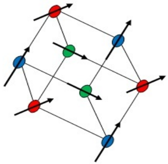
Recovery of distributed quantum information from a quantum erasure

Ankur Raina
Advisor: Prof. Shayan G. Srinivasa
Department of Electronic Systems Engineering
Indian Institute of Science, Bengaluru

Physical Nano-Memories Signal and Information Processing Lab



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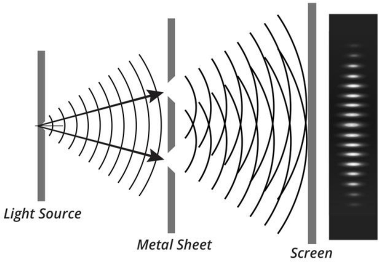
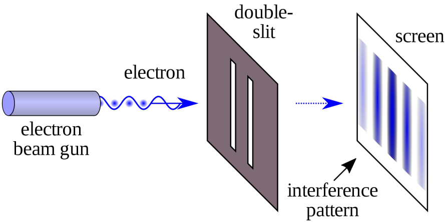
Outline

- Double slit experiment
- Superdense coding
- Teleportation
- A curious example of measurement
- The problem of node failure from a network and recovery mechanism

Double slit experiment





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- Physics allows us to manipulate information in a whole new setting.
- Encryption possible by making use of the postulates of quantum mechanics.

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- Measurement The act of measurement changes the state. This is unlike the classical world where the act of measuring the resistance of a resistor does not change the resistance.

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  - Here we say that we have performed measurement in the $\{|0\rangle, |1\rangle\}$ basis

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- The state for multiple qubits resides in the space obtained by tensor product of the individual spaces:

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \cdots \mathcal{H}_n$$
$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots |\psi_n\rangle$$

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$$|\psi\rangle_{AB} = \frac{|0\rangle_A \otimes |0\rangle_B + |1\rangle_A |1\rangle_B}{\sqrt{2}} = \frac{|0\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B}{\sqrt{2}}$$

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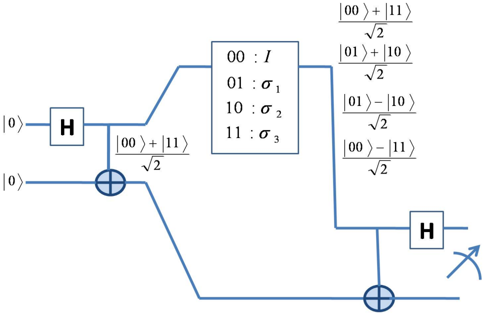
- Some sort of super luminal communication seems to be happening violating the special theory of relativity
- Einstein called this spooky action at a distance
- This phenomenon is called as entanglement



$|0\rangle$
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H
$\frac{|00\rangle+|11\rangle}{\sqrt{2}}$
00 : I
01 : σ_1
10 : σ_2
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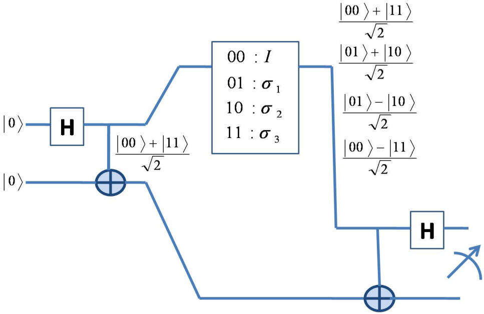
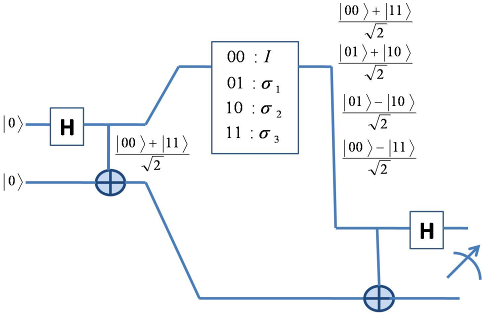
Superdense coding



- $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix};$



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- A preshared Bell pair lets A communicate two classical bits.

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$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}; |-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(\frac{|+\rangle_A + |-\rangle_A}{\sqrt{2}} |00\rangle_{BC} + \frac{|+\rangle - |-\rangle}{\sqrt{2}} |11\rangle_{BC} \right.$$

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A curious case of measurement

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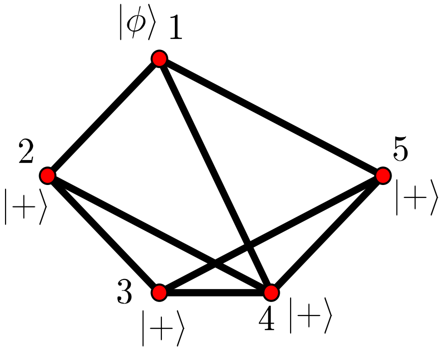
- We use the following setting for describing quantum networks via graphs.
- Nodes represented by vertices from the set V account for qubits, one at every node, while edges represent the quantum mechanical interaction that enables entangling of qubits.

The set up



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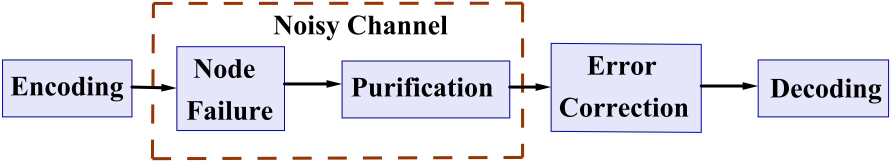

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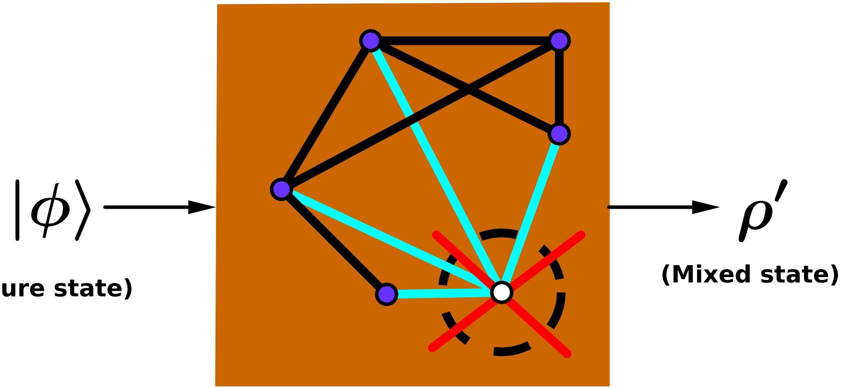
Theorem

Suppose $|\psi\rangle$ is a pure state of a composite system AB with Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_B$. Then there exist orthonormal states $|i\rangle_A$ for system A and orthonormal states $|i\rangle_B$ for system B such that

$$|\psi\rangle = \sum_i \lambda_i |i_A\rangle |i_B\rangle$$

where λ_i are non-negative real numbers satisfying $\sum_i \lambda_i^2 = 1$ known as Schmidt co-efficients.

Node failure



Purification procedure

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$$|\psi'\rangle = \lambda_0 |\theta_0\rangle |\eta_0\rangle + \lambda_1 |\theta_1\rangle |\eta_1\rangle , \quad (1)$$

or

$$|\psi'\rangle = \lambda_0 |\theta_1\rangle |\eta_0\rangle + \lambda_1 |\theta_0\rangle |\eta_1\rangle . \quad (2)$$

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- We found a procedure via the purification to recover the codeword back.
- We are investing ways of detecting and identifying the node that goes through a failure.

References

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Questions and suggestions

Thanks for your time ☺