Throughput-Optimal Discrete Rate Adaptation for Threshold-Based Feedback in OFDM Systems

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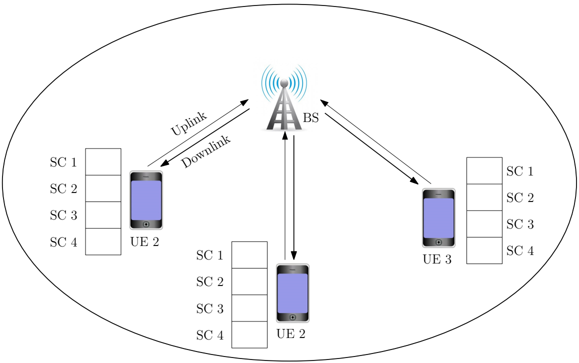
Outline

- Introduction
- Our contributions
- Throughput-optimal discrete rate adaptation:
  - Derivation
  - Simulation results
- Conclusions and future work

Introduction

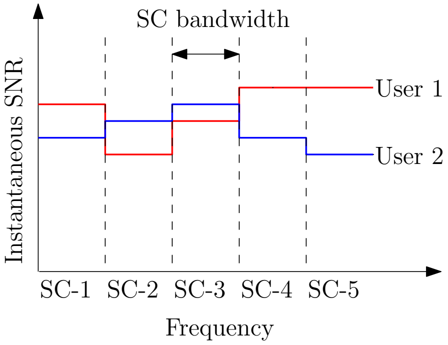
- OFDM is used in 4G Long Term Evolution (LTE) and LTE-Advanced
  - System bandwidth is divided into orthogonal subcarriers
  - Groups of adjacent subcarriers are called subchannels (SCs)

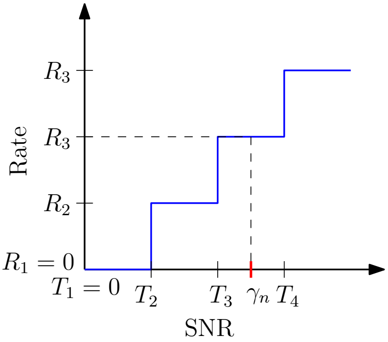
On the downlink, BS needs to determine whom to transmit to, on each SC, and with what rate



Overview of scheduling and discrete rate adaptation

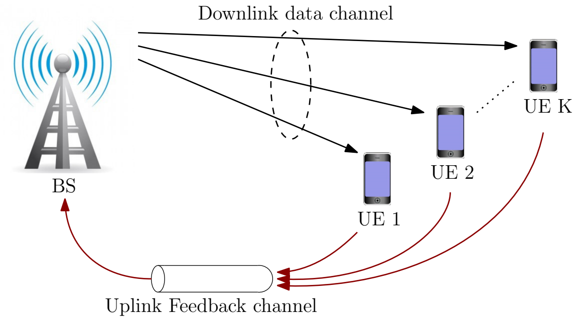
- Rate adaptation and scheduling are used to achieve high spectral efficiencies
- Scheduling: BS selects a user for each SC
- Rate adaptation: BS determines the modulation and coding scheme (MCS) for transmission to the scheduled user on each SC





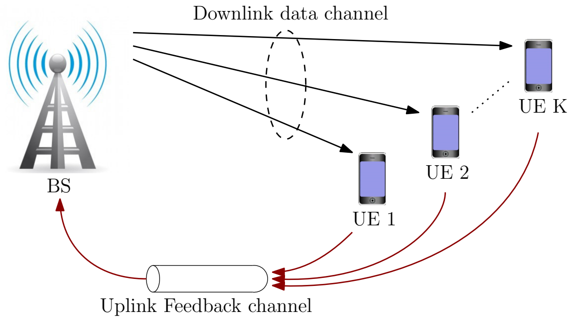
Reduced feedback schemes

- Feedback needed to make BS aware of SC gains
- Significant feedback overhead
  - K users, N SCs, B bits/SC $\Rightarrow$ KNB bits



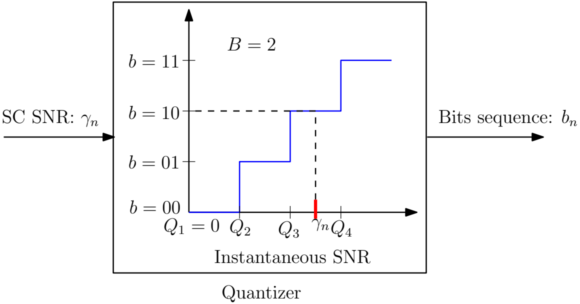
Reduced feedback schemes

- Feedback needed to make BS aware of SC gains
- Significant feedback overhead
  - K users, N SCs, B bits/SC $\Rightarrow$ KNB bits
- Limited bandwidth available in the uplink for feedback
- Reduced feedback schemes needed
  - Best-m [Svedman VTC'04]
  - Threshold-based quantized feedback [Floren Globecom'03]



Threshold based quantized feedback

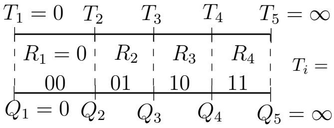
- Each user quantizes the downlink SNR for each SC using a *B*-bit quantizer that has 2^B regions



Conventional rate adaptation (CRA)

- Quantization thresholds are the same as the SNR thresholds of MCSs [Goldsmith book, Nguyen TWC'15]

$M = 4$ MCSs



$T_i = \frac{2^{R_i}-1}{\eta}$ [Baum TVT'03]

$B = 2$ bits/SC

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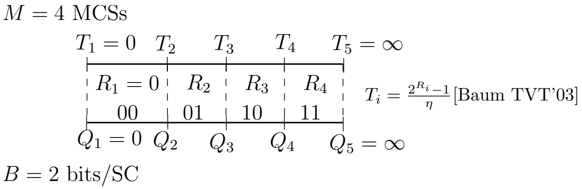

- Given the feedback for an SC
  - SC SNR lies between a lower and an upper level
  - Select the highest rate MCS whose SNR threshold does not exceed the lower level

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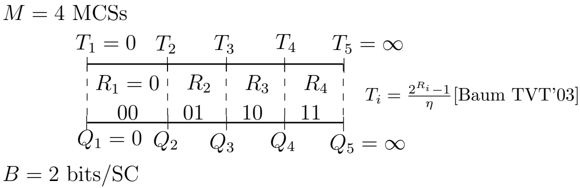

- Given the feedback for an SC
  - SC SNR lies between a lower and an upper level
  - Select the highest rate MCS whose SNR threshold does not exceed the lower level
- Feedback and rate adaptation problems are coupled

Our contributions

- A novel throughput-optimal discrete rate adaptation (TORA) scheme for OFDM systems that use threshold-based quantized feedback
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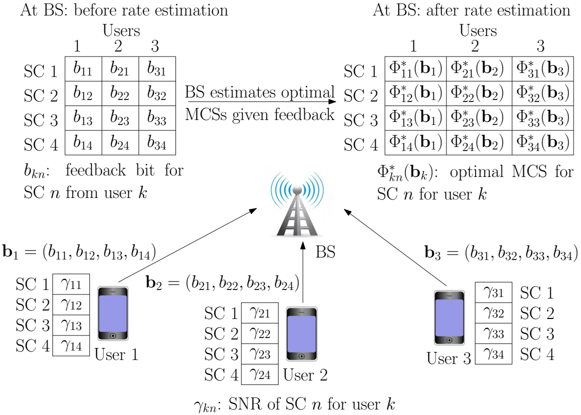
- A novel throughput-optimal discrete rate adaptation (TORA) scheme for OFDM systems that use threshold-based quantized feedback
  - Does not require any additional feedback
- Closed-form expression for feedback-conditioned goodput of an MCS for MIMO diversity modes for exponentially correlated SC SNRs
- Lower bound on the throughput gain achieved by TORA for a user with 1-bit feedback
- Study the interaction of the proposed scheme with different schedulers and the system-level throughput implications in a multi-user setting

System model

- Single cell, K users, N SCs
- Rayleigh fading
- Exponentially correlated SC SNRs for a user
- SC SNRs across users are independent.
- No. of quantization levels $\neq$ No. of MCSs



Throughput-optimal MCS

- Objective: Given the feedback, determine the MCS that maximizes fading-averaged throughput

Lemma

The throughput-optimal MCS $\Phi_n^*(\boldsymbol{b})$ for SC n given feedback $\boldsymbol{b}$ *is*

$$\Phi_n^*(\boldsymbol{b}) = \underset{1 \leq m \leq M}{\arg\max} \left\{ R_m Pr(\gamma_n \geq T_m | \boldsymbol{b}) \right\}.$$

$\Psi_n^{(m)}(\boldsymbol{b}) = R_m Pr(\gamma_n \geq T_m | \boldsymbol{b})$*: feedback-conditioned goodput of MCS m given* $\boldsymbol{b}$*.*

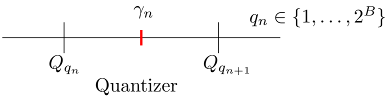
Proof:

- For MCS m, the average throughput conditioned on $\boldsymbol{b}$ is

$$\mathbb{E}_{\gamma_n}\left[R_m 1_{\{\gamma_n \geq T_m\}} | \boldsymbol{b} \right] = R_m \Pr(\gamma_n \geq T_m | \boldsymbol{b}).$$

- MCS with the largest value of $R_m \Pr(\gamma_n \geq T_m | \boldsymbol{b})$ must be chosen

Feedback-conditioned goodput: I.I.D SCs



Result

The feedback-conditioned goodput $\Psi_n^{(m)}(\boldsymbol{b})$ *of MCS m for SC n given feedback* $\boldsymbol{b}$ *is*

$$\Psi_n^{(m)}(\boldsymbol{b}) = \begin{cases} \frac{R_m C_m(n)}{D_m(n)}, & T_m \leq Q_{q_n+1}, \\ 0, & T_m > Q_{q_n+1}. \end{cases}$$

$$C_m(n) = \exp\left(-\frac{\max\left(T_m, Q_{q_n}\right)}{\bar{\gamma}}\right) - \exp\left(-\frac{Q_{q_n+1}}{\bar{\gamma}}\right),$$

$$D_m(n) = \exp\left(-\frac{Q_{q_n}}{\bar{\gamma}}\right) - \exp\left(-\frac{Q_{q_n+1}}{\bar{\gamma}}\right).$$

- Can be extended to MIMO diversity modes also

Conditional probability expression for MCS m ($\rho \neq 0$)

- For $n = 1$, numerator term $C_m(1)$ is given by

$$C_m(1) = \sum_{s=0}^{\infty} \rho^{2s} \sum_{\substack{0 \le w_1, \ldots, w_{N-1} \le s, \\ w_1 + w_2 + \ldots + w_{N-1} = s}} \left\{ \left[U\left(\frac{\max\{T_m, Q_{q_1}\}}{\bar{\gamma}(1-\rho^2)}, w_1 + 1 \right) - U\left(\frac{Q_{q_1+1}}{\bar{\gamma}(1-\rho^2)}, w_1 + 1 \right) \right] \right.$$

$$\times \left[U\left(\frac{Q_{q_N}}{\bar{\gamma}(1-\rho^2)}, w_{N-1} + 1 \right) - U\left(\frac{Q_{q_N+1}}{\bar{\gamma}(1-\rho^2)}, w_{N-1} + 1 \right) \right]$$

$$\left. \times \left[\prod_{j=2}^{N-1} \binom{w_{j-1} + w_j}{w_j} \frac{\left[U\left(\frac{Q_{q_j}(1+\rho^2)}{\bar{\gamma}(1-\rho^2)}, w_{j-1} + w_j + 1 \right) - U\left(\frac{Q_{q_j+1}(1+\rho^2)}{\bar{\gamma}(1-\rho^2)}, w_{j-1} + w_j + 1 \right) \right]}{(1+\rho^2)^{w_{j-1}+w_j+1}} \right] \right\}.$$

- For $1 \le n \le N$, denominator term $D_m(n)$ is given by

$$D_m(n) = \sum_{s=0}^{\infty} \rho^{2s} \sum_{\substack{0 \le w_1, \ldots, w_{N-1} \le s, \\ w_1 + w_2 + \ldots + w_{N-1} = s}} \left\{ \left[U\left(\frac{Q_{q_1}}{\bar{\gamma}(1-\rho^2)}, w_1 + 1 \right) - U\left(\frac{Q_{q_1+1}}{\bar{\gamma}(1-\rho^2)}, w_1 + 1 \right) \right] \right.$$

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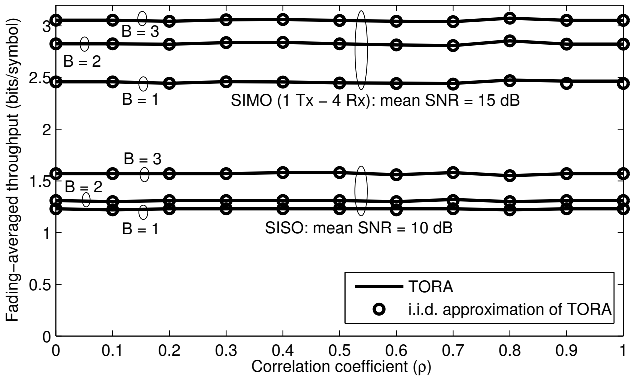
Insensitivity of average throughput to SC correlation

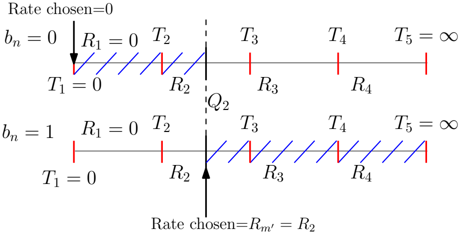


$K = 1$ user
$M = 16$ MCS

- Throughput is insensitive to ρ
  - Not much additional information is provided by other SC SNRs
- *Explanation:* For $\rho = 1$, all SC SNRs lie in the same quantization region with high probability

Contrasting CRA and TORA

- CRA for 1-bit feedback:



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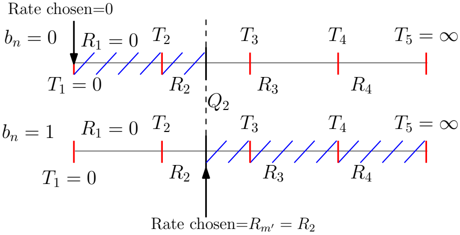

- TORA for 1-bit feedback ($\rho = 0$):

$$\Phi_n^*(\boldsymbol{b}) = \begin{cases} \underset{1 \le m \le M}{\arg\max} \left\{ R_m \left[\frac{\exp\left(-\frac{T_m}{\bar{\gamma}}\right) - \exp\left(-\frac{Q_2}{\bar{\gamma}}\right)}{1 - \exp\left(-\frac{Q_2}{\bar{\gamma}}\right)} \right] \right\}, & b_n = 0 \\ \underset{1 \le m \le M}{\arg\max} \left\{ R_m \left[\frac{\exp\left(-\frac{\max\{T_m, Q_2\}}{\bar{\gamma}}\right)}{\exp\left(-\frac{Q_2}{\bar{\gamma}}\right)} \right] \right\}, & b_n = 1 \end{cases}$$

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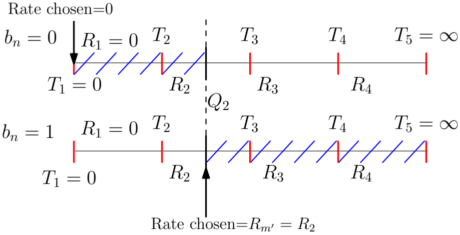

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- CRA is conservative

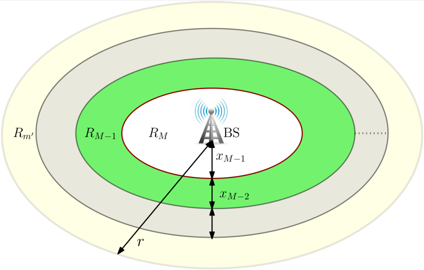
Lower bound for throughput gain (1-bit feedback)

Lemma

The fading-averaged throughput gain $\Delta R(x)$ of a user at distance x from the BS is lower bounded by

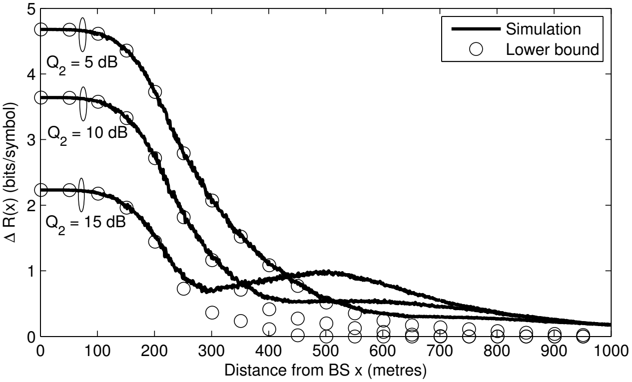
$$\Delta R(x) \geq \begin{cases} R_i \exp\left(-\frac{T_i \sigma^2 x^\alpha}{P_T \beta}\right) - R_{m'} \exp\left(-\frac{Q_2 \sigma^2 x^\alpha}{P_T \beta}\right), & x_i < x \leq x_{i-1}, \\ 0, & x_{m'} < x \leq r, \end{cases}$$

where $x_i = \left[\frac{P_T \beta \eta \ln\left(\frac{R_i}{R_{i-1}}\right)}{\sigma^2\left(2^{R_i} - 2^{R_{i-1}}\right)}\right]^{\frac{1}{\alpha}}$*, for* $i \in \{m', \ldots, M-1\}$*, and* $x_M = 0$*.*



- $R_{m'}$: Rate for SNR$= Q_2$
- x_i: Distance at which rate transition occurs
- r: Cell radius
- $\bar{\gamma}_k = \frac{P_T \beta x^{-\alpha}}{\sigma^2}$

Evaluation of $\Delta R(x)$ & its lower bound



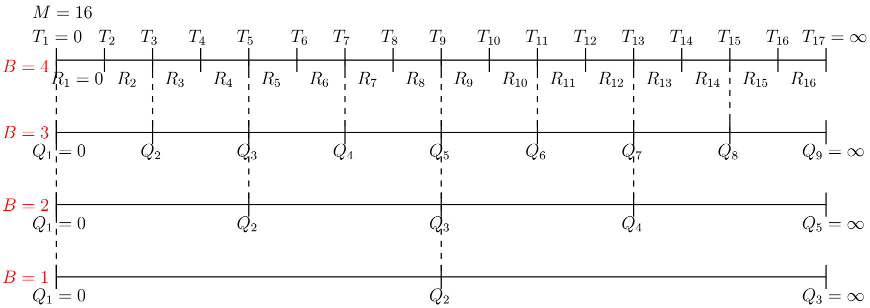
Cell radius $= 1$ km
$M = 16$ MCSs (LTE)
i.i.d. SCs

- CRA is sub-optimal

Simulation Settings

Goal: Study interaction between MCSs at BS, quantizers at users, and scheduler in a multi-user cell

- MCS tables at BS: $M = 2, 4, 8, 16$ (LTE).
- MCS threshold-based feedback: quantization threshold generation



- Schedulers used: greedy, CDF, and round robin (RR) schedulers

LTE rate table [Sesia LTE book]

CQI index	Modulation	Approximate code rate	Efficiency (information bits per symbol)
0	'Out-of-range'	—	—
1	QPSK	0.076	0.1523
2	QPSK	0.12	0.2344
3	QPSK	0.19	0.3770
4	QPSK	0.3	0.6016
5	QPSK	0.44	0.8770
6	QPSK	0.59	1.1758
7	16QAM	0.37	1.4766
8	16QAM	0.48	1.9141
9	16QAM	0.6	2.4063
10	64QAM	0.45	2.7305
11	64QAM	0.55	3.3223
12	64QAM	0.65	3.9023
13	64QAM	0.75	4.5234
14	64QAM	0.85	5.1152
15	64QAM	0.93	5.5547

MCS threshold-based feedback

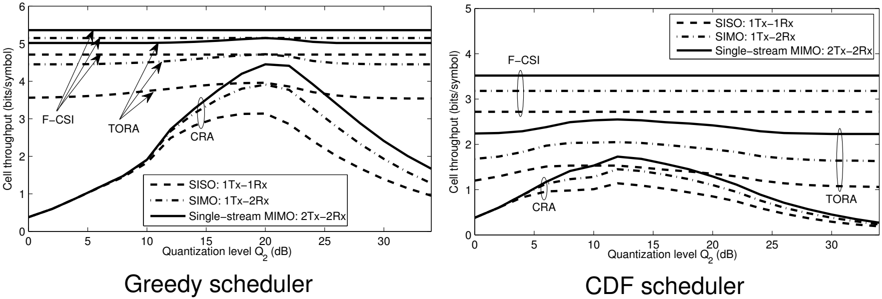
- SISO, $\rho = 0.86$, 10 users, 10 SCs, 1000 drops, 2000 fades/drop

No. of feedback bits (B)	No. of MCSs at BS (M)	Cell throughput (bits/symbol)								
		Greedy scheduler			CDF scheduler			RR scheduler		
		CRA	TORA	F-CSI	CRA	TORA	F-CSI	CRA	TORA	F-CSI
1	2	1.88	1.88	4.76	1.22	1.22	2.74	0.65	0.65	1.64
1	4	1.88	3.10	4.76	1.22	1.51	2.74	0.65	1.05	1.64
1	8	1.88	3.58	4.76	1.22	1.58	2.74	0.65	1.13	1.64
1	16	1.88	3.72	4.76	1.22	1.60	2.74	0.65	1.16	1.64
2	4	3.51	3.51	4.76	2.01	2.01	2.74	1.16	1.16	1.64
2	8	3.51	3.96	4.76	2.01	2.07	2.74	1.16	1.27	1.64
2	16	3.51	4.10	4.76	2.01	2.08	2.74	1.16	1.30	1.64
3	8	4.40	4.40	4.76	2.51	2.51	2.74	1.48	1.48	1.64
3	16	4.40	4.52	4.76	2.51	2.52	2.74	1.48	1.50	1.64
4	16	4.76	4.76	4.76	2.74	2.74	2.74	1.64	1.64	1.64

- Can reduce feedback overhead and compensate for it by having more MCSs at BS

1-bit feedback: Optimizing quantization threshold

- TU channel, 10 users, 10 SCs, 1000 drops, 2000 fades/drop



- TORA is much less sensitive to the variation in Q_2
- Significant throughput gains over CRA

Conclusions

- Proposed throughput-optimal discrete rate adaptation for threshold-based quantized feedback in OFDM
  - Enables decoupling between No. of feedback bits & No. of MCSs at BS
  - Can reduce feedback on uplink as required
  - Outperforms CRA

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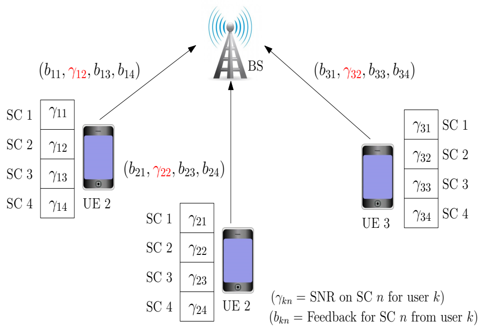
  - Can reduce feedback on uplink as required
  - Outperforms CRA
- Developed analytical expressions for the feedback-conditioned goodput for exponential correlation model
- Throughput can be increased with better quantization threshold generation methods
  - Percentile threshold-based feedback

Upcoming publications

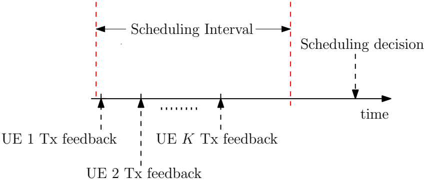
- Vineethkumar V, Neelesh B. Mehta, “Base Station-Side Rate Estimation for Threshold-Based Feedback, and Design Implications in Multi-User OFDM Systems,” Accepted in IEEE Trans. on Wireless Communications, 2017

Focus on reduced feedback schemes

- Rate adaptation for differential feedback



- Rate adaptation with differently outdated and reduced feedback



B1: Conditional goodput for MIMO diversity modes

- SIMO: 1 Tx - N_r Rx; MISO: N_t Tx - 1 Rx; MIMO: N_t Tx - N_r Rx ($N_t, N_r \geq 1$)
- With ideal beamforming, SNR on SC n, can be approximated as a chi-square RV [Li TSP'11]
- $f_{\gamma_n}(v) = \frac{1}{\Gamma(D)a^D} v^{D-1} \exp\left(\frac{v}{a}\right)$, where $a = \frac{\bar{\gamma}_k}{2}\left(\frac{N_t+N_r}{N_tN_r+1}\right)^{\frac{2}{3}}$ & $D = 2N_tN_r$.

Result

*The conditional goodput $\Psi_n^{(m)}(\boldsymbol{b})$ of MCS m for SC n given **b** is*

$$\Psi_n^{(m)}(\boldsymbol{b}) = \begin{cases} \frac{R_m\left[U\left(\frac{\max(T_m,Q_{q_n})}{2a\bar{\gamma}},\frac{D}{2}\right)-U\left(\frac{Q_{q_{n+1}}}{2a\bar{\gamma}},\frac{D}{2}\right)\right]}{U\left(\frac{Q_{q_n}}{2a\bar{\gamma}},\frac{D}{2}\right)-U\left(\frac{Q_{q_{n+1}}}{2a\bar{\gamma}},\frac{D}{2}\right)}, & T_m \leq Q_{q_i+1}, \\ 0, & T_m > Q_{q_i+1}. \end{cases}$$

B2: Derivation of conditional goodput for $\rho = 0$

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- $\Pr(\gamma_n \geq T_m \mid b_n) = \Pr\left(\gamma_n \geq T_m \mid Q_{q_n} \leq \gamma_n < Q_{q_{n+1}}\right)$.
- Upon applying Bayes' theorem,

$$\Pr\left(\gamma_n \geq T_m \mid Q_{q_n} \leq \gamma_n < Q_{q_{n+1}}\right) = \frac{\Pr\left(\gamma_n \geq T_m, Q_{q_n} < \gamma_n < Q_{q_{n+1}}\right)}{\Pr\left(Q_{q_n} \leq \gamma_n < Q_{q_{n+1}}\right)},$$

$$= \frac{\int_{\max\{T_m, Q_{q_n}\}}^{Q_{q_{n+1}}} f_{\gamma_n}(v)\,dv}{\int_{Q_{q_n}}^{Q_{q_{n+1}}} f_{\gamma_n}(v)\,dv}.$$

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$$= \frac{\int_{\max\{T_m, Q_{q_n}\}}^{Q_{q_{n+1}}} f_{\gamma_n}(v)\, dv}{\int_{Q_{q_n}}^{Q_{q_{n+1}}} f_{\gamma_n}(v)\, dv}.$$

- Rayleigh fading: $f_{\gamma_n}(v) = \frac{1}{\bar{\gamma}} \exp\left(-\frac{1}{\bar{\gamma}}\right)$.

B2: Derivation of conditional goodput for $\rho = 0$

- $\Pr(\gamma_n \geq T_m \mid \boldsymbol{b}) = \Pr(\gamma_n \geq T_m \mid b_n)$ for $\rho = 0$
- $\Pr(\gamma_n \geq T_m \mid b_n) = \Pr\left(\gamma_n \geq T_m \mid Q_{q_n} \leq \gamma_n < Q_{q_{n+1}}\right)$.
- Upon applying Bayes' theorem,

$$\Pr\left(\gamma_n \geq T_m \mid Q_{q_n} \leq \gamma_n < Q_{q_{n+1}}\right) = \frac{\Pr\left(\gamma_n \geq T_m, Q_{q_n} < \gamma_n < Q_{q_{n+1}}\right)}{\Pr\left(Q_{q_n} \leq \gamma_n < Q_{q_{n+1}}\right)},$$

$$= \frac{\int_{\max\{T_m, Q_{q_n}\}}^{Q_{q_{n+1}}} f_{\gamma_n}(v)\, dv}{\int_{Q_{q_n}}^{Q_{q_{n+1}}} f_{\gamma_n}(v)\, dv}.$$

- Rayleigh fading: $f_{\gamma_n}(v) = \frac{1}{\bar{\gamma}} \exp\left(-\frac{1}{\bar{\gamma}}\right)$.
- Carrying out the integration using $f_{\gamma_n}(v)$ gives the required result.

B3: Derivation of conditional goodput for $\rho \neq 0$

- For MCS m, the conditional probability can be evaluated as:

$$\Pr(\gamma_n \geq T_m \mid \boldsymbol{b}) = \Pr(\gamma_n \geq T_m \mid Q_{q_1} \leq \gamma_1 < Q_{q_1+1}, \ldots, Q_{q_N} \leq \gamma_N < Q_{q_N+1})$$

$$= \frac{\int_{Q_{q_1}}^{Q_{q_1+1}} \cdots \int_{\max\{T_m, Q_{q_n}\}}^{Q_{q_n+1}} \cdots \int_{Q_{q_N}}^{Q_{q_N+1}} f_{\gamma}(\boldsymbol{v})\, dv_1 \ldots dv_N}{\int_{Q_{q_1}}^{Q_{q_1+1}} \cdots \int_{Q_{q_n}}^{Q_{q_n+1}} \cdots \int_{Q_{q_N}}^{Q_{q_N+1}} f_{\gamma}(\boldsymbol{v})\, dv_1 \ldots dv_N}$$

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- Exponential correlation model: $\mathbb{E}\left[\gamma_{kn}\gamma_{kl}^*\right] = \bar{\gamma}_k \rho^{2|n-l|}$, for $1 \leq l \leq N$, and $0 \leq \rho \leq 1$

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- Exponential correlation model: $\mathbb{E}[\gamma_{kn}\gamma_{kl}^*] = \bar{\gamma}_k \rho^{2|n-l|}$, for $1 \leq l \leq N$, and $0 \leq \rho \leq 1$
- $f_\gamma(\boldsymbol{v})$ for exponentially correlated vector $\boldsymbol{v} = (v_1, \ldots, v_N)$ of SC SNRs [Mallik TIT'03]:

$$f_{\gamma}(\boldsymbol{v}) = \frac{\exp -\left(\frac{\left[v_1+v_N+\left(1+\rho^2\right)\sum_2^{N-1} v_i\right]}{\bar{\gamma}(1-\rho^2)}\right)}{\bar{\gamma}^N \left(1-\rho^2\right)^{N-1}} \sum_{n=0}^{\infty} \left(\frac{\rho^2}{\bar{\gamma}\left(1-\rho^2\right)^2}\right)^n$$

$$\times \sum_{\substack{0 \leq w_1, \ldots, w_{N-1} \leq s, \\ w_1+w_2+\ldots+w_{N-1}=s}} \frac{{x_1}^{l_1} {x_2}^{l_1+l_2} \ldots {x_{N-1}}^{l_{N-1}+l_{N-2}} {x_N}^{l_{N-1}}}{(l_1! l_2! \ldots l_{N-1}!)^2}$$

B4: Lower bound on rate gain averaged over fading and UE-location

Result

When the UE-location is uniformly distributed over the cell area, the fading-averaged and user location-averaged cell-throughput gain ΔR_{cell} is lower bounded by:

$$\Delta R_{cell} \geq \frac{2\Gamma\left(\frac{2}{\alpha}\right)}{\alpha R^2} \sum_{i=m'+1}^{M} \left(R_i c_i^{\frac{2}{\alpha}} \left[U\left(\frac{2}{\alpha}, \frac{x_i^{\alpha}}{c_i}\right) - U\left(\frac{2}{\alpha}, \frac{x_{i-1}^{\alpha}}{c_i}\right) \right] - R_{m'} c_{m'}^{\frac{2}{\alpha}} \left[U\left(\frac{2}{\alpha}, \frac{x_i^{\alpha}}{c_{m'}}\right) - U\left(\frac{2}{\alpha}, \frac{x_{i-1}^{\alpha}}{c_{m'}}\right) \right] \right),$$

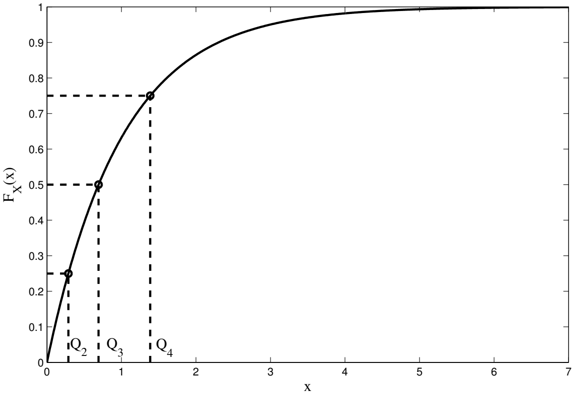
where $c_{m'} = \frac{Q_{th}\sigma^2}{P_T\beta}$, $c_i = \frac{T_i\sigma^2}{P_T\beta}$, for $i \in \{m'+1, \ldots, M\}$, and $\Gamma(\cdot)$ is the gamma function.

B5: Adaptation of scheduling algorithms

- Notation: Throughput optimal MCS $\Phi^*_{kn}(\boldsymbol{b})$ has an SNR threshold $T_{\Phi^*_{kn}(\boldsymbol{b})}$, rate $R_{\Phi^*_{kn}(\boldsymbol{b})}$, and conditional goodput $\Psi^*_{kn}(\boldsymbol{b}) = R_{\Phi^*_{kn}(\boldsymbol{b})} \Pr\left(\gamma_{kn} > T_{\Phi^*_{kn}(\boldsymbol{b})} | \boldsymbol{b}\right)$
- Greedy scheduler: $i_n = \arg\max_{1 \le k \le K} \{\Psi^*_{kn}(\boldsymbol{b})\}$
- CDF scheduler [Park TCom'06]:
  - Step 1: Compute $\Psi^*_{kn}(\boldsymbol{b})$.
  - Step 2: Generate U_{kn} uniformly distributed in $\left[F_{\Psi^*_{kn}(\boldsymbol{b})}(s_{j-1}), F_{\Psi^*_{kn}(\boldsymbol{b})}(s_j)\right)$.
  - Step 3: $i_n = \arg\max_{1 \le k \le K} \left\{ (U_{kn})^{\frac{1}{w_k}} \right\}$.
- RR scheduler: Periodic scheduling of users.

B6:Percentile threshold-based feedback

- Divide the CDF of SNR into regions such that SNR lies in each region with equal probability



B7: Percentile threshold-based feedback

- SISO, $\rho = 0.86$, 10 users, 10 SCs, 1000 drops, 2000 fades/drop

Feed-back bits (B)	No. of MCSs at BS (M)	Throughput (bits/symbol/Hz)									Jain's Fairness Index	
		Greedy Scheduler			CDF Scheduler			RR Scheduler			Greedy	
		Conv.	Proposed	Full CSI	Conv.	Proposed	Full CSI	Conv.	Proposed	Full CSI	Conv.	Proposed
1	2	1.59	1.85	4.82	0.62	0.77	2.84	0.31	0.65	1.71	0.26	0.25
1	4	2.71	3.25	4.82	1.14	1.38	2.84	0.57	1.03	1.71	0.30	0.20
1	8	3.37	3.91	4.82	1.47	1.66	2.84	0.73	1.19	1.71	0.30	0.19
1	16	3.66	4.15	4.82	1.64	1.75	2.84	0.82	1.24	1.71	0.30	0.18
1	31	3.74	4.18	4.82	1.71	1.77	2.84	0.85	1.26	1.71	0.30	0.19
2	4	3.19	3.38	4.82	1.56	1.63	2.84	0.86	1.09	1.71	0.22	0.20
2	8	3.98	4.15	4.82	1.94	2.01	2.84	1.10	1.31	1.71	0.21	0.19
2	16	4.31	4.45	4.82	2.13	2.19	2.84	1.23	1.41	1.71	0.21	0.19
2	31	4.39	4.50	4.82	2.21	2.25	2.84	1.28	1.44	1.71	0.21	0.19
3	8	4.20	4.27	4.82	2.17	2.21	2.84	1.29	1.39	1.71	0.20	0.20
3	16	4.55	4.60	4.82	2.40	2.43	2.84	1.43	1.51	1.71	0.20	0.19
3	31	4.62	4.65	4.82	2.48	2.50	2.84	1.50	1.56	1.71	0.20	0.19
4	16	4.66	4.68	4.82	2.55	2.56	2.84	1.53	1.57	1.71	0.19	0.19
4	31	4.72	4.73	4.82	2.64	2.65	2.84	1.60	1.63	1.71	0.19	0.19