathbf{y} - \mathbf{s}||_2^2 + \lambda ||\mathbf{s}||_{\mathcal{A}}$

Lasso

- If we discretize the parameter space (grid of frequencies), AST formulation boils down to Lasso

$$\Phi : N \times M \text{ Fourier matrix}$$

$$||\mathbf{s}||_{\mathcal{A}_M} = \min\{||c||_1 : \mathbf{s} = \Phi c\}$$

$$\text{minimize}_{\mathbf{s}} ||\mathbf{y} - \mathbf{s}||_2^2 + \lambda ||\mathbf{s}||_{\mathcal{A}_M}$$

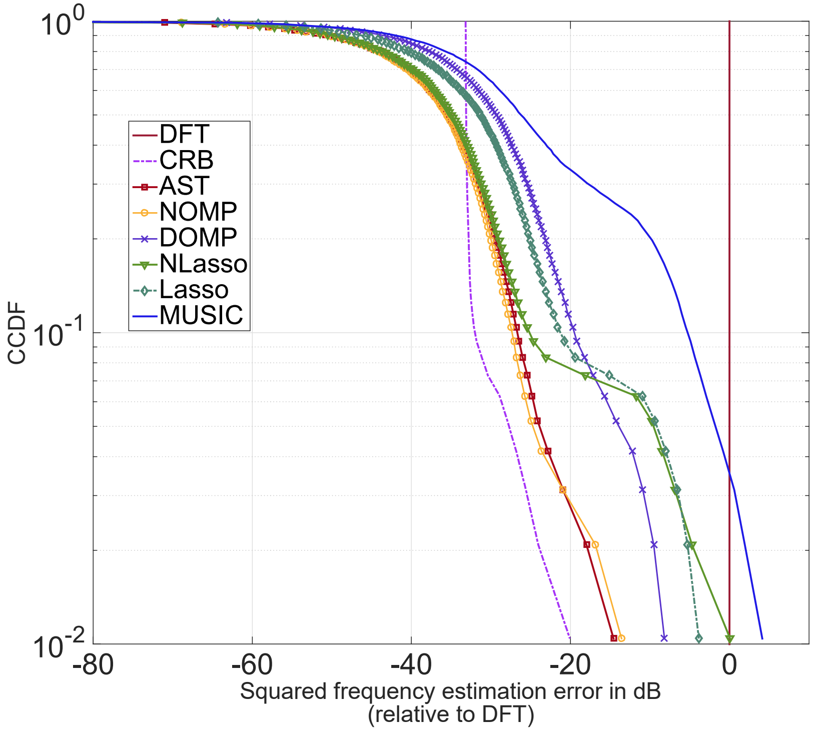
$$\Downarrow$$

$$\text{minimize}_c ||\mathbf{y} - \Phi c||_2^2 + \lambda ||c||_1$$

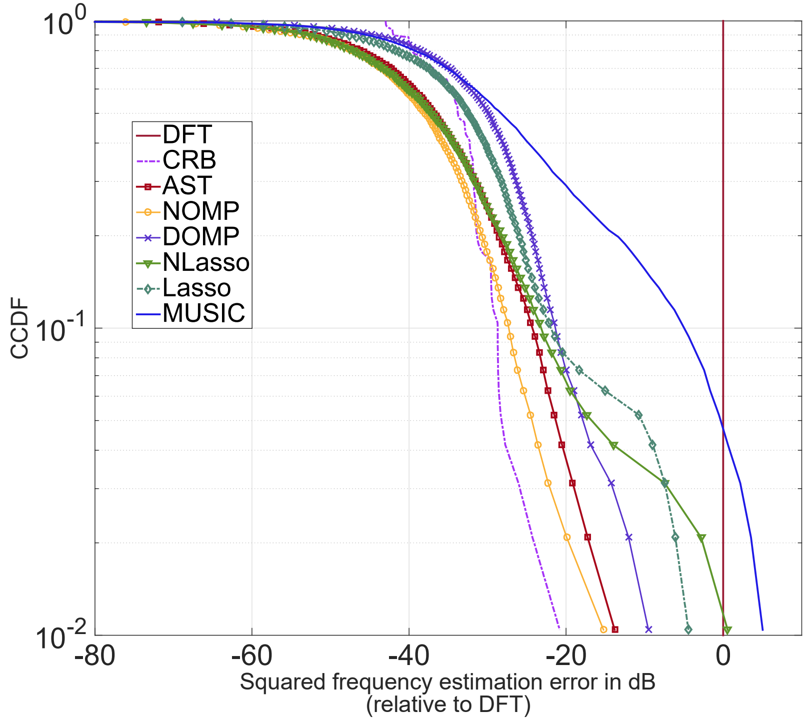
$$\text{Lasso} \rightarrow \ \ell_1\text{-regularized } \ell_2 \text{ minimization}$$

Performance – Accuracy

Fixed SNR: 25 dB



SNR ~ Uniform[15,35)



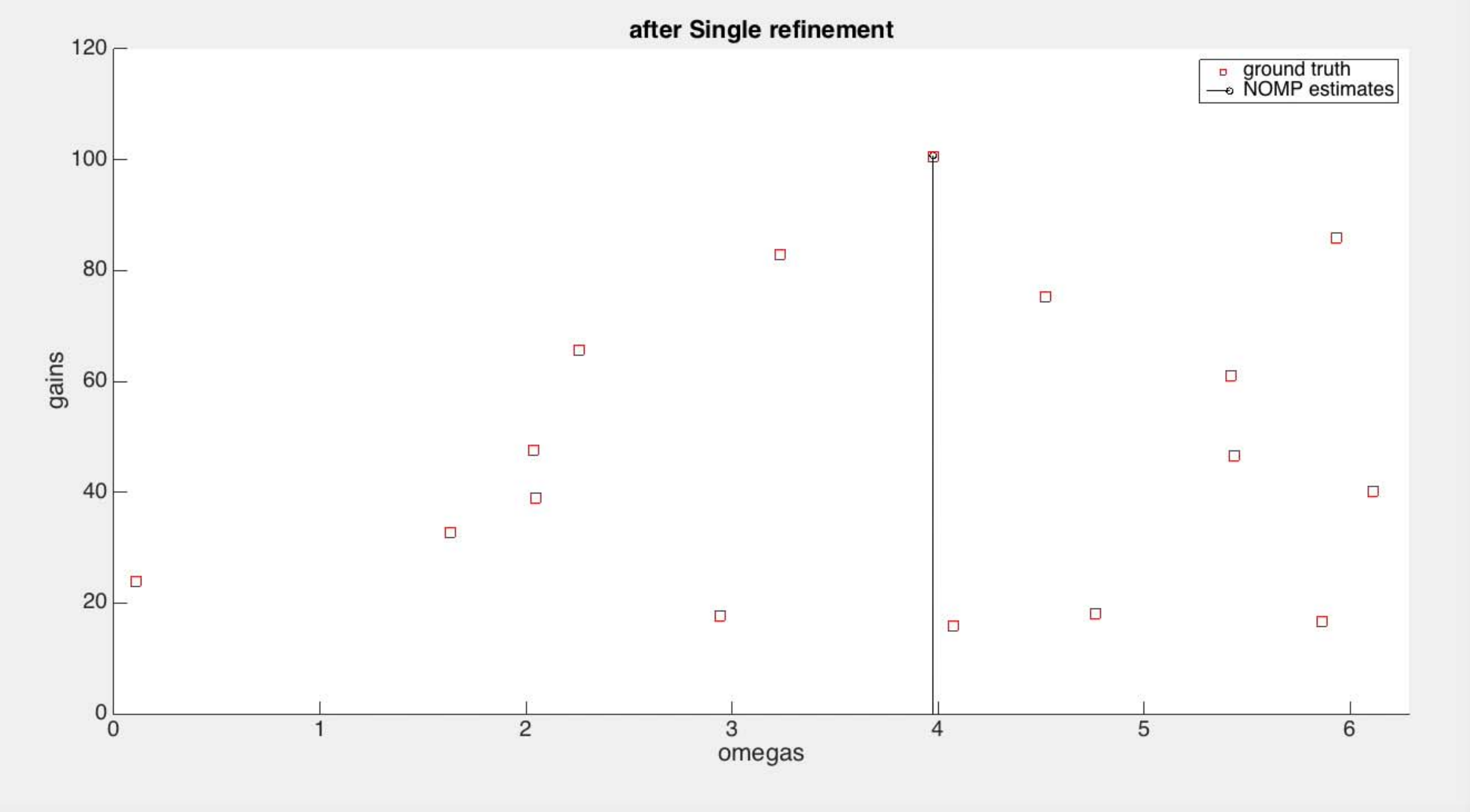
Performance – speed

Run time of various algorithms over 300 runs (#sinusoids =16)

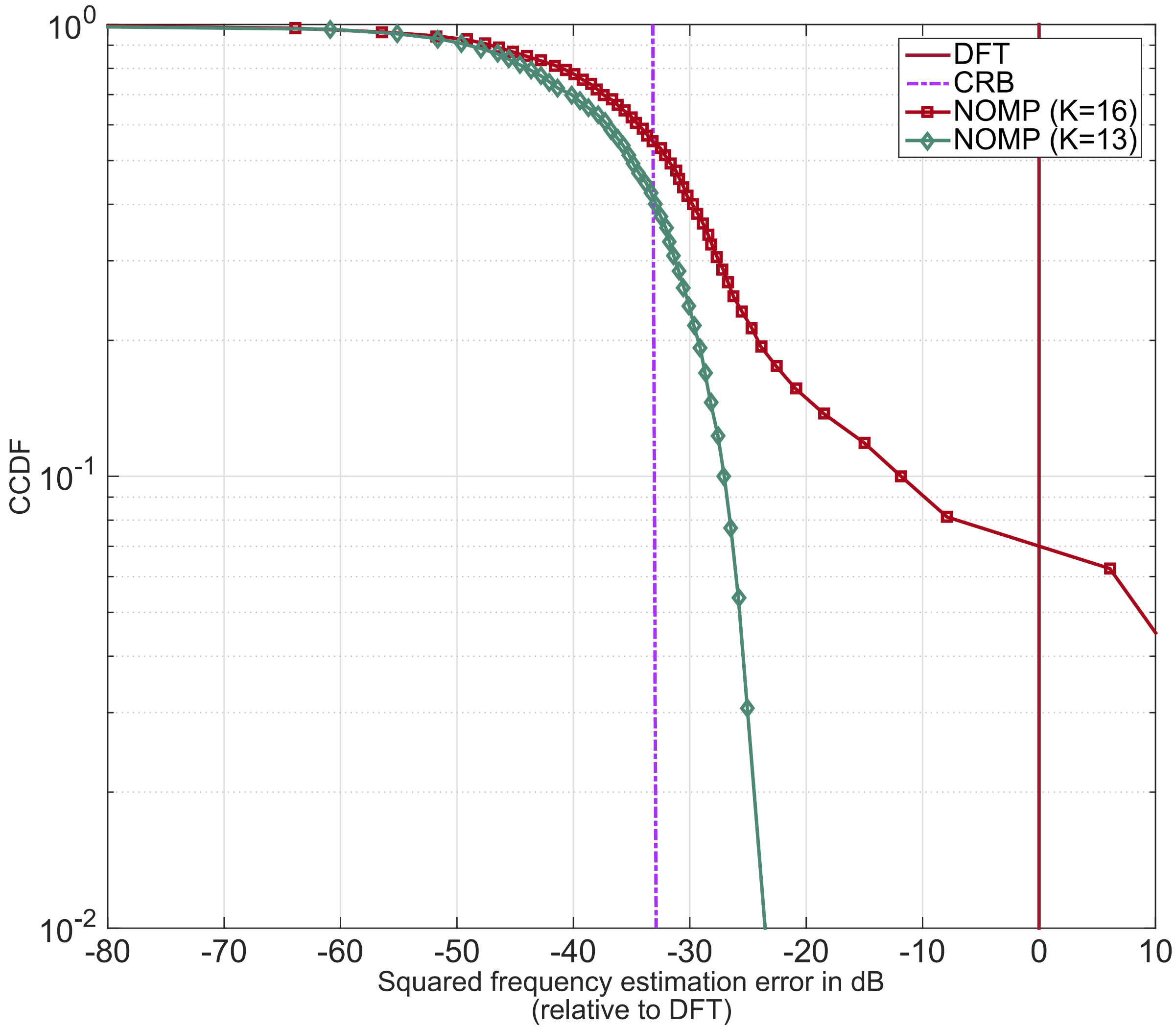
Scenarios	SNR (dB)	$\Delta\omega_{\min}/\Delta_{\mathrm{dft}}$
1	25	2.5
2	25	0.5
3	Uniform$[15, 35]$	2.5
4	Uniform$[15, 35]$	0.5

Time [sec]	NOMP	AST	NLasso	Lasso	DOMP	MUSIC
Scenario 1	22.79	1.27e3	26.62	23.68	2.69	3.26
Scenario 2	22.05	1.26e3	26.42	24.55	2.87	3.27
Scenario 3	22.40	1.38e3	32.37	29.28	2.82	3.17
Scenario 4	22.45	1.45e3	32.02	30.10	2.80	3.36

Performance – Animation



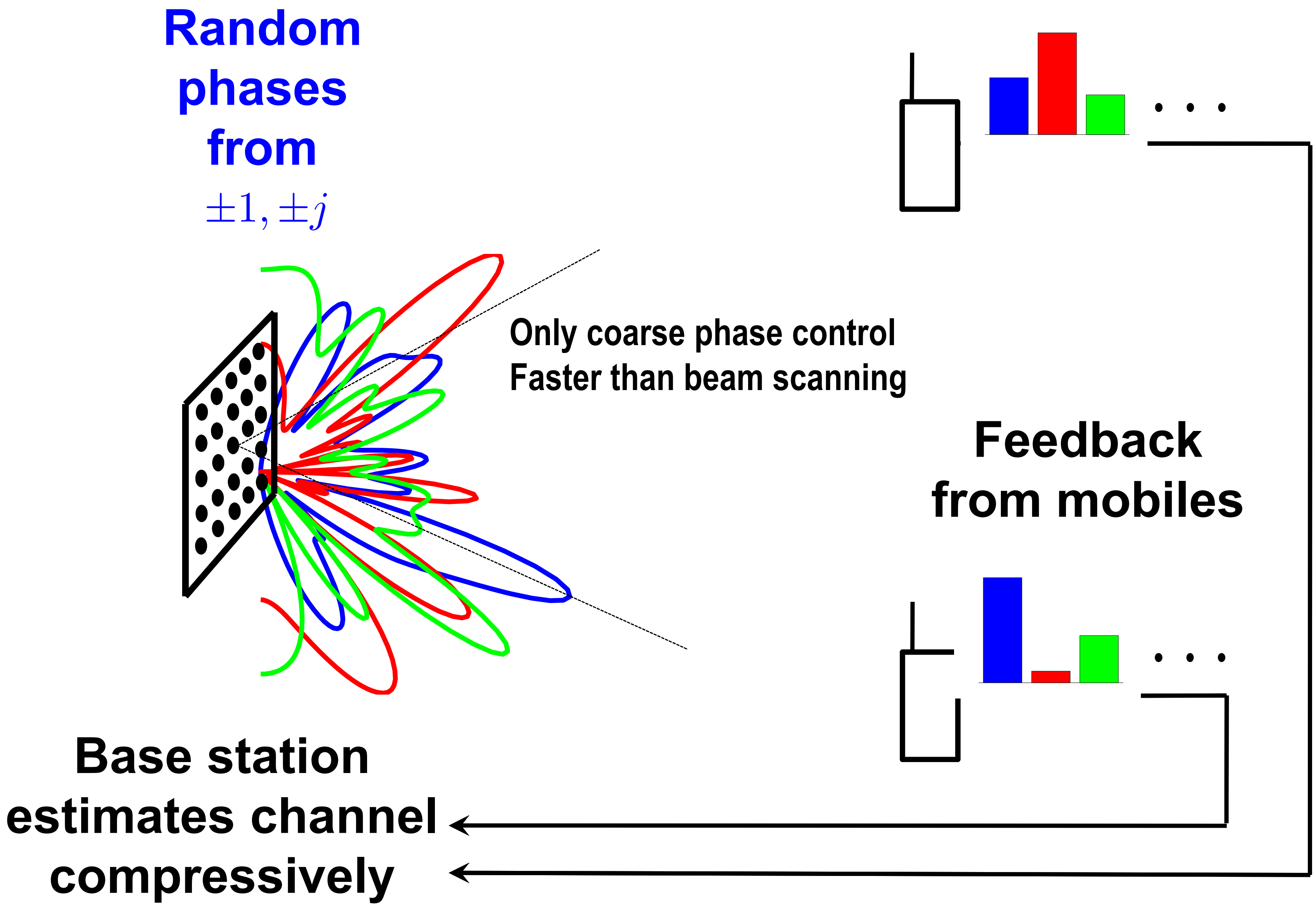
Compressive Acquisition



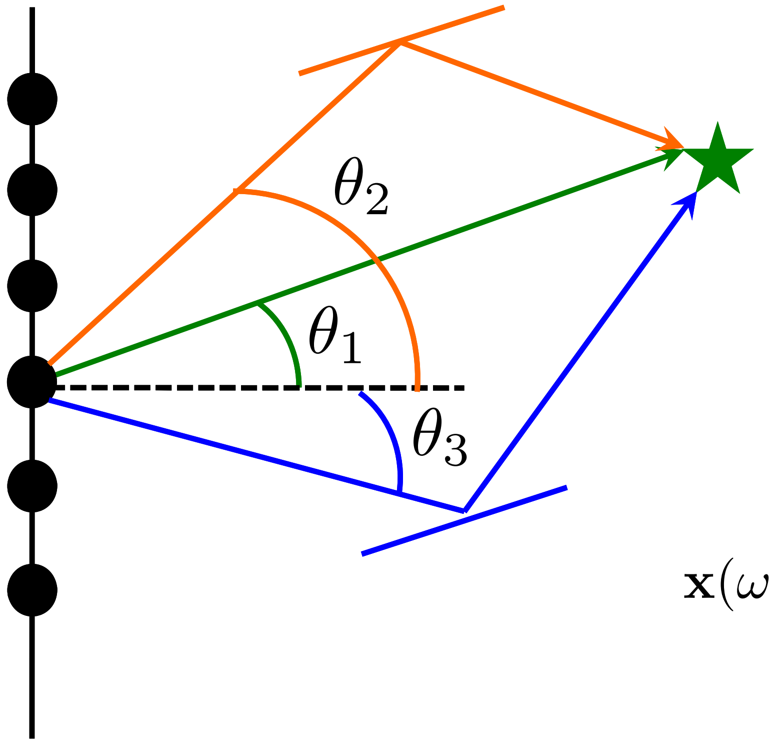
Applying NOMP to picocells

How to estimate a 1000-dimensional spatial channel?

Compressive adaptation



Estimation problem



Channel is a sum of a few sinusoids

$$\mathbf{h} = g_1\mathbf{x}(\omega_1) + g_2\mathbf{x}(\omega_2) + g_3\mathbf{x}(\omega_3)$$

$$\mathbf{x}(\omega) = \begin{pmatrix} 1 \\ e^{j\omega} \\ e^{j2\omega} \\ \vdots \\ e^{j(N-1)\omega} \end{pmatrix} \quad \omega_i = \frac{2\pi d}{\lambda}\sin\theta_i$$

Mobile makes compressive measurements

$$y_i = \mathbf{a}_i^T\mathbf{h}, i = 1, 2, \ldots, M$$

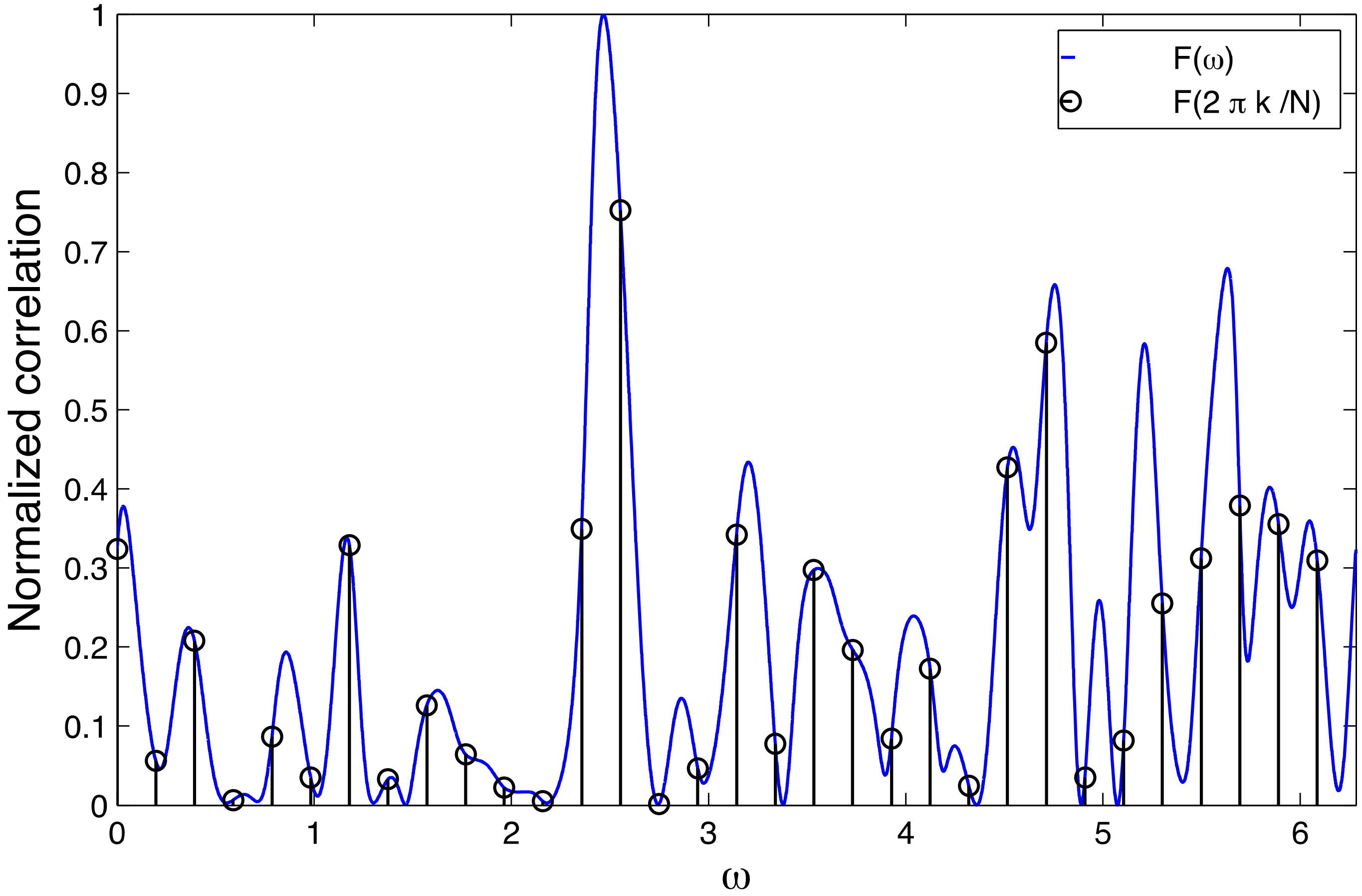
Estimate gains and spatial frequencies from compressive measurements

Algorithm

- Acquisition
  - No knowledge of spatial frequencies whatsoever
- Tracking
  - Leverage frequency estimate from previous round
  - Refine based on new measurements

Acquisition: Coarse Estimate

$$\text{maximize } F(\omega) = \left|\langle \mathbf{Ax}(\omega), \mathbf{y}\rangle\right|^2$$
$$\omega = 0, \frac{2\pi}{2N}, 2\left(\frac{2\pi}{2N}\right), \ldots, (2N-1)\left(\frac{2\pi}{2N}\right)$$



Acquisition: Coarse Estimate

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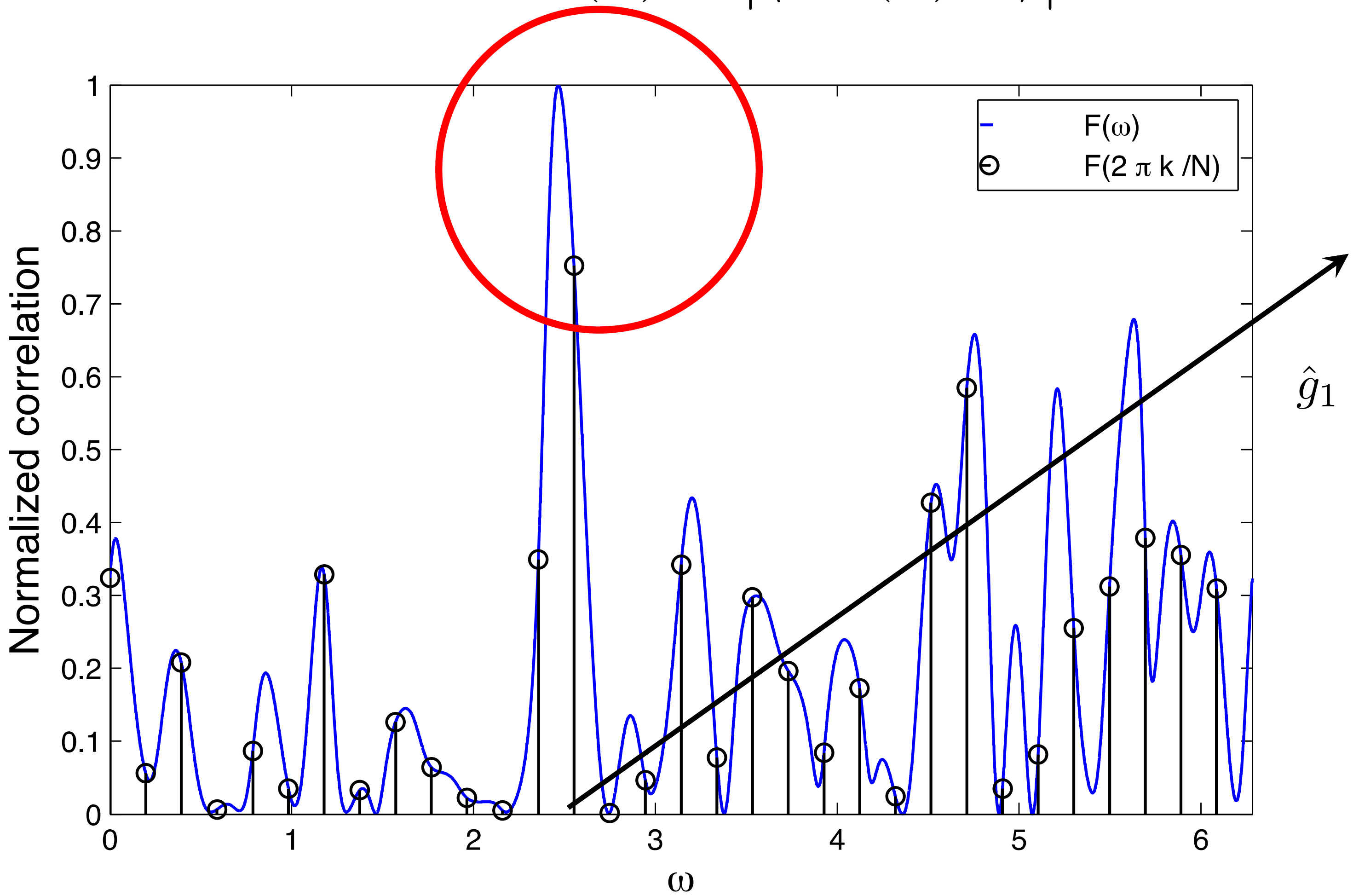
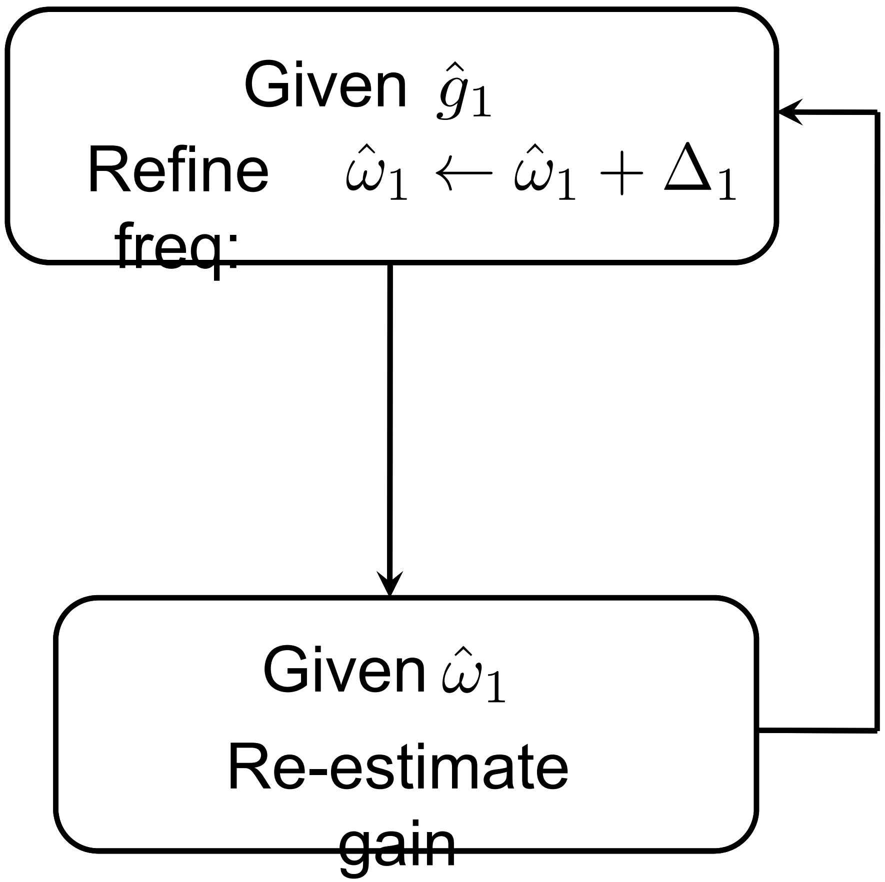


$$\hat{\omega}_1$$

$$\hat{g}_1 = \frac{\langle \mathbf{Ax}(\hat{\omega}_1), \mathbf{y} \rangle}{\|\mathbf{Ax}(\hat{\omega}_1)\|^2}$$

Acquisition: Refine Estimate



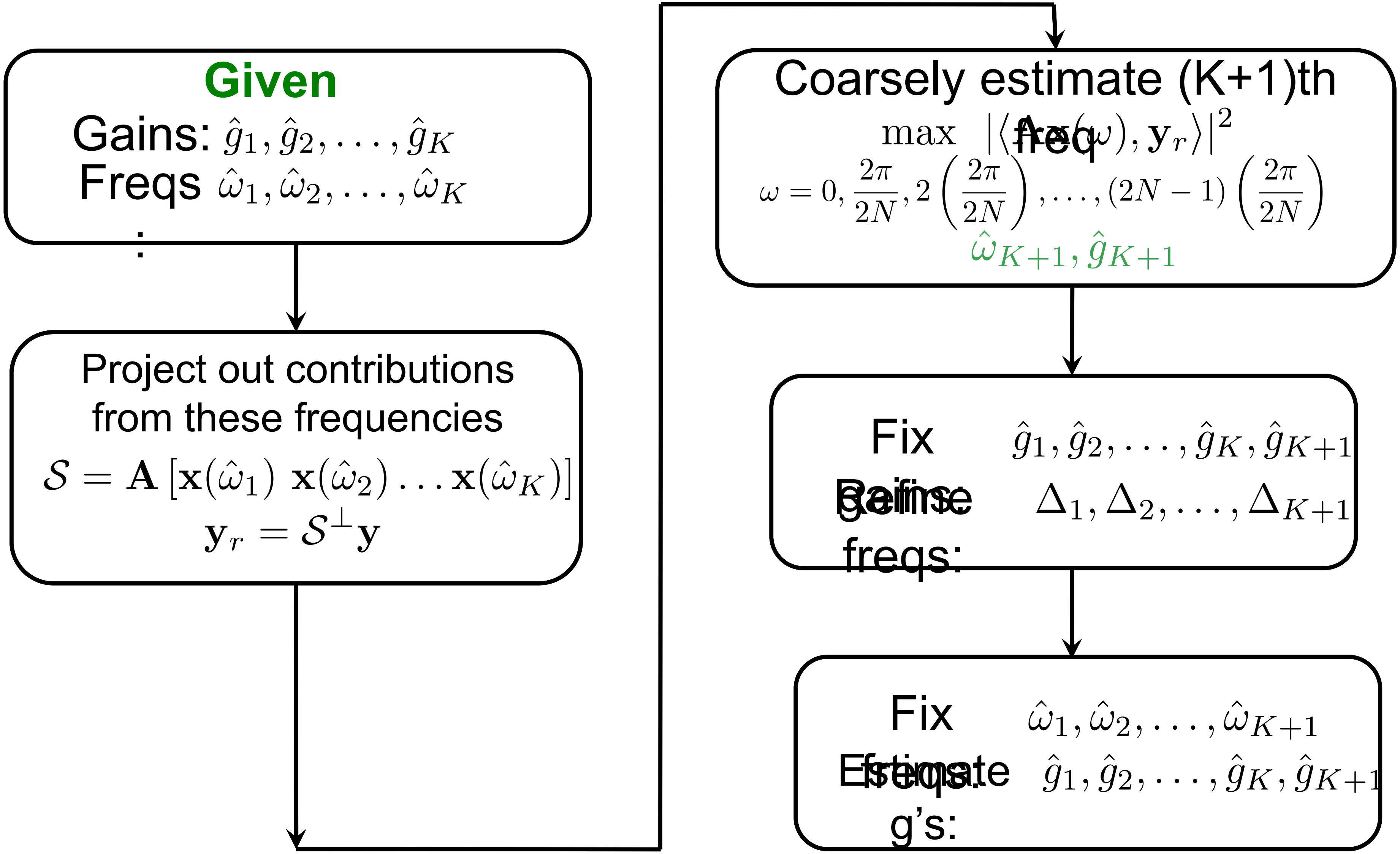
$$\mathbf{y} = \hat{g}_1 \mathbf{A}\mathbf{x}(\hat{\omega}_1 + \Delta_1) + \mathbf{n}$$

$$[\mathbf{y} - \hat{g}_1 \mathbf{A}\mathbf{x}(\hat{\omega}_1)] = \left[\hat{g}_1 \mathbf{A} \frac{d\mathbf{x}(\hat{\omega}_1)}{d\omega}\right] \Delta_1 + \mathbf{n}$$

$$\Delta_1 \in \mathbb{R}$$

$$\mathbf{y} = g_1 \mathbf{A}\mathbf{x}(\hat{\omega}_1) + \mathbf{n}$$

Multiple Frequencies



Multiple Frequencies

$$\mathbf{y} - \sum_{l=1}^{K+1} \hat{g}_l \mathbf{A}\mathbf{x}(\hat{\omega}_l)$$

$$||$$

$$\mathbf{A}\left[\hat{g}_1 \frac{d\mathbf{x}(\hat{\omega}_1)}{d\omega} \;\; \ldots \;\; \hat{g}_{K+1} \frac{d\mathbf{x}(\hat{\omega}_{K+1})}{d\omega}\right] \begin{pmatrix} \Delta_1 \\ \Delta_2 \\ \vdots \\ \Delta_{K+1} \end{pmatrix}$$

Coarsely estimate (K+1)th freq

$$\max_{\omega = 0, \frac{2\pi}{2N}, 2\left(\frac{2\pi}{2N}\right), \ldots, (2N-1)\left(\frac{2\pi}{2N}\right)} |\langle \mathbf{A}\mathbf{x}(\omega), \mathbf{y}_r \rangle|^2$$

$$\hat{\omega}_{K+1}, \hat{g}_{K+1}$$

Fix g's: $\hat{g}_1, \hat{g}_2, \ldots, \hat{g}_K, \hat{g}_{K+1}$

Refine freqs: $\Delta_1, \Delta_2, \ldots, \Delta_{K+1}$

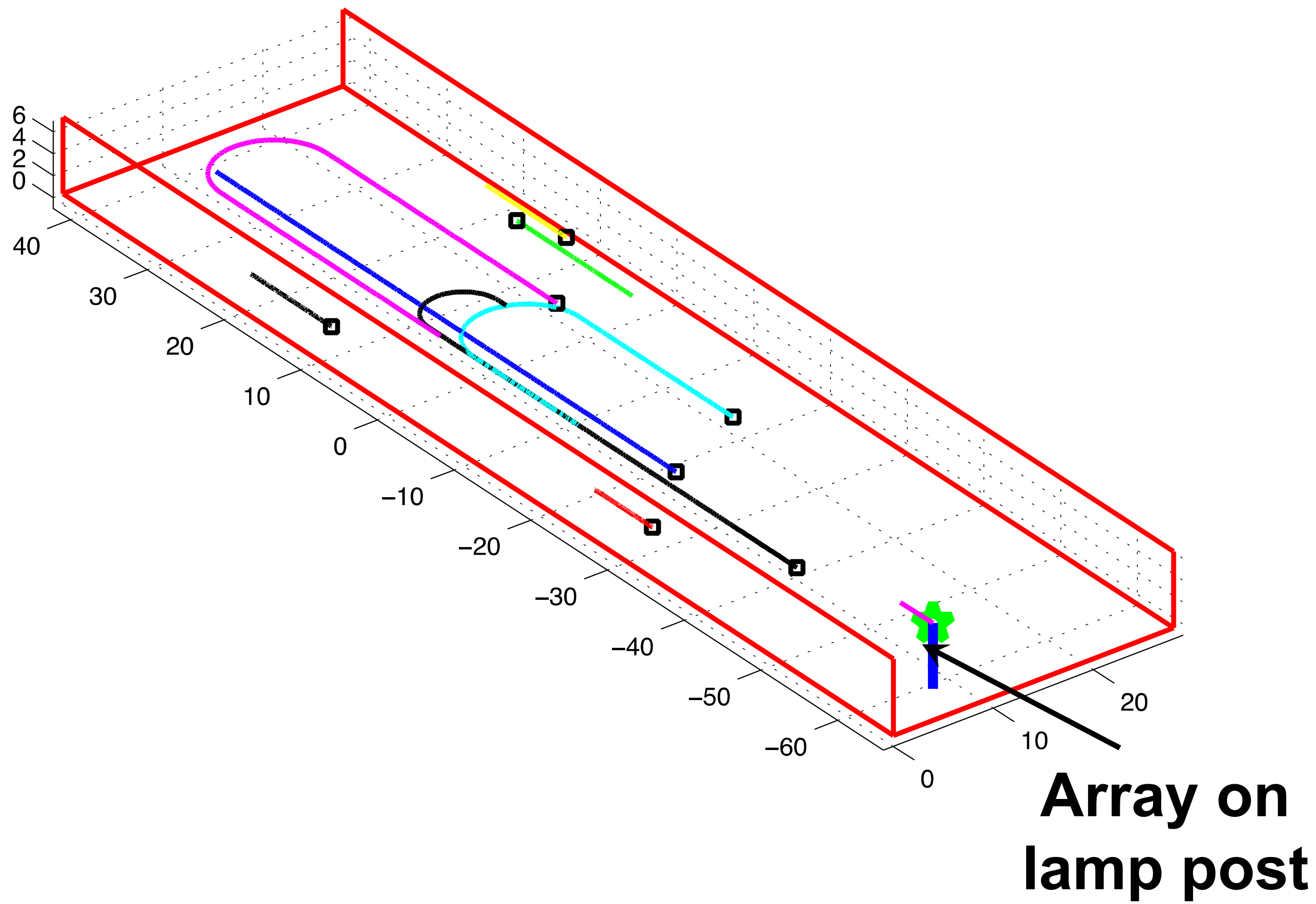
Fix freqs: $\hat{\omega}_1, \hat{\omega}_2, \ldots, \hat{\omega}_{K+1}$

Estimate g's: $\hat{g}_1, \hat{g}_2, \ldots, \hat{g}_K, \hat{g}_{K+1}$

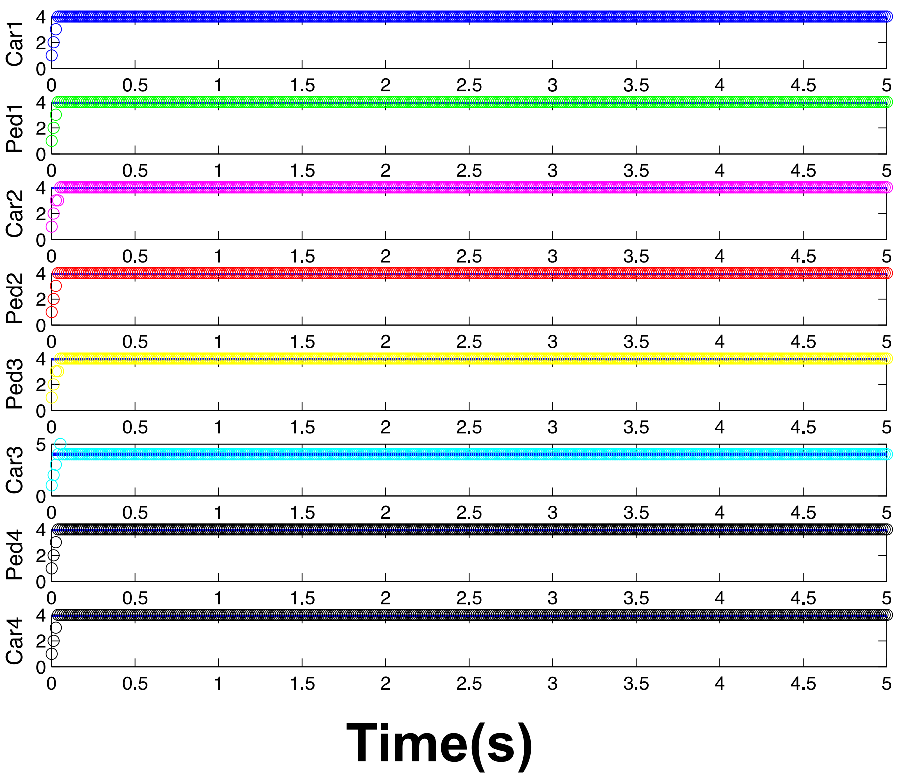
$$\mathbf{y} = \mathbf{A}\left[\mathbf{x}(\hat{\omega}_1) \;\; \ldots \;\; \mathbf{x}(\hat{\omega}_{K+1})\right] \begin{pmatrix} g_1 \\ g_2 \\ \vdots \\ g_{K+1} \end{pmatrix}$$

Same algorithm works for tracking, just bootstrap with estimate from prior round

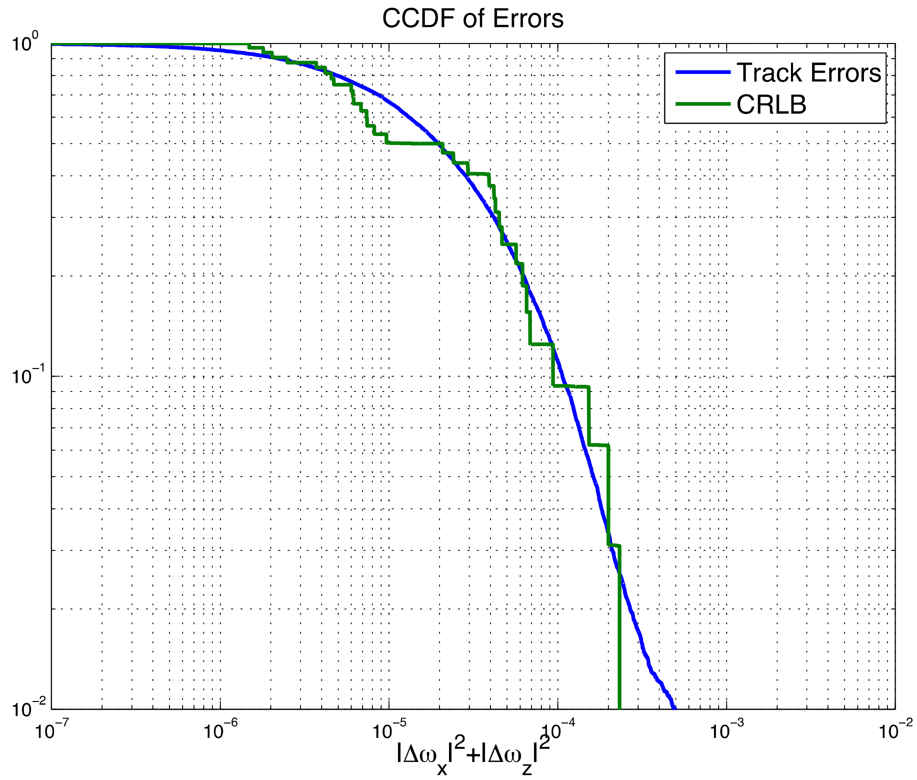
Simulation Setup



Results



Estimated number of beams



Estimation errors close to CRB

Take-aways

- Unique challenges of adapting large mm wave arrays
- Compressive adaptation approach
- New theory of compressive estimation
- New insight on algorithms attaining CRB
  - Coarse grid, then gradient or Newton based refinement does work
  - (If SNR is high enough to get past ZZB threshold)
- Specific motivating application, but leads to rather general techniques

Beyond the basics

Z. Marzi, D. Ramasamy, U. Madhow, “Compressive channel estimation and tracking for large arrays in mm wave picocells,” IEEE J. Selected Topics in Signal Processing, April 2016.

Concept system design for urban picocells

Number of beacons (isometry, SNR),

beacon management and reuse,

how to incorporate receive antenna arrays, …

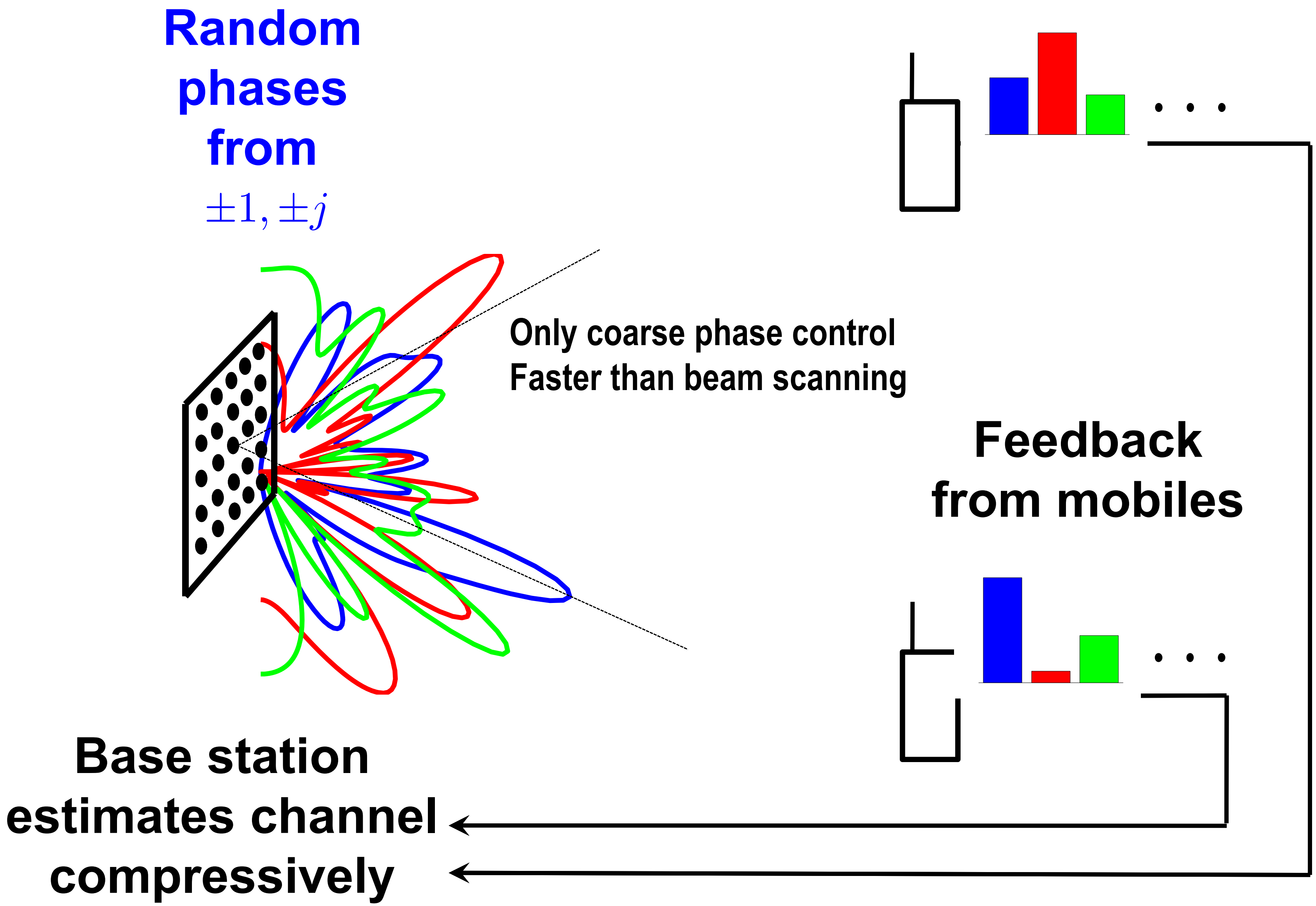
Compressive estimation in AWGN

Ramasamy, Venkateswaran, Madhow, "Compressive Parameter Estimation in AWGN," IEEE Transactions on Signal Processing, April 2014

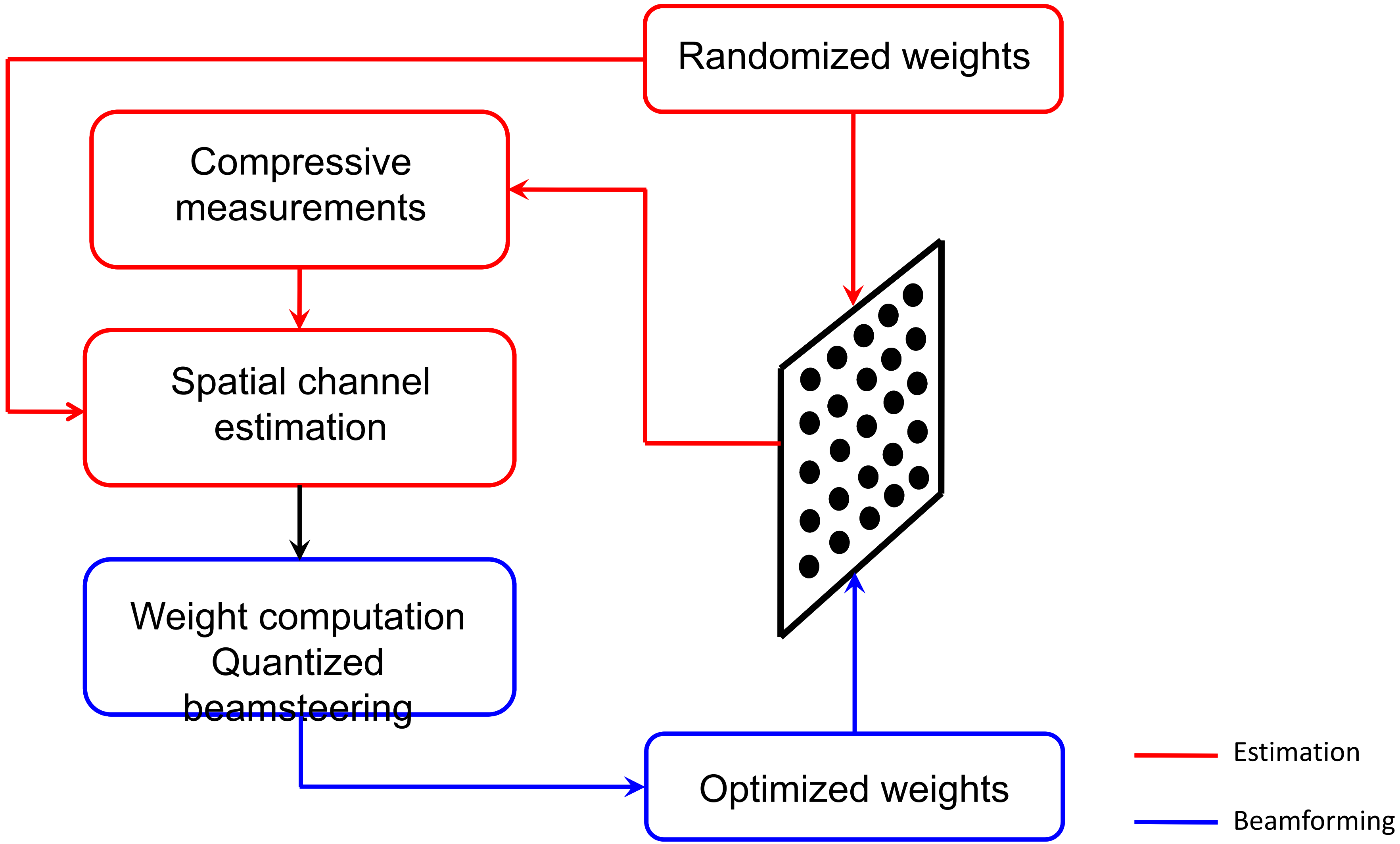
Quick recall of original motivation

(compressive picocellular architectures)

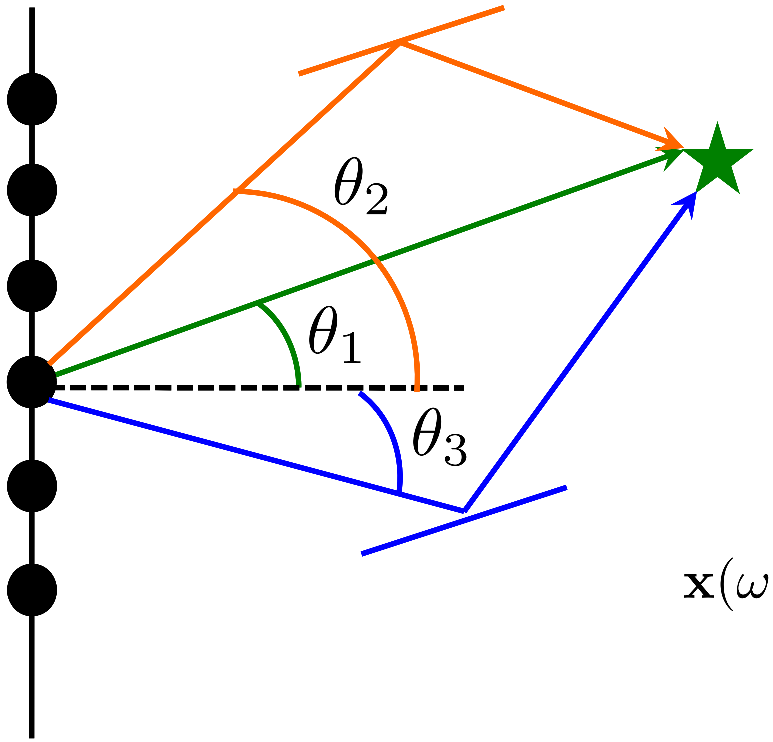
Compressive adaptation



Compressive Adaptation Architecture



Estimation problem



Channel is a sum of a few sinusoids

$$\mathbf{h} = g_1\mathbf{x}(\omega_1) + g_2\mathbf{x}(\omega_2) + g_3\mathbf{x}(\omega_3)$$

$$\mathbf{x}(\omega) = \begin{pmatrix} 1 \\ e^{j\omega} \\ e^{j2\omega} \\ \vdots \\ e^{j(N-1)\omega} \end{pmatrix} \quad \omega_i = \frac{2\pi d}{\lambda}\sin\theta_i$$

Mobile makes compressive measurements

$$y_i = \mathbf{a}_i^T\mathbf{h}, i = 1, 2, \ldots, M$$

Estimate gains and spatial frequencies from compressive measurements

We have see the algorithms

Compressive NOMP (greedy + refinement)

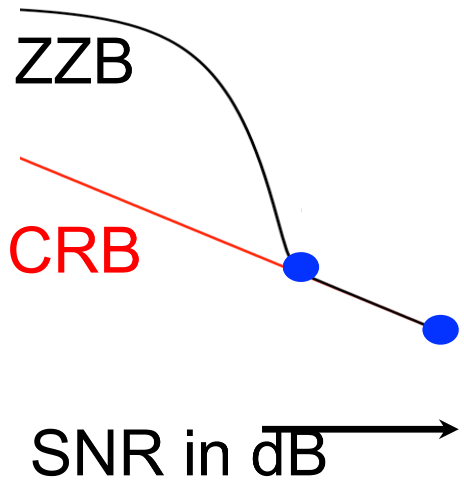
Quantized beamforming (greedy sequential)

Back to the theory

Standard parameter estimation

$$\mathbf{y} = \mathbf{s}(\boldsymbol{\theta}) + \mathbf{z}, \ \mathbf{z} \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbb{I}_M)$$

$$\hat{\theta}_{\mathbf{ML}} = \arg\min_{\theta} \| \mathbf{y} - \mathbf{s}(\theta) \|$$



Performance measures

Cramer-Rao Bound (CRB) when close to truth

Ziv-Zakai bound (ZZB) more generally
(are you in the right bin? How close, once in the right bin?)

ZZB tends to CRB at high SNR (high prob of right bin).
This is when estimation can be expected to "work well."

Performance depends on Euclidean distances

CRB depends on Fisher Information Matrix

$$F_{m,n}(\boldsymbol{\theta}) = \frac{2}{\sigma^2}\Re\left\{\left(\frac{\partial \mathbf{s}(\boldsymbol{\theta})}{\partial \theta_m}\right)^H \frac{\partial \mathbf{s}(\boldsymbol{\theta})}{\partial \theta_n}\right\}$$

Depends on changes in signal geometry for small changes in parameter

Ziv-Zakai bound is based on an associated detection problem

$$H_1 : \mathbf{y} = \mathbf{s}(\boldsymbol{\theta}_1) + \mathbf{z},\ \Pr(H_1) = \frac{p(\boldsymbol{\theta}_1)}{p(\boldsymbol{\theta}_1) + p(\boldsymbol{\theta}_2)}$$

$$H_2 : \mathbf{y} = \mathbf{s}(\boldsymbol{\theta}_2) + \mathbf{z},\ \Pr(H_2) = \frac{p(\boldsymbol{\theta}_2)}{p(\boldsymbol{\theta}_1) + p(\boldsymbol{\theta}_2)}.$$

Depends on changes in signal geometry for general changes in parameter

$$d(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2) = \|\mathbf{s}(\boldsymbol{\theta}_1) - \mathbf{s}(\boldsymbol{\theta}_2)\|$$

Compressive measurements: model

High-dimensional signal space $\mathbf{x}(\boldsymbol{\theta}) \in \mathbb{R}^N$

(but unknown parameter lies in low-dimensional space)

M compressive measurements

$$y_i = \langle \mathbf{w}_i , \mathbf{x}(\boldsymbol{\theta}_t) + \mathbf{z}_i \rangle$$

$$\boldsymbol{A} = [\mathbf{w}_1 \ \cdots \ \mathbf{w}_M]^T$$

$$\mathbf{y} = \boldsymbol{A}\mathbf{x}(\boldsymbol{\theta}_t) + \mathbf{z}$$

$$\mathbf{z}_i \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbb{I}_N)$$

Noise power is same

$$\mathbf{z} \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbb{I}_M)$$

When does this provide the "same" performance as standard estimation?

Compressive estimation works well when:

1) Signal space geometry is preserved
(similar to RIP for compressive sensing)

2) “Effective SNR” is high enough

The structure of compressive estimation

GENERAL STRUCTURE

1) Required isometries
CRB: Preserve distance changes under small perturbations
ZZB: Preserve distance changes generally

2) SNR penalty (➔ "effective SNR")
Dimension reduction from *N* to *M* ➔ SNR reduction by *M/N*

3) Definition of "working well"
ZZB tends to CRB (coarse errors highly unlikely)

PROBLEM-SPECIFIC ANALYSIS

How many observations needed to preserve isometries?

Isometries needed

Tangent plane isometry (for CRB)

$$\sqrt{\frac{M}{N}}(1-\epsilon) \leq \frac{\|\boldsymbol{A}\sum a_m(\partial \mathbf{x}(\boldsymbol{\theta})/\partial\theta_m)\|}{\|\sum a_m(\partial \mathbf{x}(\boldsymbol{\theta})/\partial\theta_m)\|} \leq \sqrt{\frac{M}{N}}(1+\epsilon)$$

$$\forall\,[a_1, a_2, \ldots, a_K]^T \in \mathbb{R}^K \backslash \{\mathbf{0}\},\ \forall \boldsymbol{\theta} \in \Theta$$

Pairwise ε-isometry (for ZZB)

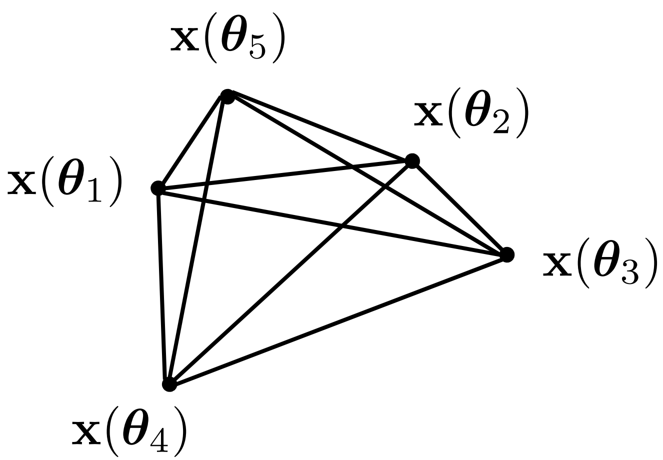
$$\sqrt{\frac{M}{N}}(1-\epsilon) \leq \frac{\|\boldsymbol{A}\mathbf{x}(\boldsymbol{\theta}_1) - \boldsymbol{A}\mathbf{x}(\boldsymbol{\theta}_2)\|}{\|\mathbf{x}(\boldsymbol{\theta}_1) - \mathbf{x}(\boldsymbol{\theta}_2)\|} \leq \sqrt{\frac{M}{N}}(1+\epsilon)$$

$$\forall\, \boldsymbol{\theta}_1, \boldsymbol{\theta}_2 \in \Theta.$$

What geometry preservation looks like

All measurements

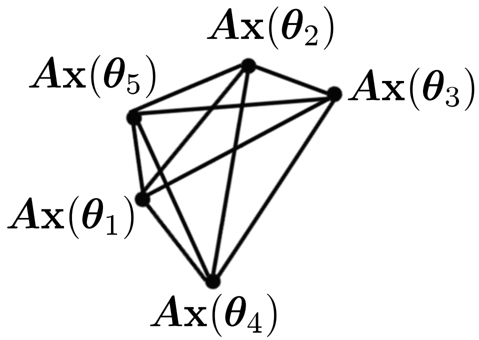
$$\mathbf{y} = \mathbf{x}(\boldsymbol{\theta}_t) + \mathbf{z} \quad \mathbf{z} \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbb{I}_N)$$



$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}_i} \|\mathbf{y} - \mathbf{x}(\boldsymbol{\theta}_i)\|^2$$

Compressive measurements

$$\mathbf{y} = \boldsymbol{A}\mathbf{x}(\boldsymbol{\theta}_t) + \mathbf{z} \quad \mathbf{z} \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbb{I}_M)$$



$\in \mathbb{R}^M$

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}_i} \|\mathbf{y} - \boldsymbol{A}\mathbf{x}(\boldsymbol{\theta}_i)\|^2$$

Why we can hope for geometry preservation

$$\mathbf{v} = \mathbf{x}(\boldsymbol{\theta}_i) - \mathbf{x}(\boldsymbol{\theta}_j)$$

- Random projections must preserve norm of

$$\frac{1}{M}\|\boldsymbol{A}\mathbf{v}\|^2 = \frac{1}{M}\sum_{i=1}^{i=M} |\mathbf{w}_i^T \mathbf{v}|^2 \xrightarrow[\text{concentrates}]{M \text{ large enough}} \text{Mean} \quad (1/N)\|\mathbf{v}\|^2$$

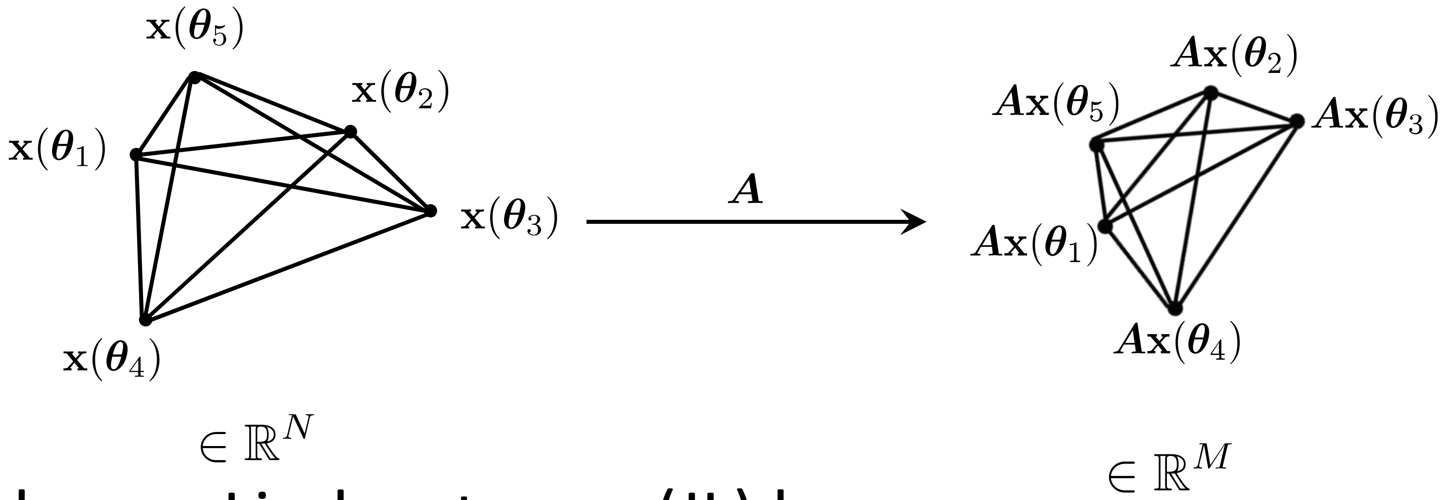
i.i.d. with mean $(1/N)\|\mathbf{v}\|^2$

- Chernoff bound on deviations from the mean (with tolerance ε) + Union bound (for all pairwise differences)

Johnson-Lindenstrauss (JL) Lemma

Achlioptas, "Database-friendly Random Projections", 2001

How many measurements?



Johnson-Lindenstrauss (JL) lemma:
Pairwise ε-isometry for finite signal model $\mathcal{H} = \{\mathbf{x}(\boldsymbol{\theta}_i)\}$
when the number of random projections is

$$M = O\left(\epsilon^{-2} \log |\mathcal{H}|\right)$$

K signals, M measurements

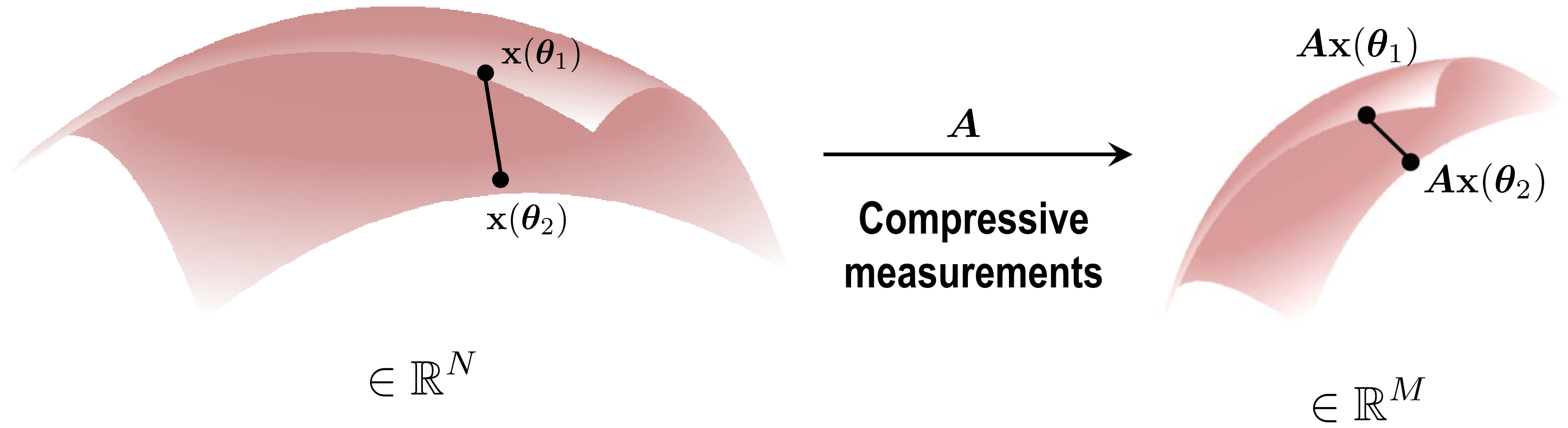
Chernoff bound + Union bound $\sim K^2 e^{-\alpha M}$

$\Rightarrow M = O(\log K)$

Continuous signal model

Parameters come from a continuum $\boldsymbol{\theta} \in \mathbb{R}^K$

Need pairwise isometries for all $(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2)$ **pairs**



Cannot directly use JL lemma

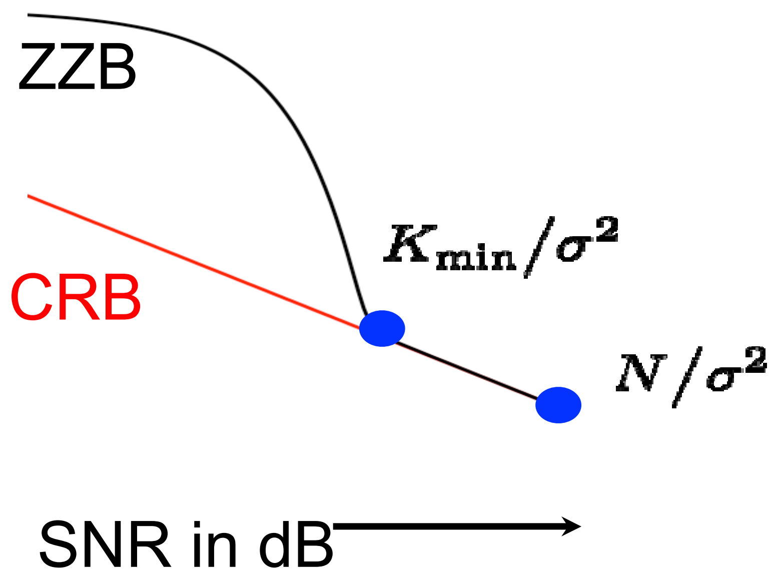
But discretization, JL lemma, and smoothness can be used to do the trick

How many measurements for good performance?

- If pairwise isometry holds, then both CRLB and ZZB go through

➔ Only effect of compressive measurements is SNR reduction

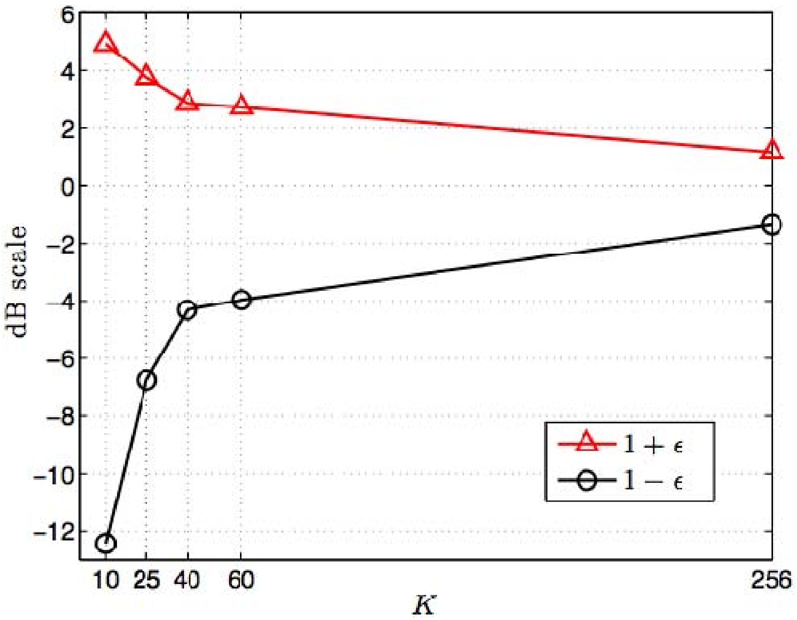
- Number of measurements must satisfy two criteria for good performance
  - Should be enough to provide pairwise isometry
  - Effective SNR should be such that ZZB tends to CRLB

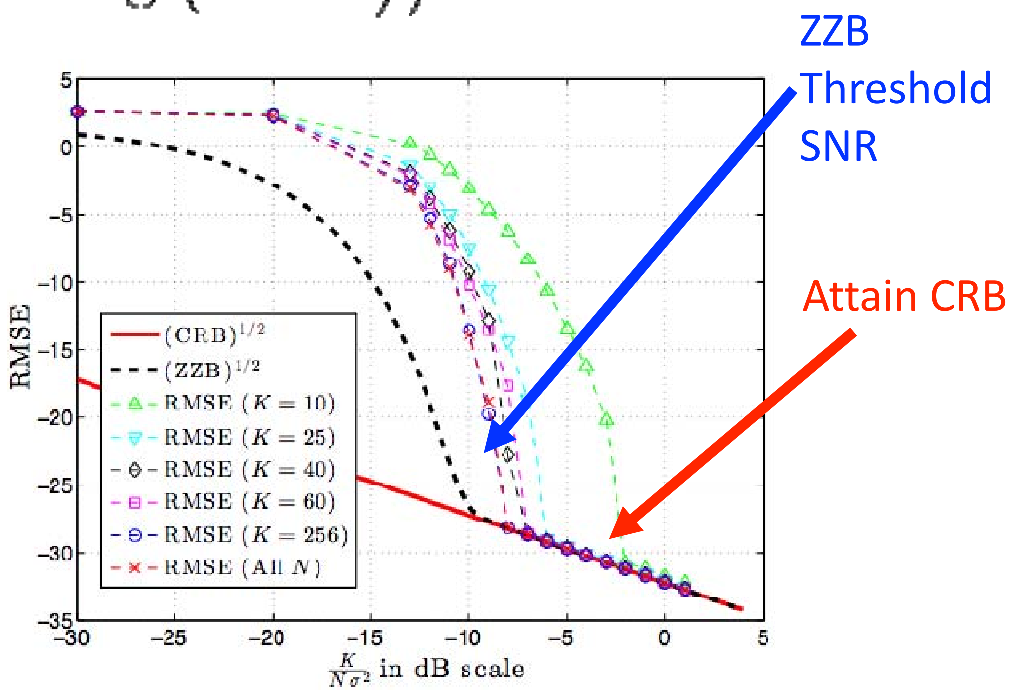


Attaining the CRB for a sinusoid

Problem-specific analysis→ Pairwise isometry requires

$$K = O\left(\epsilon^{-2}\log\left(N\epsilon^{-1}\right)\right)$$





More random projections
Better isometry constants

Effective SNR

RMSE performance for 40+ measurements closely follows that for all N=256 measurements
Isometry constants good for 40+ measurements

$$K = \min(40, \text{ZZB threshold SNR} \times N\sigma^2)$$

Beamforming and nullforming with drastically quantized weights

U. Madhow

ECE Department, UCSB

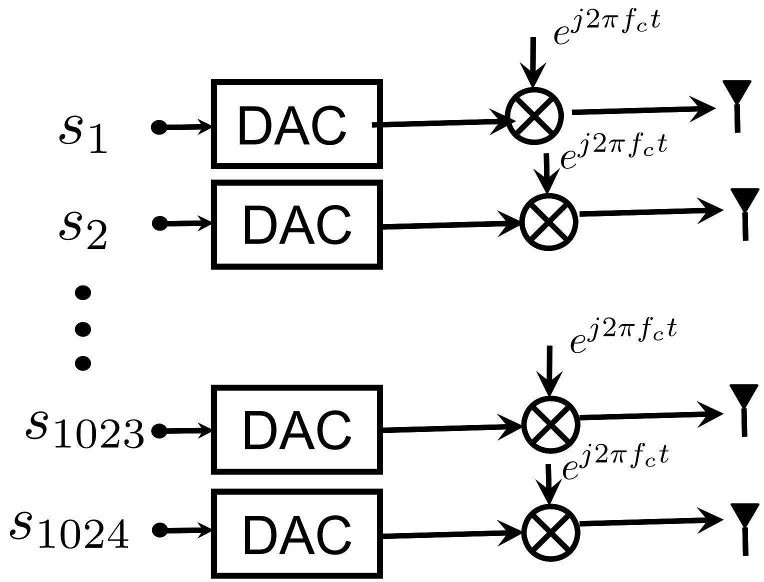
Summer school, June 27-July 1, 2016, IISc, Bangalore

A reminder of the overall context

Adapting very large mm wave arrays

Traditional Digital Beamforming

Easy to implement:
Zero Forcing
MMSE
Codebook based approaches

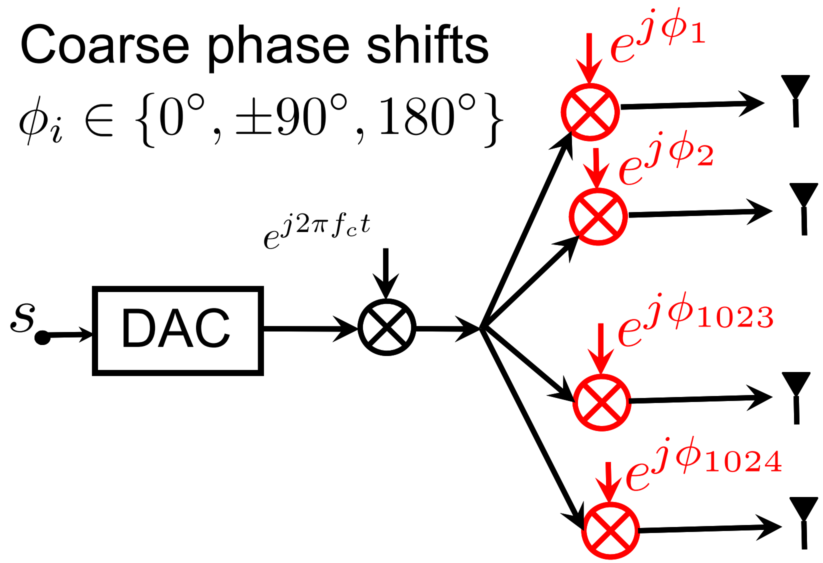


What's the problem?

Digital beamfoming does not scale to 1000 elements!

(Cost and power consumption)

RF Beamforming with hardware constraints

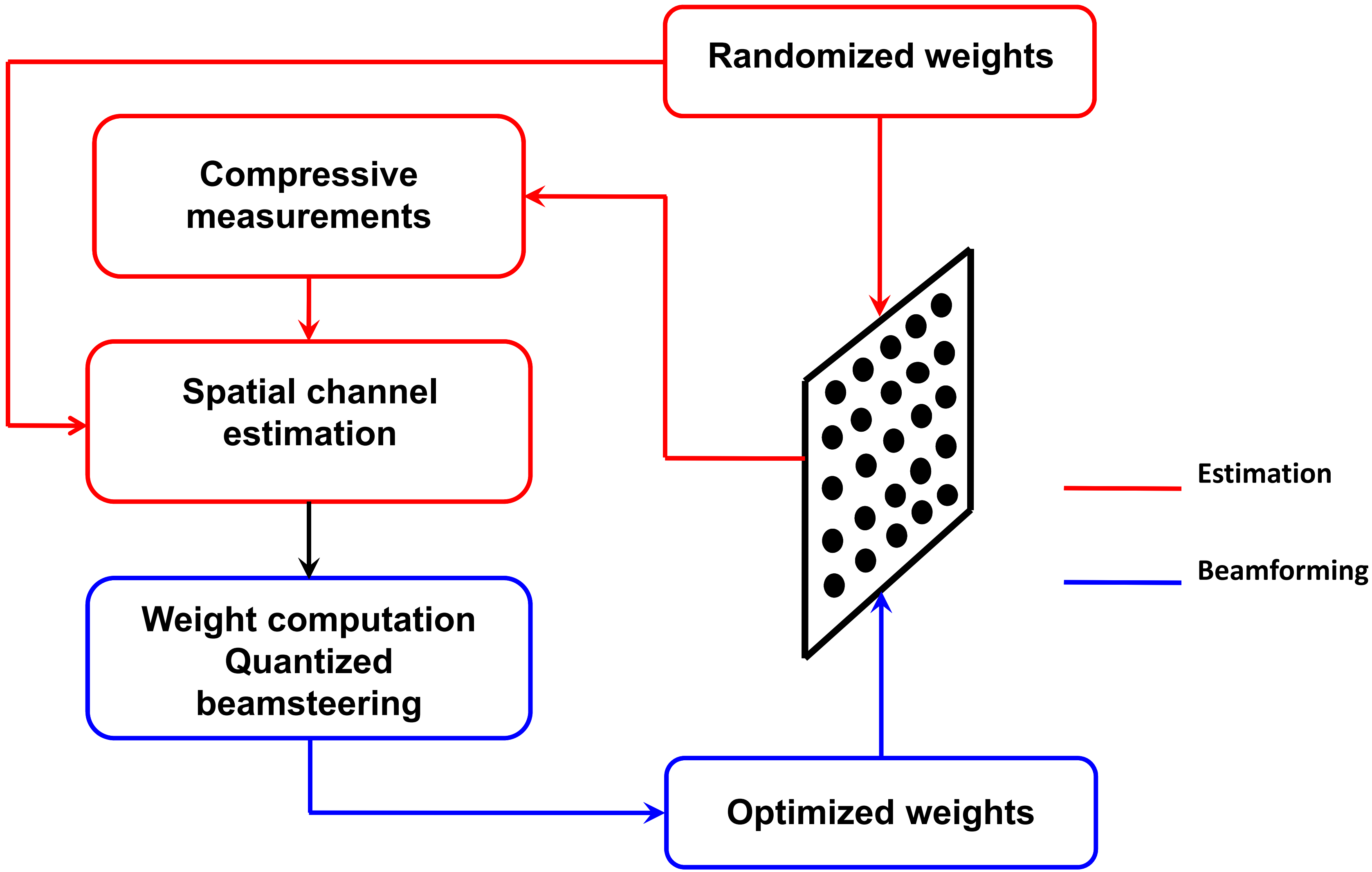


Much more feasible

But how do we adapt it?

No access to individual elements ➔ least squares does not work

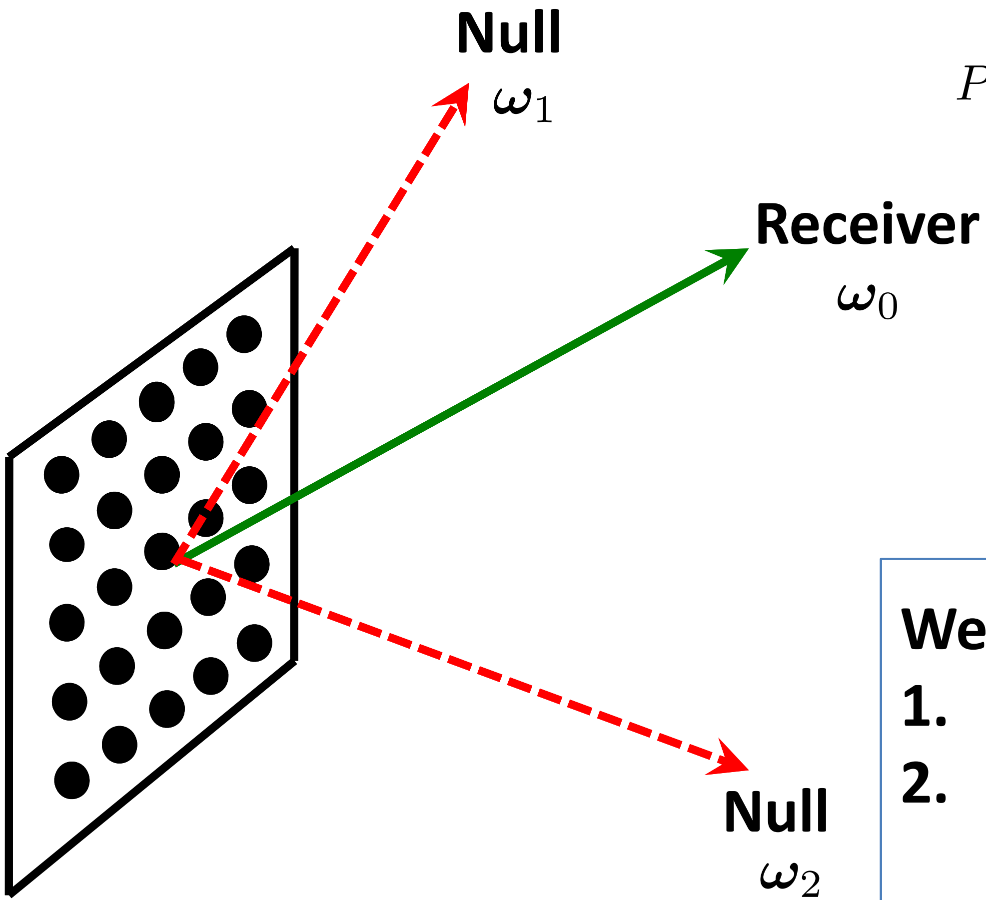
Compressive Adaptation Architecture



Quantized Beamsteering

Slides prepared by Zhinus Marzi

Large arrays with coarse weights



$$P = \left| \sum_{m,n} g_{mn} e^{j\beta_{mn}} \exp\left(j\left(m\omega_x + n\omega_y\right)\right) \right|^2$$

$$\omega_x = \frac{2\pi d}{\lambda} \cos\theta \cos\phi$$
$$\omega_y = \frac{2\pi d}{\lambda} \cos\theta \sin\phi$$

We are limited to:

1. **Fixed gains**
2. **Coarse control over phases (2 bits: ±1, ±j)**

Approach

1. Compute a good starting point by relaxing constraints
   - Zero-forcing solution with no constraint on amplitude/phase

2. Quantize phases to nearest among (±1,±j)

3. Sequentially alter phases to improve Signal-to-Null Ratio

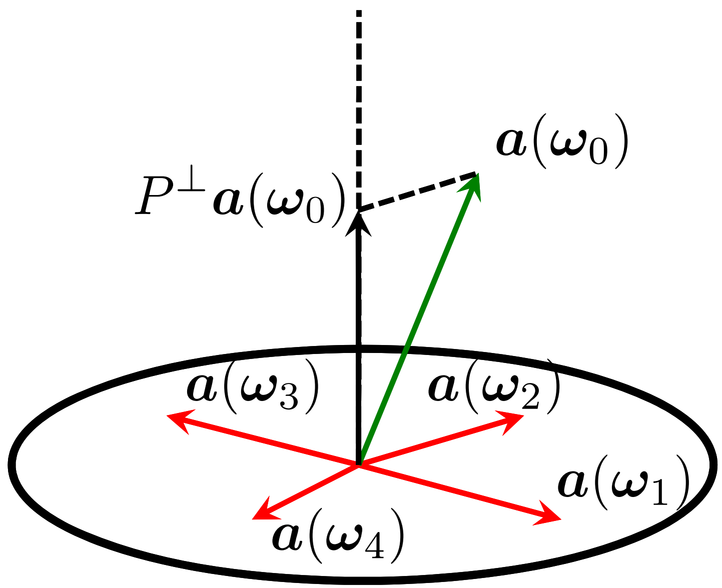
Step 1: Zero Forcing Solution

$$P = \left| \sum_{m,n} g_{mn} e^{j\beta_{mn}} \exp\left(j\left(m\omega_x + n\omega_y\right)\right) \right|^2$$

$$\boldsymbol{\psi} = \mathrm{vec}\left[e^{j\beta_{mn}}\right]$$

$$\boldsymbol{a}(\boldsymbol{\omega}) = \mathrm{vec}\left[g_{mn} \exp\left(-j\left(m\omega_x + n\omega_y\right)\right)\right]$$

$$P(\boldsymbol{\omega}) = \left|\boldsymbol{a}(\boldsymbol{\omega})^H \boldsymbol{\psi}\right|^2$$



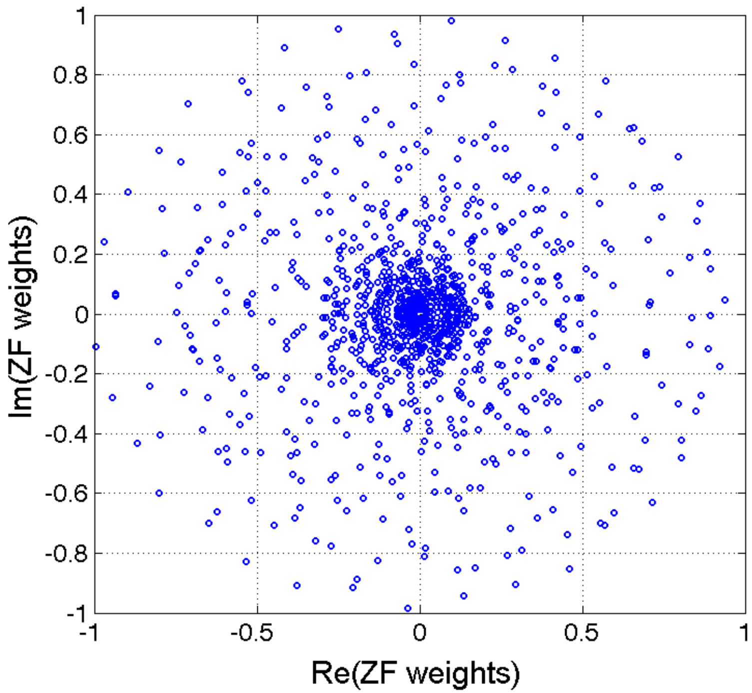
Goals

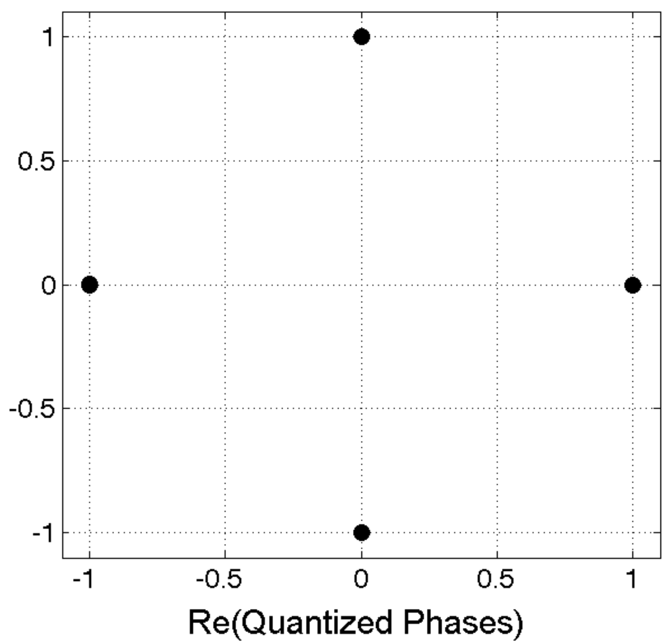
Steer towards $\boldsymbol{\omega}_0$

Nulls along $\boldsymbol{\omega}_1, \boldsymbol{\omega}_2, \ldots, \boldsymbol{\omega}_Q$

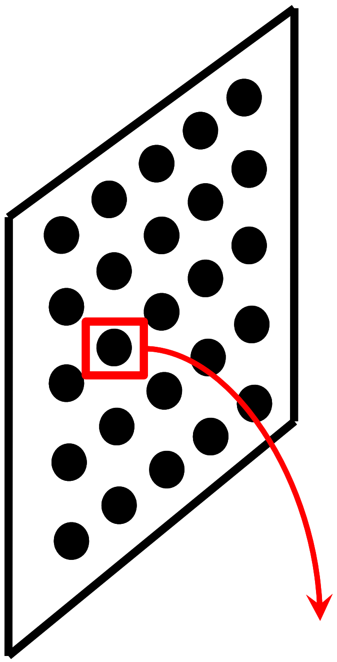
Step 2: Naïve quantization

Quantize each phase to nearest among $\{\pm 1, \pm j\}$





Step 3: Sequential Optimization



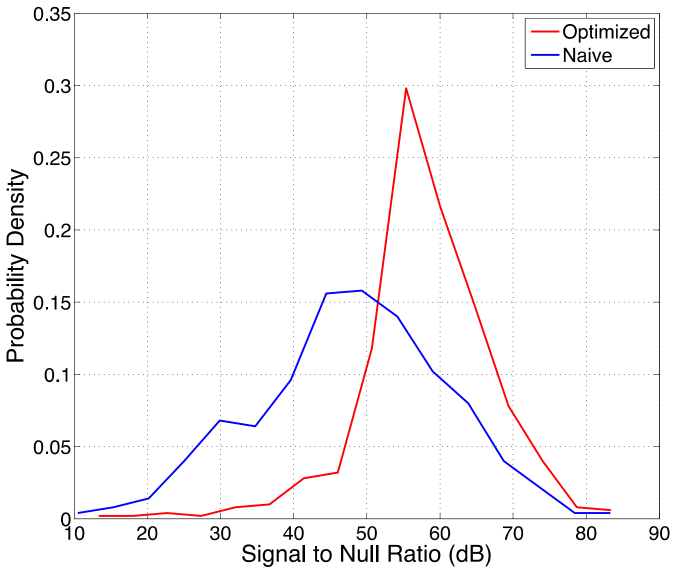
Element *k*

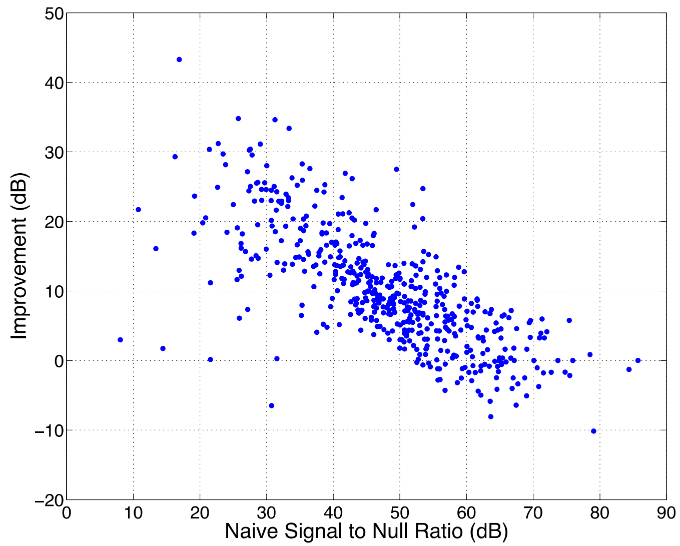
- For each k
  - Fix phases at all elements but k
  - Change phases at k to 1,j,-1,-j
  - S-to-Null Ratios are s_0, s_1, s_2, s_3
  - Pick maximum and set phase at k appropriately

$$s = \frac{|\boldsymbol{a}(\boldsymbol{\omega}_0)^H \boldsymbol{\psi}|^2}{\sum_{i=1}^{Q} |\boldsymbol{a}(\boldsymbol{\omega}_i)^H \boldsymbol{\psi}|^2}$$

Use integrals over small bands for robustness to estimation error

Signal-to-Null Ratio





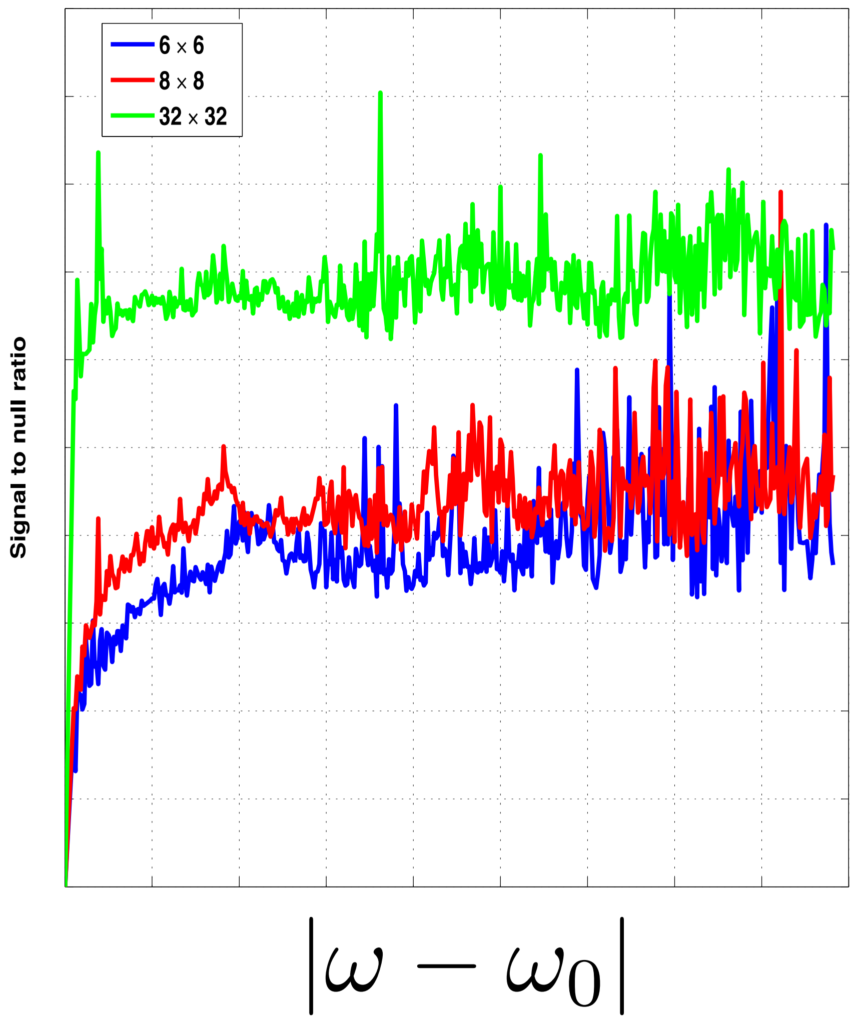
Mean Signal-to-Null ratio: 58 dB

Mean Improvement over naïve: 10 dB

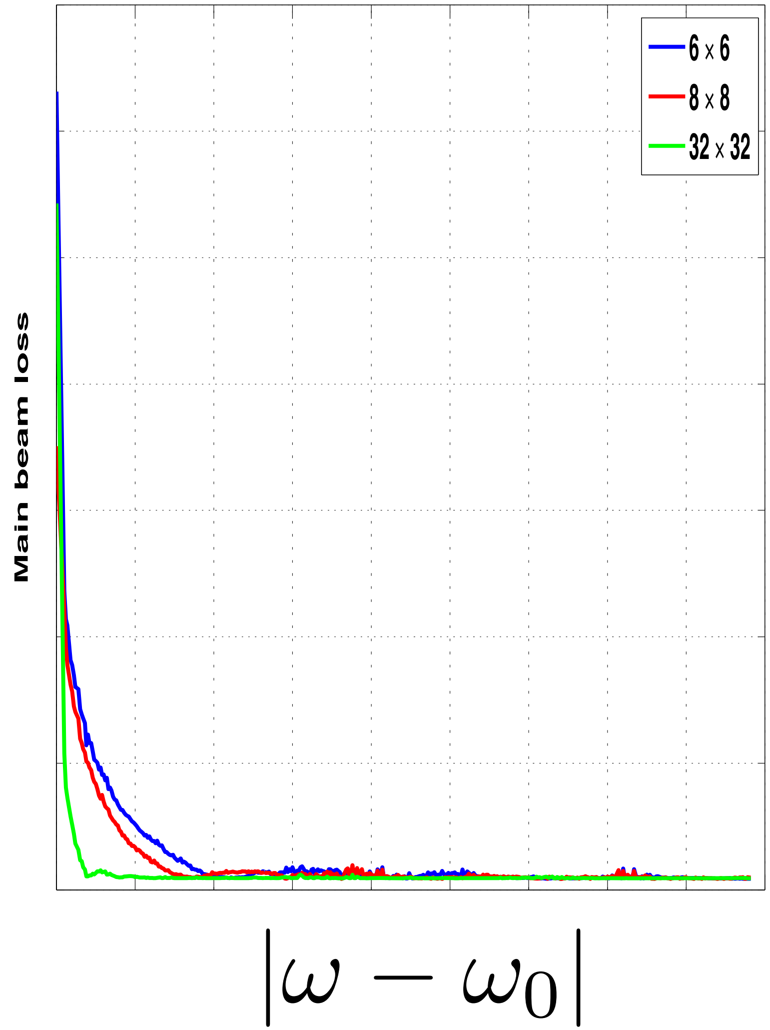
Big improvements (~30 dB) when it really counts!

Simulation results

Signal to null ratio



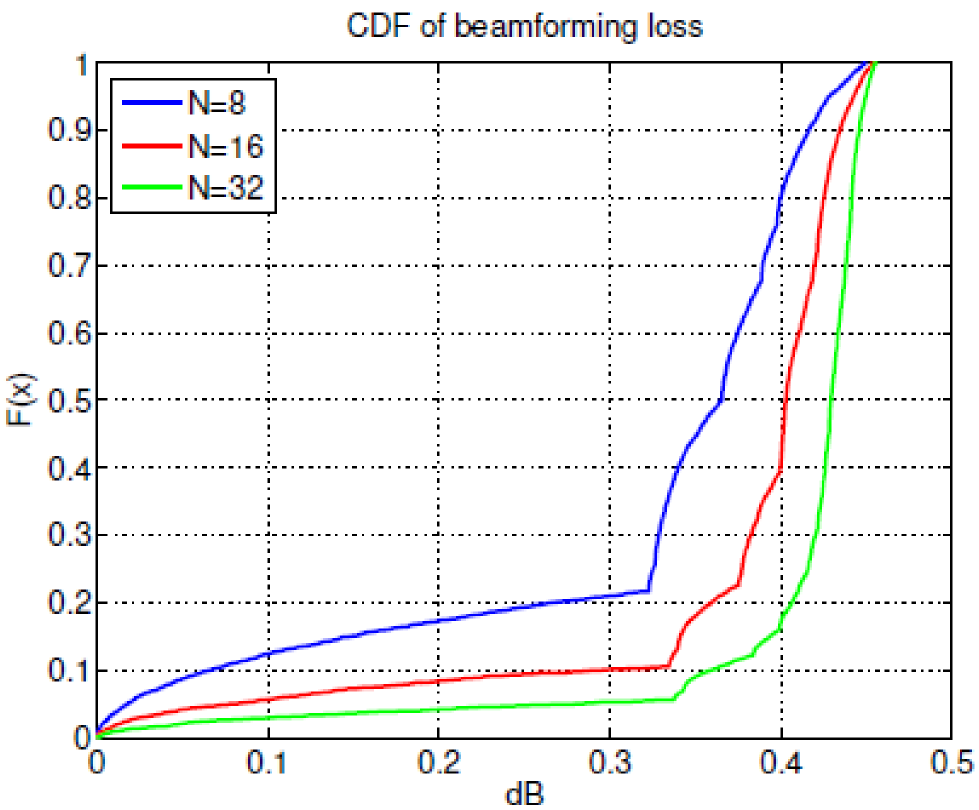
Main beam loss



Beamforming loss (No nullforming)

- **What if we only beamform toward desired direction?**
  - **Only need to change the cost function for sequential optimization**

N: size of linear antenna



Take-aways

- Can perform both beamforming and nullforming effectively with coarsely quantized weights
- Empirical observation: Loss relative to unquantized weights decreases as number of antennas increases
- Theorems?

References

Sequential algorithm for quantized beamforming first discussed in:
Ramasamy, Venkateswaran, Madhow, Compressive adaptation of large steerable arrays, ITA 2012.

Since then used in other publications, but theory still missing.

Short-range mm wave radar

Upamanyu Madhow

ECE Department, UCSB

Slides prepared by Babak Mamandipoor, with input from Anant Gupta

Summer school, IISc, Bangalore, June 27-July 1, 2016

References

Builds on an invited presentation at ITA 2016.

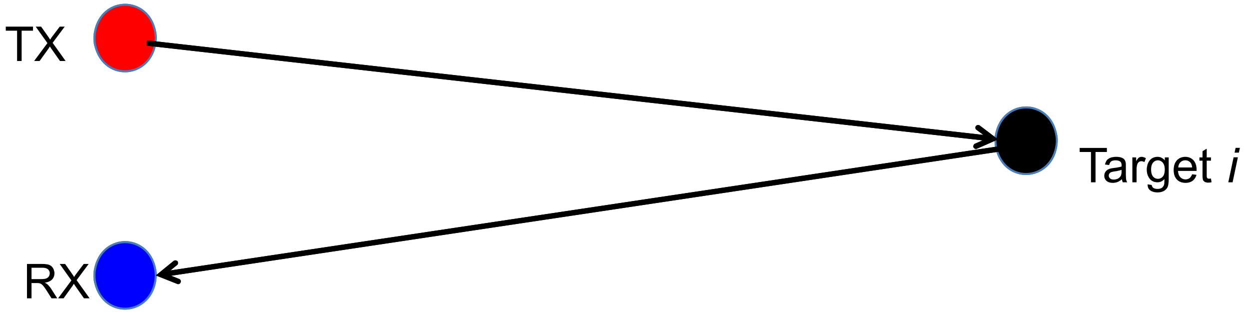
IMS 2016 paper:

B. Mamandipoor, M. Fallahpour, G. Malysa, K. Noujeim, U. Madhow and A. Arbabian, "Spatial-Domain Technique to Overcome Grating Lobes in Sparse Monostatic mm-Wave Imaging Systems", MTT-S International Microwave Symposium (IMS), San Francisco, CA, May 2016.

Journal paper in preparation.

Classical long-range radar

$$s(t)$$



$$\alpha_i s(t-\tau_i)e^{j2\pi f_i t}$$

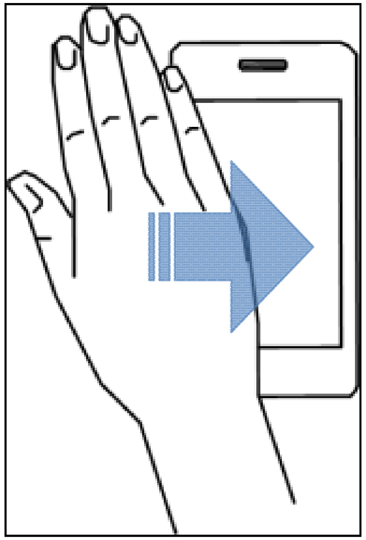
Signal received from multiple targets

$$y(t) = \sum_{i=1}^{K} \alpha_i s(t-\tau_i)e^{j2\pi f_i t} + n(t)$$

Typical approach (e.g., SAR): matched filter against expected response (point scatterer target model)

Emerging short-range radar applications

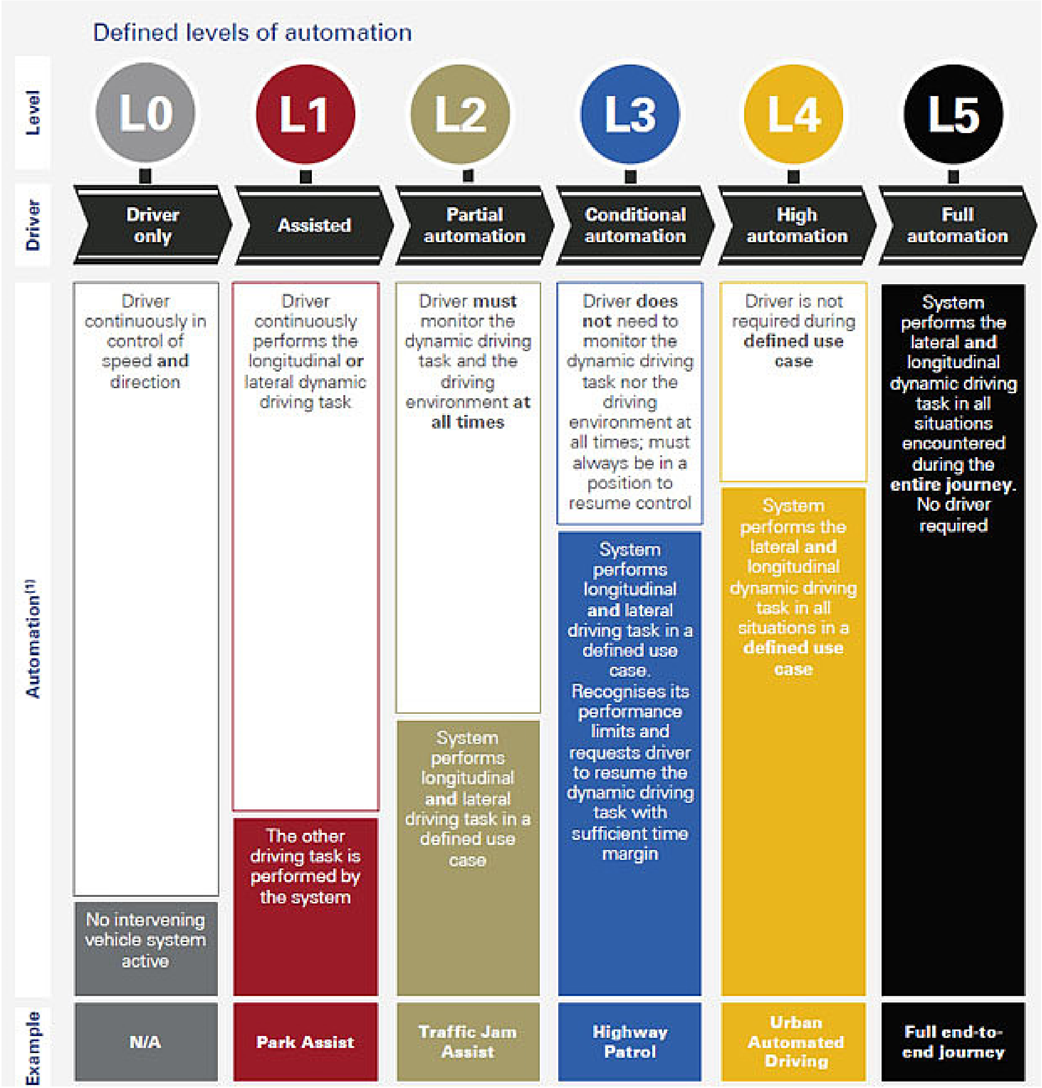
Vehicular situational awareness

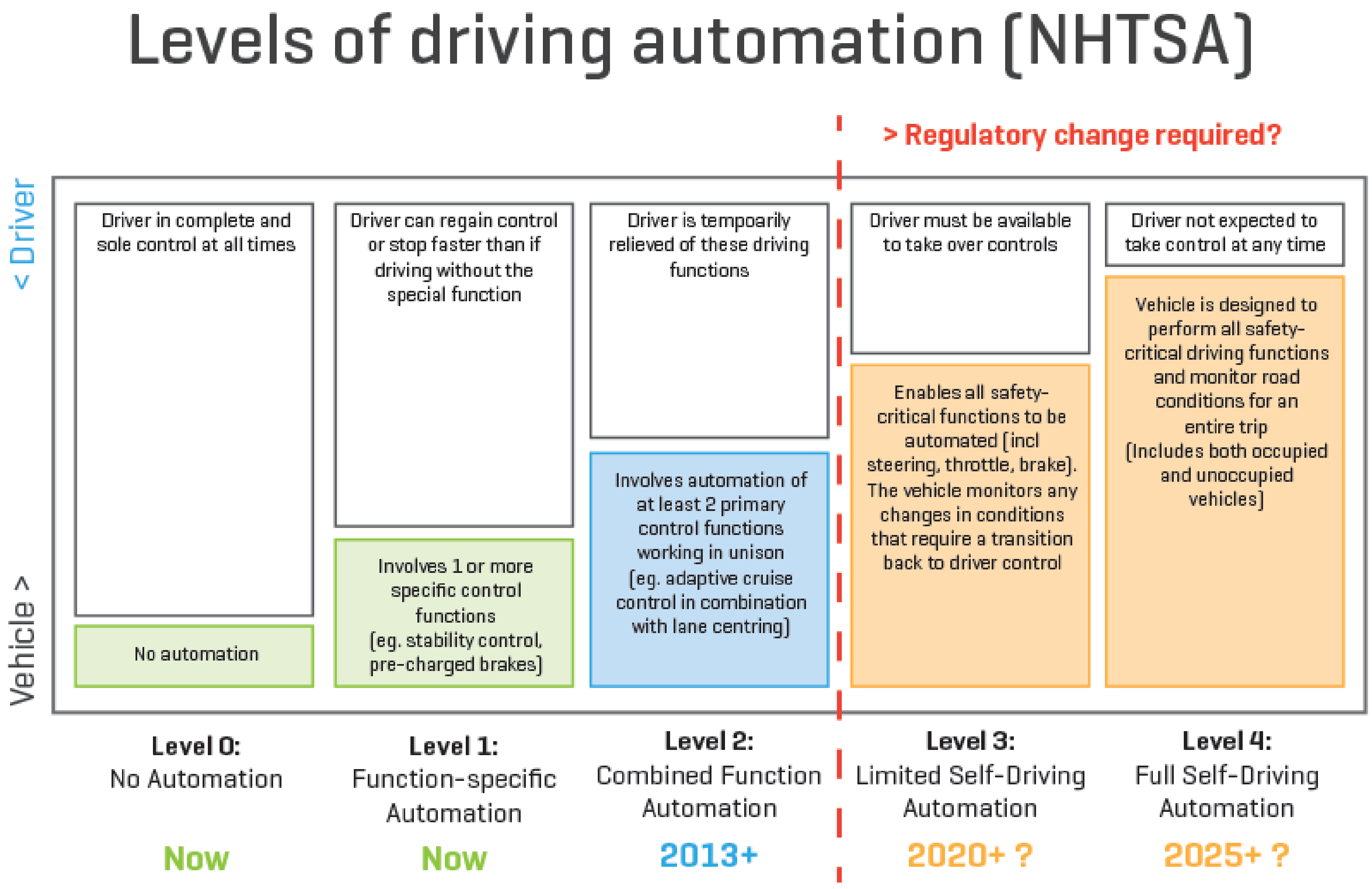


Gesture recognition

Designs constrained by cost, complexity and geometry

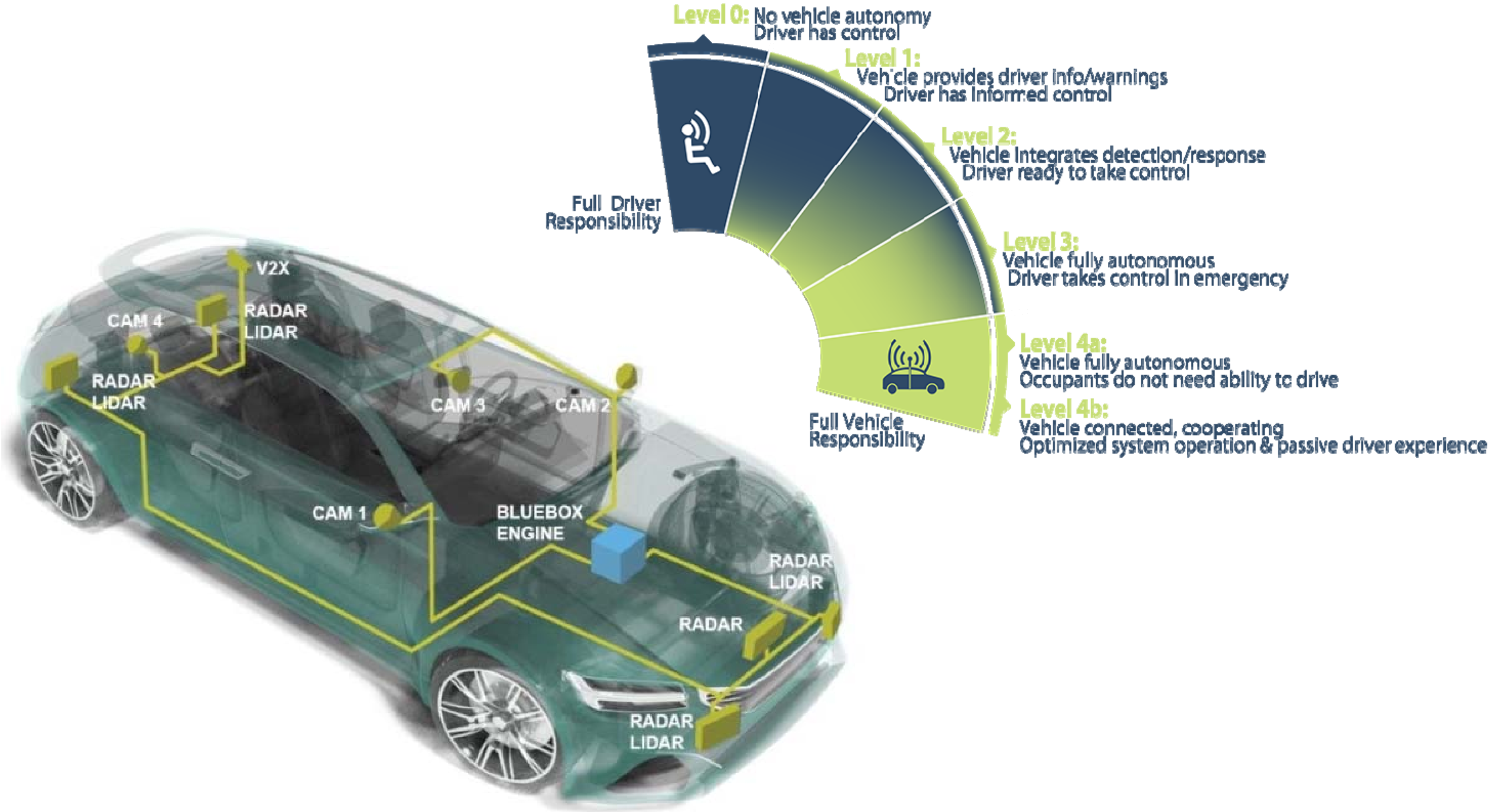
NHSTA policy for vehicle Automation





Source: NHTSA (Modified)

NXP Blue box for "Level 4" autonomous vehicles



Sensing modalities comparison

Table 1. Typical strengths and weaknesses of automotive sensors available today.

	Short Range Radar	Long Range Radar	Lidar	Ultrasound	Video Camera	3D-Camera	Far IR Camera
Range Measurement < 2m	o	o	o	++	-	++	-
Range Measurement 2..30m	+	++	++	-	-	o	-
Range Measurement 30..150m	n.a.	++	+	- -	-	-	-
Angle Measurement < 10 deg	+	+	++	-	++	+	++
Angle Measurement > 30 deg	o	-	++	o	++	+	++
Angular Resolution	o	o	++	-	++	+	++
Direct Velocity Information	++	++	- -	o	- -	- -	- -
Operation in Rain	++	+	o	o	o	o	o
Operation in Fog or Snow	++	++	-	+	-	-	o
Operation if Dirt on Sensor	++	++	o	++	- -	- -	- -
Night vision	n.a.	n.a.	n.a.	n.a.	-	o	++

++ : Ideally suited / + : Good performance / o : Possible, but drawbacks to be expected;

- : Only possible with large additional effort / - - : Impossible / n.a. : Not applicable

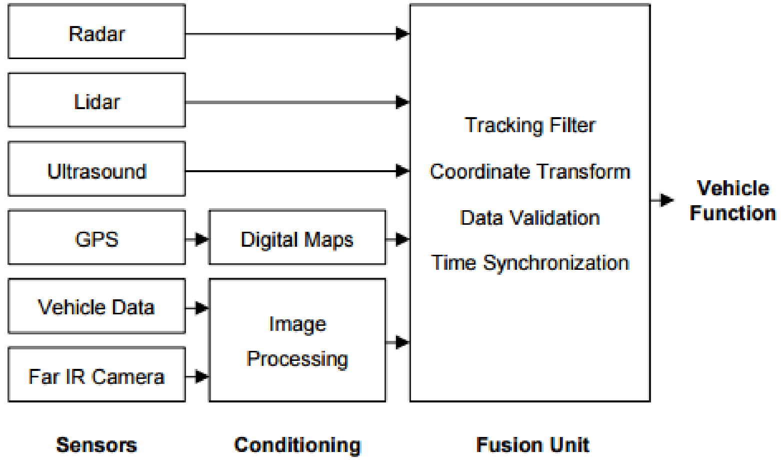
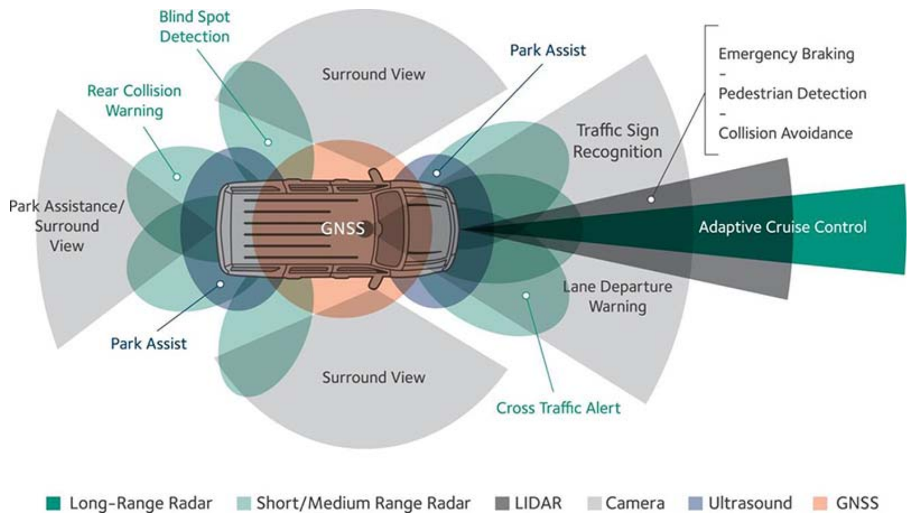
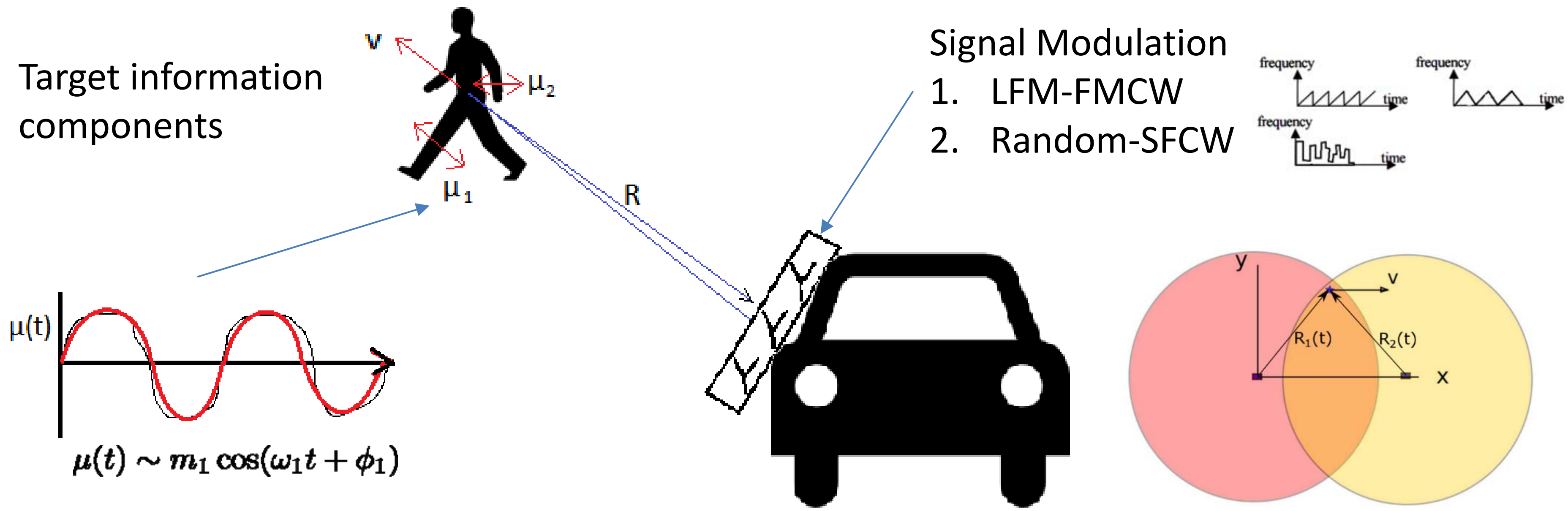


Fig. 7. Typical automotive sensor data fusion architecture.



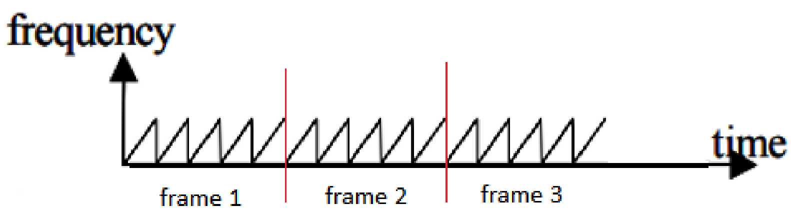
Rasshofer, R. H. and Gresser, K.: Automotive Radar and Lidar Systems for Next Generation Driver Assistance Functions, Adv. Radio Sci., 3, 205-209, doi:10.5194/ars-3-205-2005, 2005.

Short to medium range radar



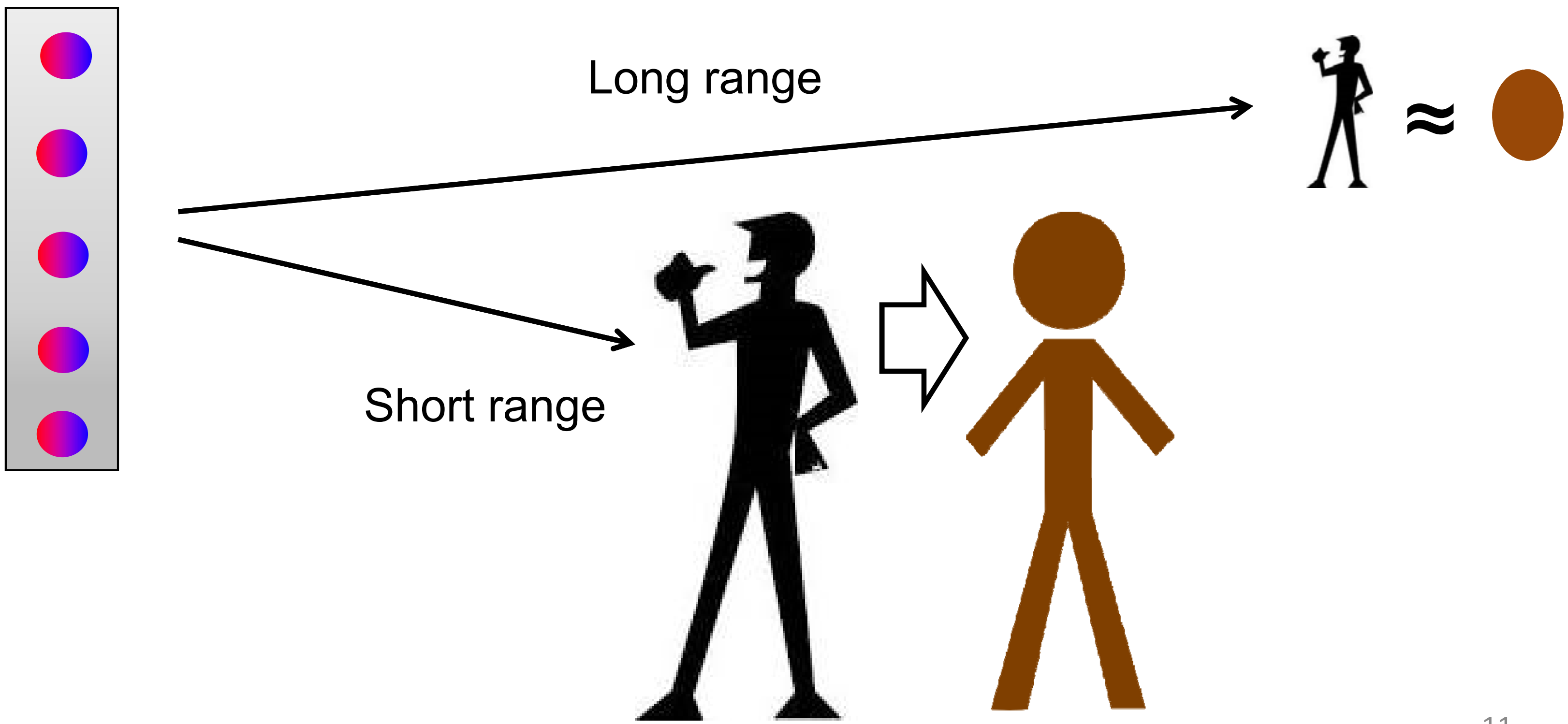
- Time varying range given by:

$$R(t) = \underbrace{R_0 + v_r t + \mu(t)}_{R_0,\ v_r,\ \mu \text{ remain constant in frame}}$$



New models are needed at short ranges

Targets look bigger at short range

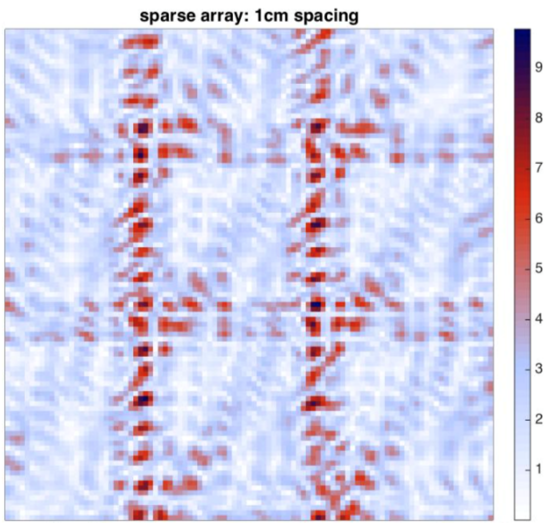


Need to revisit classical models

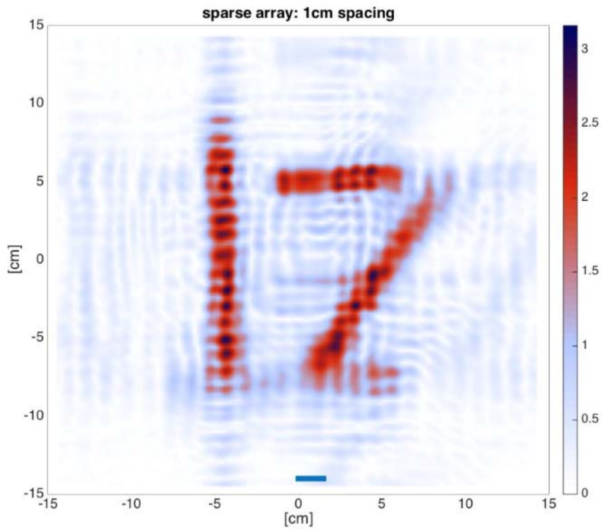
Results from new model

Target

Standard approach



New approach

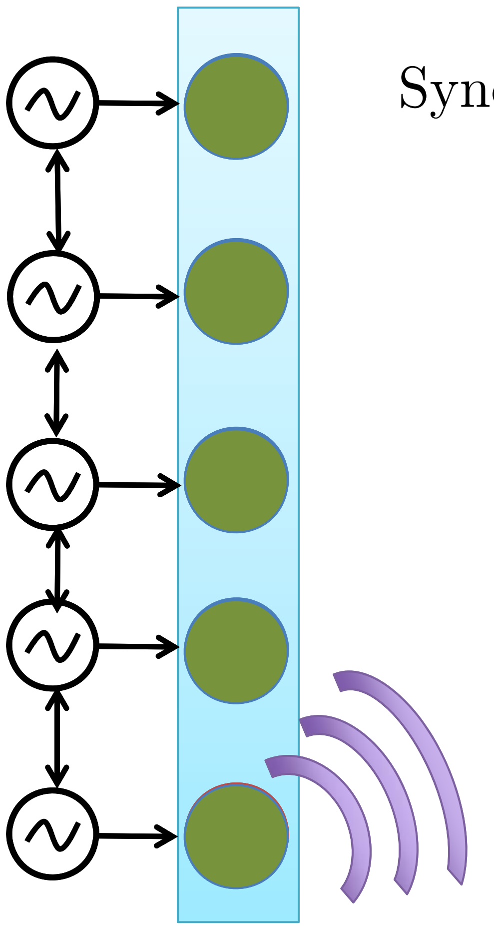


Overview

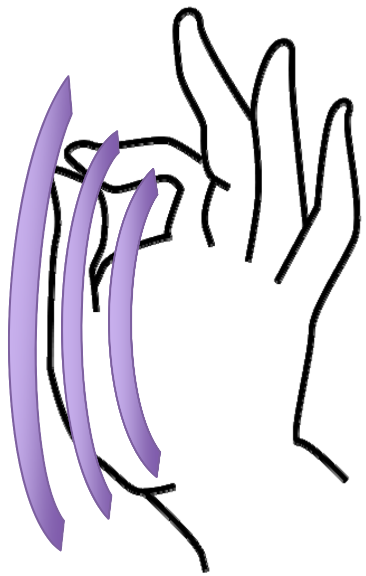
- Big picture considerations
  - Target models
  - Monostatic versus multistatic
- Fundamentals of monostatic arrays
  - Degrees of freedom as a function of scene and array geometry
  - Sparse array ➔ grating lobes
- Changing the dictionary to suppress grating lobes
  - From classical point scatterers to patches
  - The role of estimation-theoretic bounds in dictionary design
- Sparse reconstruction
  - Super-resolution by combining Newton with greedy pursuit

Architectural choice

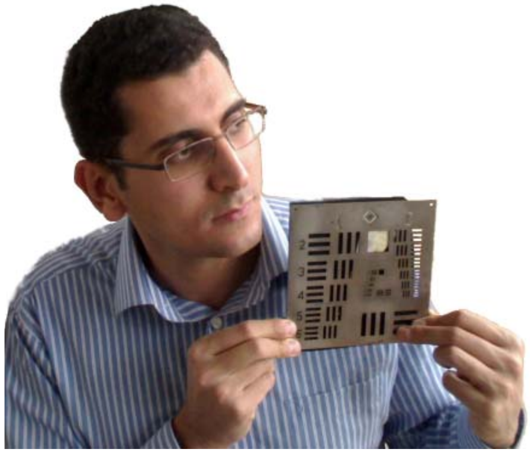
Multistatic



Synchronization $\rightarrow$ hard to achieve at mm-wave



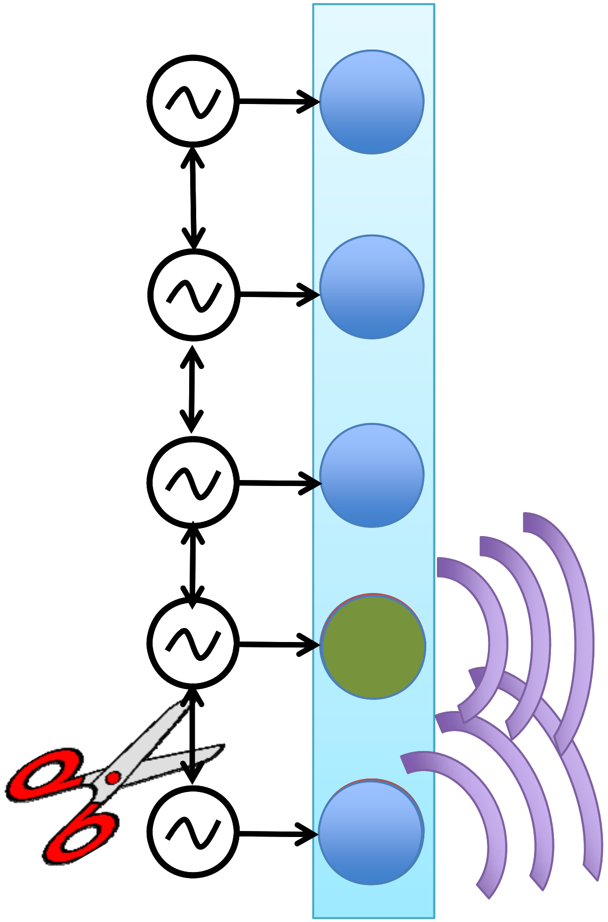
Rhode & Schwarz Imager: Big and multistatic



- MM-Wave imager from Rhode & Schwarz
- 736 Tx/Rx elements in 0.5mx0.5m aperture
- All elements are synchronized across the array

S. A. Ahmed, A. Schiessl, and L. Schmidt, “A Novel Fully Electronic Active Real-Time Imager Based on a Planar Multistatic Sparse Array” IEEE Transactions on microwave theory and techniques, vol. 59, NO. 12, December 2011.

Monostatic



- ✓ No synchronization needed
- ✓ Modularity
- ✓ Parallel data acquisition
- ✓ Low cost

Focus today: Array of monostatic elements

Degrees of freedom

Mathematical formulation

$L_1 =$ size of the aperture

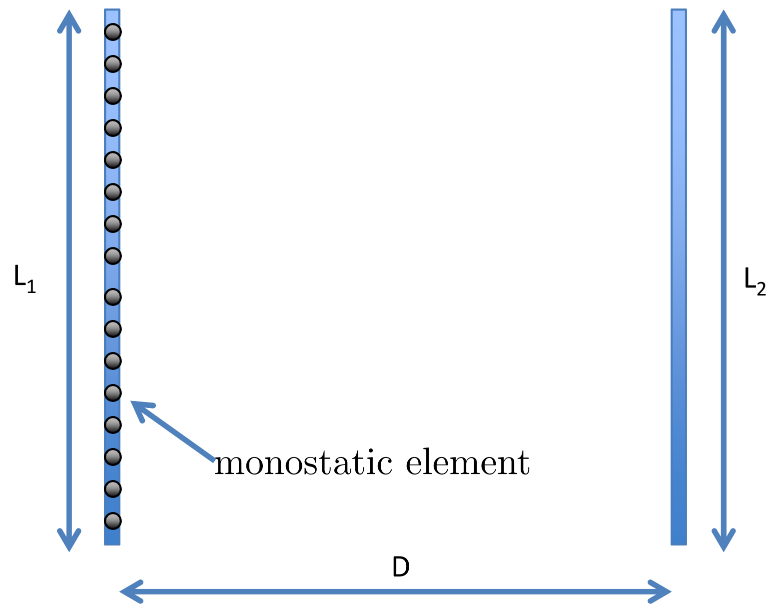
$L_2 =$ extent of the imaged object

$D =$ distance of the object from the array

$d =$ inter-element spacing

$\lambda =$ wavelength

$k = \frac{2\pi}{\lambda} =$ wavenumber



response of mono static array at n^{th} element

$$r[n] = \int_{x \in \Psi} \gamma(x) e^{-j2kR(x,x_n)} dx \qquad R(x, x_n) = \sqrt{D^2 + (x - x_n)^2} \approx D + \frac{(x - x_n)^2}{2D}$$

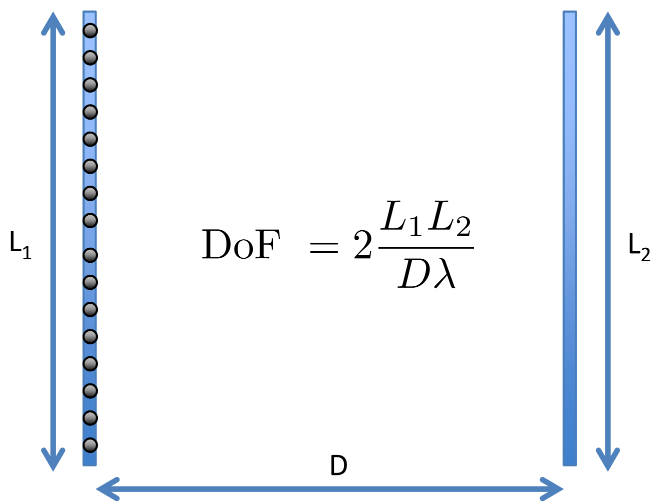
$$r[n] = e^{-j2kD} \int_{x \in \Psi} \gamma(x) e^{-j\frac{k}{D}(x - x_n)^2} dx$$

DoF: fundamental limits

$$r[n] = e^{-j2kD} \int_{x \in \Psi} \gamma(x) \underbrace{e^{-j\frac{k}{D}(x-x_n)^2}}_{\text{Integral kernel has been studied before}} dx$$

Eigenfunctions: Prolate Spheroidal Wave Functions

Eigenvalues: remains approximately constant until we reach a critical value (DoF)



$$\text{DoF} = 2\frac{L_1 L_2}{D\lambda}$$

$$\left.\begin{aligned} L_1 = L_2 &= 15 \text{ cm} \\ D &= 30 \text{ cm} \\ \lambda &= 0.5 \text{ cm} \end{aligned}\right\} \ \text{DoF} = 30$$

E. Torkildson, U. Madhow, M. Rodwell, "Indoor Millimeter Wave MIMO: Feasibility and Performance, " IEEE Transactions on Wireless Communications, vol. 10, no. 12, pp. 4150-4160, December 2011.

DoF: a practical interpretation

$r_1 \in \mathbb{C}^N$: response to a point scatterer located @x_1
$r_2 \in \mathbb{C}^N$: response to a point scatterer located @x_2

$$r_1^H r_2 = \sum_n \gamma_1^* \gamma_2 \; e^{j\frac{k}{D}(x_1 - x_n)^2} e^{-j\frac{k}{D}(x_2 - x_n)^2}$$

$$= \gamma_1^* \gamma_2 \; \exp\left(j\frac{k}{D}(x_1^2 - x_2^2)\right) \sum_n \exp\left(j2\frac{k}{D}(x_2 - x_1)x_n\right)$$

$$= C\sum_n \exp(j\omega n) = C\underbrace{\frac{\sin(N\omega/2)}{\sin(\omega/2)}}_{\substack{\text{periodic Dirichlet kernel} \\ \text{period} = 2\pi}}$$

$$\boxed{\omega \triangleq \frac{2k}{D}(x_2 - x_1)d}$$

To avoid aliasing: visible range of $\omega < 2\pi$

$$\frac{2k}{D}L_2 d < 2\pi \;\Rightarrow\; \frac{2}{\lambda D}L_2 d < 1 \;\Rightarrow\; \frac{2}{\lambda D}L_2 L_1 < (N-1) \;\Rightarrow\; \frac{2}{\lambda D}L_2 L_1 + 1 < N$$

$$\#\text{array elements} \geq \text{DoF} + 1$$

DoF ➔ geometry-based limit on resolution

For $N \gg \dfrac{2L_1L_2}{D\lambda}$,

$$\frac{|r_1^H r_2|}{||r_1||.||r_2||} = \frac{1}{N}\frac{\sin\left(\frac{kL_1N}{D(N-1)}(x_2-x_1)\right)}{\sin\left(\frac{kL_1}{D(N-1)}(x_2-x_1)\right)}$$

$$\approx \frac{1}{N}\frac{\sin\left(\frac{kL_1}{D}(x_2-x_1)\right)}{\frac{kL_1}{D(N-1)}(x_2-x_1)}$$

$$\approx \operatorname{sinc}\left(\frac{kL_1}{\pi D}(x_2-x_1)\right) \longrightarrow \boxed{\text{Independent of } N}$$

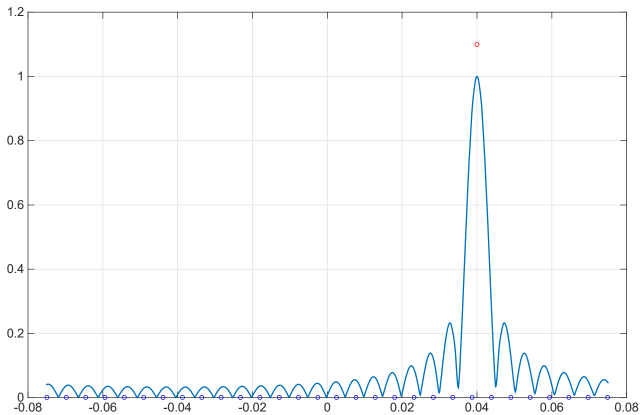
Increasing the number of array elements beyond DoF:

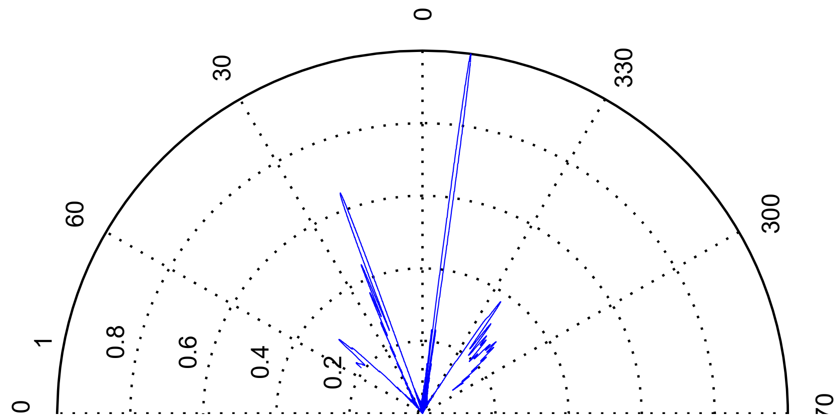
- Does not improve the ambiguity function for locating a point scatterer
- Only leads to an increase in the effective signal to noise ratio (SNR)

Going below DoF → Grating lobes

Grating lobes appear when #elts < DoF

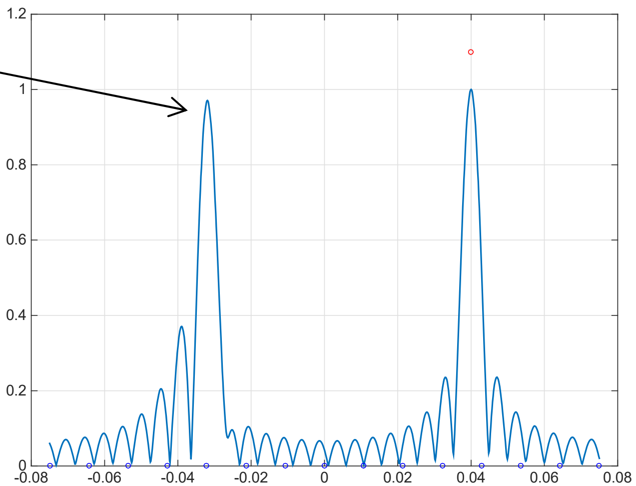
30 elements

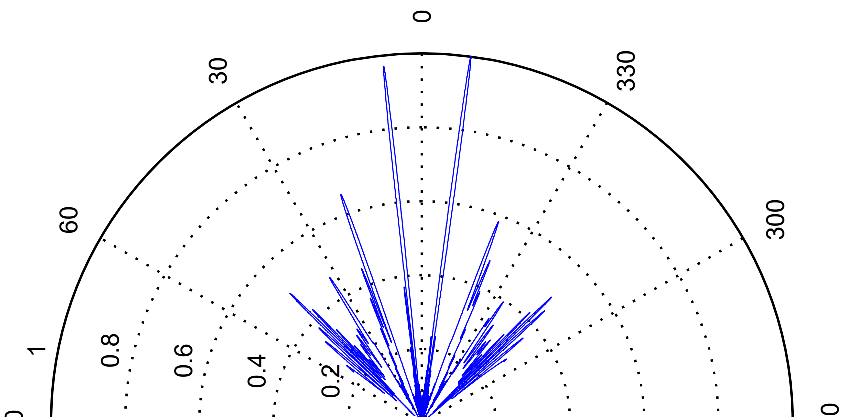




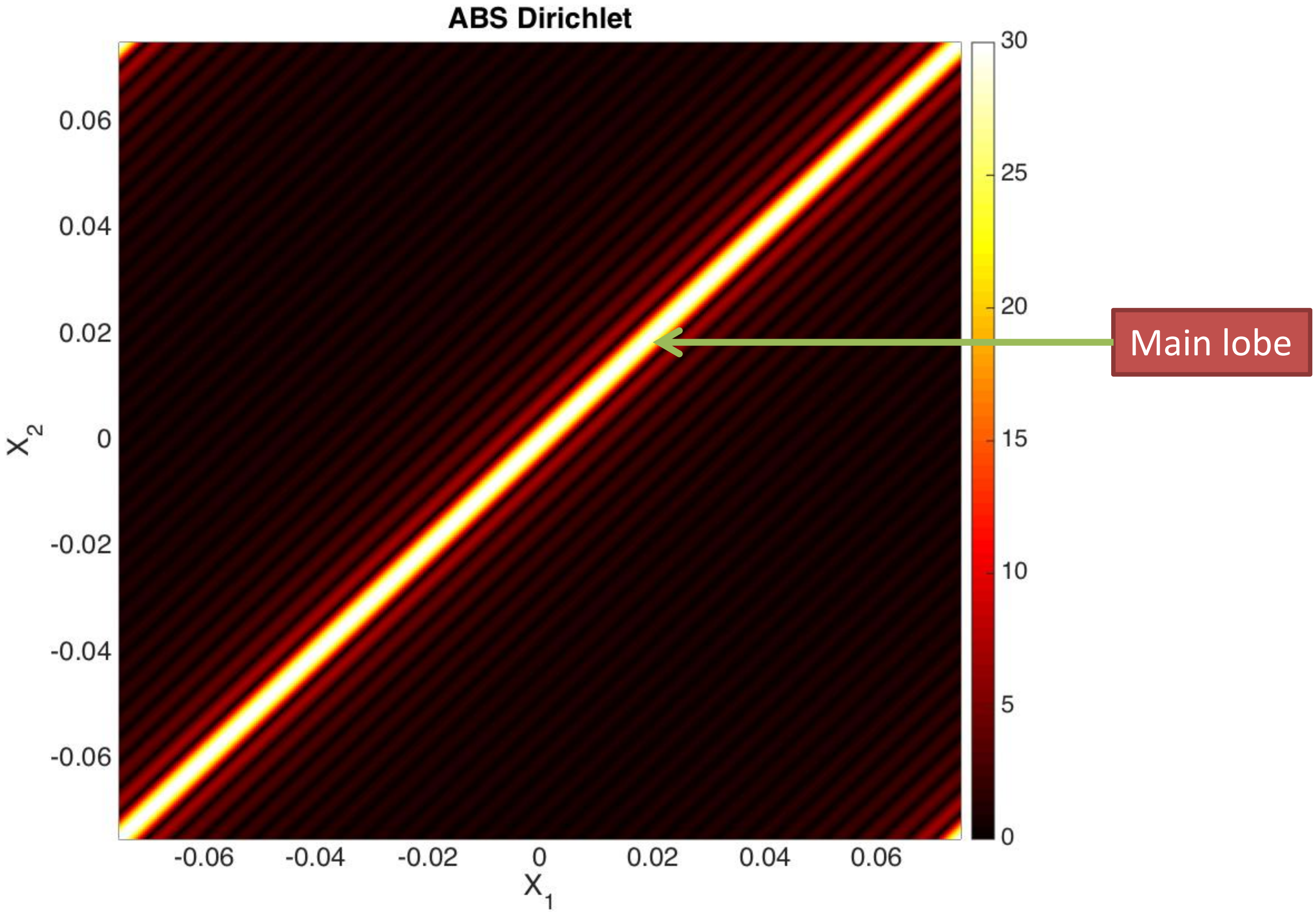
15 elements

Grating lobe (ambiguity)

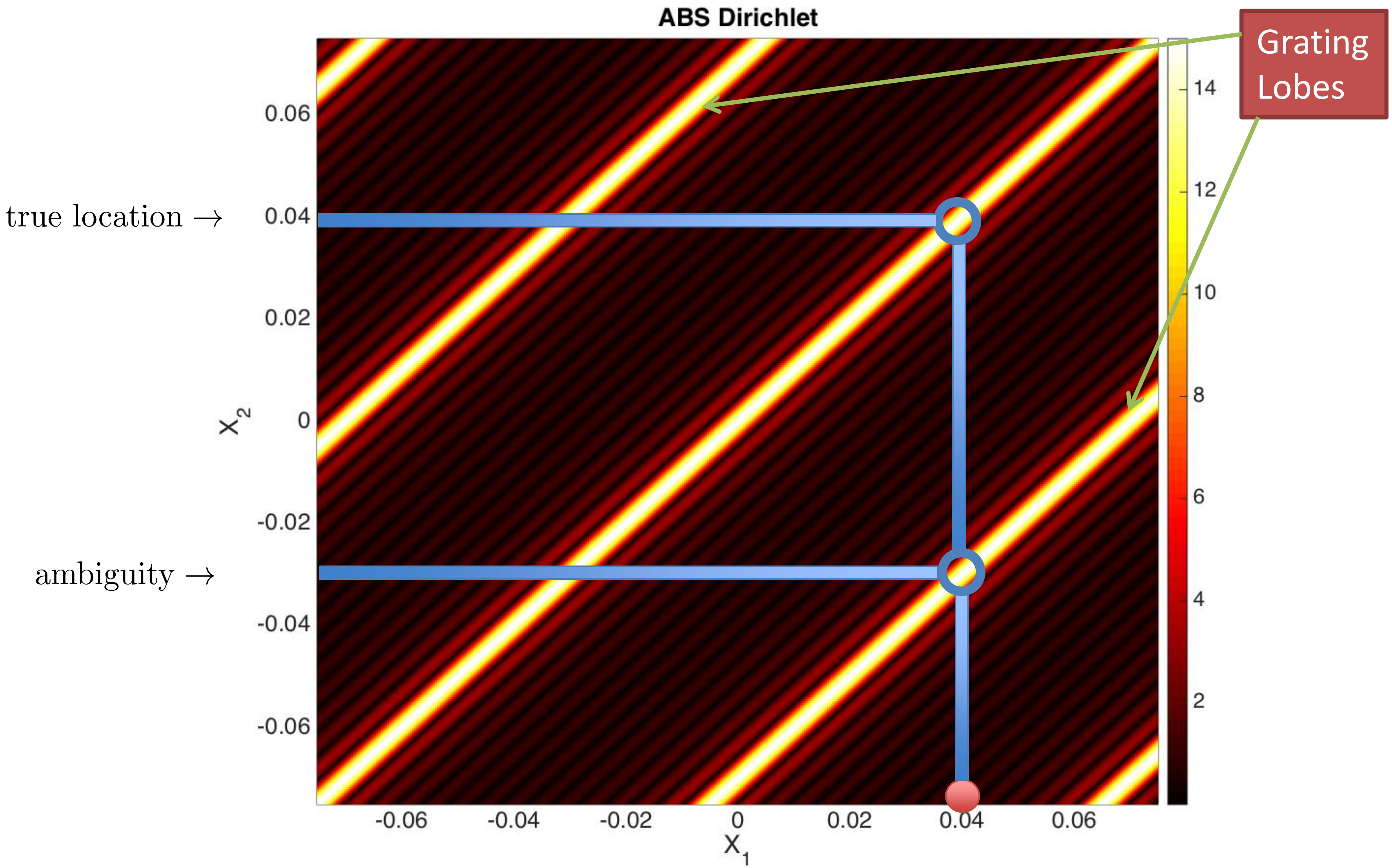




2D Dirichlet – N=31



2D Dirichlet – N=15



Spatial aggregation, or the patch model

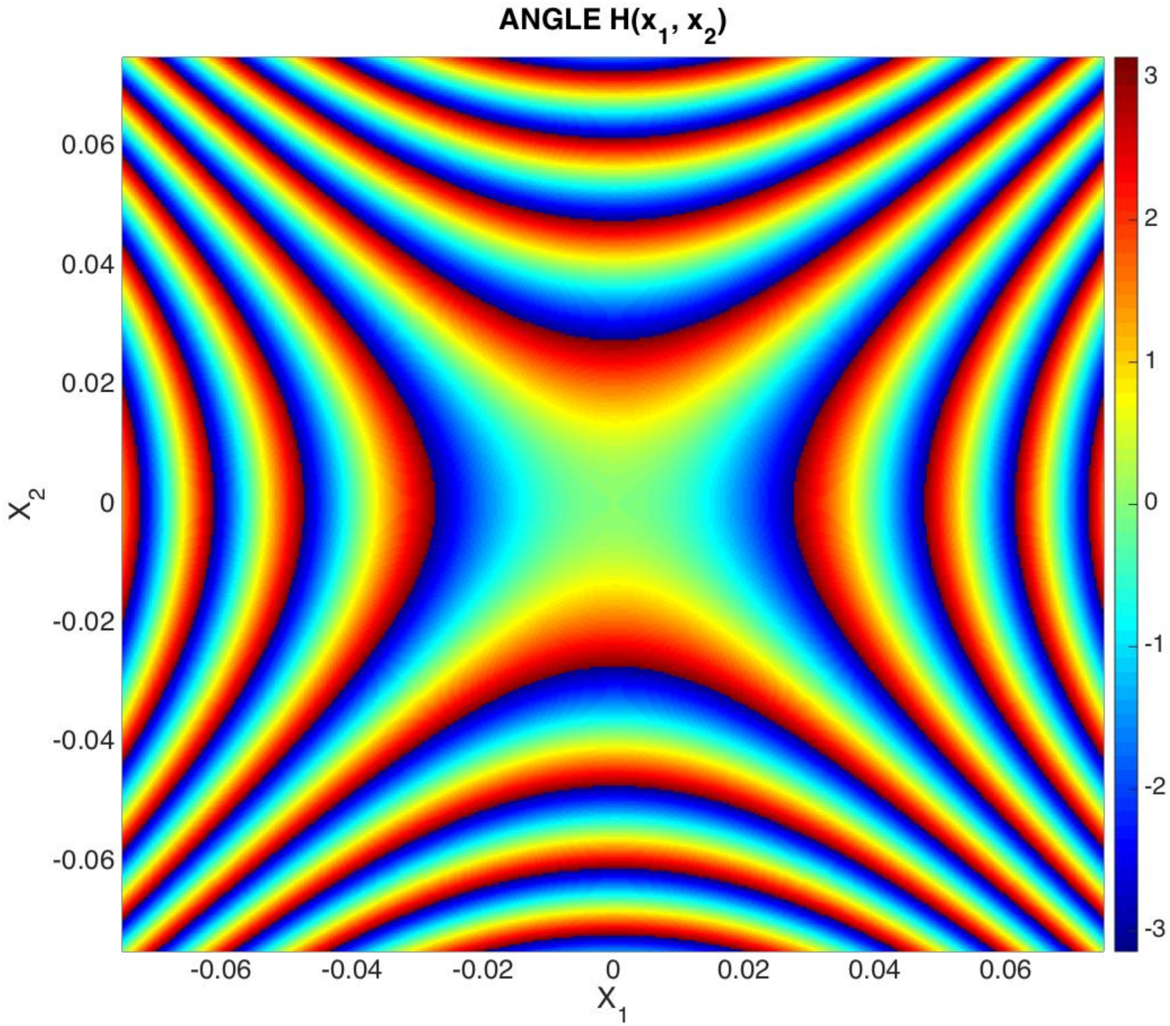
Spatial Aggregation: mathematical formulation

$$r_1 \in \mathbb{C}^N = \text{ response to the collection of point scatterers denoted by } \Psi_1$$
$$r_2 \in \mathbb{C}^N = \text{ response to the collection of point scatterers denoted by } \Psi_2$$

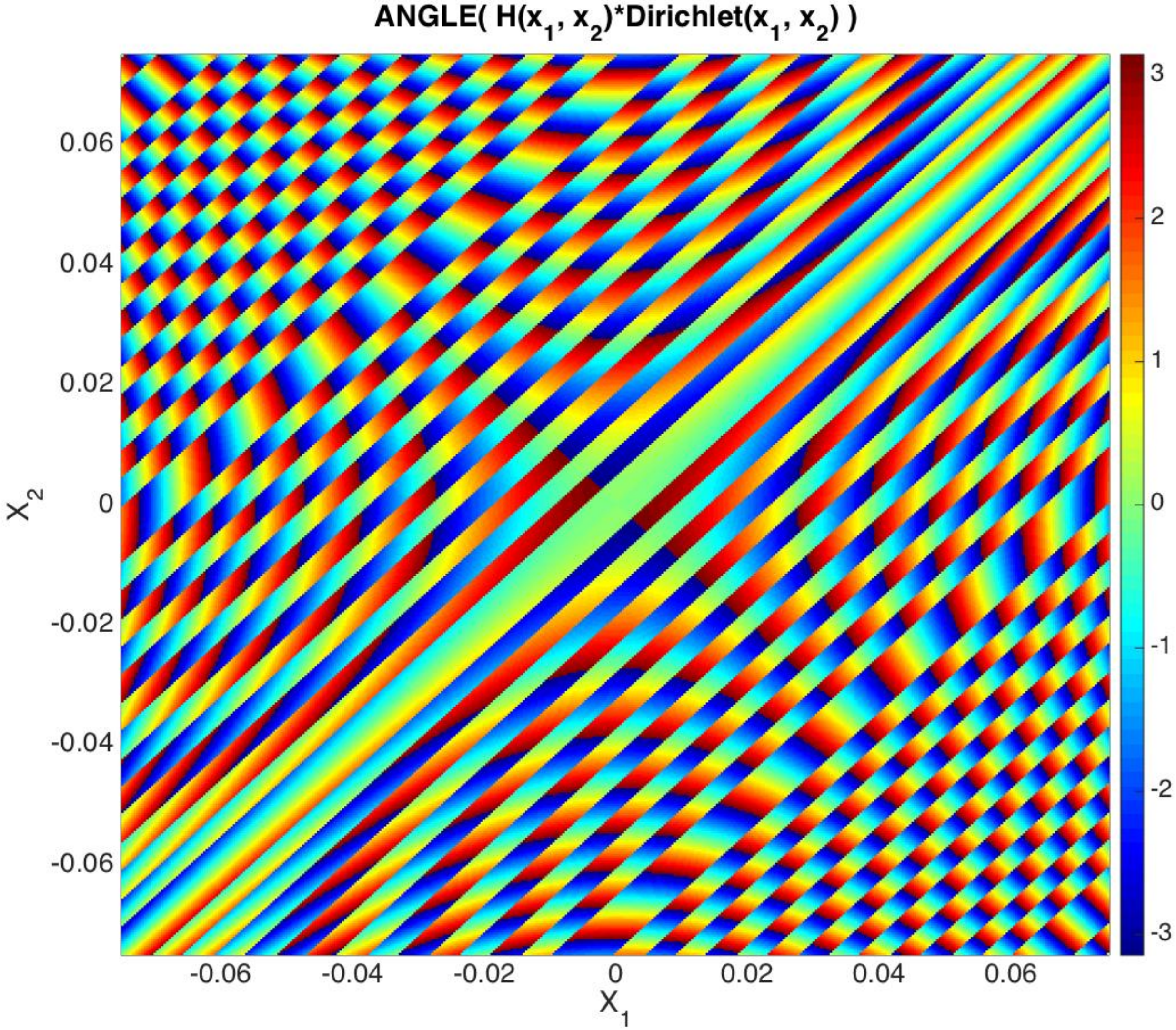
$$r_1^H r_2 = \sum_n \left(\int_{x_1 \in \Psi_1} \gamma^*(x_1) e^{j2kR(x_1,x_n)} dx_1 \int_{x_2 \in \Psi_2} \gamma(x_2) e^{-j2kR(x_2,x_n)} dx_2 \right)$$
$$= \sum_n \left(\int_{x_1 \in \Psi_1} \int_{x_2 \in \Psi_2} \gamma^*(x_1)\gamma(x_2) e^{j2k\left(\frac{(x_1-x_n)^2-(x_2-x_n)^2}{2D}\right)} dx_2 dx_1 \right)$$
$$= \int_{x_1 \in \Psi_1} \int_{x_2 \in \Psi_2} \gamma^*(x_1)\gamma(x_2) \underbrace{e^{j\frac{k}{D}(x_1^2-x_2^2)}}_{H(x_1,x_2)} \underbrace{\sum_n e^{j\frac{2k}{D}(x_2-x_1)x_n}}_{\text{Dirichlet}} dx_2 dx_1$$
$$= \int_{x_1 \in \Psi_1} \int_{x_2 \in \Psi_2} \gamma^*(x_1)\gamma(x_2) H(x_1,x_2) \mathrm{Dir}(x_1,x_2) dx_2 dx_1$$

$H(x_1, x_2) =$ spatial filter that causes destructive summation of the collection response

$H(x_1, x_2)$

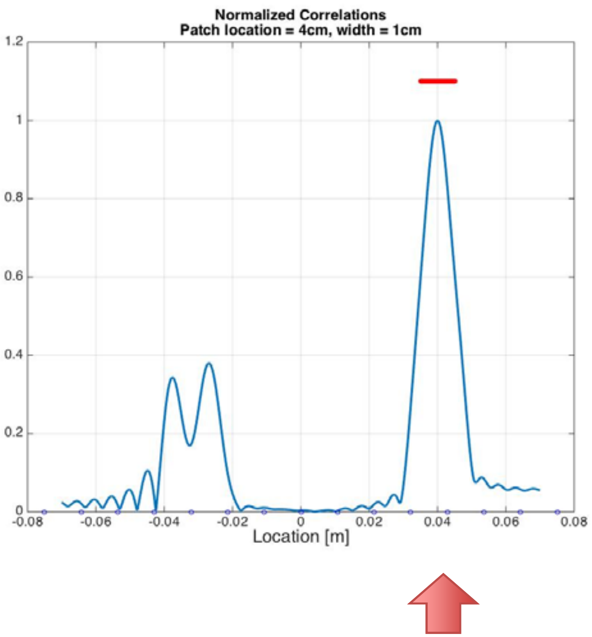

ANGLE H(x1, x2)
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
X2
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
X1
3
2
1
0
-1
-2
-3

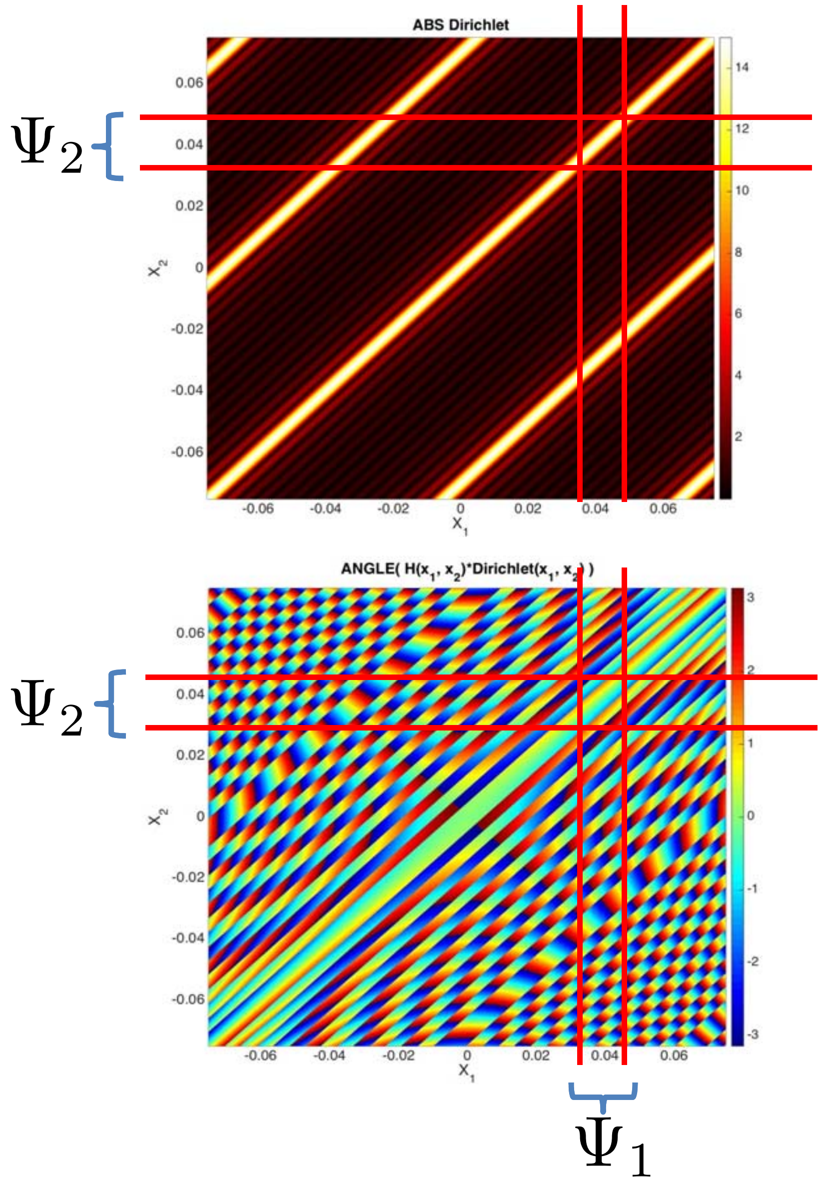
$H(x_1 , x_2)$*Dirichlet(x_1 , x_2)



Integration in the main lobe

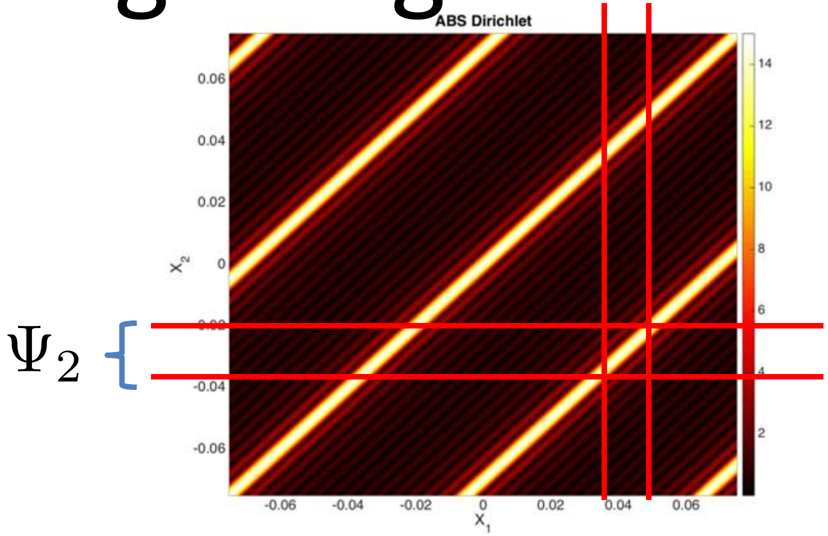
Patch width = 1cm
Patch location = 4cm
elements = 15

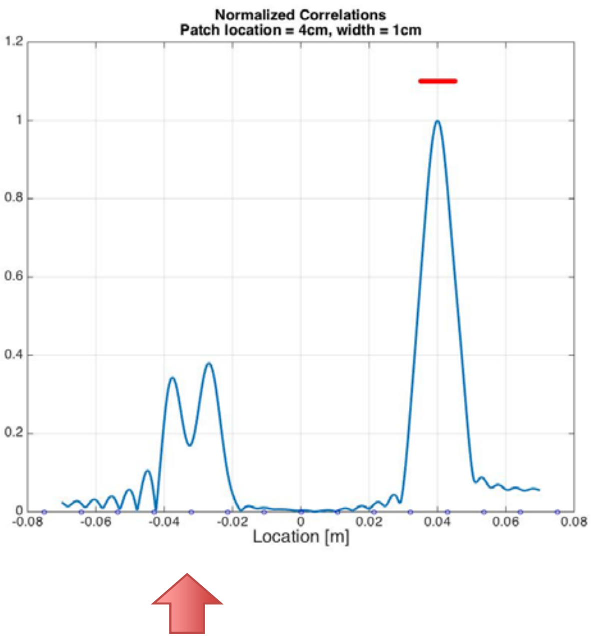


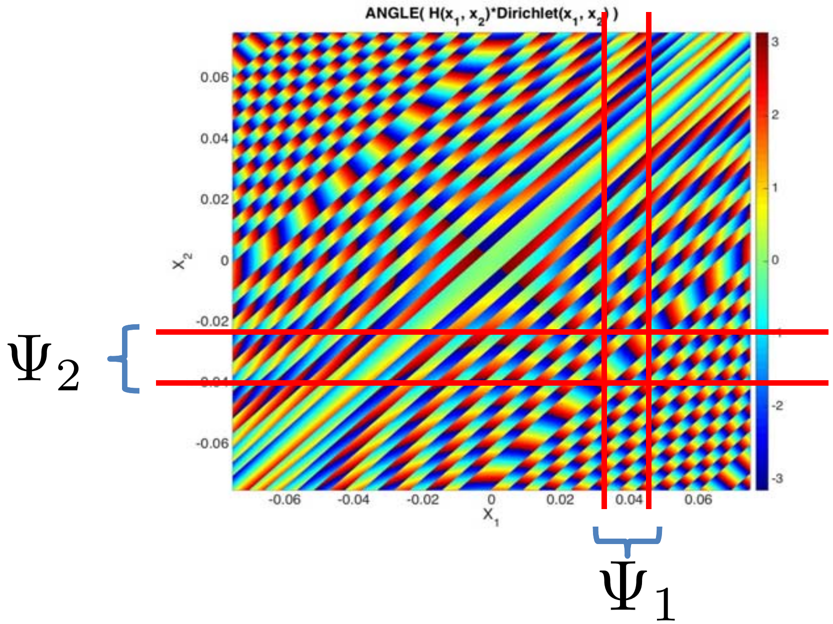


Integration in the grating lobe

Patch width = 1cm
Patch location = 4cm
elements = 15







Choosing patch size via estimation theory

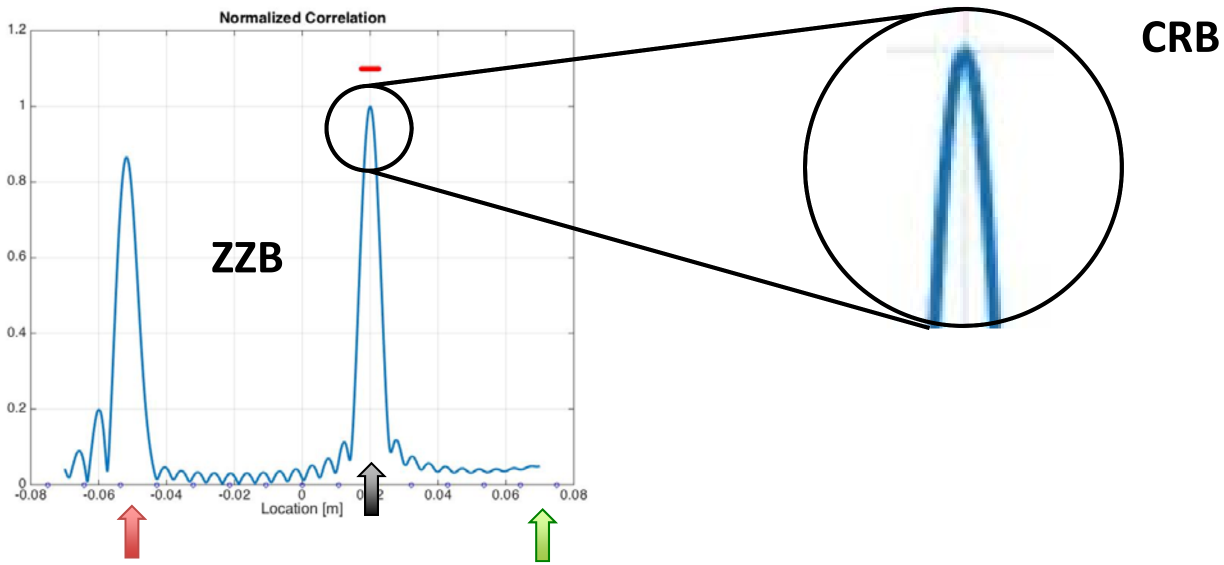
$$\mathbf{y} = \mathbf{s}(\theta) + \mathbf{z}$$

target parameter $\quad \sim CN(\mathbf{0}, \sigma^2 \mathbb{I})$

$$\left.\begin{aligned} L_1 = L_2 &= 15 \text{ cm} \\ D &= 30 \text{ cm} \\ \lambda &= 0.5 \text{ cm} \end{aligned}\right\} \text{DoF} = 30$$

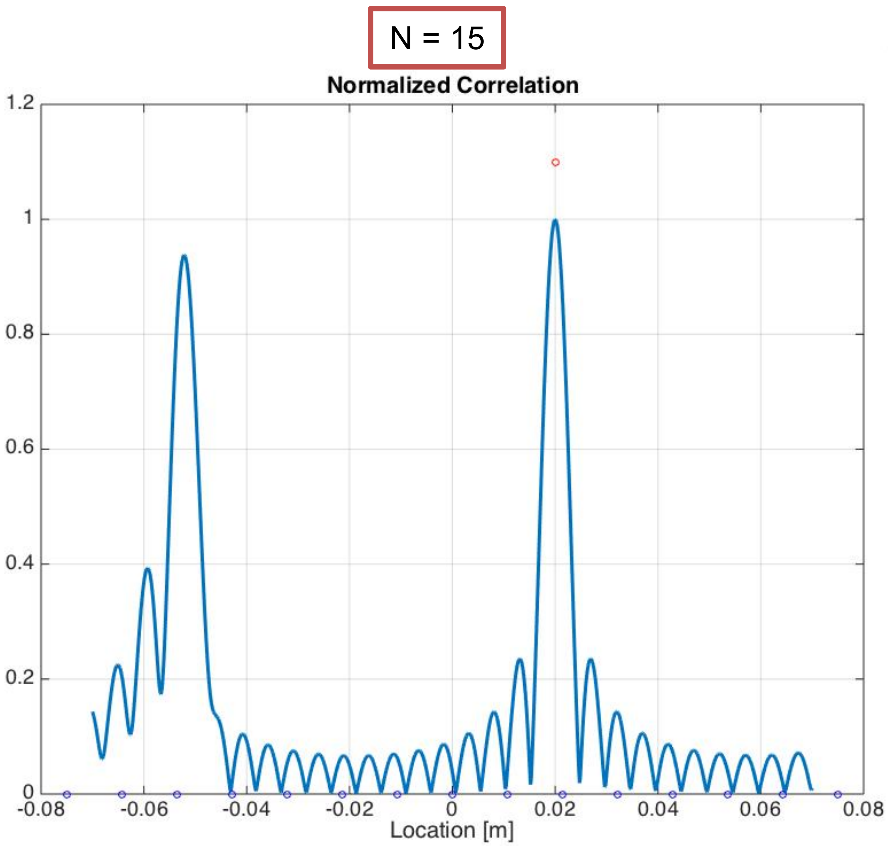
Cramer-Rao Bound (CRB): Lower bound on variance assuming estimate is in the right bin

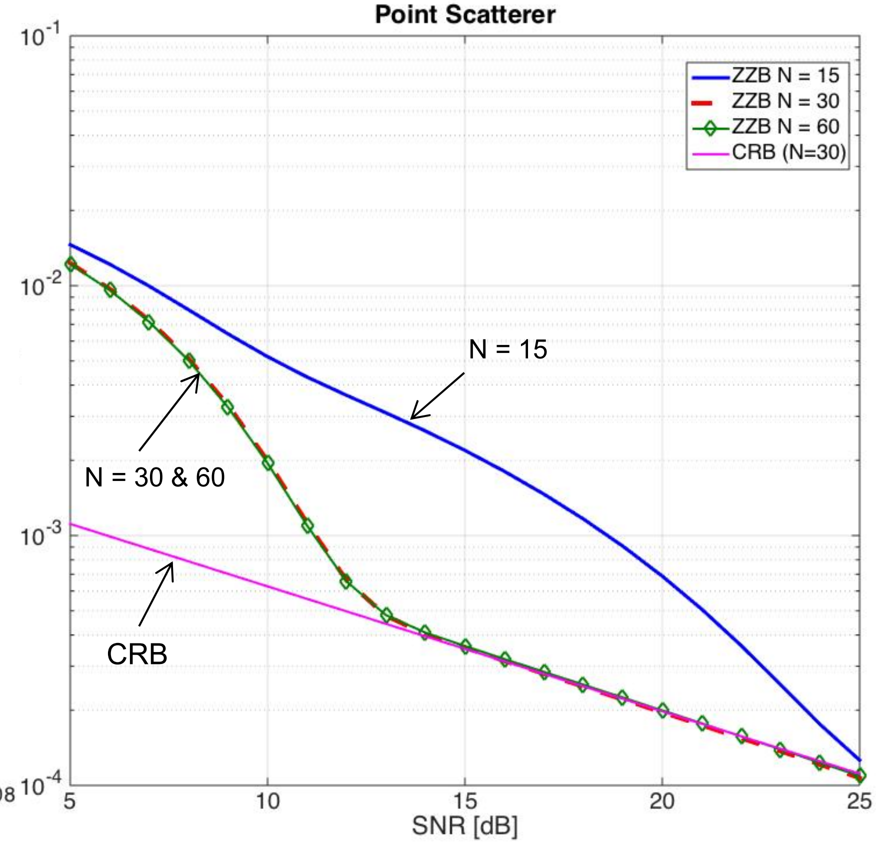
Ziv-Zakai Bound (ZZB): Tighter than CRB, accounts for estimate falling in wrong bin (converges to CRB at high SNR if correlation plot approximately unimodal)



Choose the smallest patch size such that ZZB "behaves well"

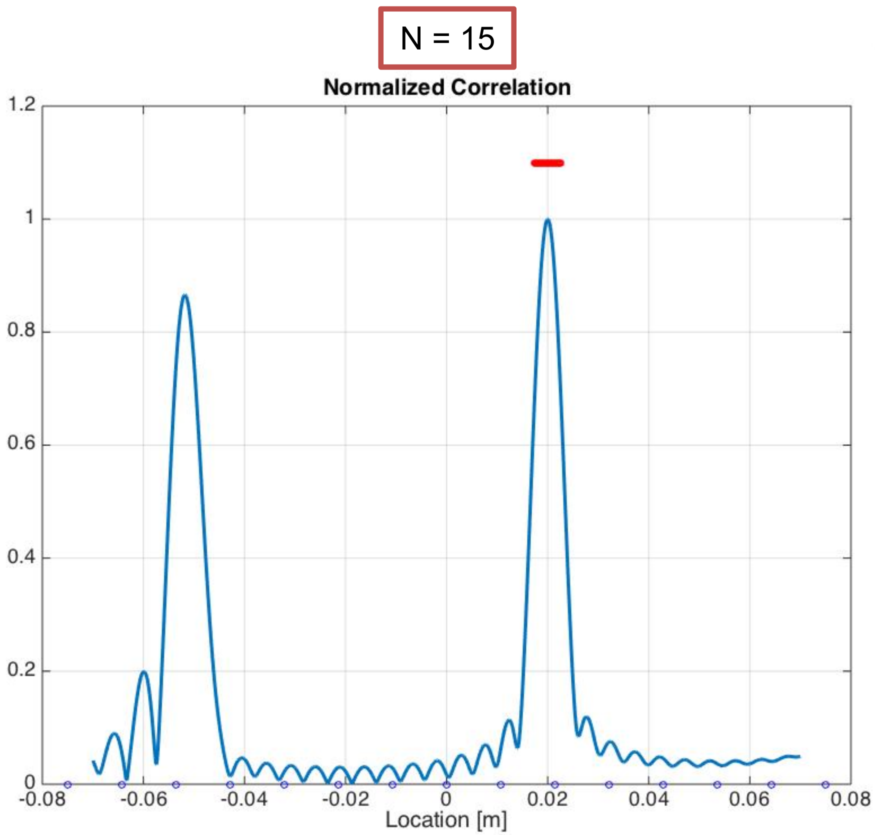
Point Scatterer

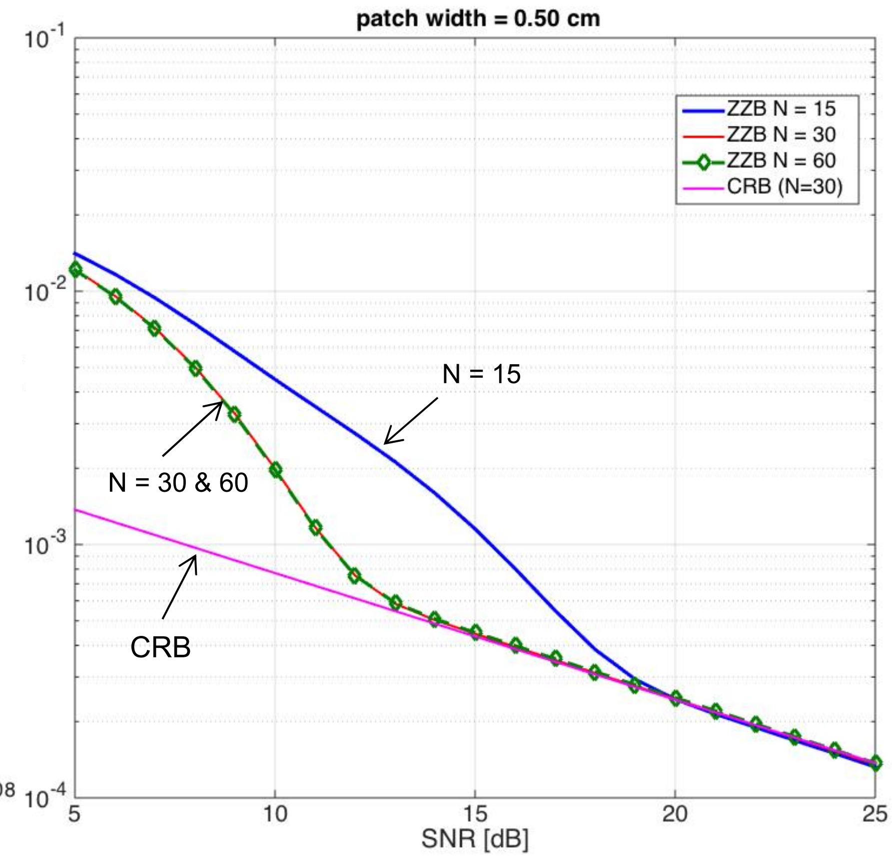




Grating lobes ➔ ZZB behaves badly

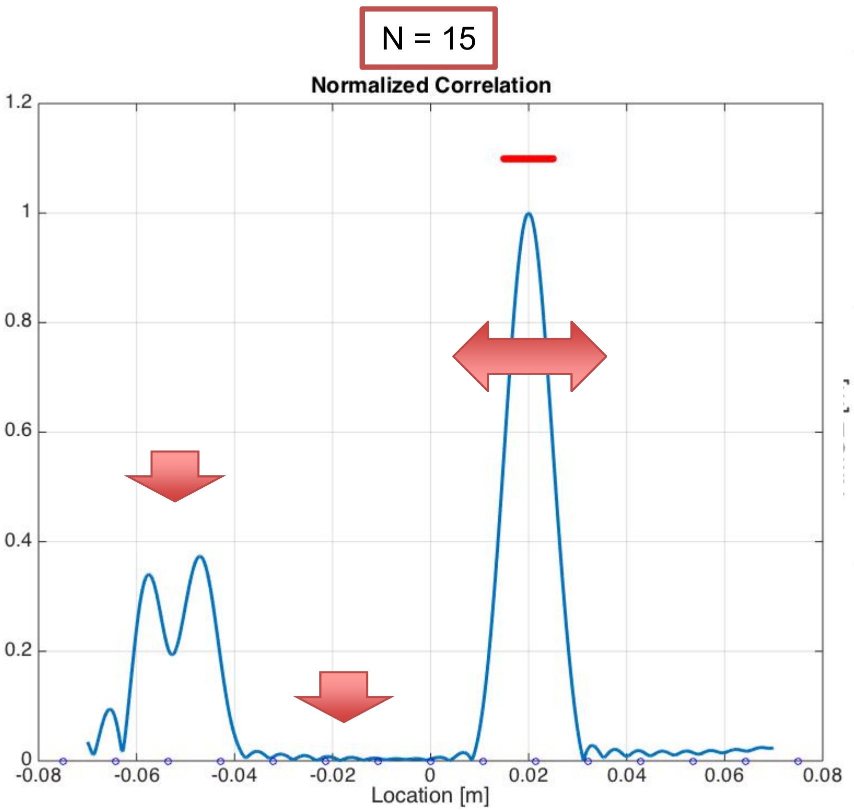
0.5 cm Patch

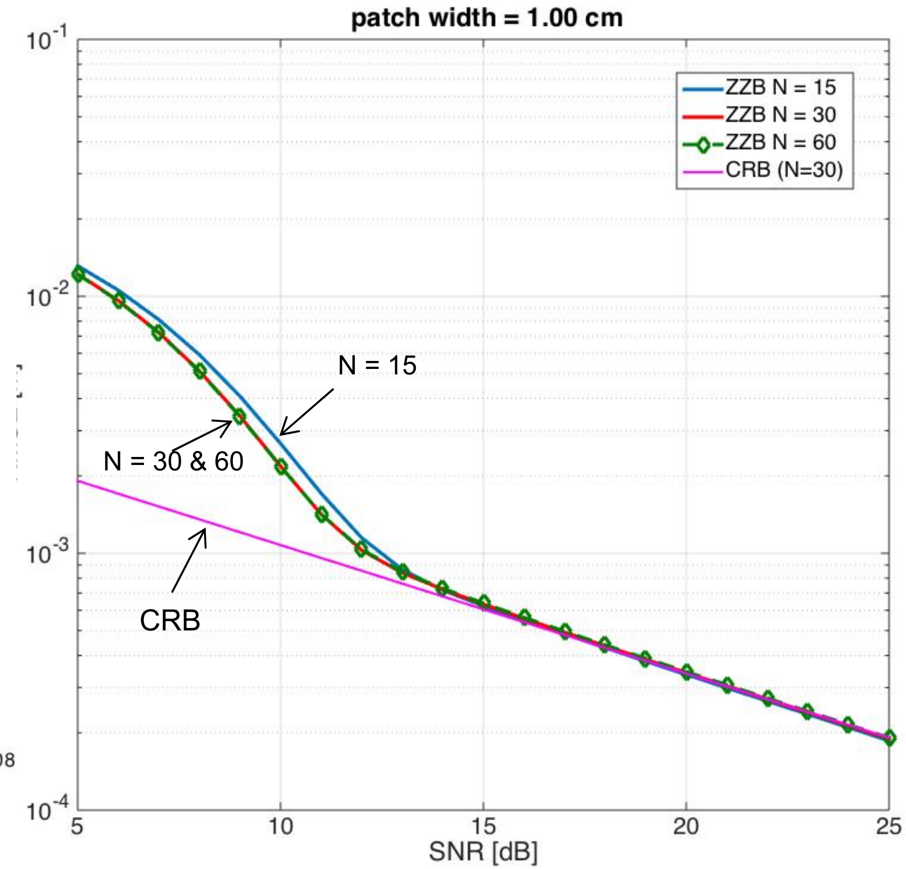




Better, but ZZB still behaves worse than with DoF # elements

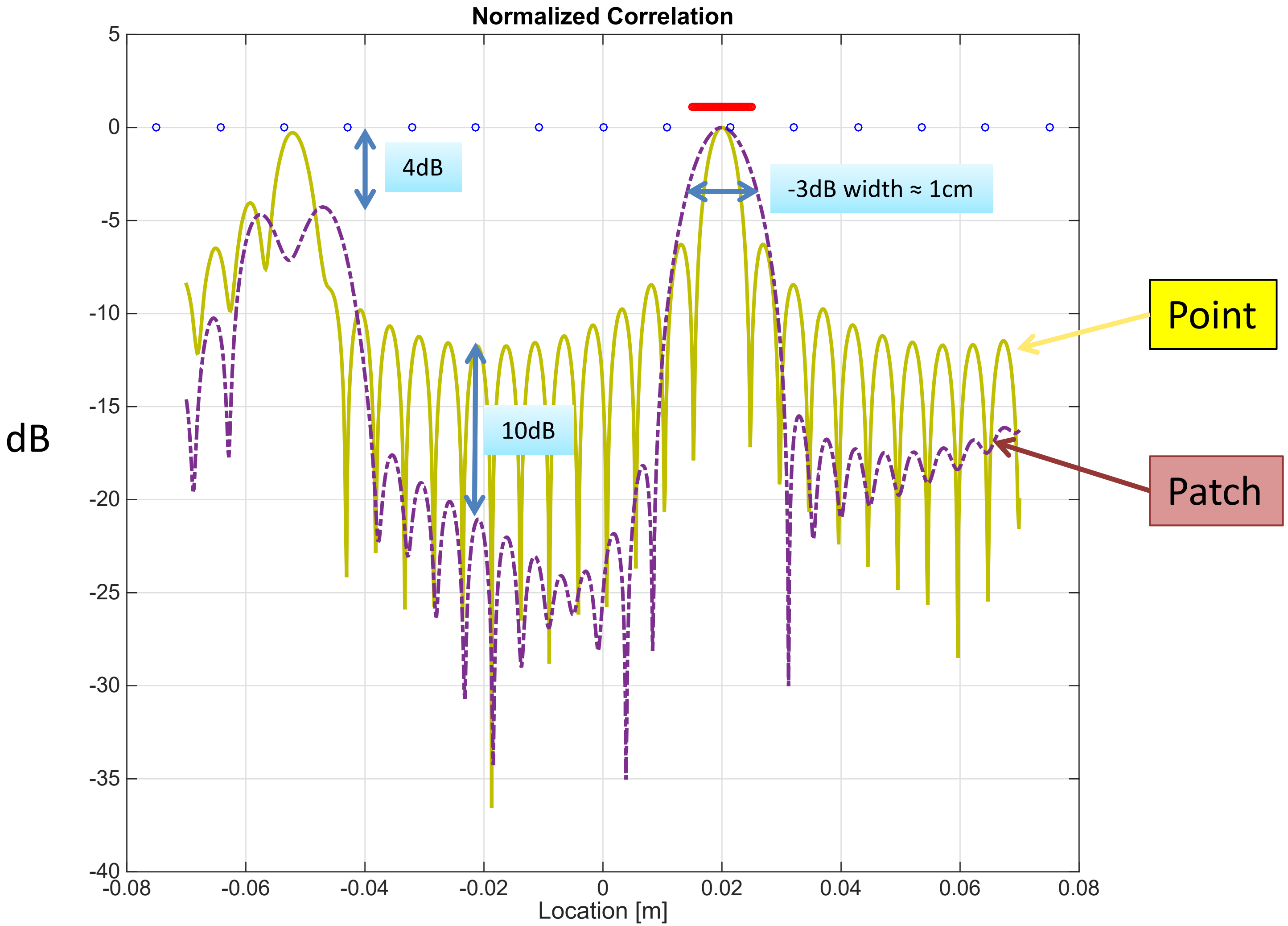
1 cm Patch





Large enough patch ➔ ZZB behavior same as with DoF # elements

Normalized Correlations

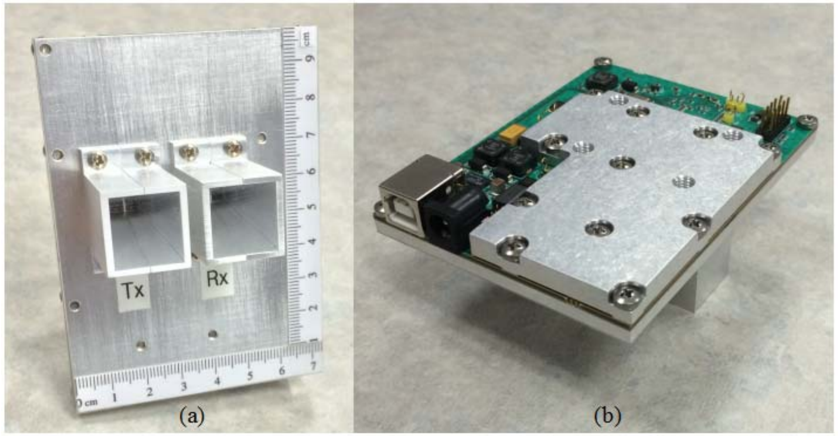


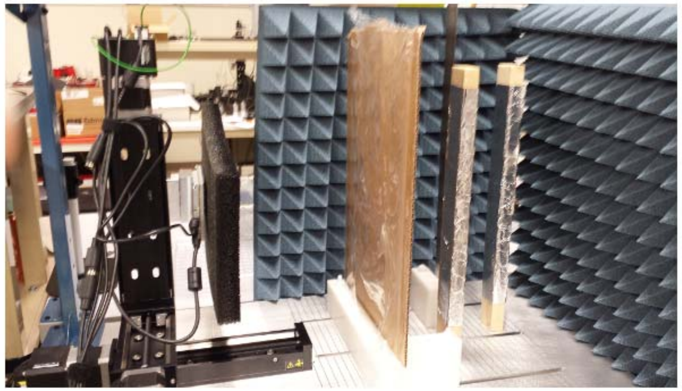
Patch model suppresses both grating lobes and sidelobes

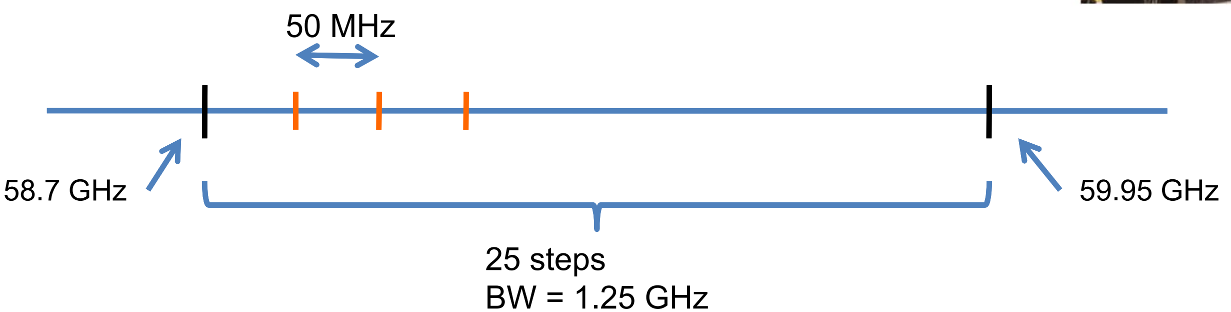
Experimental results

SFCW Radar Prototype

- 60 GHz Wideband SFCW radar
- Quasi-monostatic architecture
- Moving platform emulates 2D array
- Software controlled step size and frequency order







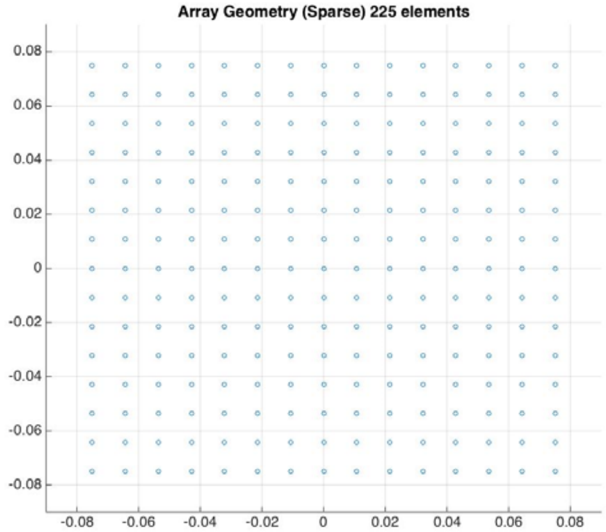
Built by:
Karam Noujeim (Anritsu)
Measurements taken by:
Greg Malysa (now at TI)

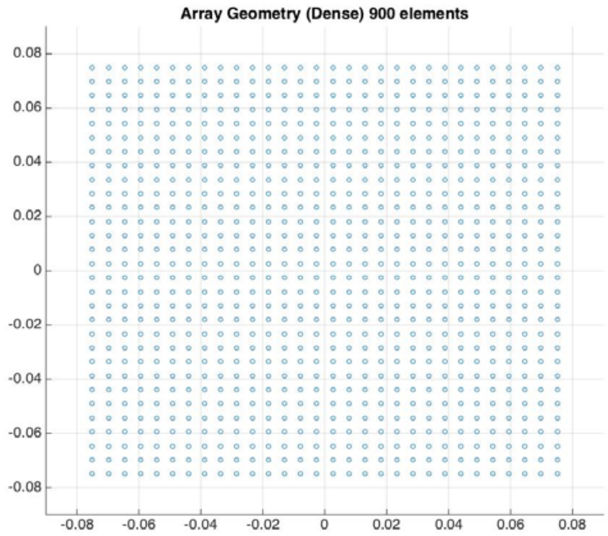
Emulated Array Geometry

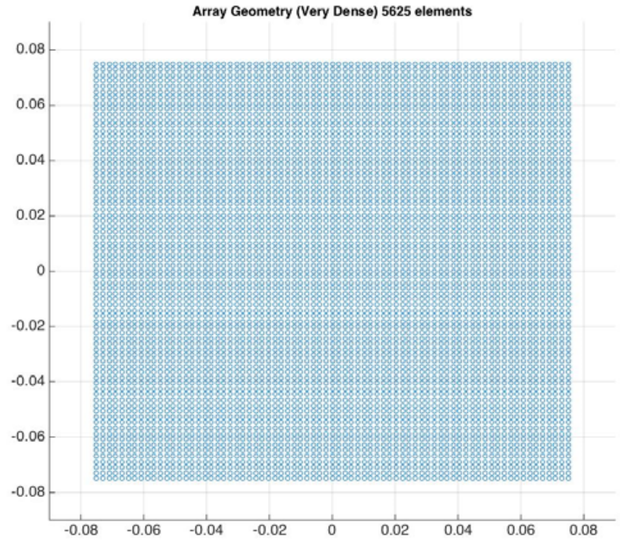
Sparse Array: 225 elements
Inter-element spacing 1cm

Dense Array: 900 elements
Inter-element spacing 5mm

Very Dense Array: 5625 elements
Inter-element spacing 2mm





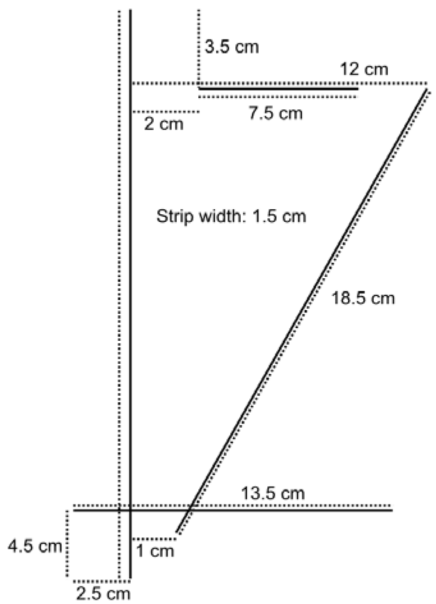


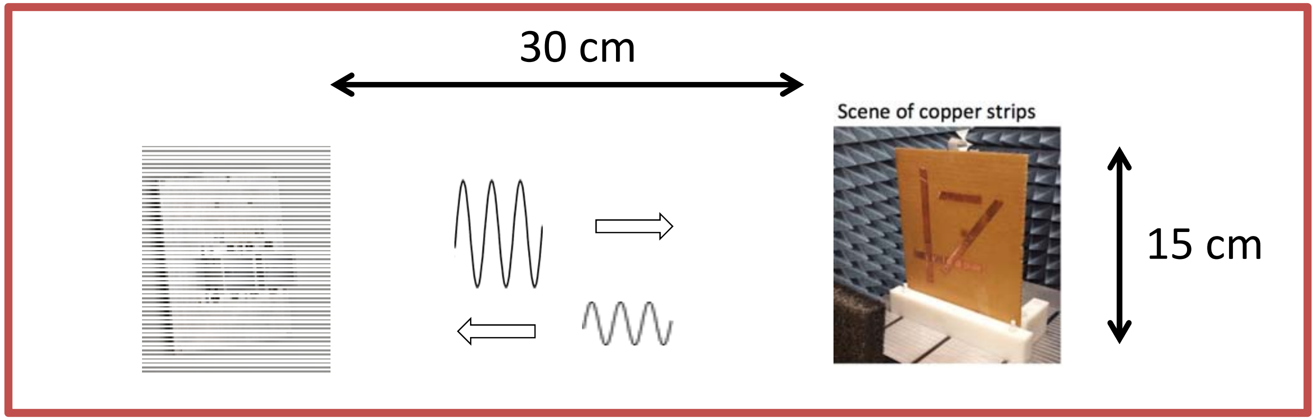
15 elements
2λ-spacing

30 elements
λ-spacing

75 elements
0.4λ-spacing

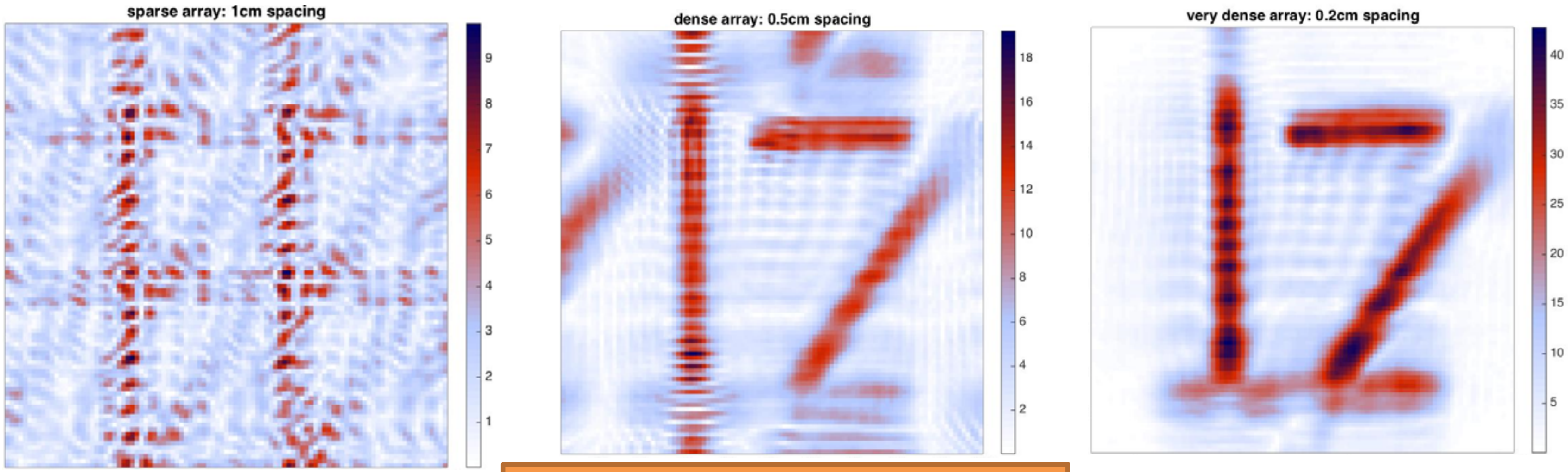
Scene with copper strips



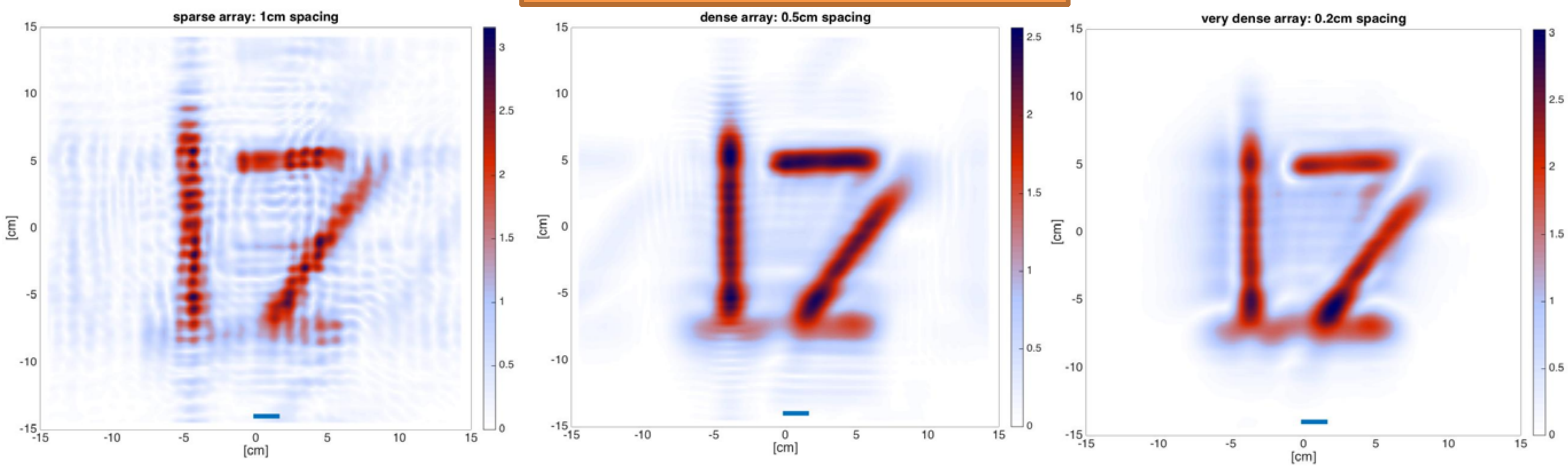


Matched Filter: Point v/s Patch

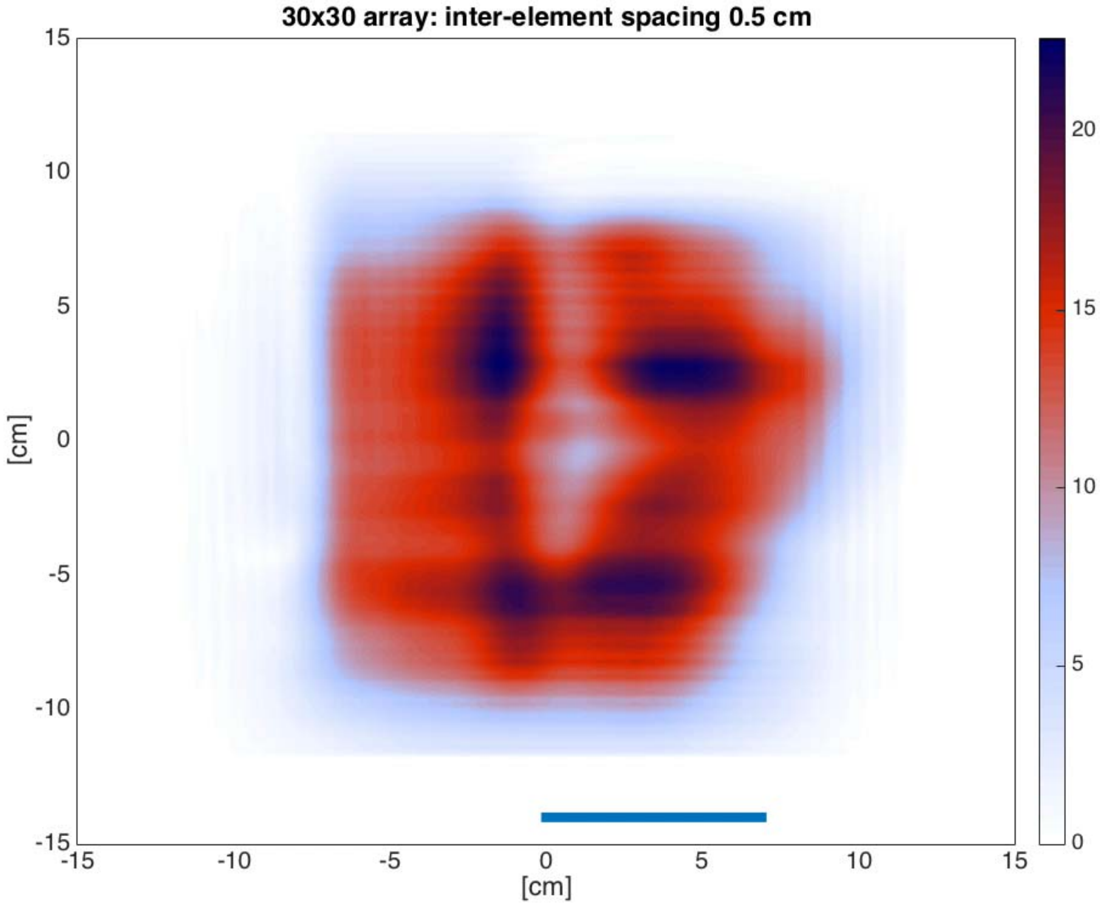
Matched Filter (point)



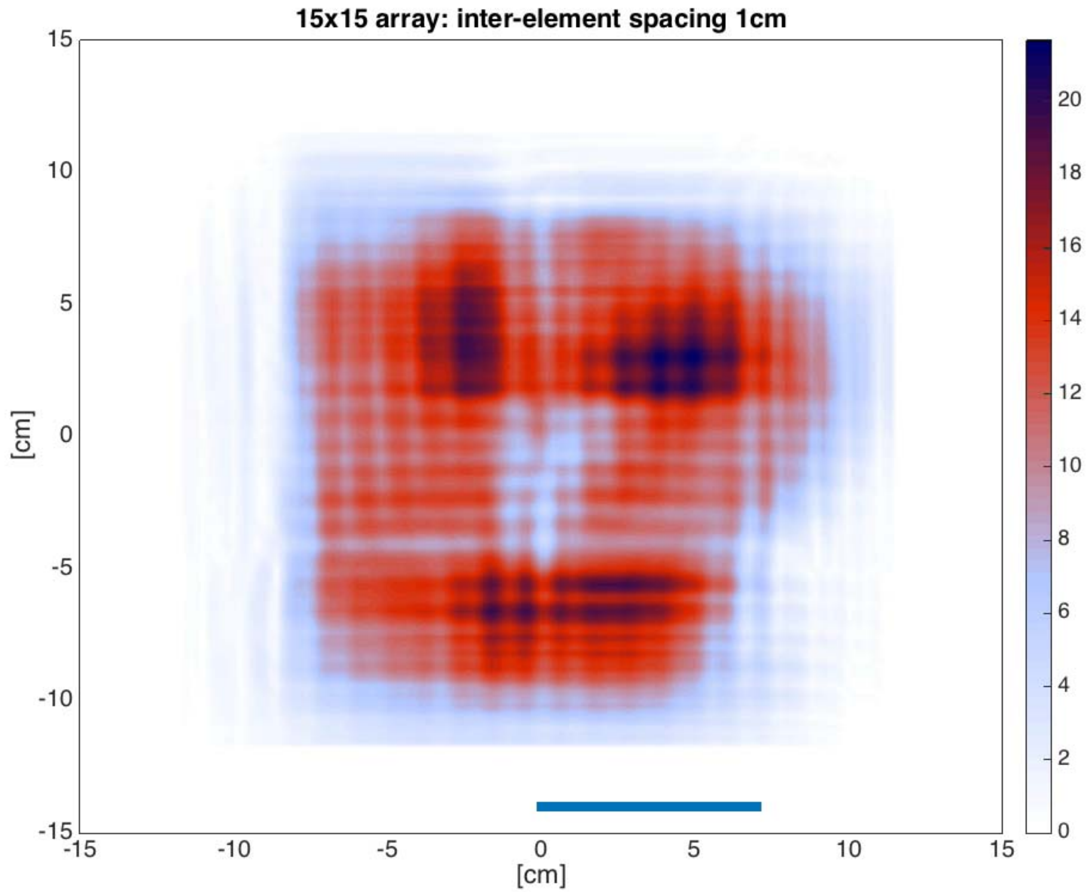
Matched Filter (1.5cm patch)



Dense Array – Changing the nominal patch size



Sparse Array – Changing the nominal patch size



Sparse Reconstruction

For a scene of $\mathcal{K}$ patches, the overall response is

$$y = \sum_{i=1}^{\mathcal{K}} g_{\alpha_i} r_{\alpha_i} + z$$

$\in \mathbb{C}^N$

$\in \mathbb{C}$

$\sim CN(\mathbf{0}, \sigma^2 \mathbb{I})$

Goal: Estimate patch locations (& possibly sizes) and gains

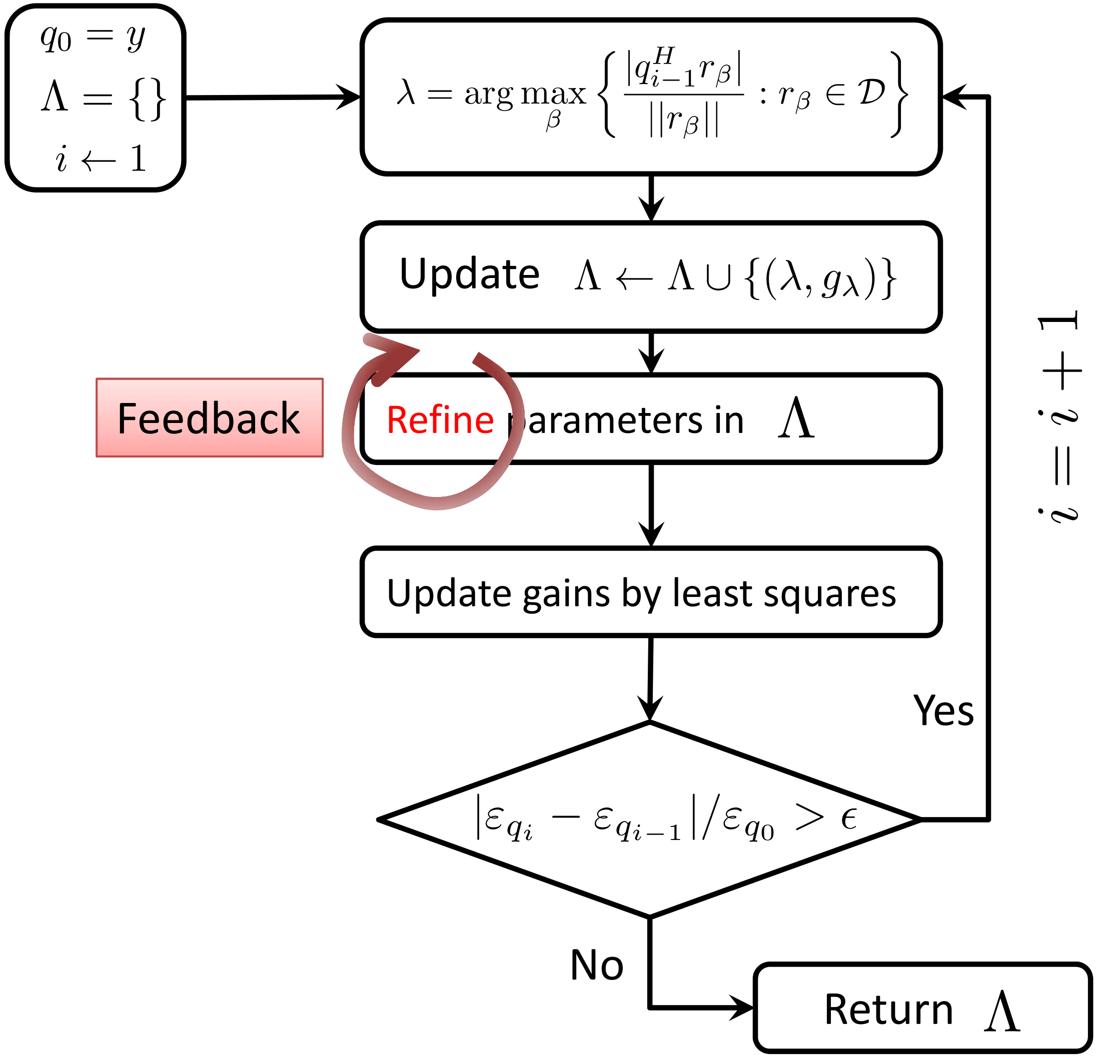
Newtonized OMP (NOMP)

GLRT cost function

$$G_q(\beta) \triangleq \frac{|q^H r_\beta|}{||r_\beta||}$$

Residual response

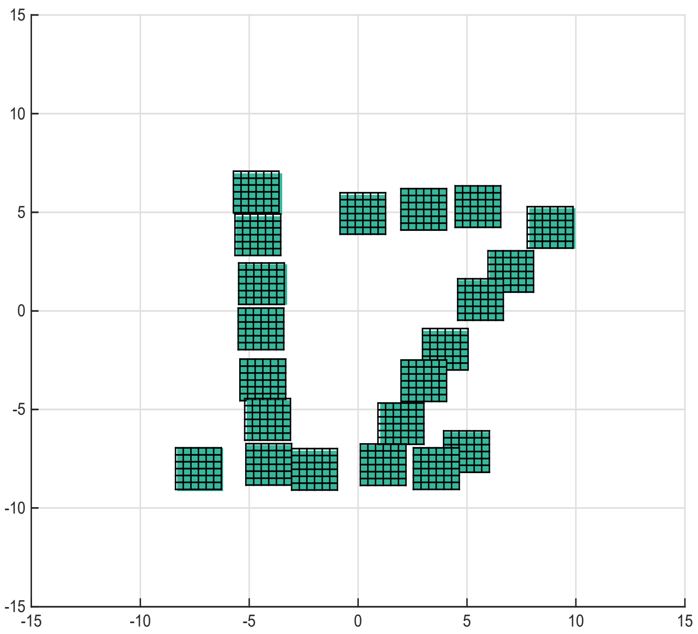
$$q_i = y - \sum_{\lambda \in \Lambda} g_\lambda r_\lambda$$



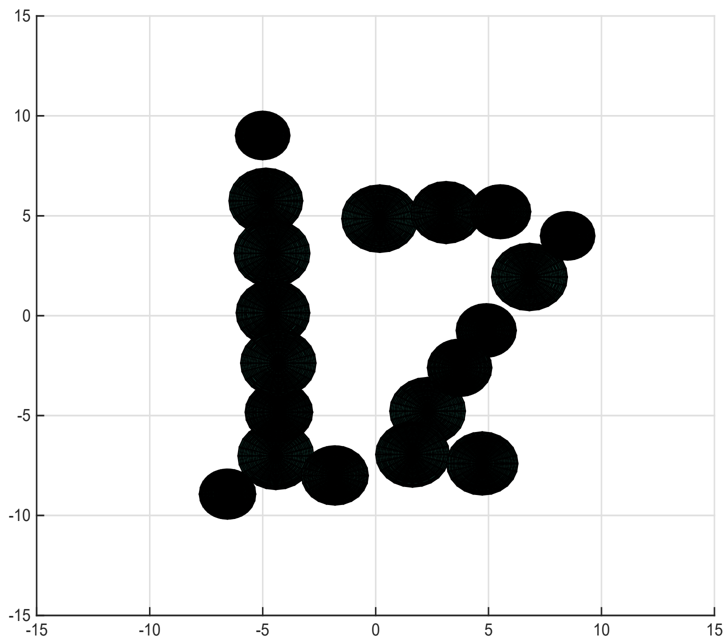
B. Mamandipoor, D. Ramasamy, U. Madhow, “Newtonized Orthogonal Matching Pursuit: Frequency Estimation over the Continuum,”arXiv preprint arXiv:1509.01942, 2015.

NOMP for Patch Detection

- Sparse array → applying NOMP algorithm for detection in the dictionary of square and circular patches



Fixed patch width = 1.5cm
Refine patch center



Refine both center and radius

Conclusions

- Undersampling via sparse arrays ➔ additional assumptions required for scene reconstruction
- Spatial aggregation suppresses grating/side lobes
  - Physically plausible, since natural scenes are spatially lowpass
- Dictionary adapted to:
  - Nature of the aperture: sparsity level and geometry
  - Nature of the scene: size/shape/type of targets
- NOMP + patch model is an effective approach for sparse reconstruction
- Natural next steps
  - Joint delay-Doppler estimation with patches
  - Interface with machine learning algorithms

Mm-wave mesh network design

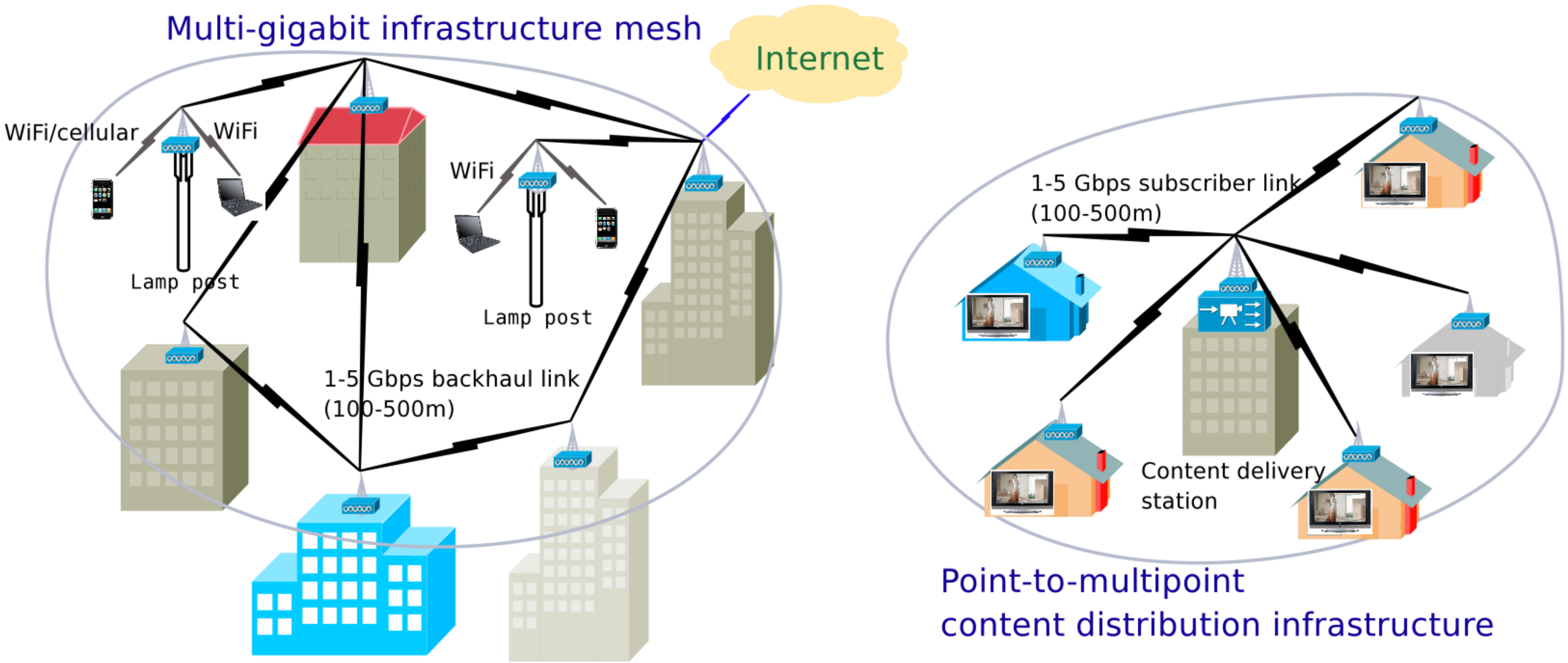
Upamanyu Madhow

ECE Department, UCSB

Thanks to Maryam Eslami Rasekh for compiling these slides.

2016 Summer School, IISc Bangalore

Mesh for backhaul and last mile



Fundamentals are different from those at lower carrier freqs
(Directionality, blockage)

Two models

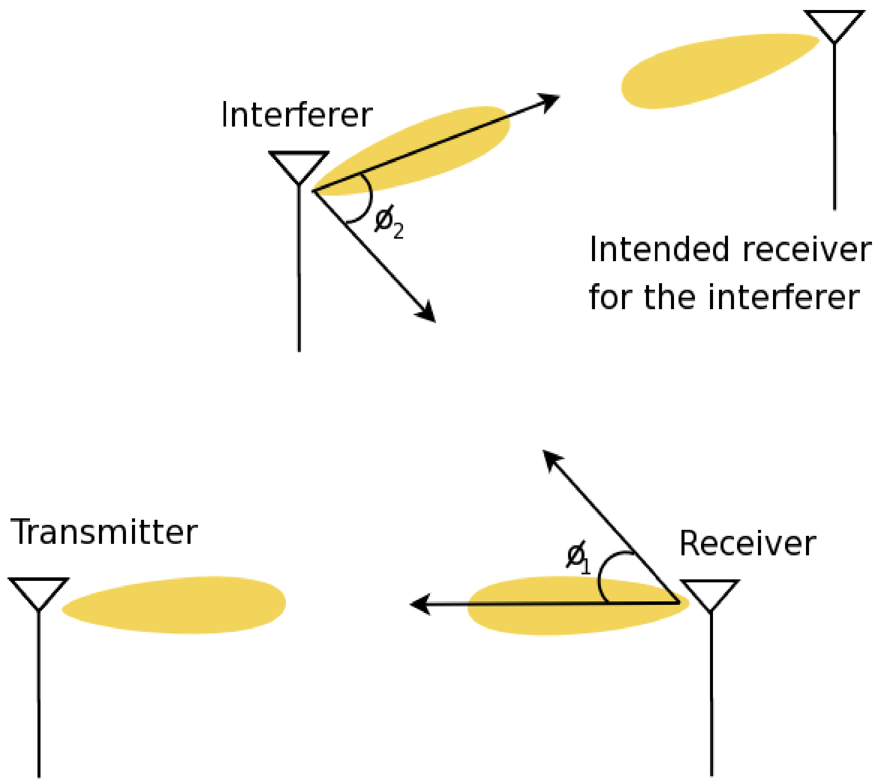
- Decentralized mesh networking
  - Is it possible, given deafness due to directionality?
  - Simple model: randomly dispersed nodes in 2D plane, no blockage
  - New interference analysis and MAC
- Mesh backhaul in urban canyons
  - Optimization framework for joint resource allocation and routing to gateway nodes
  - Determine backhaul link speeds relative to access link speeds

Decentralized, highly directional networking (Model 1)

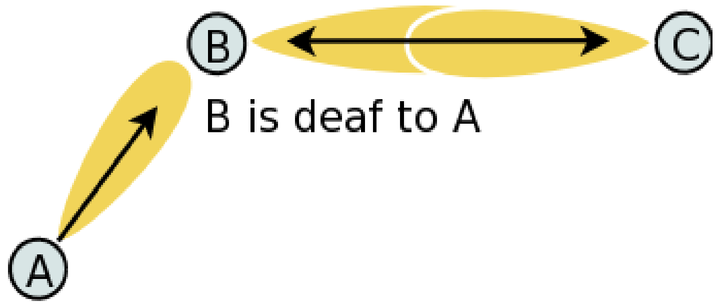
Singh, Mudumbai, Madhow, *Interference analysis for highly directional 60-GHz mesh networks: the case for rethinking medium access control,* IEEE/ACM Trans. Networking, October 2011.
Singh, Mudumbai, Madhow, *Distributed coordination with deaf neighbors: efficient medium access for 60 GHz mesh networks,* IEEE Infocom 2010.

Interference and Deafness

Interference with directional links



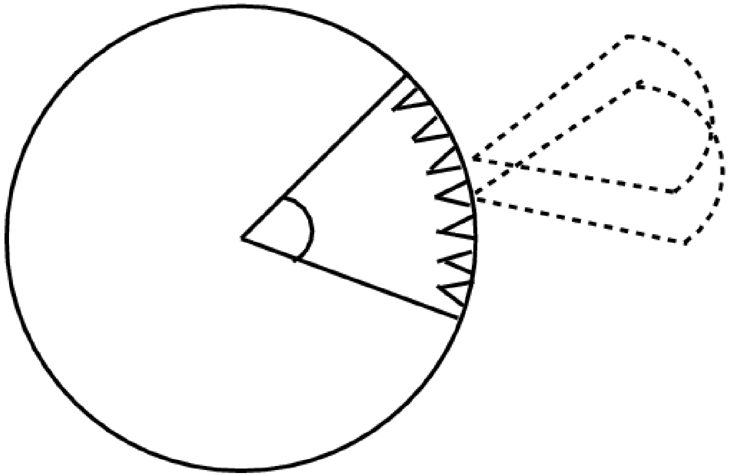
Deafness



Networking in outdoor P2P networks

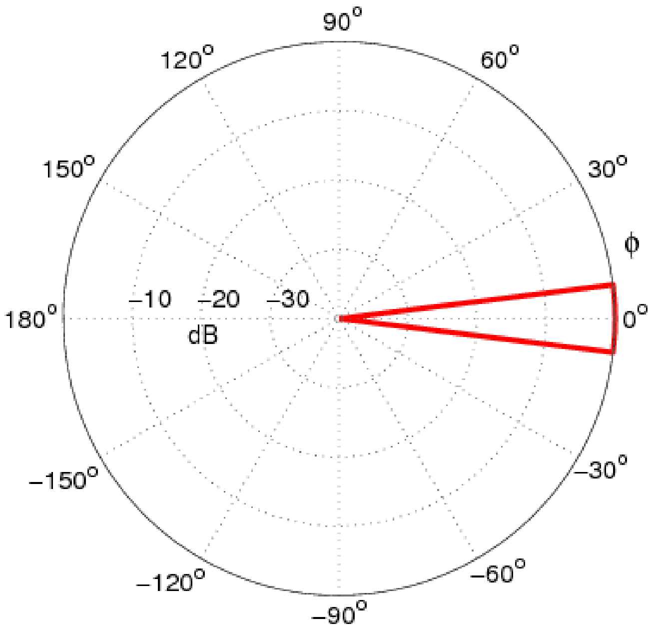
- No ``omnidirectional mode'' for MAC
  - Must use directionality to attain link budget
  - Directional-only mode also simplifies PHY
- Are directional links like wires?
  - A qualified yes
- How do we exploit ``wire-like'' characteristics for MAC?
  - Carrier sense is out, but interference is much reduced
- Many other details
  - Network discovery
  - Synchronization maintenance (if used in MAC)
- Step 1: Understand spatial interference

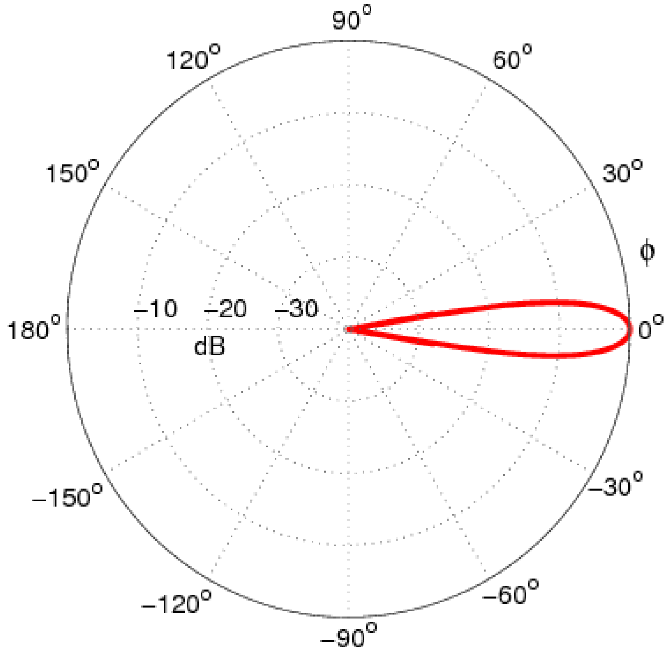
Modeling beam patterns



Approximating a circular array of slot antennas as a uniform linear array of flat-top elements.

Gain pattern for a flat-top antenna (beam angle 14.4 degrees) and a 12 element linear array of flat-top elements, each of sector size 20 degrees. Antenna gain in both cases: 24 dBi

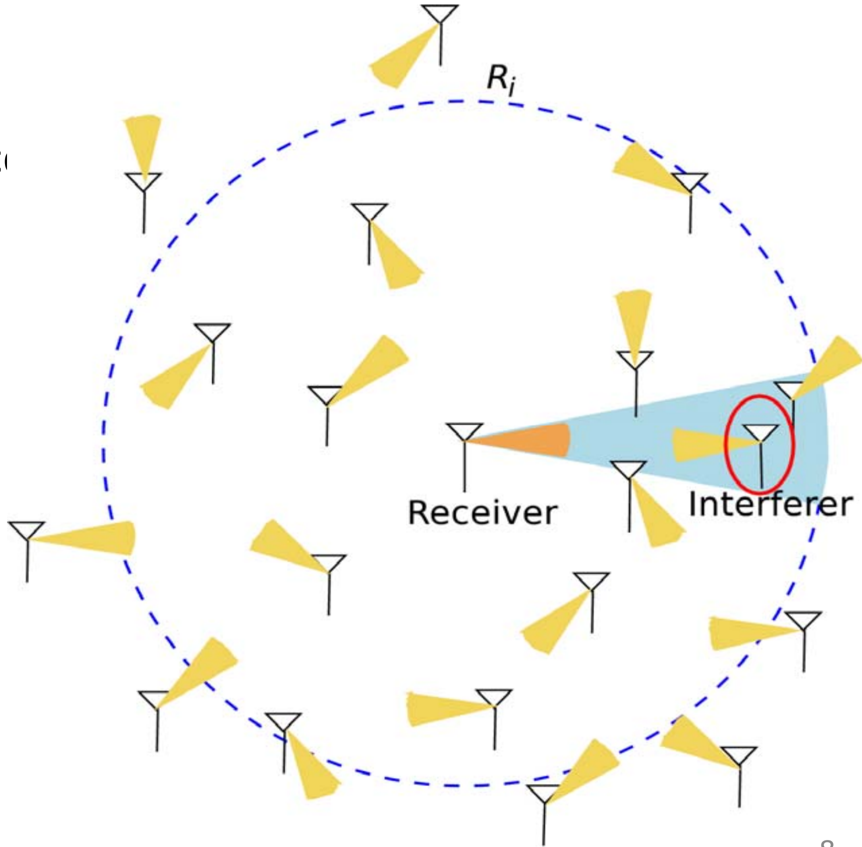




7

Interference under the protocol model

- Flat top antenna, randomly placed transmitters, random orientation wrt desired receiver
- Collision iff there exists at least one interferer
  - within the interference range
  - within the receiver beamwidth
  - pointing in the direction of the rec



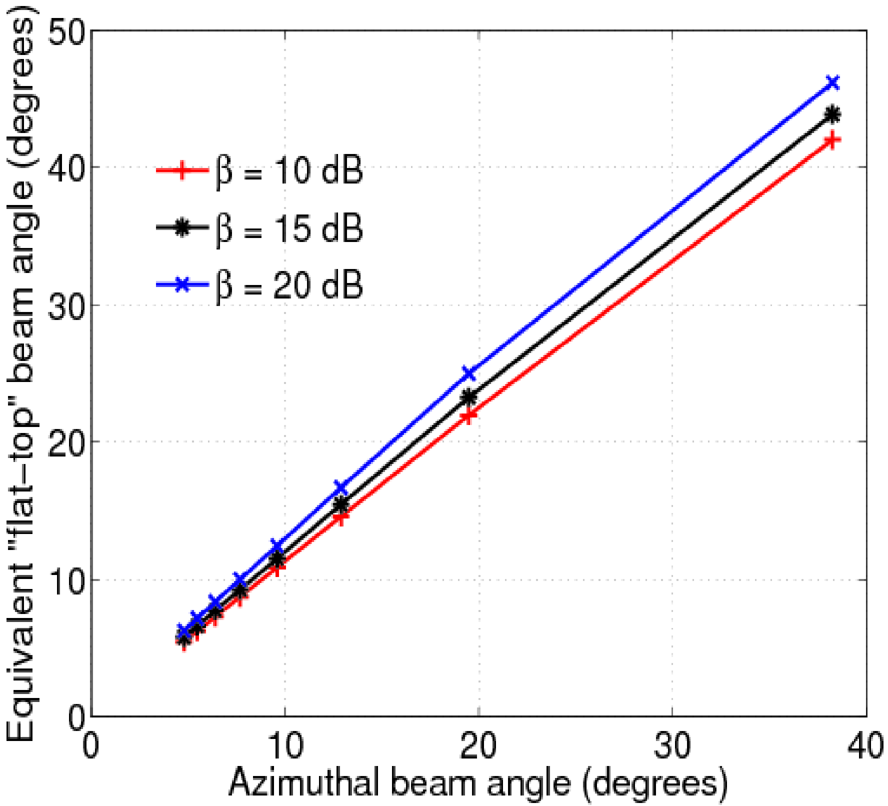
Collision Probability $\boxed{1-e^{-\lambda\beta A_c}}$

$$A_c = \frac{(R_0\Delta\Phi)^2}{4\pi} e^{-\alpha(R_i - R_0)}$$

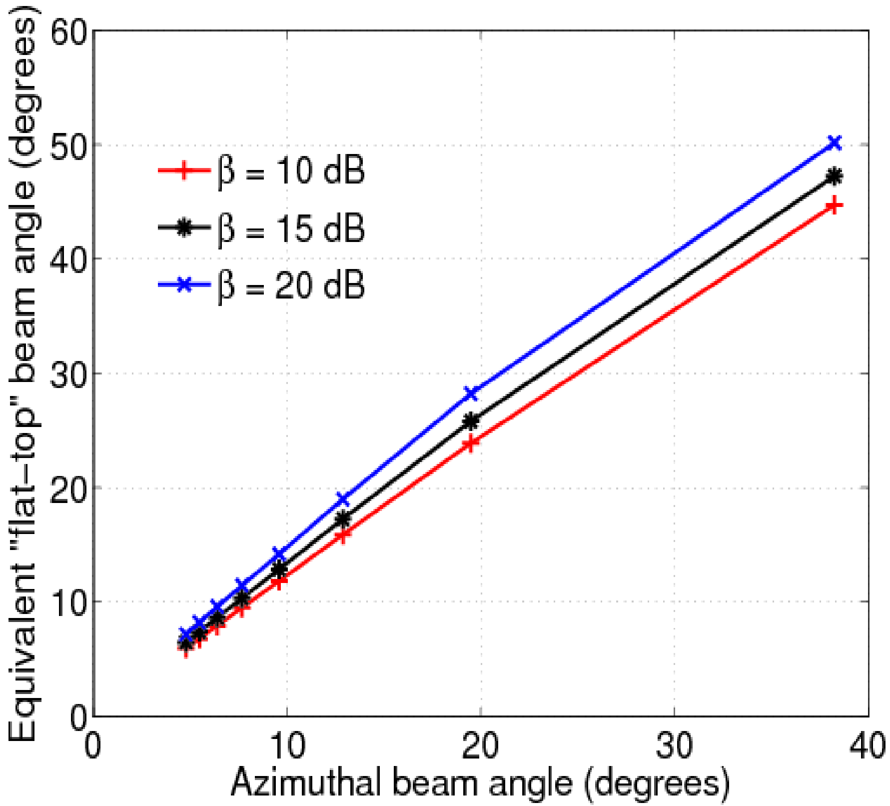
β : SINR threshold
λ : density of transmitting nodes
ΔΦ: (azimuthal) beamwidth
R_0 : nominal link range
R_i : interference range
α : atmospheric absorption coefficient

8

Generalizes to arbitrary antenna patterns



Nominal link 100m

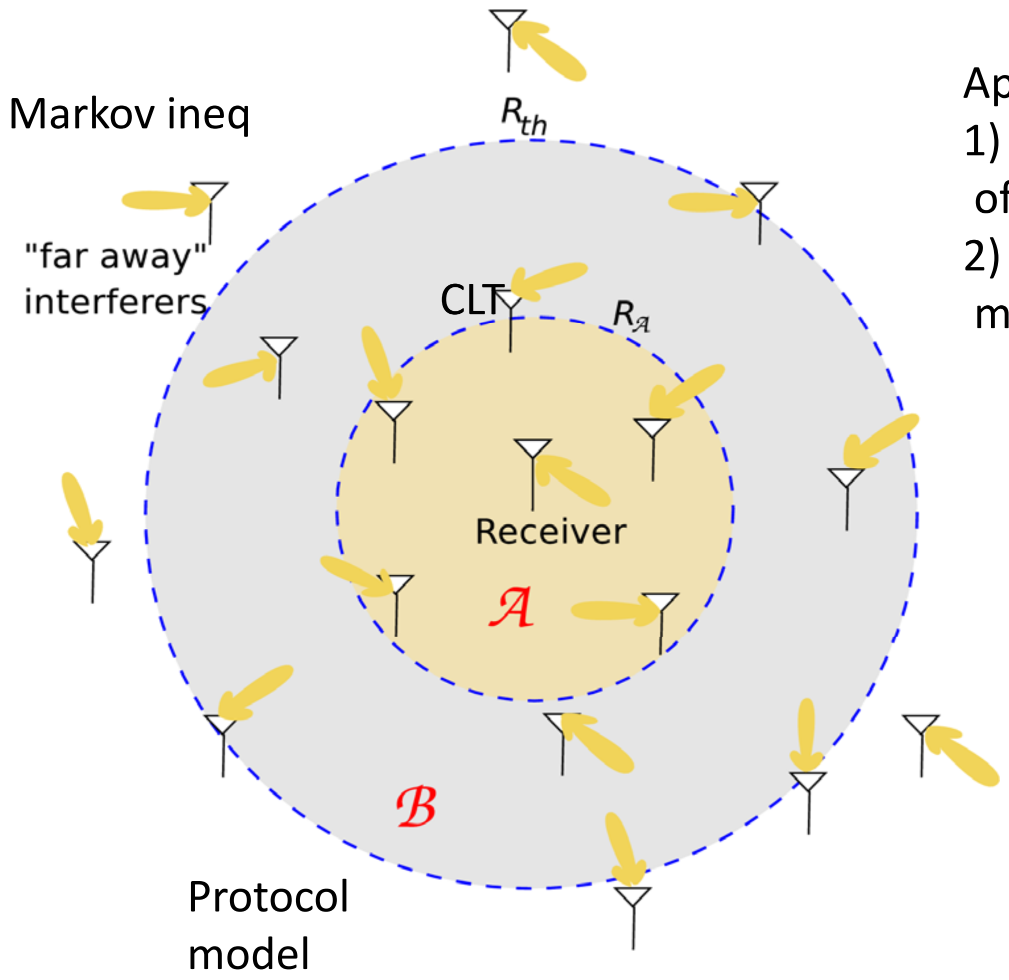


Nominal link 200m

General antenna patterns can be modeled using equivalent flat top beam angle

Physical model

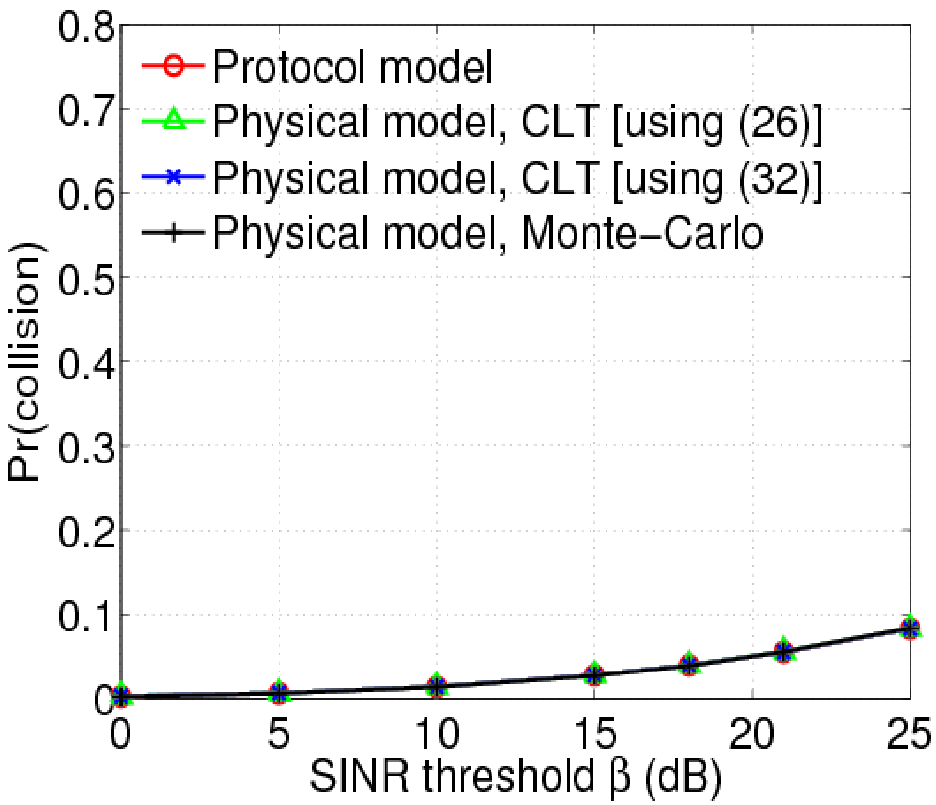
Collision prob = P[sum interference exceeds threshold]



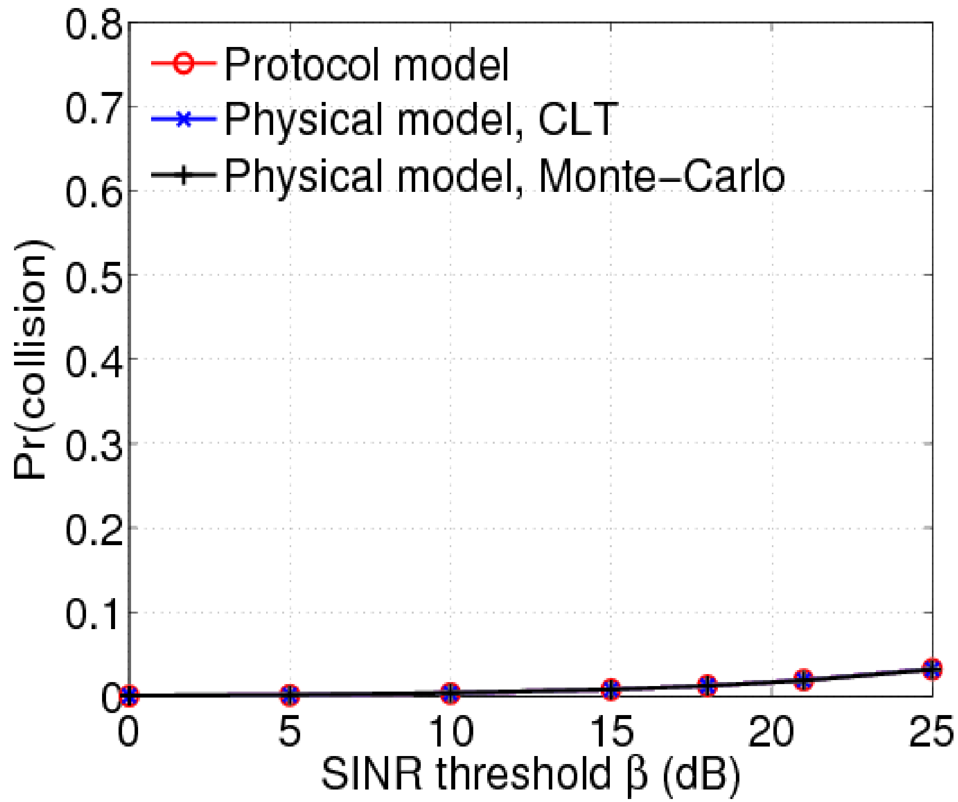
Approach:
1) Exploit oxygen absorption to bound effect of far-away interferers using Markov ineq
2) Use CLT or Chernoff bound plus protocol model for nearby interferers

10

Collision probabilities (sparse network)



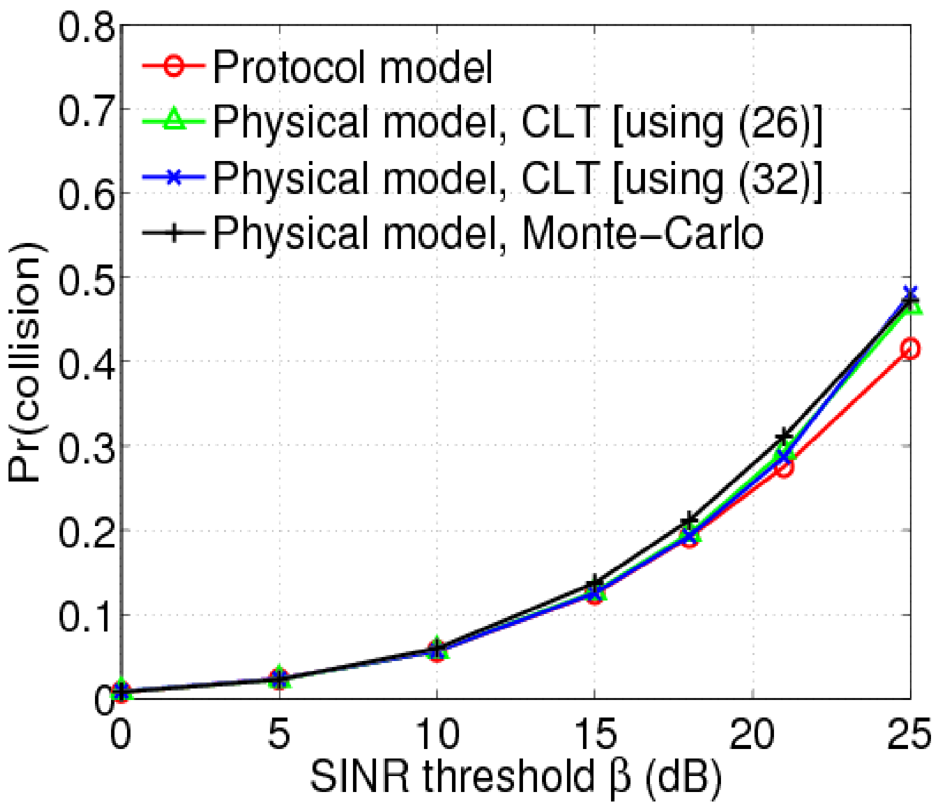
Flat-top antenna



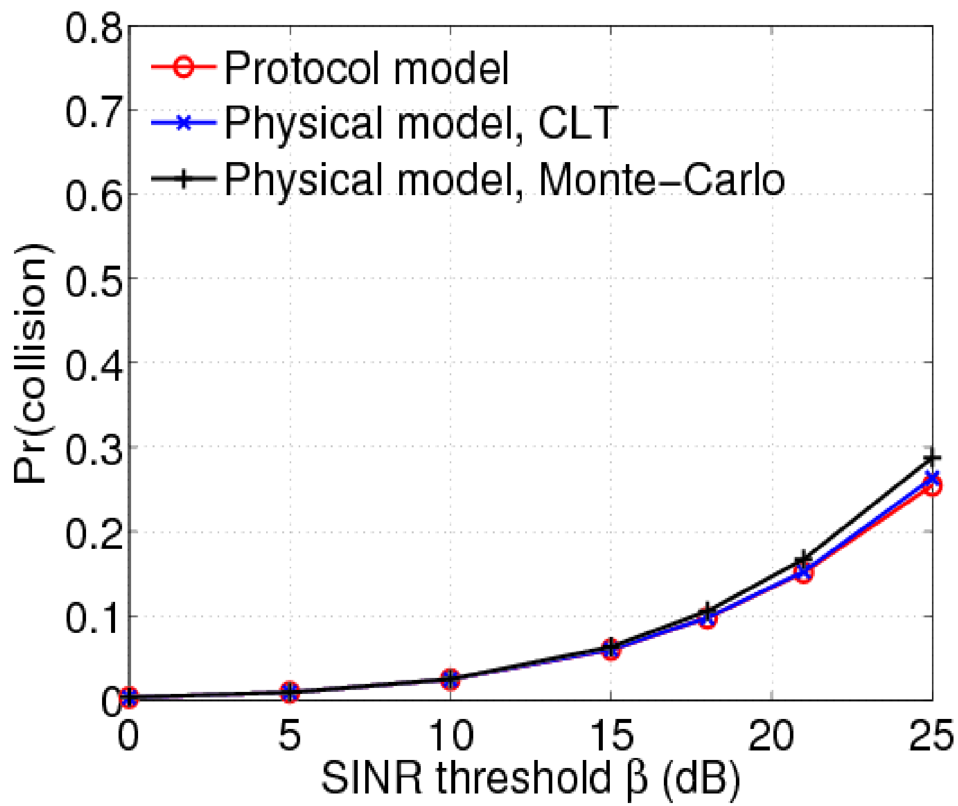
Linear array

Link range R =200m, $\pi\rho R^2=\pi$

Collision probabilities (dense network)



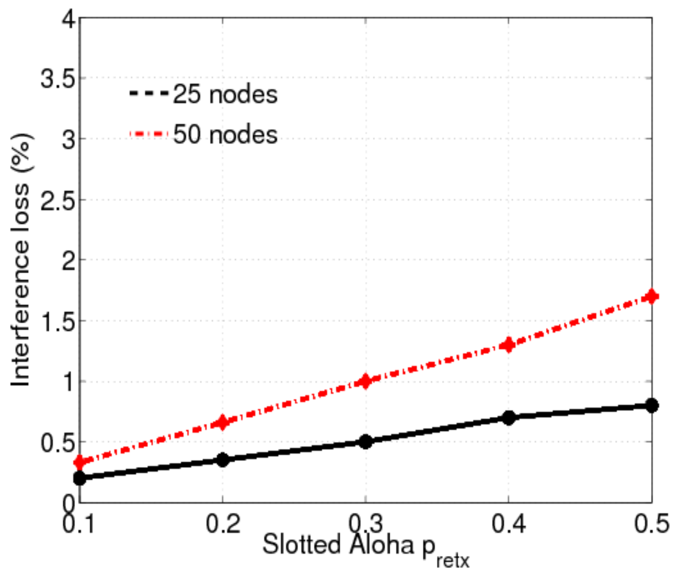
Flat-top antenna

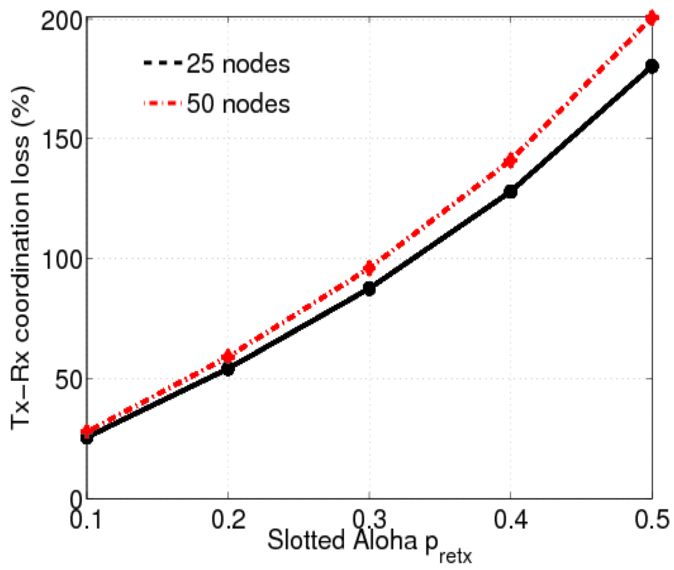


Linear array

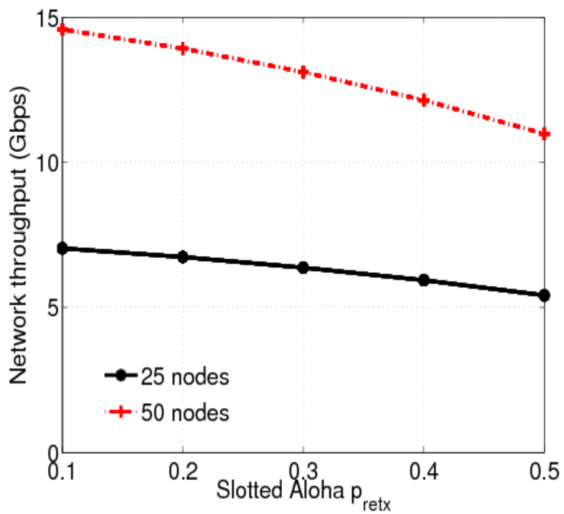
Link range R =100m, $\pi\rho R^2$=5.2 (Pr(connected network) = 0.99)

Coordination is the bottleneck





Collision losses order of magnitude smaller than losses due to failed coordination

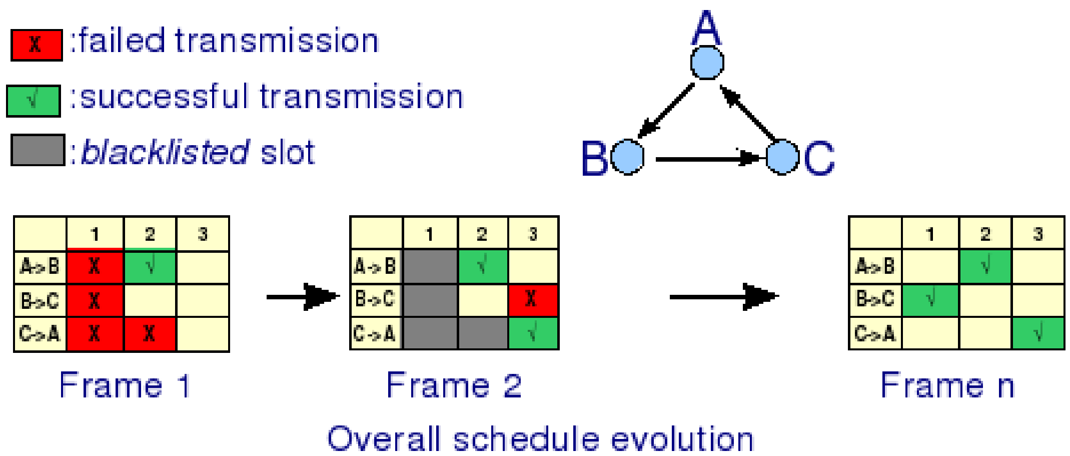


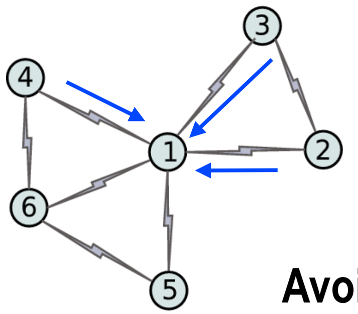
MAC Design: Approach and Issues

- Different transmitters do not coordinate with each other
  - *Wire-like* links, *deaf* neighbors
- Transmitter tries to coordinate with intended receiver
  - Half-duplex constraint
  - Receiver can only receive successfully from one node at a time
- Novel design approach needed for pseudowired links
  - MAC emphasis shifts from interference management/avoidance to scheduling
  - Distributed learning vs. centralized scheduling

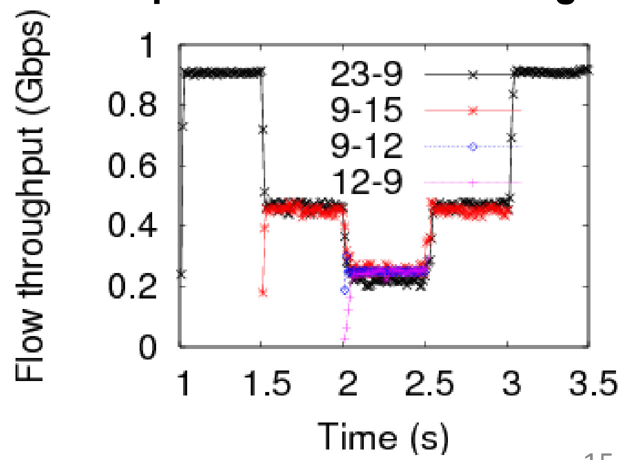
Memory-guided directional MAC (MDMAC)

Stigmergic evolution of TDM-like schedule

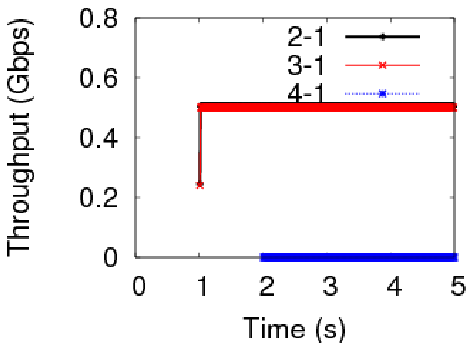


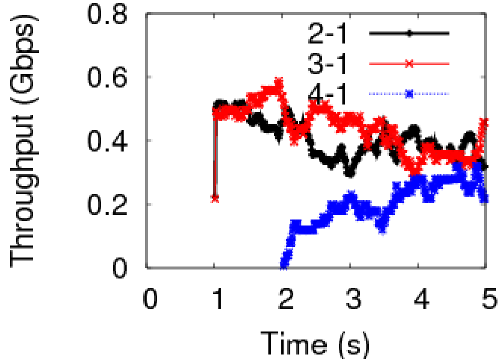


Adaptation to traffic changes

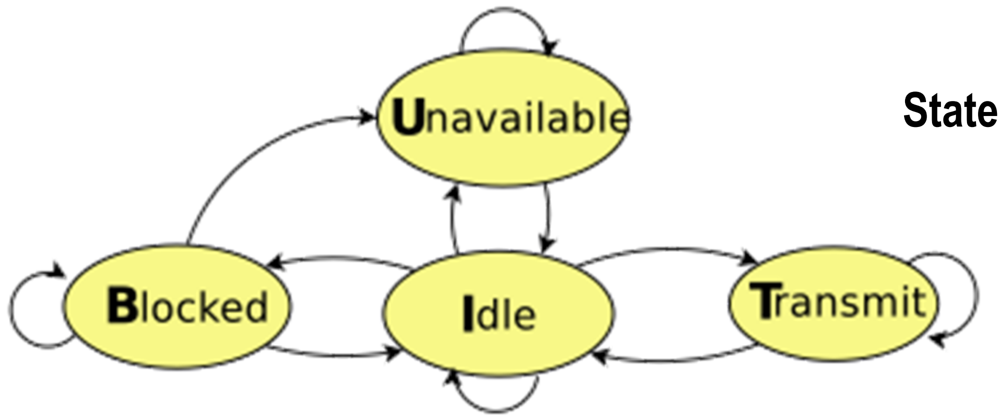


Avoiding lockout via randomization





Design guidelines from fixed point analysis



State diagram for a typical outgoing link

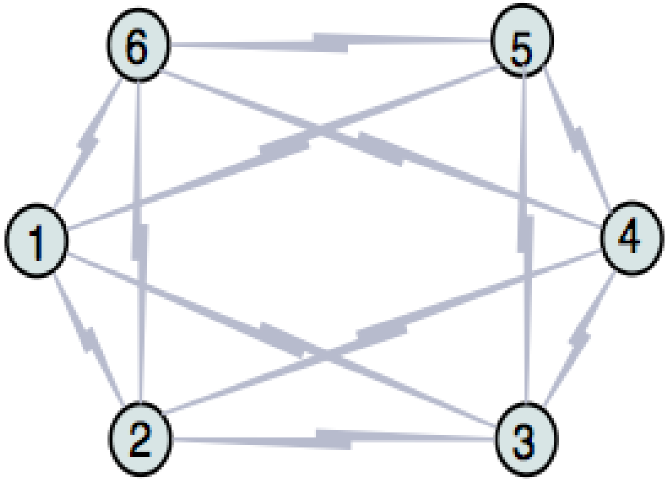
Randomized holding time for slot governs **P[Transmit → Idle]** and **P[Unavailable → Idle]**
Randomized holding time for blacklisted slot governs **P[Blocked → Idle]**

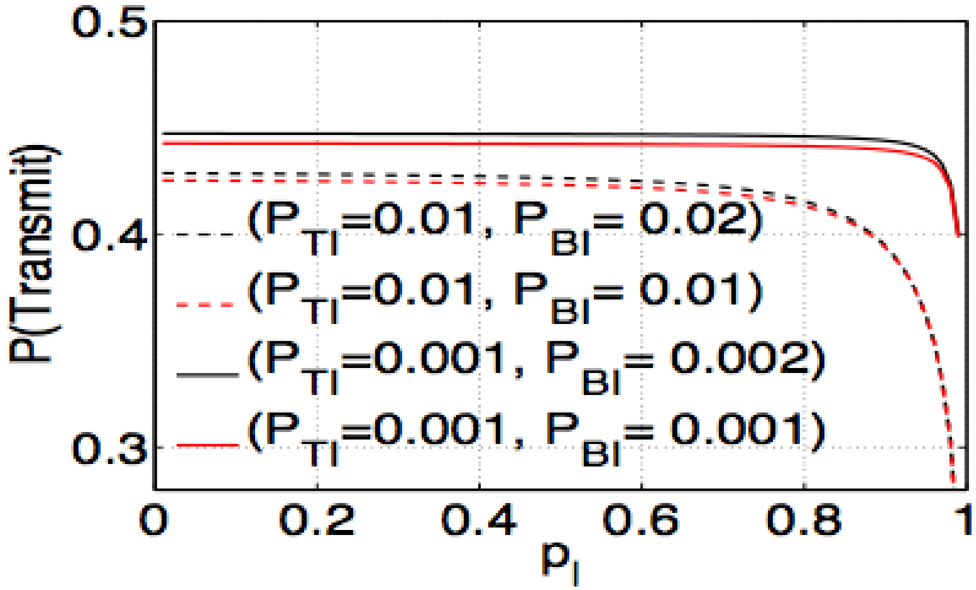
$$P_{IT} = p_{tx}\left(\frac{P_I}{P_I + P_B}p_l + \frac{P_B}{P_I + P_B}\right)$$

Transition probs for a 2-node network

$$P_{IU} = \frac{p_{tx}p_l P_I}{P_I + P_B}, P_{BU} = \frac{p_{tx}P_I}{P_I + P_B}, P_{IB} = \frac{p_{tx}^2 P_I}{P_I + P_B}$$

Guidelines for a 4 node network

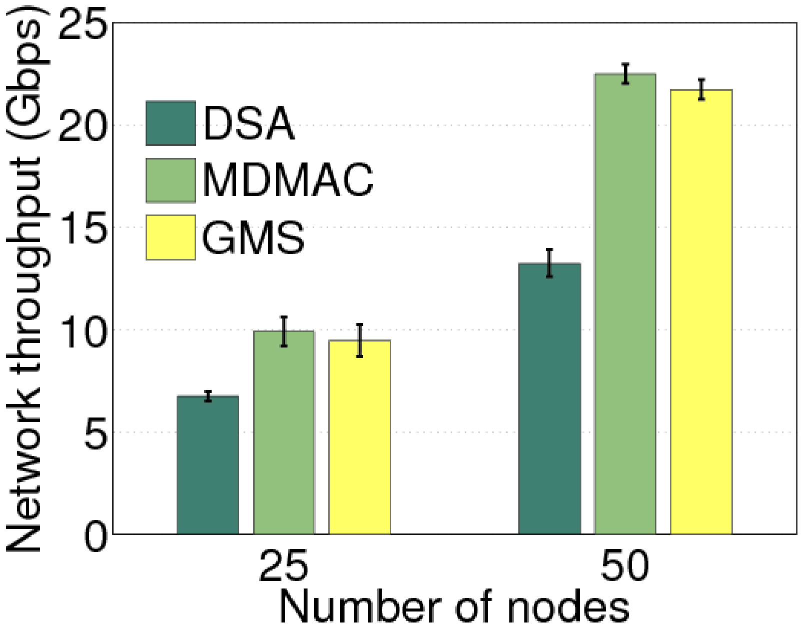




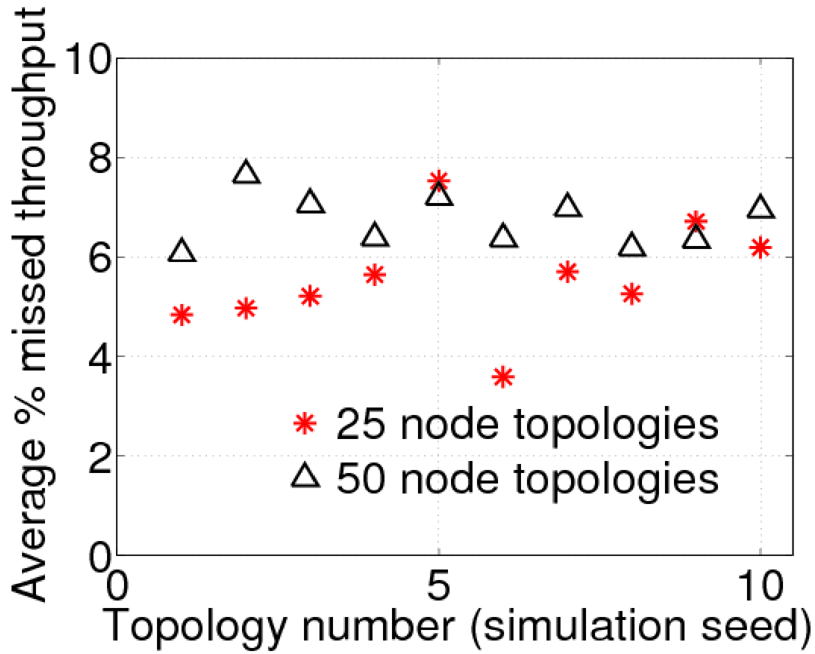
Long holding times (~500-1000 frames) give better throughput, but shorter holding times are also OK

Performance insensitive to listening probability

MDMAC performance: saturated traffic



Aggregate network throughput



Missed transmit opportunities

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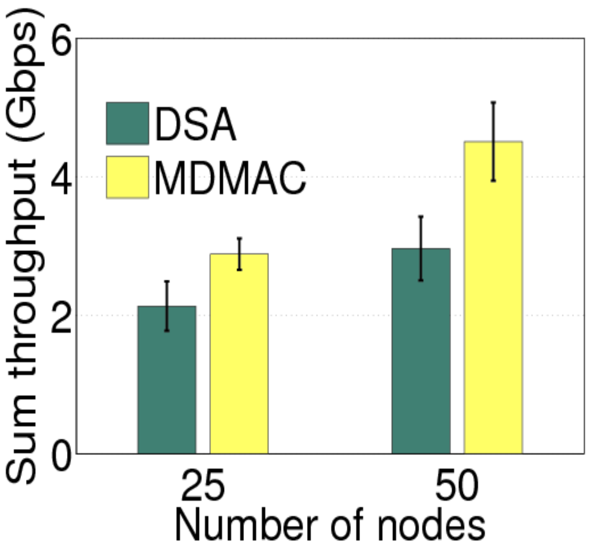
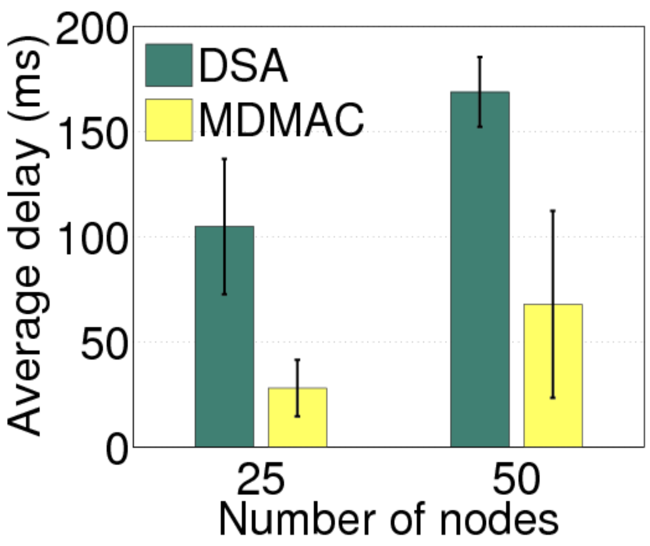
TDM-like performance on a mesh network

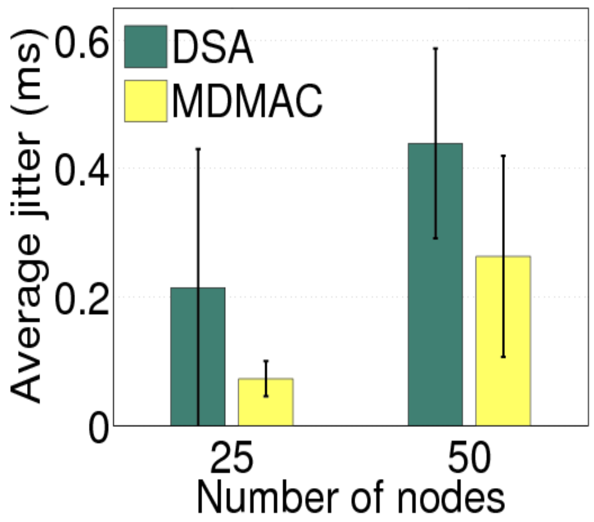
4-5% “missed transmit opportunities”
Significantly better than benchmark directional slotted aloha



Aggregate network throughput



End-to-end delay

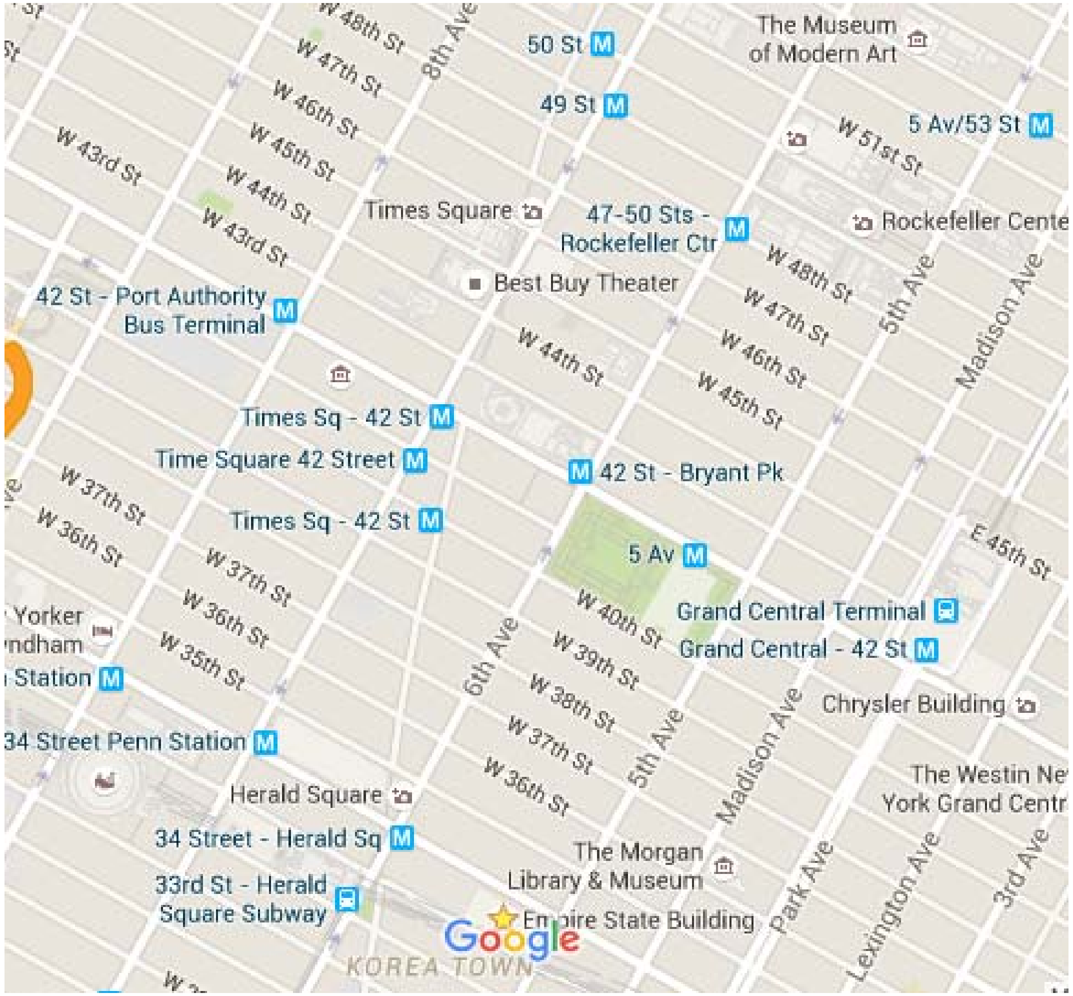


Delay-jitter

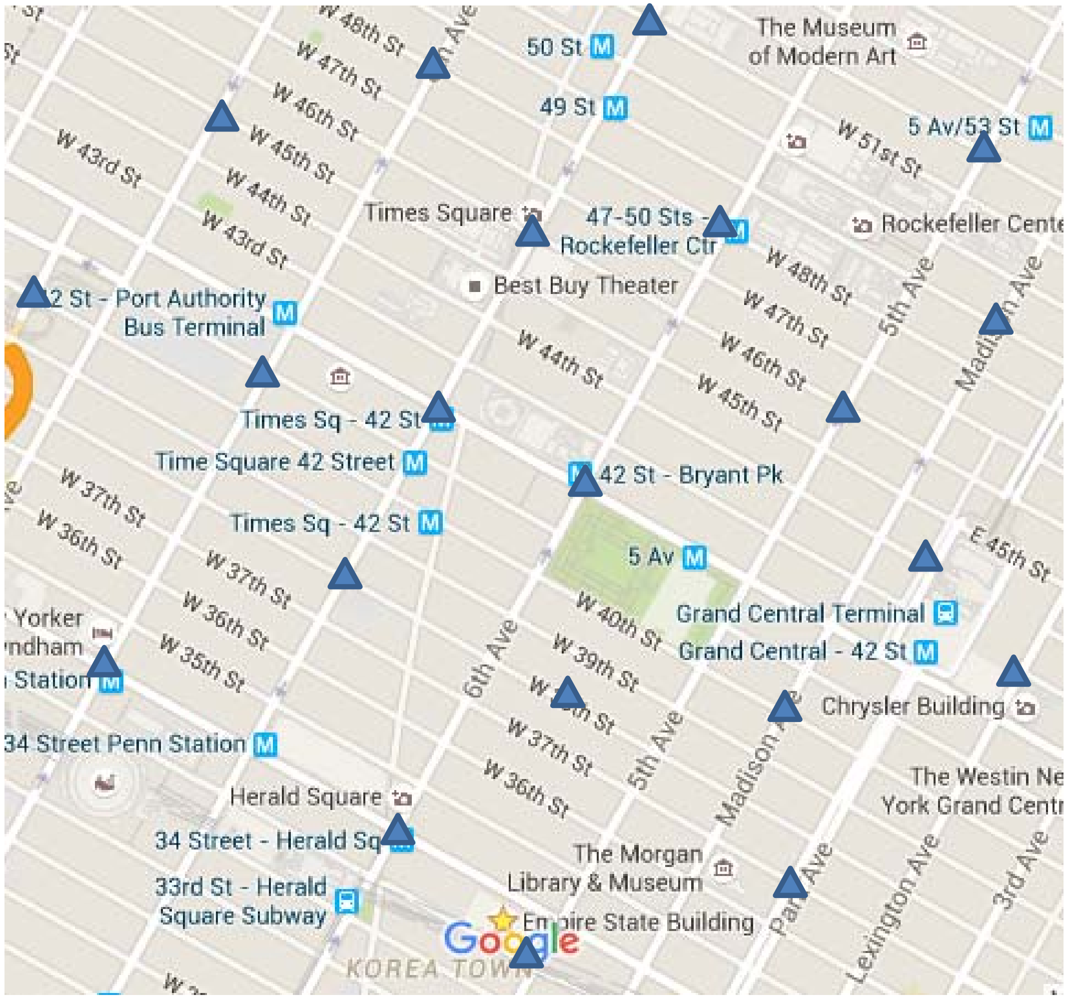
Joint resource allocation and routing for mm-wave backhaul (Model 2)

Maryam E. Rasekh, Dongning Guo, U. Madhow, *Interference-aware routing and spectrum allocation for millimeter wave backhaul in urban picocells,* Allerton 2015.

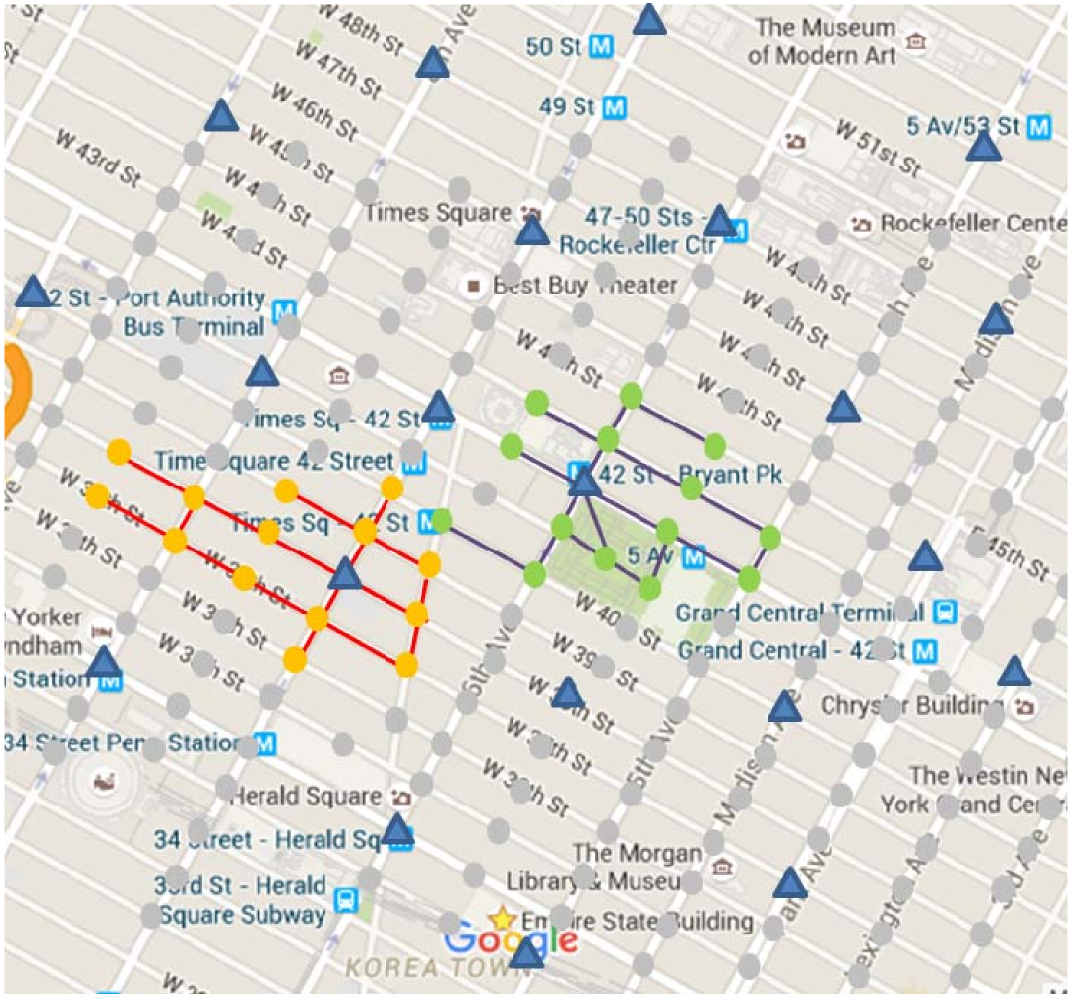
Inside a dense city structure



Extend current cell centers…

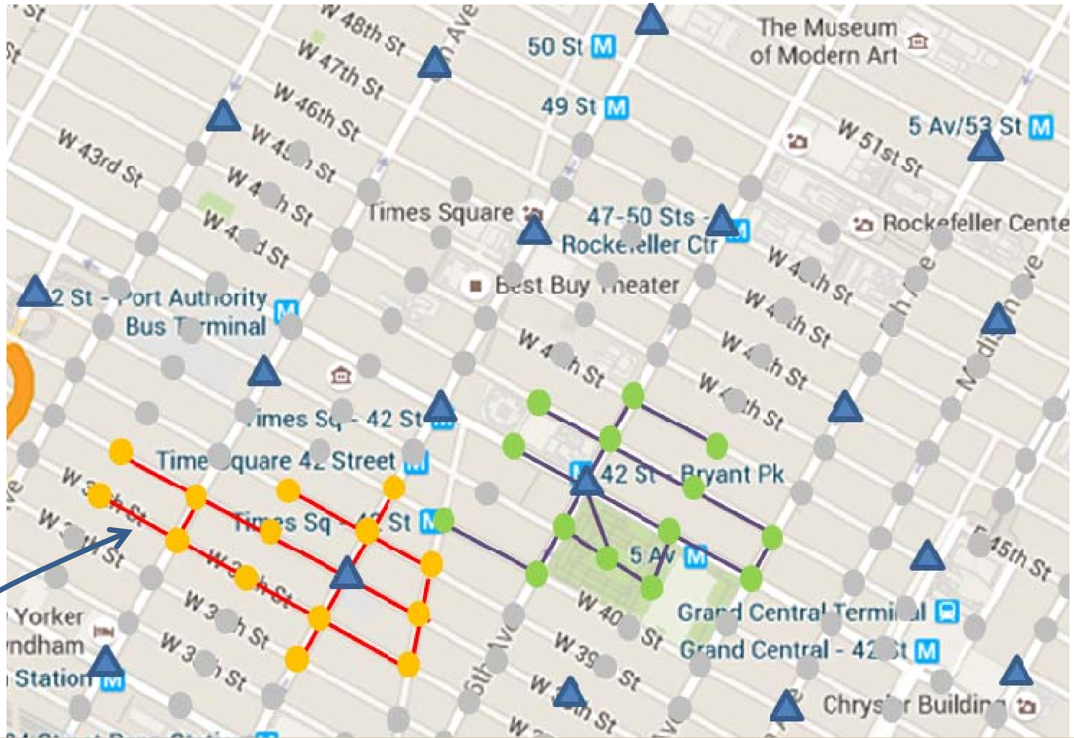


…to a much denser deployment



Old base stations act as gateway nodes for the new network

…to a much denser deployment



Old base stations act as gateway nodes for the new network

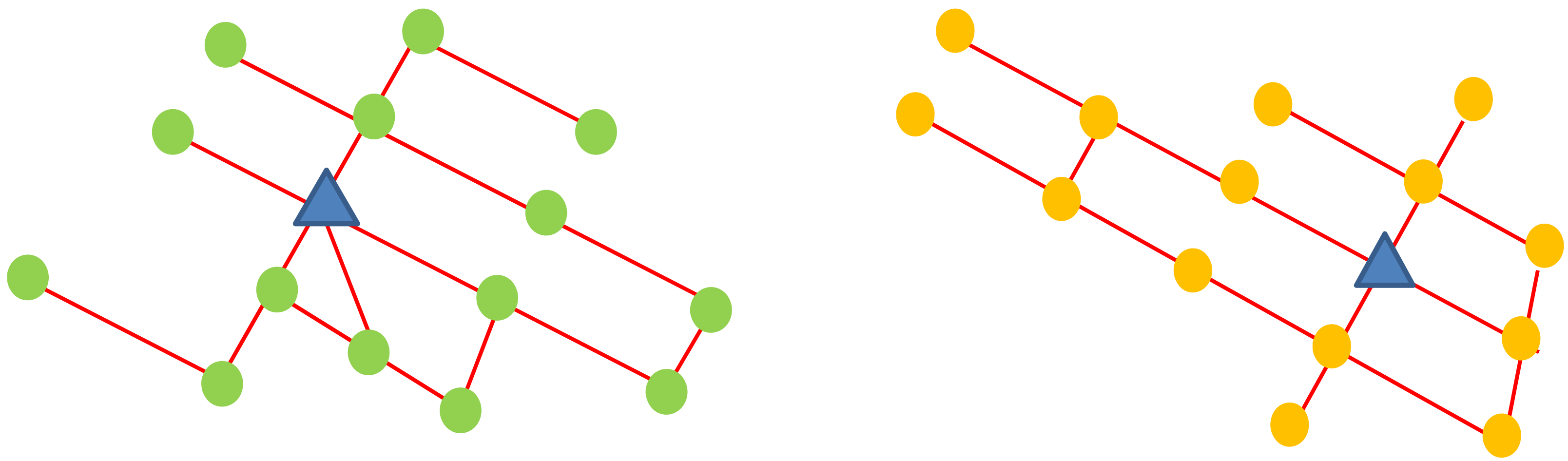
Each node is connected to its neighbors through a LOS mmwave link

The resulting mesh network acts as backhaul for the new picocells

Resource allocation

- Each node uses nearest gateway: relatively independent clusters of nodes around gateways
- Problem of resource allocation and routing limited to small clusters

$\Rightarrow$ **Centralized allocation**



Interference model

- Allocate resources considering limitations:

  (1) No simultaneous transmit and receive on any node

  (2) Possible interference between aligned links

Interference model

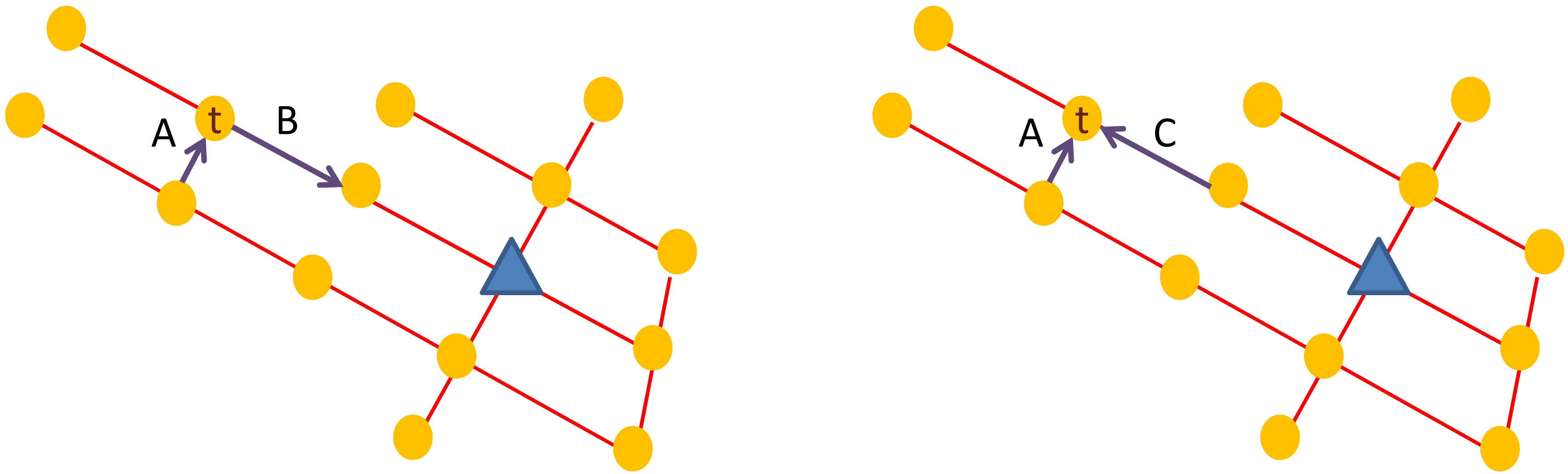
- Allocate resources considering limitations:
  (1) No simultaneous transmit and receive on any node

(2) Possible interference between aligned links

Interference model

Example:

- Links A and B cannot be active simultaneously – node t would have to transmit and receive simultaneously
- Links A and C can be active simultaneously

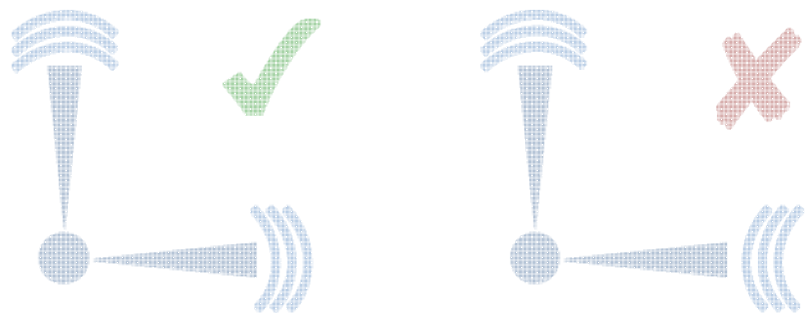


Note that each line represents **two** links, one in each direction

Interference model

- Allocate resources considering limitations:

  (1) No simultaneous transmit and receive on any node

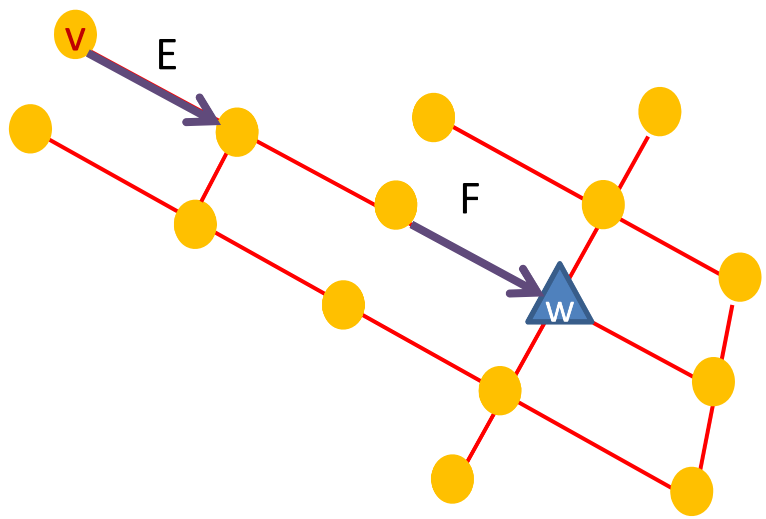


  (2) Possible interference between aligned links

Interference model

Example:

- Link E can cause interference on link F – since receiver at node w is aligned with transmitter at node v



Resource allocation

- Resources (designated parts of available bandwidth) are allocated to links
- Throughput of each link depends on the interference caused by links using the same bandwidth (Shannon capacity)

$$r_l = \int_{B_l} \log_2 \left(1 + \frac{S_l}{N_0 + \sum_{\substack{q \text{ using} \\ \text{band } df}} I_{ql}}\right) df$$

- Network level throughput: bits per second transferred from gateway to each node (over one or more hops)

Resource allocation

- Resources (designated parts of available bandwidth) are allocated to links
- Throughput of each link depends on the interference caused by links using the same bandwidth (Shannon capacity)

data rate of link l (bit/s) $$r_l = \int_{B_l} \log_2\left(1 + \frac{S_l}{N_0 + \sum_{\substack{q \text{ using} \\ \text{band } df}} I_{ql}}\right) df$$

bandwidth assigned to link l

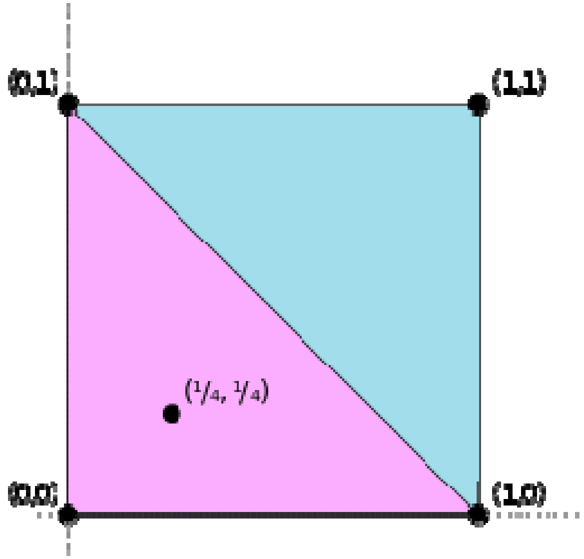
Interference of link q on link l

- Network level throughput: bits per second transferred from gateway to each node (over one or more hops)

Approach

- Caratheodory theorem:

  if a point *x* of r^d lies in the convex hull of a set *P*, there is a subset *P*′ of *P* consisting of *d*+1 or fewer points such that *x* lies in the convex hull of *P*′



- Blow up problem size
- Simple convex optimization formulation

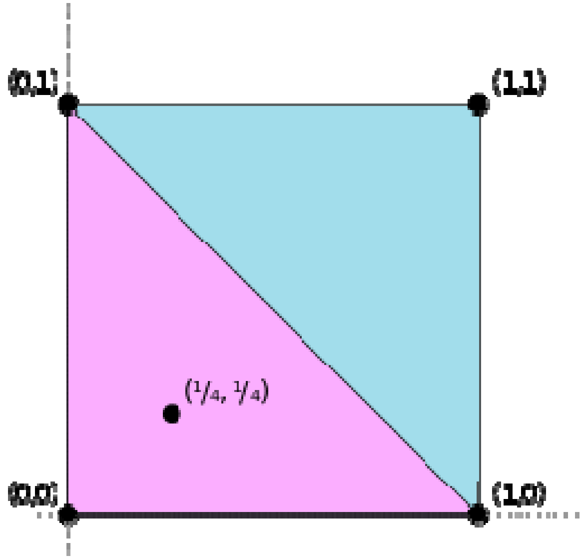
⇒Result size will be small

B. Zhuang, D. Guo, and M. L. Honig, "Traffic-driven spectrum allocation in heterogeneous networks," IEEE J. Sel. Areas Commun. Special Issue on Recent Advances in Heterogeneous Cellular Networks, 2015

Approach

- Caratheodory theorem:

  if a point x of r^d lies in the convex hull of a set P, there is a subset P' of P consisting of d+1 or fewer points such that x lies in the convex hull of P'



- Allocation to links → Allocation to *subsets* of links
  - L links   N nodes
  - $2^L - 1$ possible subsets of links

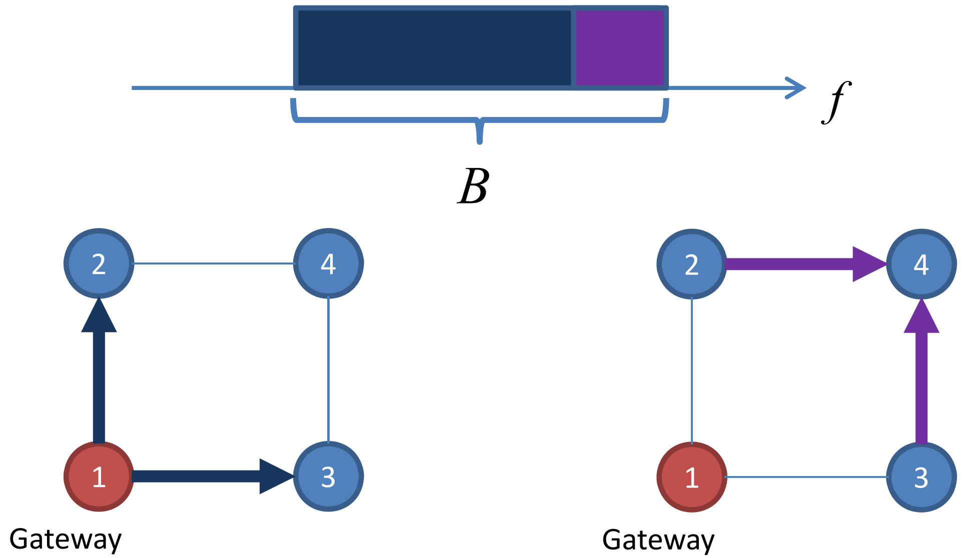
B. Zhuang, D. Guo, and M. L. Honig, "Traffic-driven spectrum allocation in heterogeneous networks," IEEE J. Sel. Areas Commun. Special Issue on Recent Advances in Heterogeneous Cellular Networks, 2015

Approach

- Non-overlapping portions of available bandwidth allocated to subsets of links

$$x_P = \text{spectrum allocated to subset } P \subset \{1,2,\ldots,L\}$$

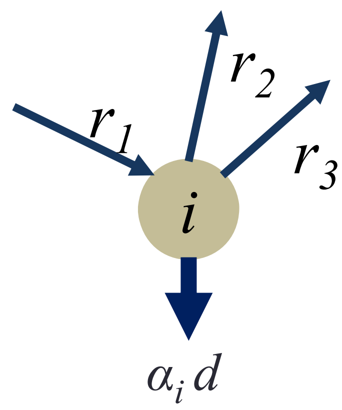
$$\sum_P x_P = B = 1$$



Problem formulation

- Goal: provide date rate $\alpha_i d$ at node i

- Flow conservation: $r_1 - r_2 - r_3 \geq \alpha_i d$



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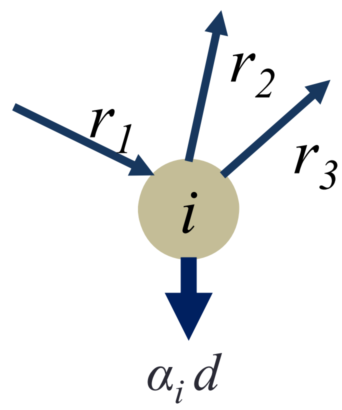


- Flow conservation: $r_1 - r_2 - r_3 \ge \alpha_i d$

Direction/connectivity

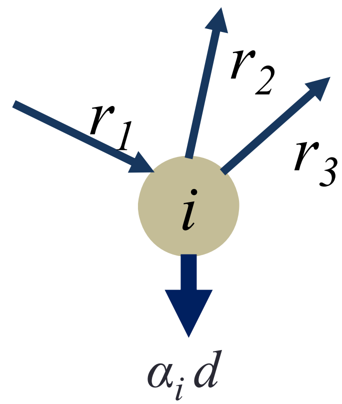
bandwidth allocated to subset P

$$\sum_{P\subset\{1,\dots,L\}} x_P \sum_{l\in P} r_{l,P}\, f_{li} \ge \alpha_i d, \qquad i = 2,3,\dots,N$$

spectral efficiency of link l when subset P is active

Problem formulation

- Goal: provide date rate $\alpha_i d$ at node i
- Flow conservation: $r_1 - r_2 - r_3 \geq \alpha_i d$



$$\sum_{P\subset\{1,\ldots,L\}} \overbrace{x_P}^{\text{bandwidth allocated to subset } P} \sum_{l\in P} \underbrace{r_{l,P}}_{\text{spectral efficiency of link } l \text{ when subset } P \text{ is active}} \overbrace{f_{li}}^{\text{Direction/connectivity}} \geq \underbrace{\alpha_i d},$$

$$f_{li} = \begin{cases} +1 & \text{link } l \text{ runs into node } i \\ -1 & \text{link } l \text{ runs out of node } i \\ 0 & \text{link } l \text{ is not connected to node } i \end{cases}$$

$$r_{l,P} = \log\left(1 + \frac{\gamma_l}{1 + \sum_{k\in P\setminus\{l\}} I_{k\to l}}\right)$$

Problem formulation

First attempt:

$$\underset{\{x_P\}}{\text{maximize}} \quad d$$ (delivered rate)

$$\text{subject to} \quad \sum_{P \subset \{1,..,L\}} x_P = 1$$ (resource constraint)

$$\sum_{P \subset \{1,..,L\}} x_P \sum_{l \in P} r_{l,P} f_{li} - \alpha_i d \geq 0, \quad i = 2, \ldots, N$$

(flow balance)

Problem formulation

- Variables: $X = [x_1, x_2, \ldots, x_{2^L - 1}, d]^T$

$$\underset{X}{\text{maximize}} \qquad [0,0,\ldots,0,1]X = cX$$

- Constraints: $AX \leq b$

$$A_{(N \times 2^L)} = \begin{bmatrix} 1 & \cdots & 1 & 0 \\ \ddots & & & \alpha_2 \\ & -A_{ij} & & \vdots \\ & & \ddots & \alpha_N \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$A_{ij} = \sum_{l \in P_j} r_{l,P_j} f_{li}$$

Problem formulation

- Variables: $X = [x_1, x_2, \ldots, x_{2^L - 1}, d]^T$

$$\underset{X}{\text{maximize}} \qquad [0,0,\ldots,0,1]X = cX$$

- Constraints: $AX \leq b$

$$A_{(N \times 2^L)} = \begin{bmatrix} 1 & \cdots & 1 & 0 \\ \ddots & & & \alpha_2 \\ & -A_{ij} & & \vdots \\ & & \ddots & \alpha_N \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

resource constraint

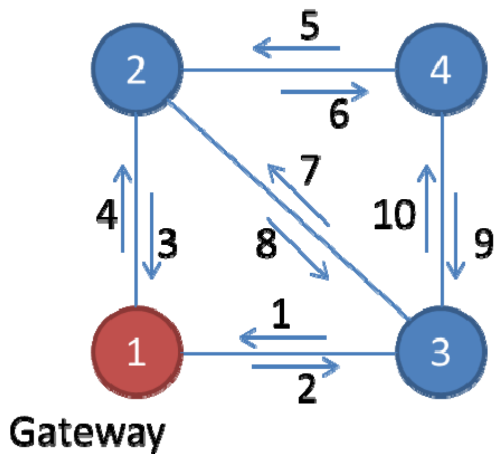
flow balance at each node

$$A_{ij} = \sum_{l \in P_j} r_{l,P_j} f_{li}$$

Formulation 1

- Insensitive to power/delay
  - Long paths
  - Redundancies

$$d = 1.7297 \text{ bps}$$
$$d / d_{link} = 0.5$$



Link index	1	2	3	4	5	6	7	8	9	10	%
Active subsets	0	**1**	0	**1**	0	0	0	0	0	0	53.21
	0	**1**	0	0	0	**1**	**1**	0	0	0	10.89
	0	**1**	0	0	0	**1**	0	0	0	0	10.89
	0	0	0	**1**	0	0	0	**1**	0	**1**	10.89
	0	0	0	**1**	0	0	0	0	0	**1**	10.89
	0	0	0	0	0	**1**	0	0	0	**1**	3.21
Total link activation (%)	0	75	0	75	0	25	10.89	10.89	0	25	

Formulation 2

- Sum of link data rates under allocation as a proxy for delay/power

$$s_k = \sum_{l \in P_k} r_{l,P_k}, \quad k = 1,\ldots,2^L - 1$$

- Penalize via modified objective function

$$\underset{\{x_P\}}{\text{maximize}} \quad (c - \lambda s)X$$

- Value of $\lambda > 0$ chosen to prioritize throughput

Choice of weighting factor λ

$$\underset{\{x_P\}}{\text{maximize}} \quad (c - \lambda s)X$$

prioritize this term

If X_2 is better than X_1 in terms of throughput we should not discard it because of delay, i.e. if

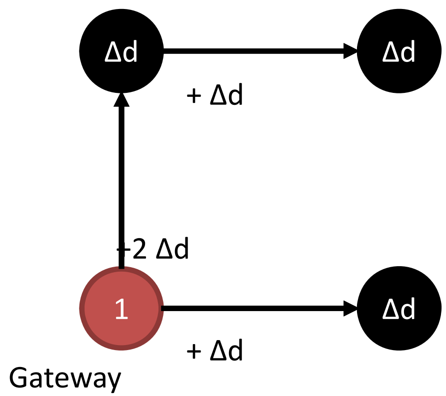
$$d_1 = cX_1 < \; d_2 = cX_2$$

Then we want

$$cX_1 - \lambda s X_1 \leq cX_2 - \lambda s X_2$$

$$d_2 - d_1 > \lambda(sX_1 - sX_2)$$

Sufficient condition: $$\boxed{\lambda < \frac{1}{(N-1)L}}$$



Choice of weighting factor λ

$$\underset{\{x_P\}}{\text{maximize}} \quad (c - \lambda s) X$$

prioritize this term

If X_2 is better than X_1 in terms of throughput we should not discard it because of delay, i.e. if

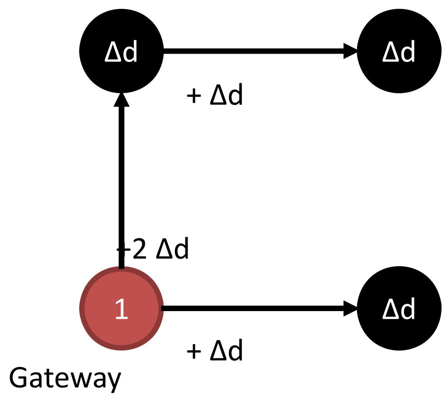
$$d_1 = cX_1 < \; d_2 = cX_2$$

Then we want

$$cX_1 - \lambda s X_1 \leq cX_2 - \lambda s X_2$$

$$d_2 - d_1 > \lambda(sX_1 - sX_2)$$

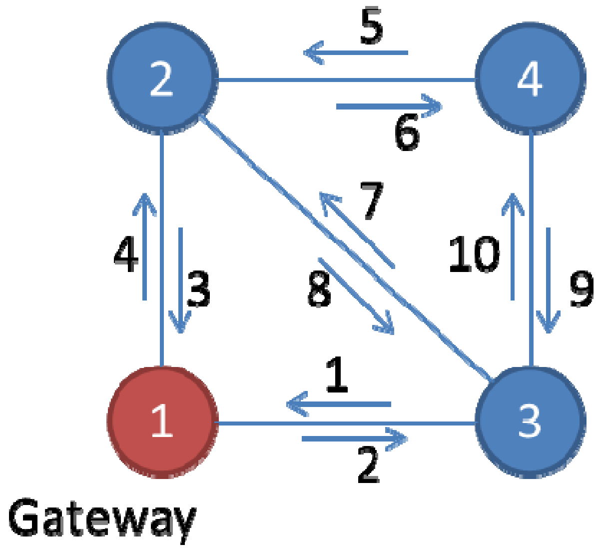
Sufficient condition: $$\boxed{\lambda < \frac{1}{\sum \alpha_i L}}$$



Formulation 2

- Prevents unnecessary link activation
- No throughput cost

- Is this the *minimal* solution..?



$$d = 1.7297 \text{ bps}$$
$$d / d_{link} = 0.5$$

Link index	1	2	3	4	5	6	7	8	9	10	%
Active subsets	0	**1**	0	**1**	0	0	0	0	0	0	57.02
	0	**1**	0	0	0	**1**	0	0	0	0	17.98
	0	0	0	**1**	0	0	0	0	0	**1**	17.98
	0	0	0	0	0	**1**	0	0	0	**1**	7.02
Total link activation (%)	0	75	0	75	0	25	0	0	0	25	

Minimal solution

Problem can be rewritten as:

$$\underset{\{x_P\}}{\text{maximize}} \qquad d = \min\{d_2, d_3, \ldots, d_N\}$$

$$\text{subject to} \qquad \sum_{P \subset \{1,..,L\}} x_P = 1$$

$$\sum_{P \subset \{1,..,L\}} x_P \sum_{l \in P} r_{l,P} f_{li} - \alpha_i d \geq 0, \quad i = 2,\ldots,N$$

$$d_i = \frac{1}{\alpha_i} \sum_{P \subset \{1,..,L\}} x_P \sum_{l \in P} r_{l,P} f_{li}, \quad i = 2,\ldots,N$$

Caratheodory: optimal solution **exists** with allocation size ≤ N

-- not necessarily unique

Minimal allocation

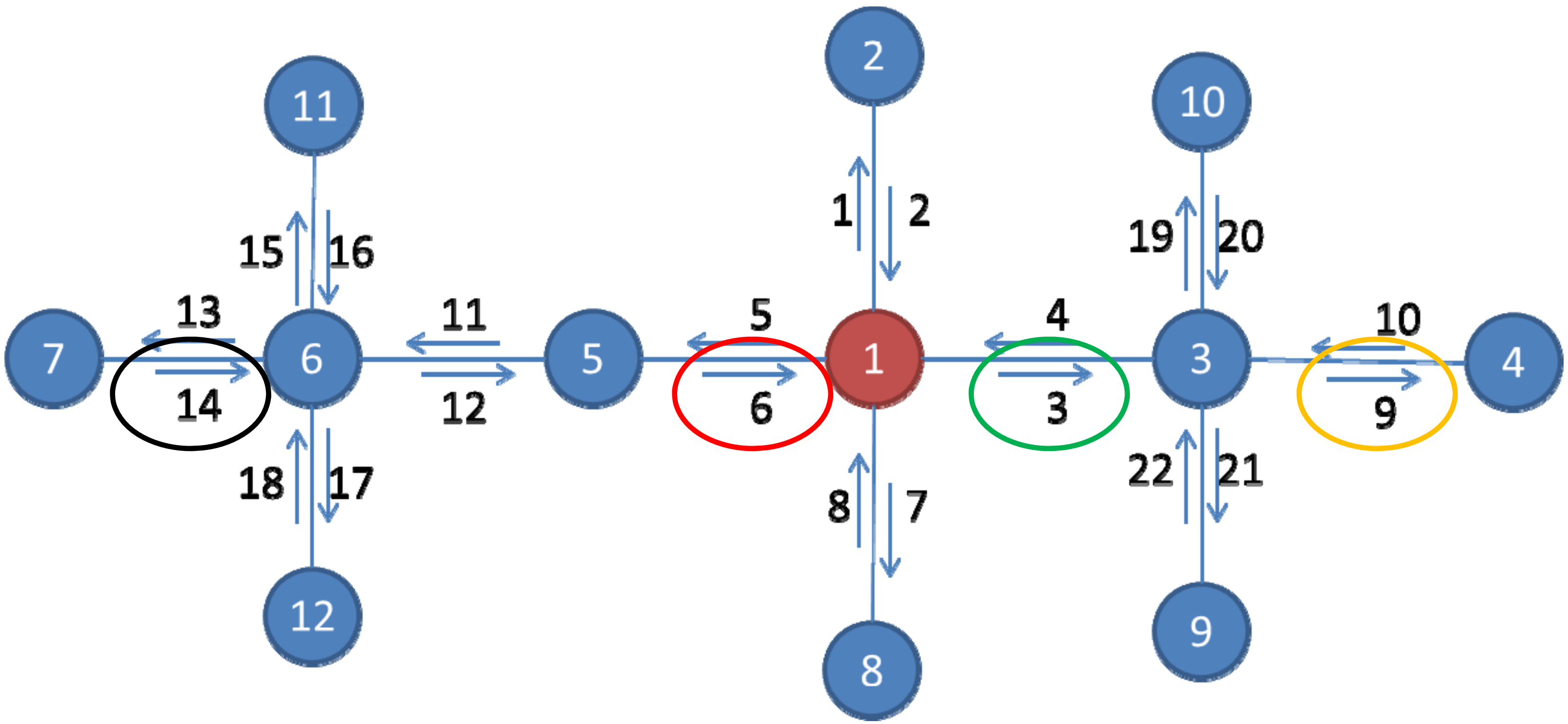
- Resulting allocation not minimal
  - Sparse interference matrix, optimal solution not unique

  → perturb interference matrix
- Optimize over resulting subsets for original problem
- Observed: allocation size ≤ N
  - Actual number depends on perturbation values

Link index	1	2	3	4	5	6	7	8	9	10	%
Active subsets	0	**1**	0	**1**	0	0	0	0	0	0	75
	0	0	0	0	0	**1**	0	0	0	**1**	25
Total link activation (%)	0	75	0	75	0	25	0	0	0	25	

Effect of aligned LOS interference

$\alpha_i = 1 \qquad \gamma_l = 10\text{dB}$



Effect of aligned LOS interference

Interference to signal ratio

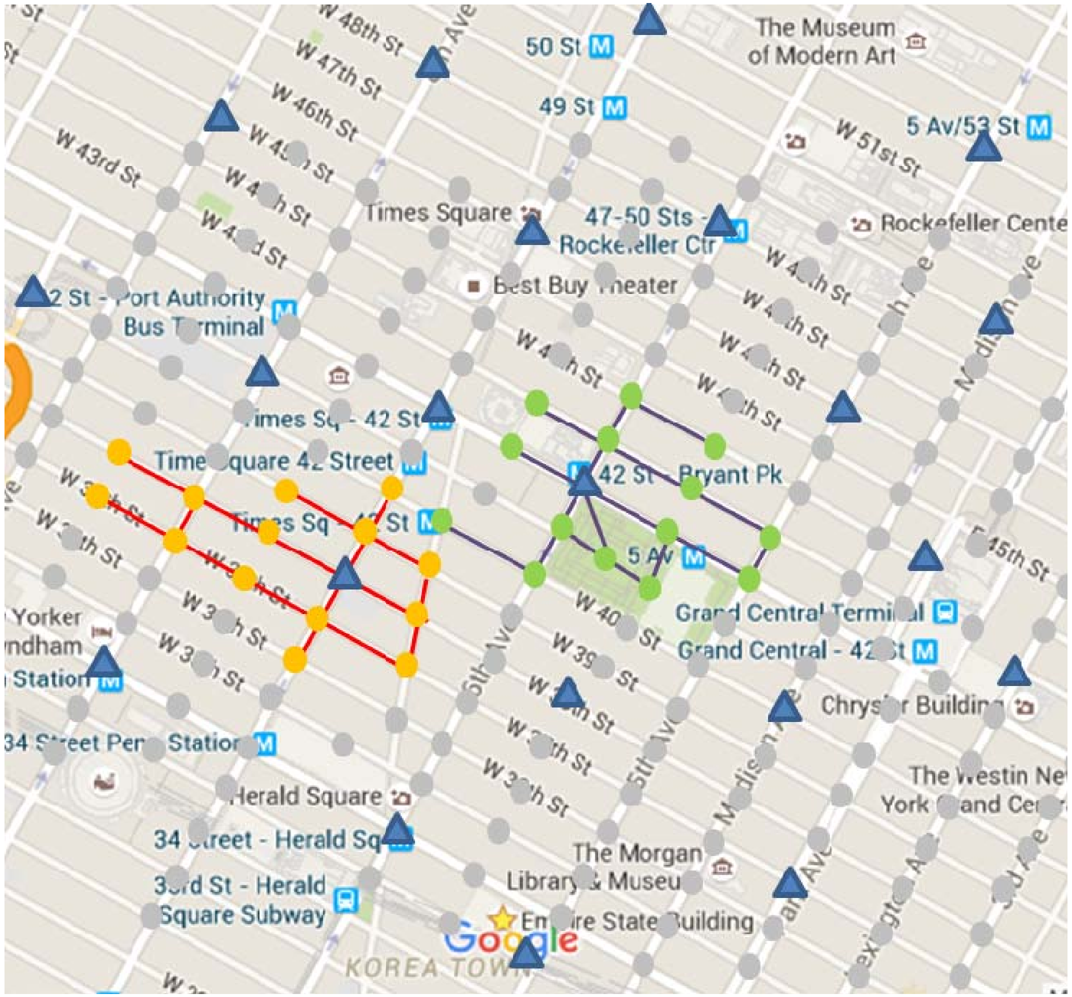
Link #	3	4	5	6	9	10	11	12	13	14
3	0	∞	0	∞	∞	0	0	**0.2**	0	**0.125**
4	∞	0	∞	0	0	∞	0	0	0	0
5	0	∞	0	∞	0	**0.2**	∞	0	0	0
6	∞	0	∞	0	0	0	0	∞	0	**0.2**
9	∞	0	0	**0.2**	0	∞	0	**0.125**	0	**0.08**
10	0	∞	0	0	∞	0	0	0	0	0
11	0	**0.2**	∞	0	0	**0.125**	0	∞	∞	0
12	0	0	0	∞	0	0	∞	0	0	∞
13	0	**0.125**	**0.2**	0	0	**0.08**	∞	0	0	∞
14	0	0	0	0	0	0	0	∞	∞	0

Effect of aligned LOS interference

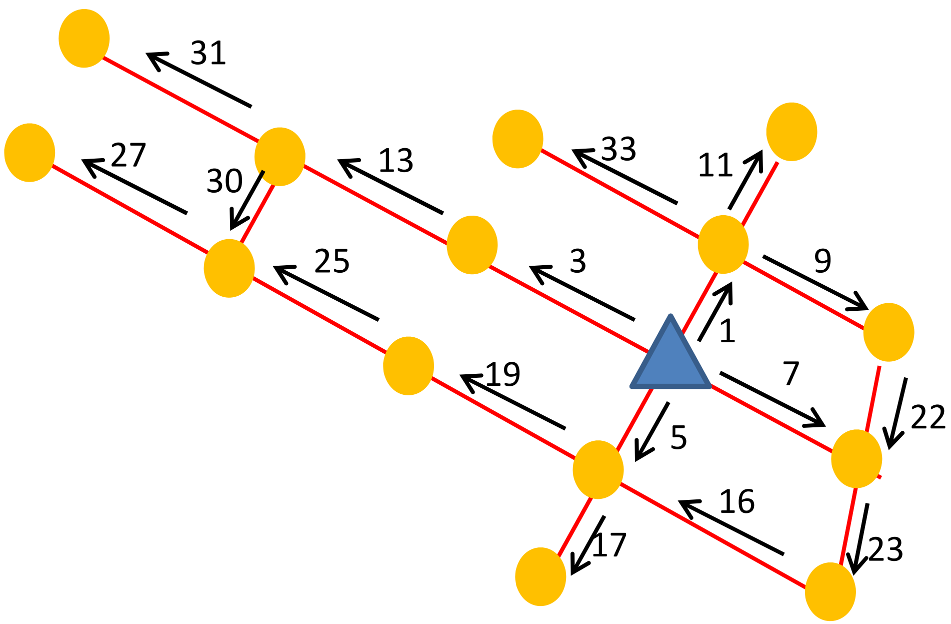
With aligned interference	
Links in subset	% allocation
6	14.99
2,6	10.63
4,6	10.63
6,10	10.63
4,12	31.88
12,20	10.63
6,8,14,16,18,22	10.63
d=0.3676 bps, d/d_{link} = 0.1063	

Without aligned interference	
Links in subset	% allocation
6	0.1111
4,6	0.1111
6,10	0.1111
4,12	0.2222
4,8,12	0.1111
6,14	0.1111
2,12,20	0.1111
6,16,18,22	0.1111
d=0.3844 bps, d/d_{link} = 0.1111	

Results – New York City

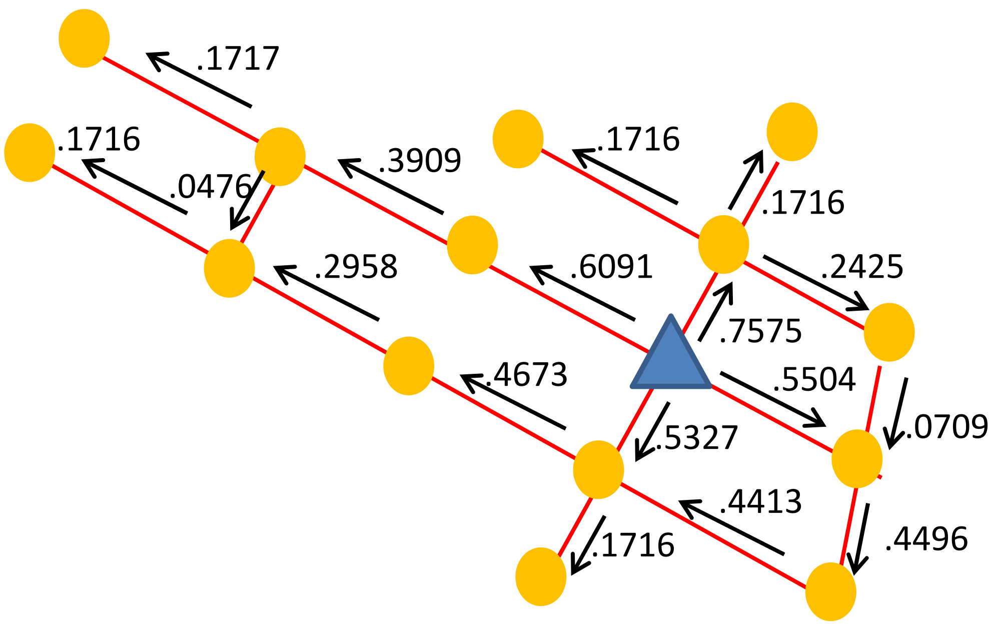


Results – New York City



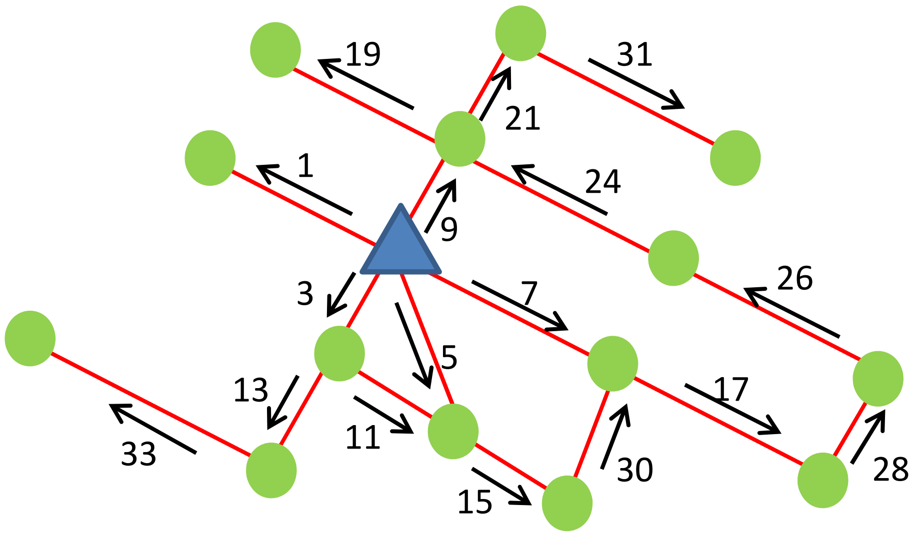
Links in subset	% allocation
1,3,5,7,16,	3.27
3,7,9,11,17,19	1.21
1,3,5,7,16,22	3.26
1,3,7,17,19,22	0.56
1,13,19,23	29.57
3,9,11,17,19,23	5.87
1,13,17,19,23	9.52
1,3,5,7,25	9.14
1,3,5,7,16,22,25	3.27
1,3,5,7,16,27,33	17.16
3,5,7,9,16,25,31	7.09
3,5,7,9,11,16,25,31	5.32
3,5,7,9,11,16,25,30,31	4.76
d=0.5938 bps,	**d/d_{link} = 0.1716**

Results – New York City



Links in subset	% allocation
1,3,5,7,16,	3.27
3,7,9,11,17,19	1.21
1,3,5,7,16,22	3.26
1,3,7,17,19,22	0.56
1,13,19,23	29.57
3,9,11,17,19,23	5.87
1,13,17,19,23	9.52
1,3,5,7,25	9.14
1,3,5,7,16,22,25	3.27
1,3,5,7,16,27,33	17.16
3,5,7,9,16,25,31	7.09
3,5,7,9,11,16,25,31	5.32
3,5,7,9,11,16,25,30,31	4.76
d=0.5938 bps, **d/d_{link} = 0.1716**	

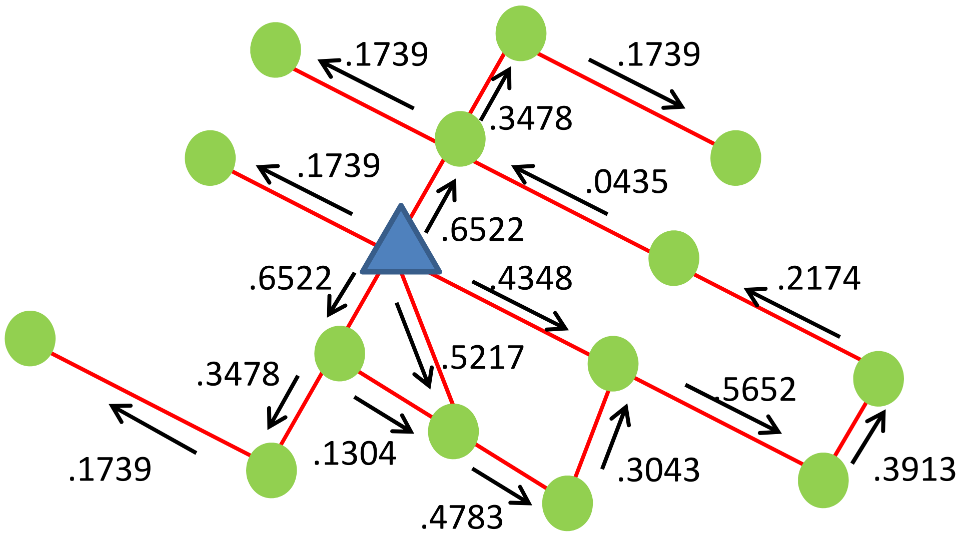
Results – New York City



- 5 Gbps link data rate
  → 500Mbps per-node rate
- 10-20 Gbps link data rate
  → 1-2Gbps per-node rate

Links in subset	% allocation
3,5,7,9	0.0435
1,3,5,9,17	0.1304
3,9,15,17,31,33	0.1739
1,3,9,15,17	0.0435
3,9,15,17,26	0.2174
7,13,15,21,28	0.0435
3,5,7,24,28	0.0435
5,7,13,21,28,30	0.1304
5,7,13,19,21,28,30	0.0435
5,7,11,13,19,21,28,30	0.1304
d=0.6016 bps,	**d/d_{link} = 0.1739**

Results – New York City



- 5 Gbps link data rate
  → 500Mbps per-node rate
- 10-20 Gbps link data rate
  → 1-2Gbps per-node rate

Links in subset	% allocation
3,5,7,9	0.0435
1,3,5,9,17	0.1304
3,9,15,17,31,33	0.1739
1,3,9,15,17	0.0435
3,9,15,17,26	0.2174
7,13,15,21,28	0.0435
3,5,7,24,28	0.0435
5,7,13,21,28,30	0.1304
5,7,13,19,21,28,30	0.0435
5,7,11,13,19,21,28,30	0.1304
d=0.6016 bps,	**d/d_{link} = 0.1739**

Millimeter wave picocells: Interference analysis and capacity

Upamanyu Madhow

ECE Department, UCSB

(slides prepared by Zhinus Marzi)

Marzi, Madhow, Zheng, *Interference analysis for mm-wave picocells,* Globecom 2015

2016 Summer School, IISc Bangalore

Mm-wave enables aggressive spatial reuse

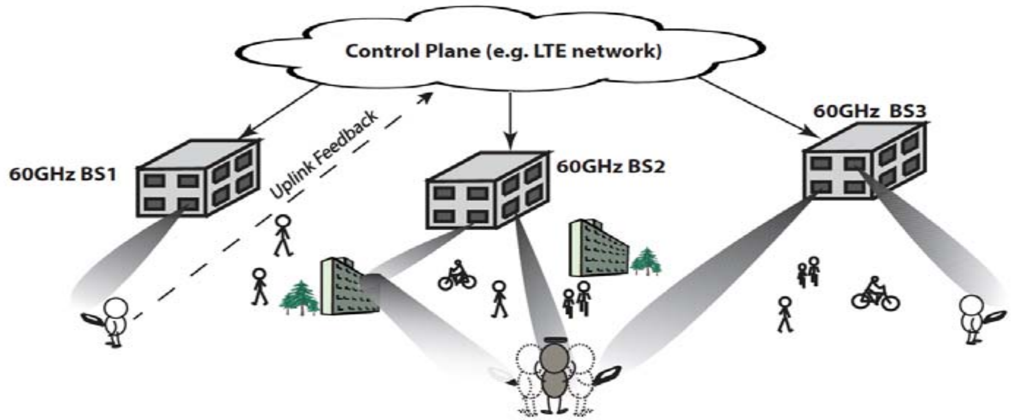
Large arrays in small form factors

Directive links

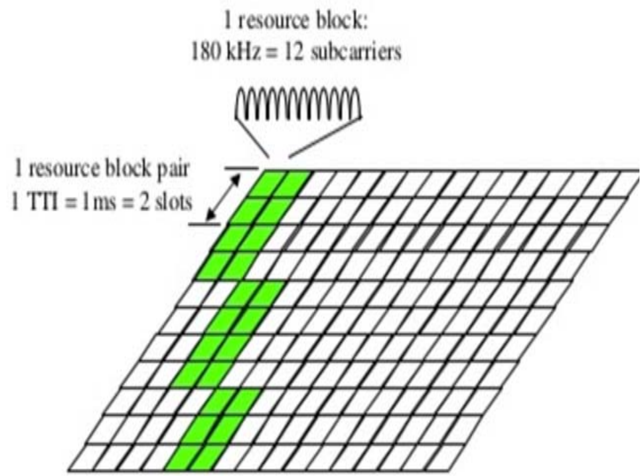
Limited interference

Dense cells / much higher spatial reuse

32 x 32
8 x 8 cm^2



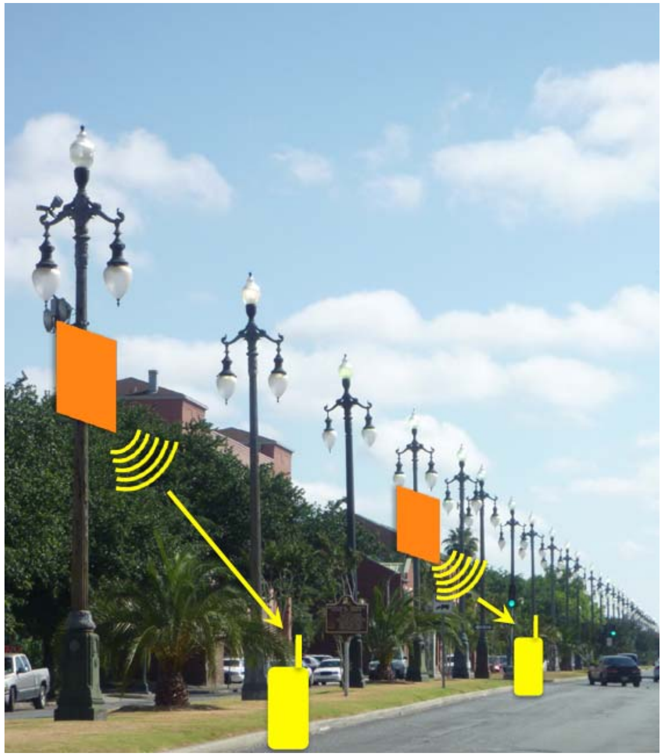
Exploiting *space*

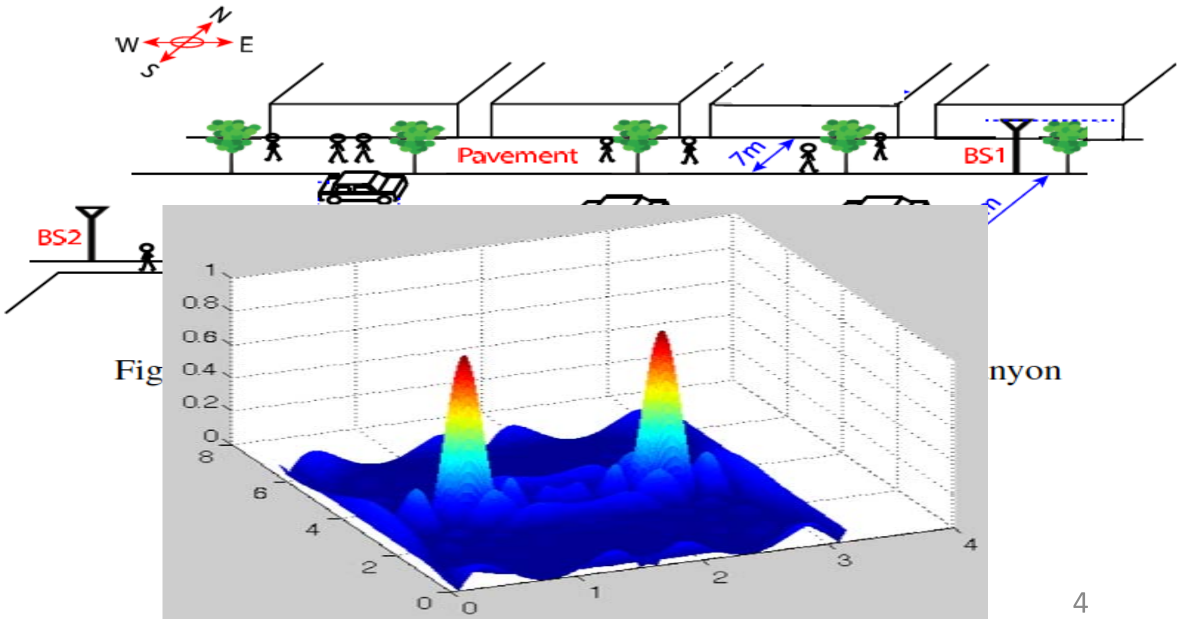


- LTE resource blocks (OFDMA):
  - Time - frequency
- Mmwave Resource blocks:
  - Time - frequency - *space*
- Increased spatial reuse is always an option, but:
  - In LTE: Increase in spatial reuse ➔ decrease in spectral efficiency (due to interference)
  - In mm-wave: Directive links allows increased spatial reuse without loss in spectral efficiency

Big picture

- Interference characterization and capacity estimate for mm-wave
- Geometric interference analysis tailored to urban canyon





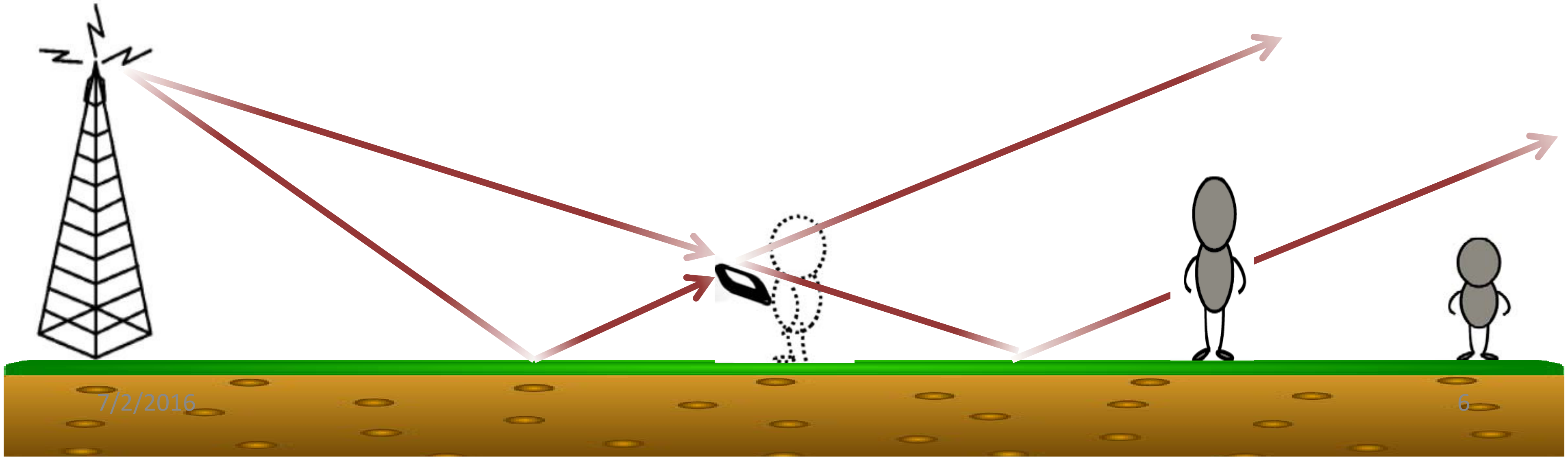
Key results

- Interference is not a big problem
  - Main lobe interference vanishes after a few cells
  - Side lobe interference is relatively small

➔Hardware/noise limited performance attainable with minimal coordination among base stations

Main lobe interference model

- For large antenna arrays, main lobe is well modeled by a single ray.



Main lobe interference escapes upward

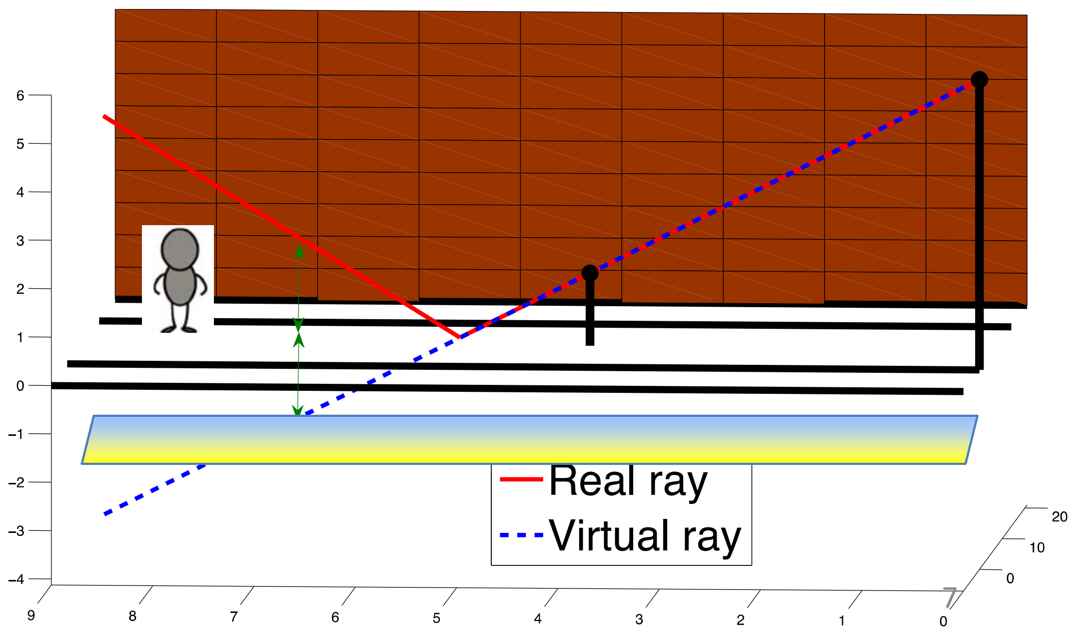
- The main beam from a face creates interference for at most N_{max} adjacent BSs in the direction it is facing.

$$N_{max} = \lceil \frac{H_{BS} + h_{max}}{H_{BS} - h_{max}} \rceil$$

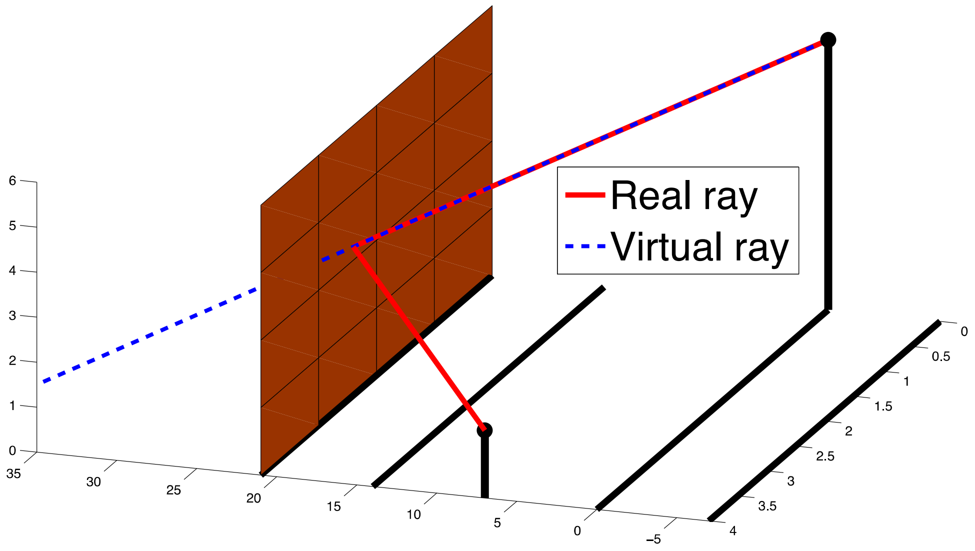
$$H_{BS} = 6m$$

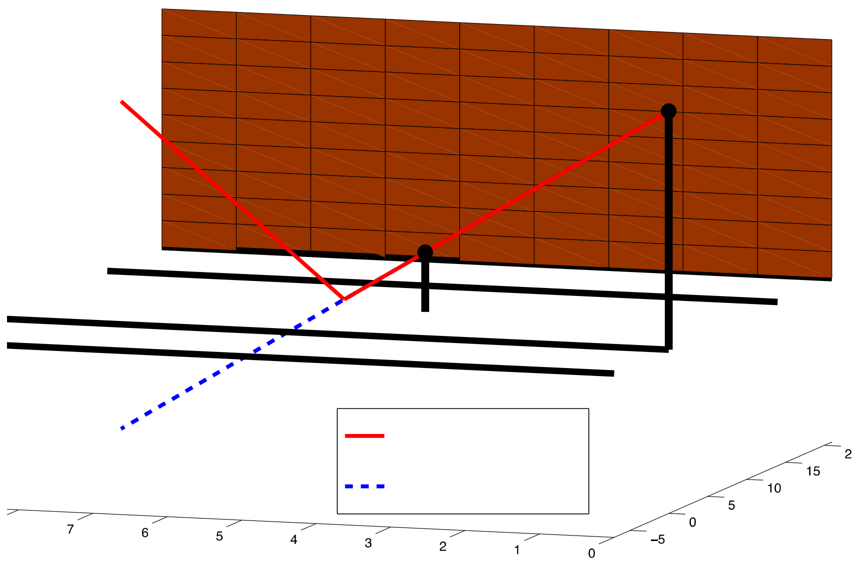
$$h_{max} = 2m$$

$$N_{max} = 2$$



Proof:





$$X = \frac{a}{c}(z - H_{BS})\big|_{z=-h_{max}} = \frac{a}{c}(-h_{max} - H_{BS})$$

$$\frac{X - X_{BS}}{a} = \frac{Y - Y_{BS}}{b} = \frac{Z - Z_{BS}}{c}$$

$$X = \max_u \frac{X_u}{\mp Z_u + H_{BS}}(h_{max} + H_{BS})$$

	LoS	NLoS1	NLoS2	NLoS3
a	X_u	X_u	X_u	X_u
c	$Z_u - H_{BS}$	$Z_u - H_{BS}$	$Z_u - H_{BS}$	$-Z_u - H_{BS}$

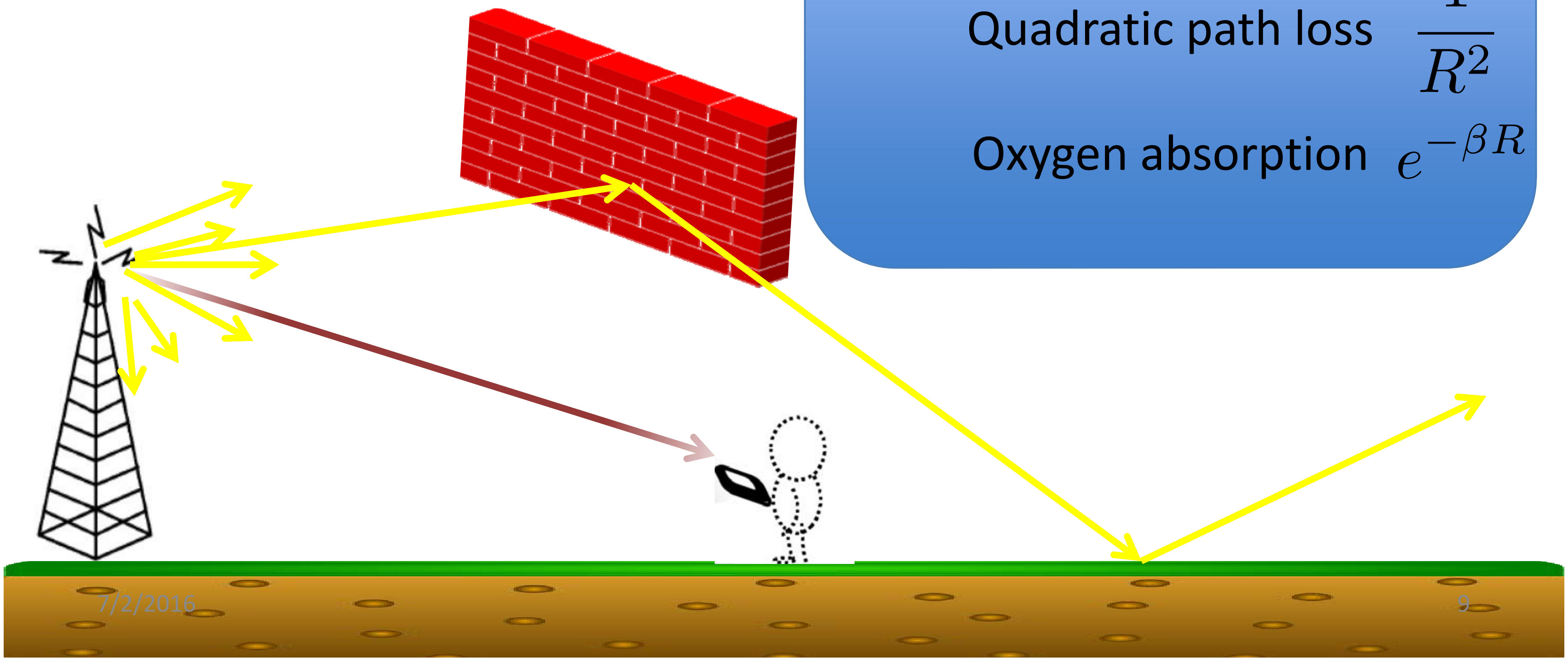
Side lobe interference

Antenna's attenuation

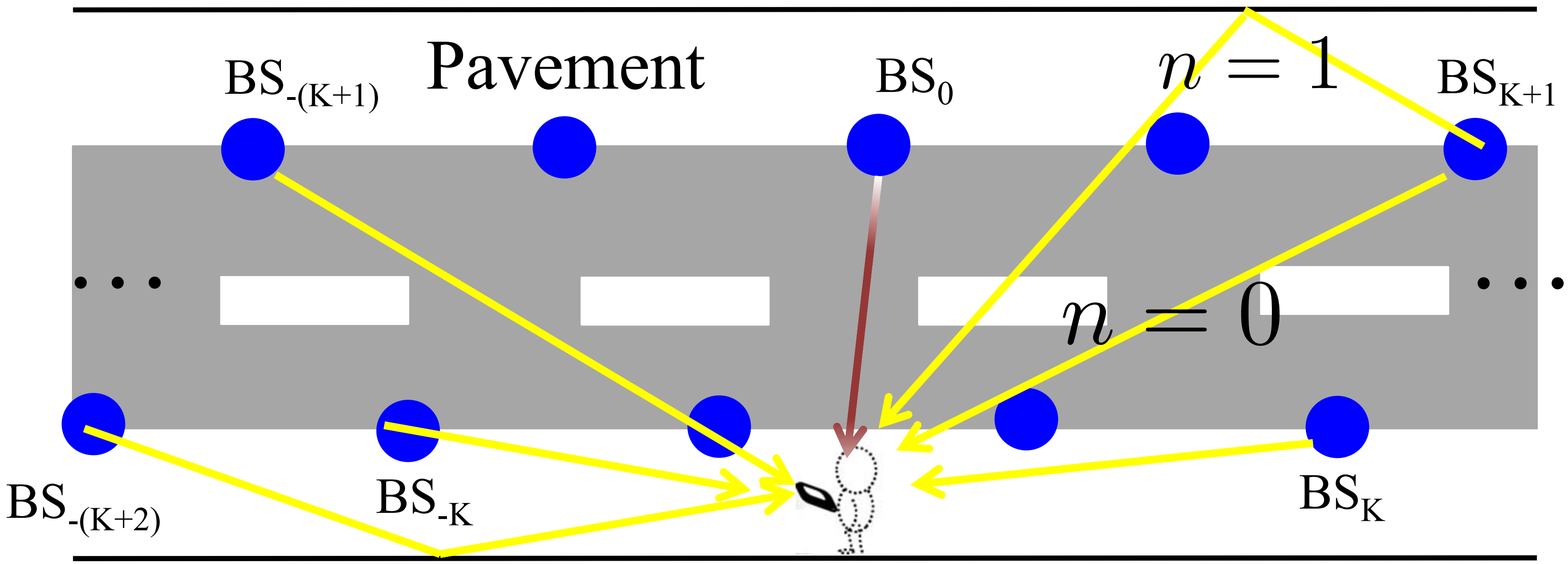
Reflection loss

Quadratic path loss $\frac{1}{R^2}$

Oxygen absorption $e^{-\beta R}$

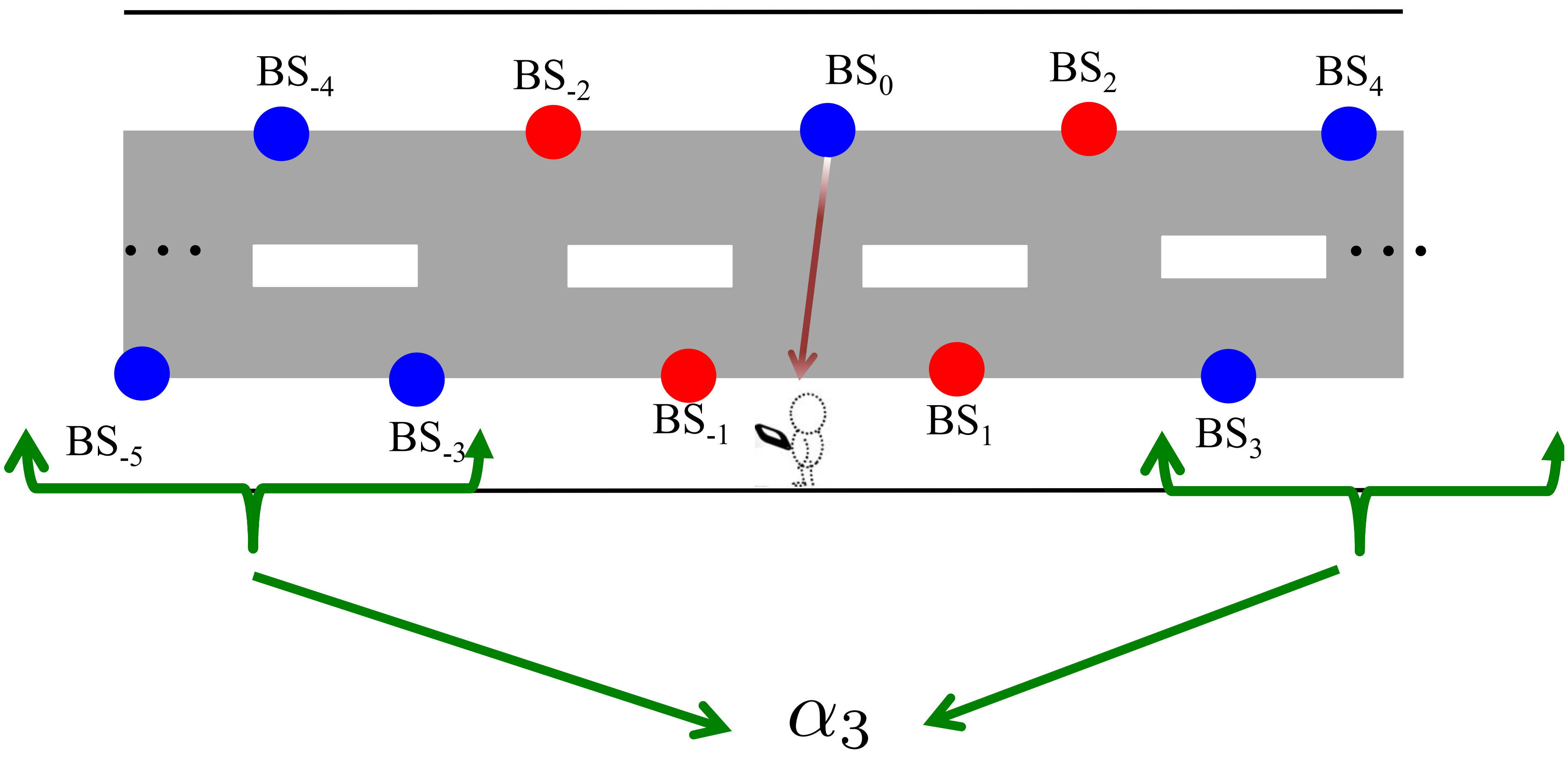


Side lobe interference



$$\alpha_K = \frac{\sum_{k=-\infty}^{K} I_k + \sum_{k=K}^{\infty} I_k}{P} < Ce^{-\beta K d}$$

Side lobe interference



derivation:

$$\alpha_K = \frac{\sum\limits_{k=K}^{\infty} I_k + \sum\limits_{n=-\infty}^{-K} I_k}{P}$$

$$I_k = \sum_{n=0}^{\infty} I_{k,n}$$

$$I_{k,n} \approx N_n P_{Tx} (g_{Tx})_{k,n} (g_{Rx})_{k,n} (\frac{\lambda}{4\pi r_{k,n}})^2 e^{-\beta r_{k,n}} (\frac{1}{l_{k,n}})^n$$

Path lengths

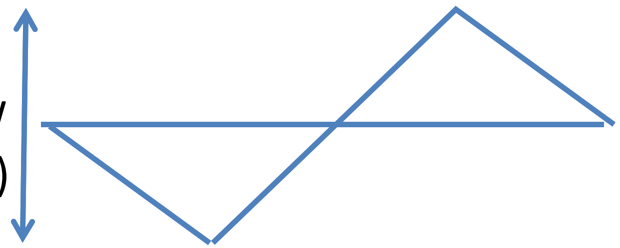
$$r_{k,n} = 2n\sqrt{(\frac{r_{k,0}}{2n})^2 + (\frac{W}{2})^2}$$

$$= \sqrt{r_{k,0}^2 + n^2 W^2}$$

$$r_{k,n}^2 \cong r_{k,0}^2 \quad + n^2 w^2$$

$$r_{k,0} \cong kd$$

W
(Distance of walls)



n=2

Proof: (cont)

$$\frac{\alpha_K}{2} = \frac{\sum_{k=K}^{\infty} I_k}{P} = \sum_{k=K}^{\infty}\sum_{n=0}^{\infty}\left(N_n \frac{(g_{Tx})_{k,n}(g_{Rx})_{k,n}}{G_{Tx}G_{Rx}}\left(\frac{r_{max}}{r_{k,n}}\right)^2 e^{-\beta(r_{k,n}-r_{max})}\left(\frac{1}{(l_{k,n})}\right)^n\right)$$

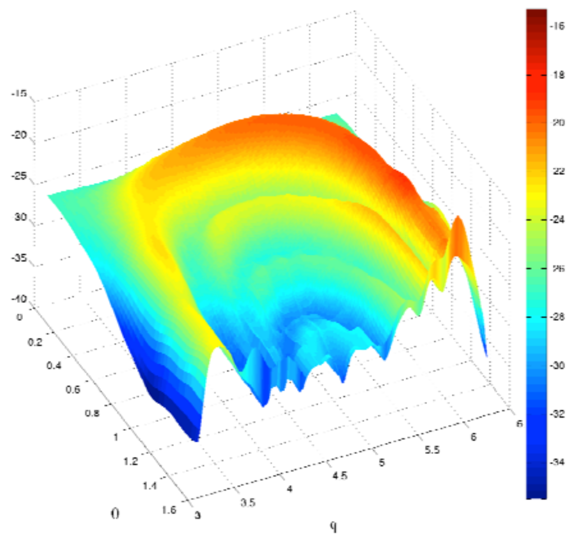
$$\frac{\sum_{k=K}^{\infty} I_k}{P} \leq 4r_{max}^2 e^{\beta r_{max}} \sum_{k=K}^{\infty}\sum_{n=0}^{\infty}\left(\frac{1}{(kd)^2+(nW)^2}\right.$$

$$\left. e^{-\beta\sqrt{(kd)^2+(nW)^2}}\left(\frac{1}{(l_{min})}\right)^n\right) < 4\sum_{k=K}^{\infty} e^{-\beta kd}\sum_{n=0}^{\infty}\frac{1}{(l_{min})^n}$$

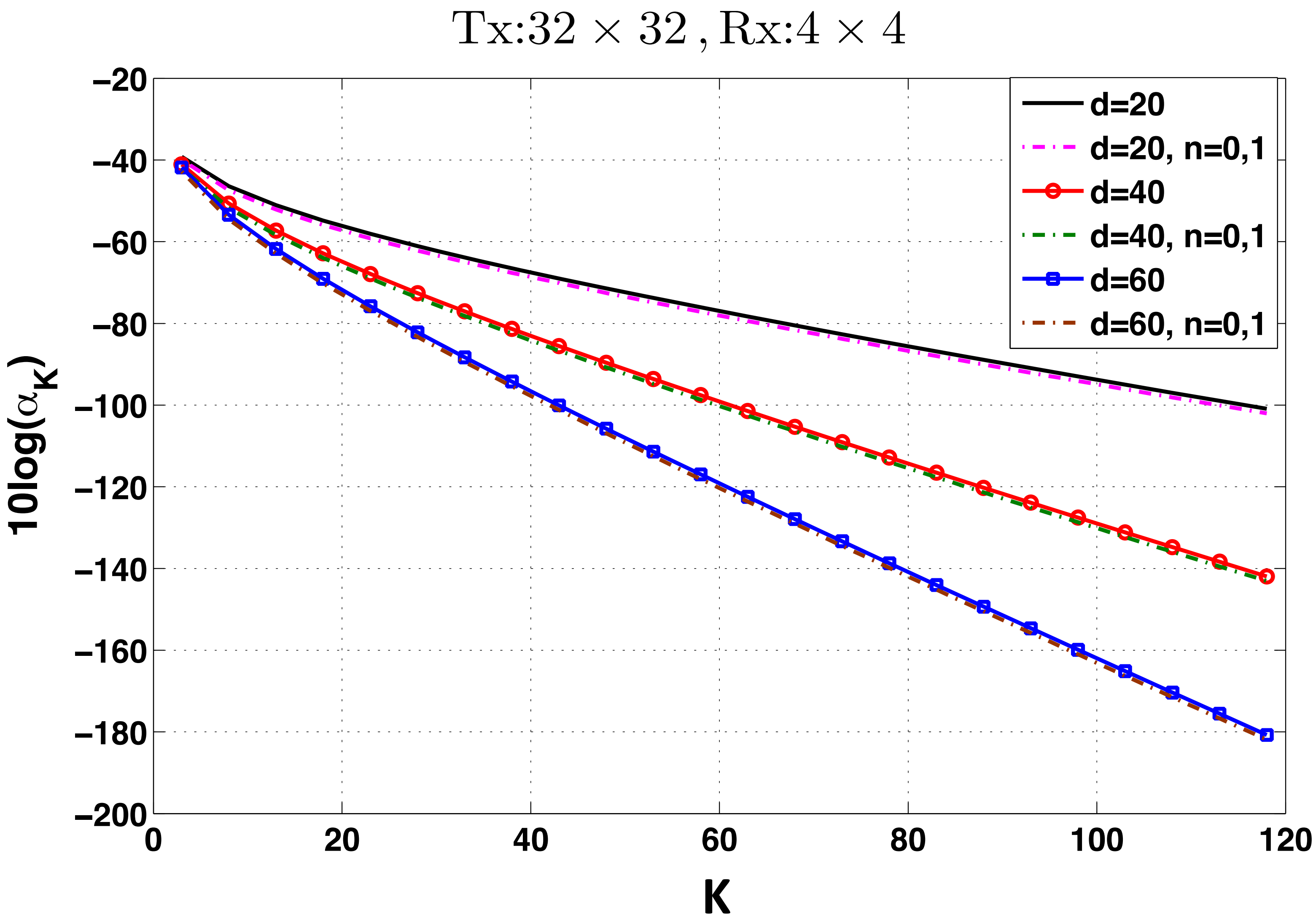
$$= 4\frac{e^{-\beta Kd}}{1-e^{-\beta d}}\frac{l_{min}}{l_{min}-1}$$

Assumptions

- Reasonable approximations for path lengths
- Reflection loss of 5 dB
- Tx: 32×32 and Rx: 4×4
- For $K > 3$ where Tx antenna gain could only be in side lobe levels.

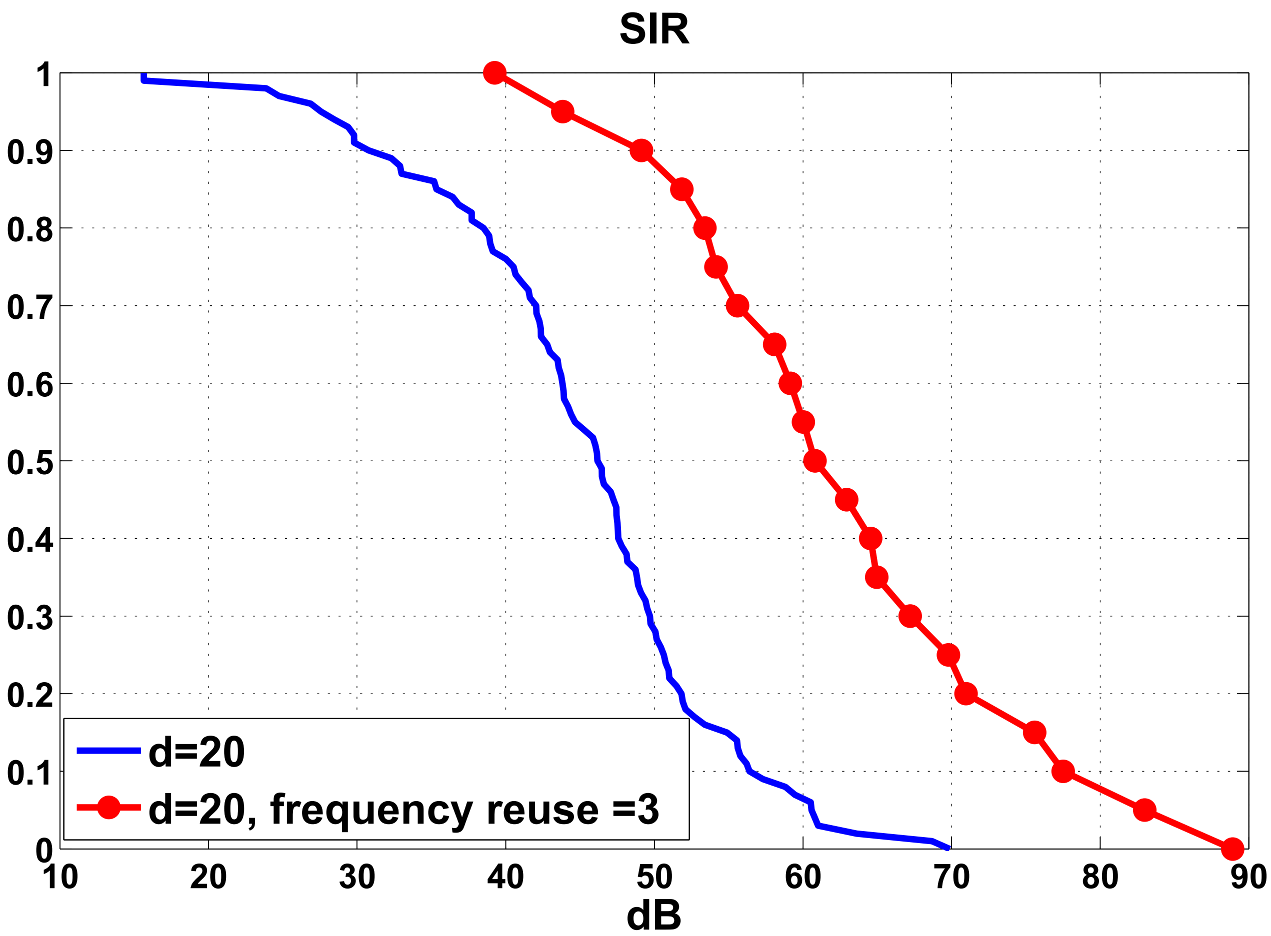


α_K: Cumulative sidelobe interference/signal ratio



Beyond the region of mainlobe interference (K>2) side-lobe interference is insignificant

CCDF of SIR



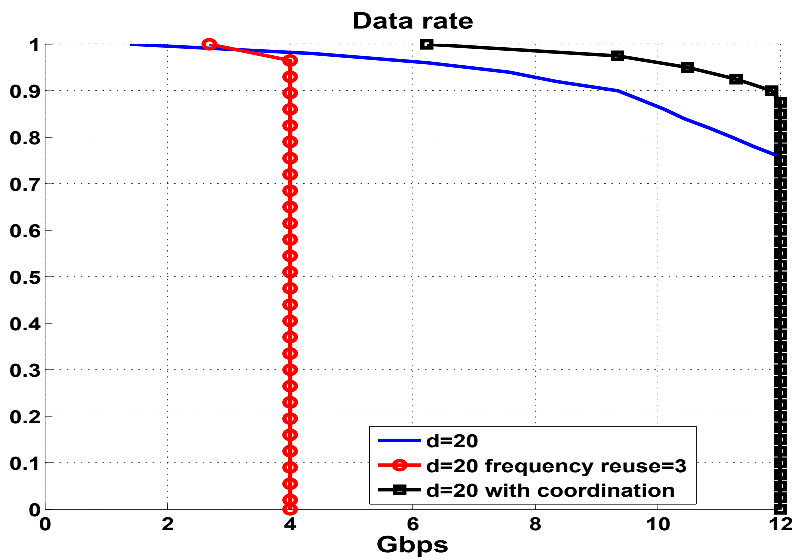
Simulation results justify our predictions of more than 40dB SIR by FR=3

Data rate limited by hardware, not interference

1. FR=1 (blue)
2. FR=1 with minimal coordination (black)
3. FR=3 (red)

$$r = \frac{1}{F}\min\left(r_{max}, log(1+SINR)\right)$$

$$r_{max} = 6\,bps/Hz \quad (64QAM)$$



$$1.2\ Tbps/km\,(BW = 2\,GHz)$$

The potential capacity is huge!

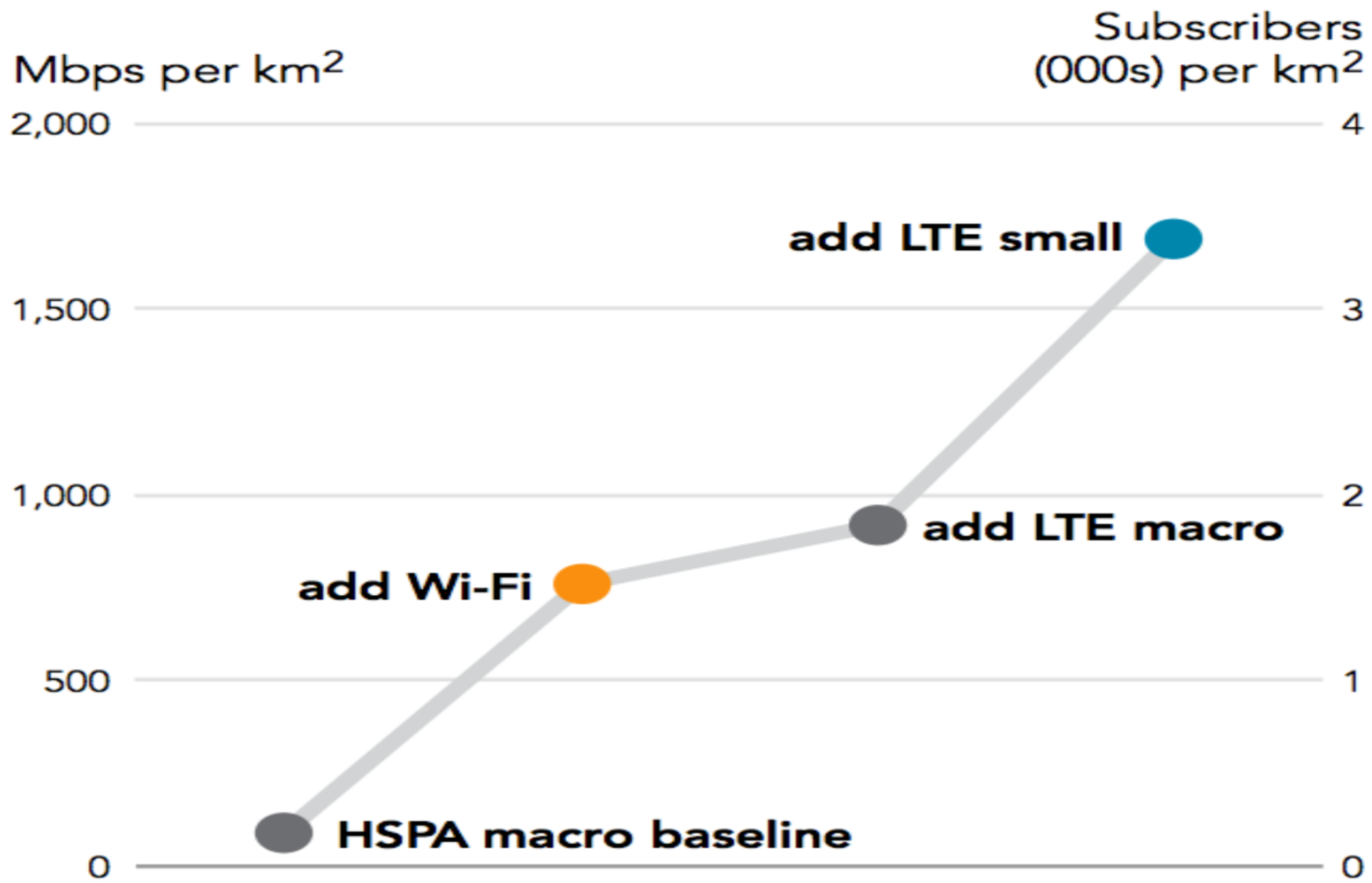
POTENTIAL CAPACITY GAINS OVER CONVENTIONAL CELLULAR NETWORKS

LTE DOWNLINK CAPACITY

Operator	Carrier and BW	Capacity / km²
LTE Operator 1	1 x 10 MHz @ 800 MHz 1 x 5 MHz @ 900 MHz 1 x 20 MHz @ 1800 MHz 2 x 5 MHz @ 2100 MHz 1 x 20 MHz @ 2600 MHz	156 Mbps
LTE Operator 2	1 x 10 MHz @ 800 MHz 1 x 5 MHz @ 900 MHz 1 x 10 MHz @ 1800 MHz 3 x 5 MHz @ 2100 MHz 1 x 20 MHz @ 2600 MHz	120 Mbps
LTE Operator 3	1 x 10 MHz @ 1800 MHz 4 x 5 MHz @ 2100 MHz 1 x 20 MHz @ 2600 MHz	120 Mbps
LTE Operator 4	1 x 5 MHz @ 900 MHz 1 x 10 MHz @ 1800 MHz 3 x 5 MHz @ 2100 MHz 1 x 20 MHz @ 2600 MHz	120 Mbps
LTE Advanced operator	1 x 40 MHz @ 3.5 GHz	96 Mbps
Total	255 MHz	612 Mbps

Sauter, M. (2013). 3G, 4G and beyond: bringing networks, devices and the web together. John Wiley & Sons.

FIGURE 1: Total RAN Capacity Density

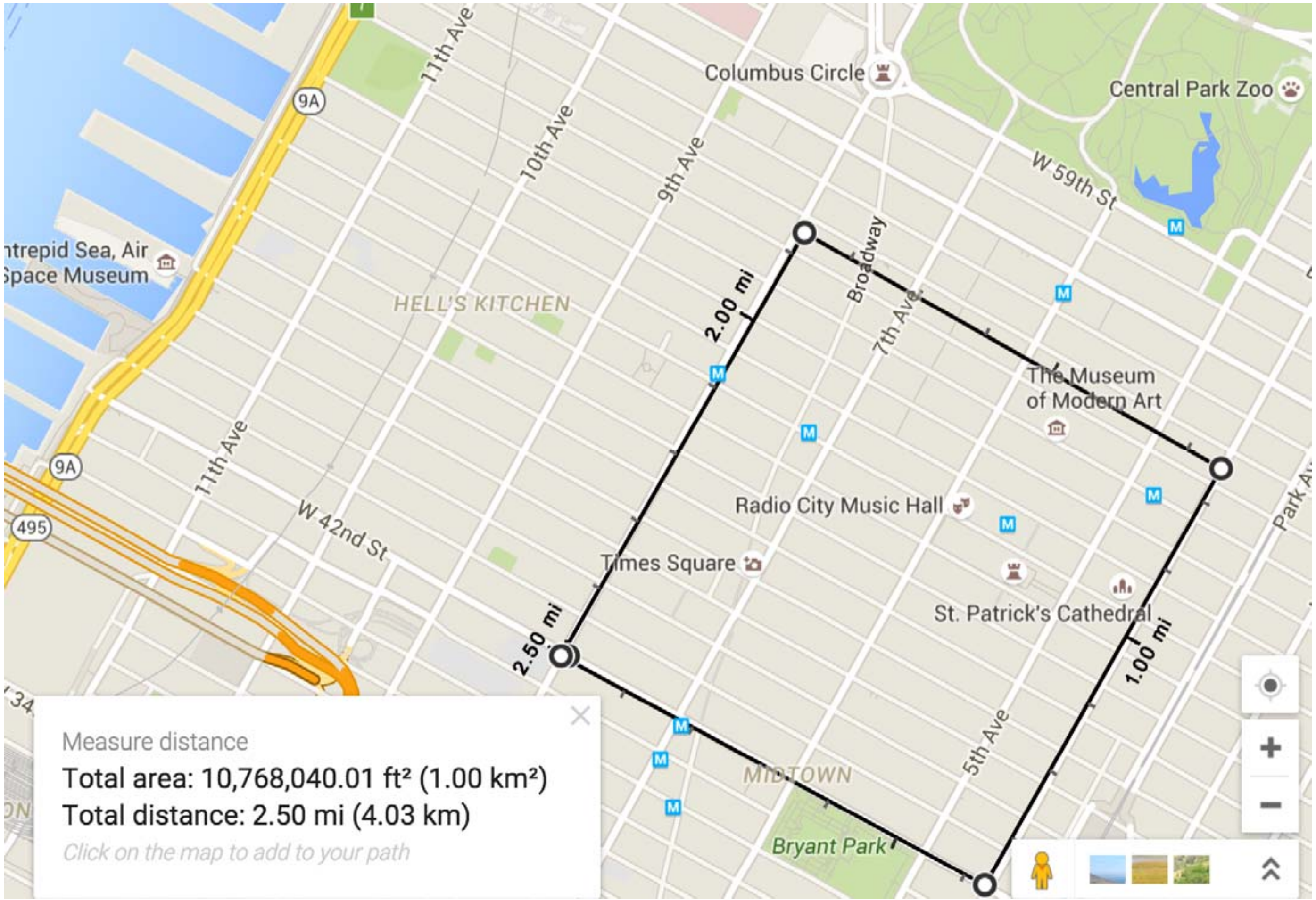


Dealing with Density: The Move to Small-Cell Architectures ruckus wireless, Ruckus wireless, white paper

Achievable capacity along a canyon (Gbps/km) in mm-wave cellular

Tx array	d=100 (m)			d=20 (m)		
	QPSK	16 QAM	64 QAM	QPSK	16 QAM	64 QAM
8 x 8	80 (60%)	160 (30%)	240 (20%)	400 (100%)	800 (100%)	1200 (88%)
32 x 32	80 (100%)	160 (90%)	240 (70%)	400 (100%)	800 (100%)	1200 (90%)

1 km^2 in Manhattan ~ 15 canyons



Capacity per unit area

Measure	LTE	mmwave	Gain
Bandwidth	255 MHz	2 GHz	~8x
Capacity / km^2	< 1.5 Gbps	18 Tbps (15 × 1.2 Tbps)	~12,000x
		1.2 Tbps (15 × 80 Gbps)	~800x
Spatial reuse	-	-	~1500x
			~100x

Well beyond Cellular 1000X is achievable (in principle)

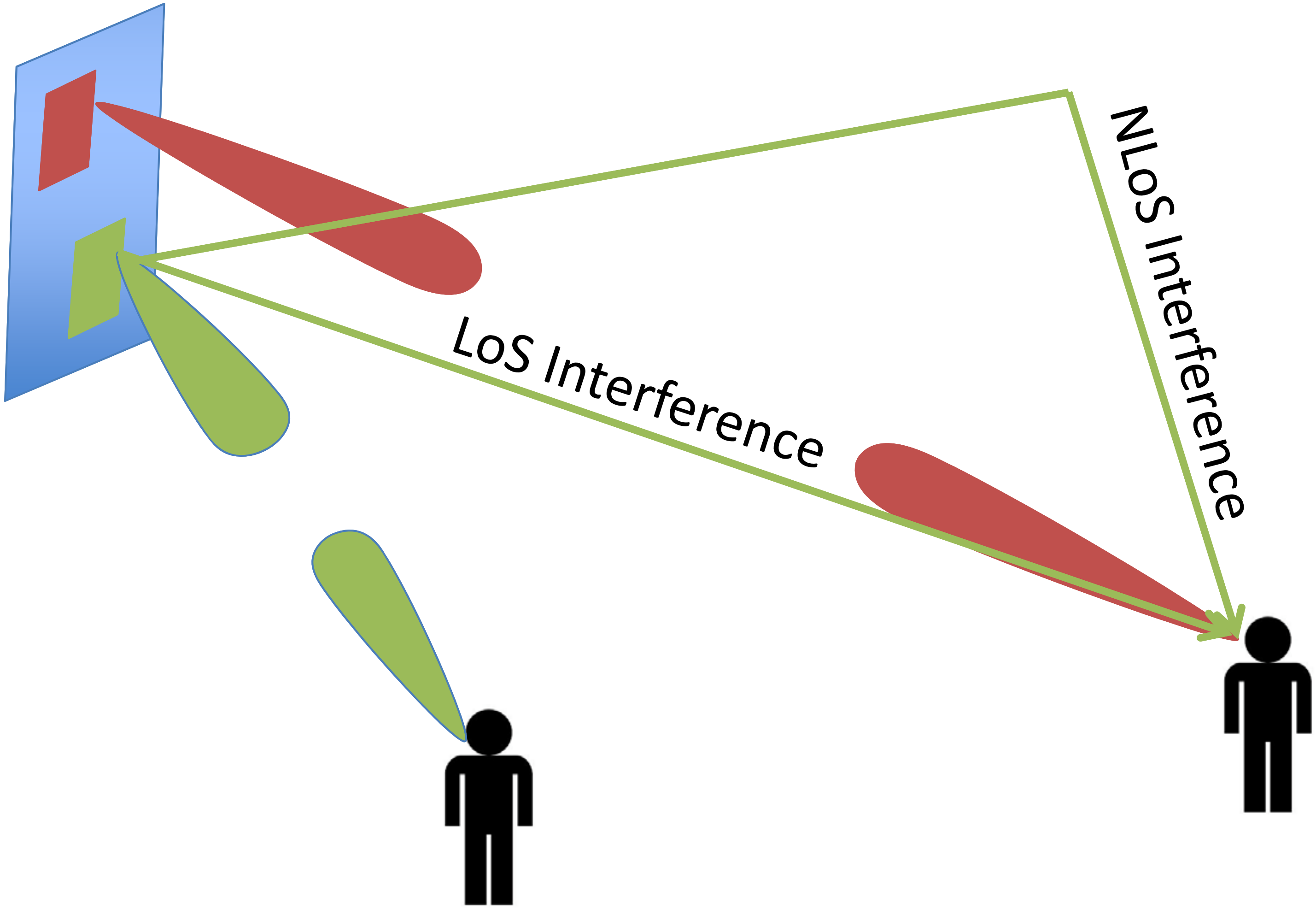
Take-aways

- Geometric analysis with some idealizations
  - **Assuming everything works, we can even get to Cellular 10000X!**

- Design implications:
  - hardware rather than interference/noise is the bottleneck even as we scale down cell size
  - Orthogonalization is wasteful when links are so highly directive
  - Coordination with nearby picocells is important

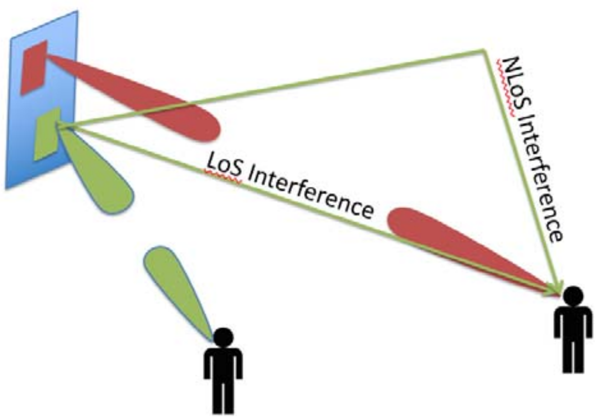
INTRA-CELL INTERFERENCE (FUTURE WORK)

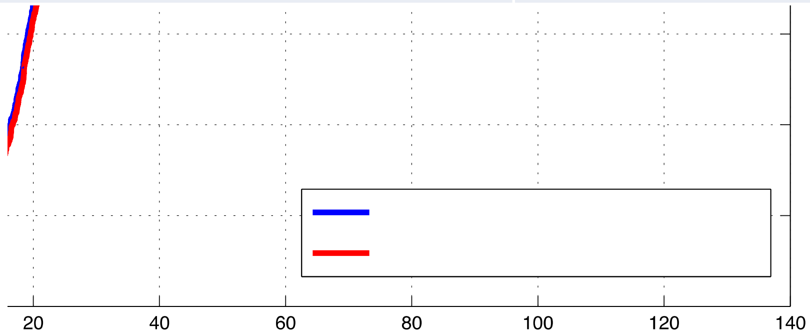
Interference characterization



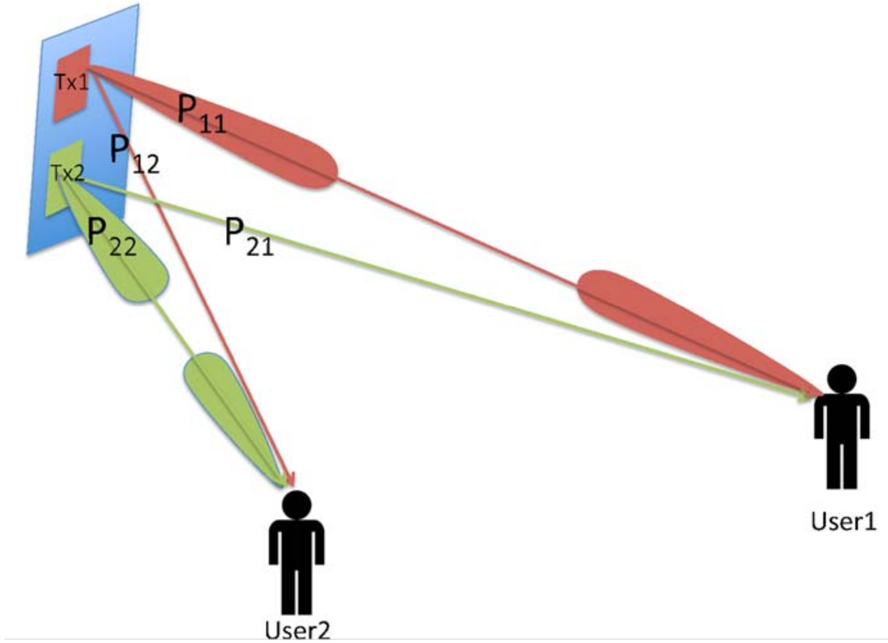
LoS vs. NLoS intra-cell interference

Factors	LoS interference	NLoS interference
Path loss	☒	☑
Rx gain	☒	☑
Reflection loss	☒	☑
Tx gain	☑	☑





SIR characterization



$$SIR_{u2} = \frac{P_{22} l G_{Rx}}{P_{12} l g_{Rx}}$$

$$\text{Signal to null ratio for Tx1} = \frac{P_{11}}{P_{12}}$$

$$SIR_{u2} \geq \text{Signal to null ratio for Tx1}$$

Null-forming

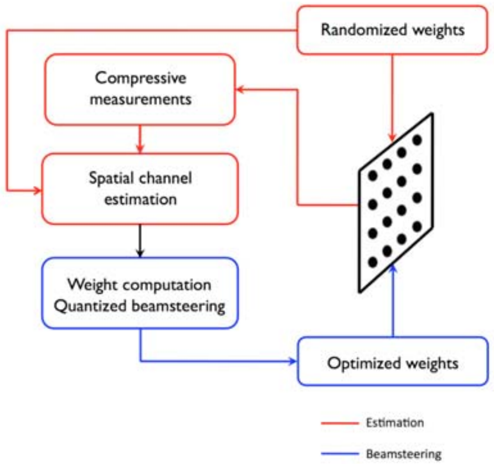
ZF Problem

$$\max_{\phi} |\phi^H \mathbf{x}_t(\omega_0)|^2$$

$$\text{subject to:} \quad ||\phi||^2 = 1$$

$$\phi^H \mathbf{x}_t(\omega_q) = 0 \quad q = 1, .., Q$$

$$P = |\phi^H \mathbf{x}_t(\omega)|^2$$



Quantized Algorithm

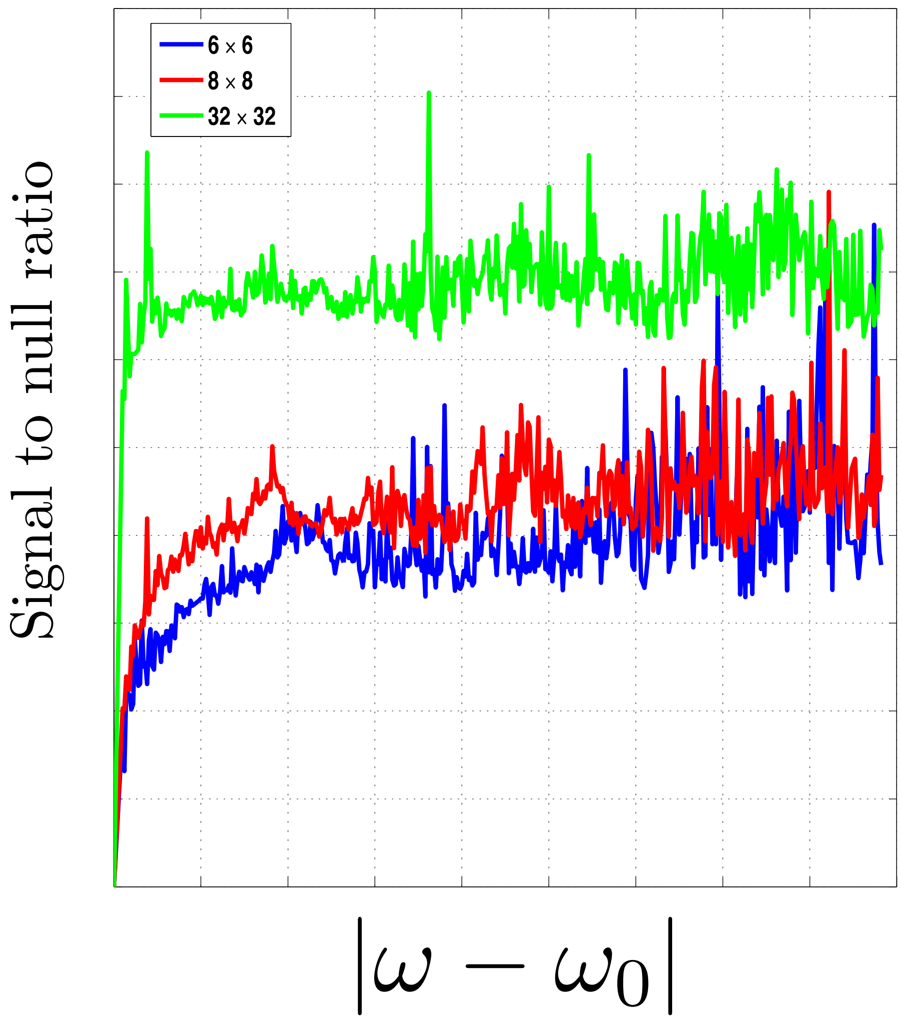
$$\text{maximize}\, \gamma = \frac{P(\omega_0)}{\sum_{q=1}^{Q} P(\omega_{\mathbf{q}})}$$

$$\text{Subject to:}\, \phi_i \in \{\pm 1 \pm j\}$$

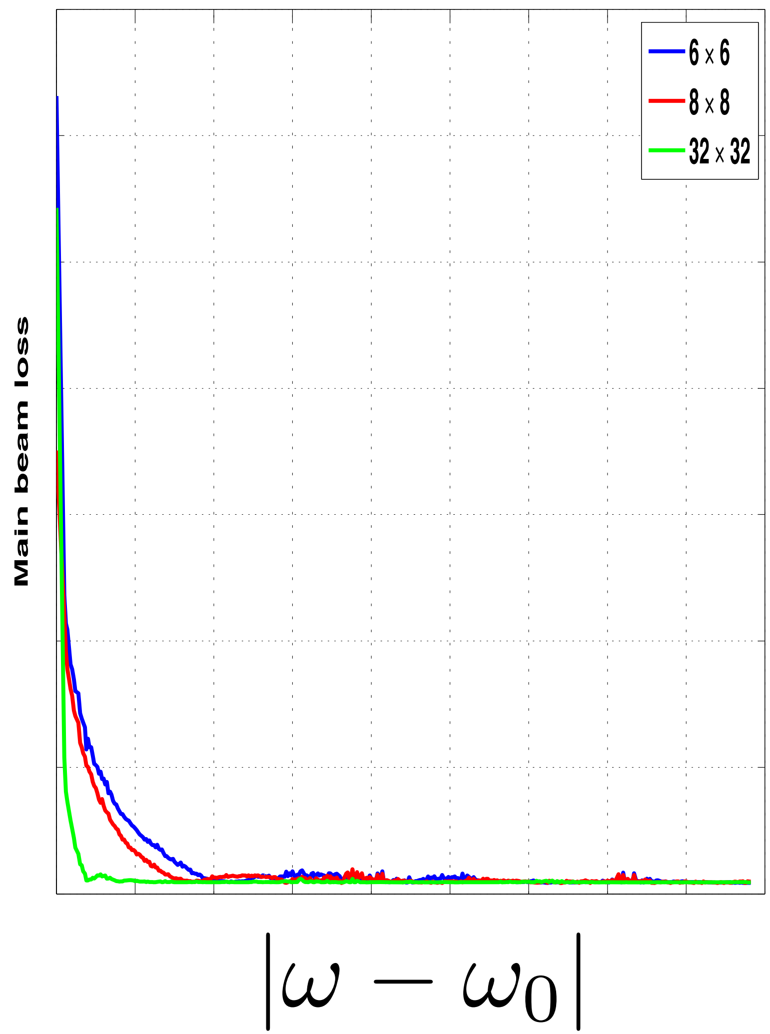
Ramasamy, D., Venkateswaran, S., & Madhow, U. (2012, February). Compressive adaptation of large steerable arrays. In *Information Theory and Applications Workshop (ITA), 2012* (pp. 234-239). IEEE.

Preliminary simulation results

Signal to null ratio



Main beam loss



Millimeter wave
References and Open Issues

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LoS MIMO

Sheldon, Seo, Torkildson, Madhow, Rodwell*, A 2.4 Gb/s millimeter-wave link using adaptive spatial multiplexing,* APS-URSI 2010.
Torkildson, Madhow, Rodwell, *Indoor millimeter wave MIMO: feasibility and performance,* IEEE Trans. Wireless Comm., Dec 2011.
Mamandipoor, Sawaby, Arbabian, Madhow, *Hardware-constrained signal processing for mm-wave LoS MIMO links,* Asilomar 2015
Irish, Quitin, Madhow, *Sidestepping the Rayleigh limit for LoS spatial multiplexing: a distributed architecture for long-range wireless fiber,* ITA 2013.
Irish, Quitin, Madhow, *Achieving multiple degrees of freedom in long-range mm-wave MIMO channels using randomly distributed relays,* Asilomar 2013.

Open Issues

- Hybrid analog/digital signal processing architectures and
- algorithms (design and evaluation under different channel
- models)
- Fundamental limits under abstractions of hardware
- constraints
- Distributed architectures
- Hardware demonstrations

Large Arrays

Ramasamy, Venkateswaran, Madhow, *Compressive adaptation of large steerable arrays,* ITA 2012.
Ramasamy, Venkateswaran, Madhow, *Compressive parameter estimation in AWGN,* IEEE Trans. Signal Proc., December 2014.
Marzi, Ramasamy, Madhow, *Compressive channel estimation and tracking for large arrays in mm wave picocells*, IEEE J. Selected Topics in Signal Processing, April 2016. (See also Allerton'12 paper by Ramasamy, Venkateswaran, Madhow).
Mamandipoor, Ramasamy, Madhow, "Newtonized Orthogonal Matching Pursuit: Frequency Estimation over the Continuum," to appear, IEEE Trans. Signal Processing (see also GlobalSIP'15 paper by same authors).

Open Issues

- Demonstrating compressive estimation for large arrays experimentally (beamforming, nullforming, tracking)
- Hybrid transceiver architectures, multi-user MIMO

Mm-wave Picocells

Marzi, Ramasamy, Madhow, *Compressive channel estimation and tracking for large arrays in mm wave picocells*, IEEE J. Selected Topics in Signal Processing, April 2016. (See also Allerton'12 paper by Ramasamy, Venkateswaran, Madhow).
Zhu et al, *Demystifying 60 GHz Outdoor Picocells,* Mobicom 2014.
Marzi, Madhow, Zheng, *Interference analysis for mm-wave picocells,* Globecom 2015

Open Issues
Demonstrating compressive picocell architecture experimentally
Abstractions for protocol design and evaluation
Base station coordination, handoffs, end-to-end performance

Mesh Networks

Singh, Mudumbai, Madhow*, Interference analysis for highly directional 60-GHz mesh networks: the case for rethinking medium access control,* IEEE/ACM Trans. Networking, October 2011.
Singh, Mudumbai, Madhow, *Distributed coordination with deaf neighbors: efficient medium access for 60 GHz mesh networks,* IEEE Infocom 2010.
Rasekh, Guo, Madhow*, Interference-aware routing and spectrum allocation for millimeter wave backhaul in urban picocells,* Allerton 2015.

Open Issues

Comprehensive design and evaluation for picocellular backhaul and last mile applications
Tractable optimization framework and interference/propagation models
Architectures and evaluation for novel system concepts (e.g., drones, satellites)
Analytical characterization and optimization of decentralized mesh networks

Millimeter wave radar

Mamandipoor et al, *Spatial-Domain Technique to Overcome Grating Lobes in Sparse Monostatic mm-Wave Imaging Systems,* IMS 2016.
Mamandipoor, Ramasamy, Madhow, "Newtonized Orthogonal Matching Pursuit: Frequency Estimation over the Continuum," to appear, IEEE Trans. Signal Processing (see also GlobalSIP'15 paper by same authors).

Open Issues

Fundamental characterization of short-range delay/Doppler imaging
Design and evaluation in specific contexts: gesture recognition, vehicular automation
Interface with machine learning algorithms
Prototyping and experimental validation

ADC-limited communication

Ponnuru, Seo, Madhow, Rodwell, *Joint mismatch and channel compensation for high-speed OFDM receivers with time-interleaved ADCs,* IEEE TCOM, August 2010.
Singh, Dabeer, Madhow, *On the limits of communication with low-precision analog-to-digital conversion at the receiver,* IEEE TCOM, December 2009.
Wadhwa, Shanbhag, Madhow, *Space-time slicer architectures for analog-to-information conversion in channel equalizers,* ICC 2014.
Roufarshbaf, Madhow, *Analog multiband: efficient bandwidth scaling for mm wave communication*, IEEE J. Selected Topics in Signal Processing, April 2016.

Open issues

Fundamental limits and architectures for various settings
-- mm-wave MIMO with large bandwidths
--low-power, short range links
Hardware demonstrations