Extra Samples can Reduce the Communication for Independence Testing

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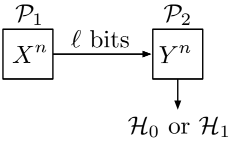

$\mathcal{H}_0$: Bits have a (known) correlation ρ

$\mathcal{H}_1$: Bits are independent

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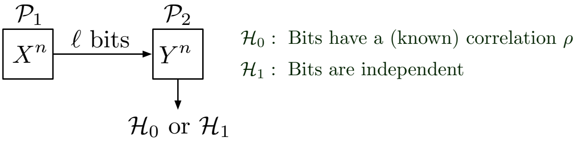

- How many bits l must $\mathcal{P}_1$ send to $\mathcal{P}_2$?
- A simple scheme – $\mathcal{P}_1$ sends X^n to $\mathcal{P}_2$.

Communication needed by the simple scheme

– via the case of "collocated" parties

- For pmfs P and Q on a finite alphabet $\mathcal{Z}$, let $n(\delta, \epsilon)$ be the minimum n such that we can find an acceptance region, $A_n \subset \mathcal{Z}^n$ so that

$$P^n(A_n) \geq 1 - \delta, \text{ and,}$$
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- It can be seen using Hoeffding's inequality that

$$n(\delta, \epsilon) = \frac{1}{D(P||Q)} \log \frac{1}{\epsilon} + O_\delta \left(\sqrt{\log \frac{1}{\epsilon}} \right).$$

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- In our problem, $\mathcal{Z} = \mathcal{X} \times \mathcal{Y} = \{0, 1\} \times \{0, 1\}$.

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– via the case of "collocated" parties

Consider $P_{XY} \equiv \texttt{BSS}(\rho)$ defined by

$$P(0,0) = P(1,1) = \frac{1}{4}(1+\rho), \text{ and, } P(0,1) = P(1,0) = \frac{1}{4}(1-\rho)$$

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For $P \equiv P_{XY} \equiv \mathtt{BSS}(\rho)$, and $Q \equiv P_X P_Y$, we get

$$n(\delta,\epsilon) = \frac{1}{1 - h\left(\frac{1-\rho}{2}\right)} \cdot \log\frac{1}{\epsilon} + O_\delta\left(\sqrt{\log\frac{1}{\epsilon}}\right)$$

For $Q \equiv P_{XY} \equiv \mathtt{BSS}(\rho)$, and $P \equiv P_X P_Y$, we get

$$n(\delta,\epsilon) = \frac{2}{\log\frac{1}{1-\rho^2}} \cdot \log\frac{1}{\delta} + O_\epsilon\left(\sqrt{\log\frac{1}{\delta}}\right)$$

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- Suppose, no constraint on the number of samples observed by $\mathcal{P}_1, \mathcal{P}_2$.
- Then, can we test for independence by communicating fewer bits?

A "less costly" communication scheme

We will show that we can test independence of bit sequences using

$$C(\delta, \epsilon) = \frac{1}{\rho^2} \cdot \log\frac{1}{\epsilon} + O_\delta\left(\sqrt{\log\frac{1}{\epsilon}}\right) \text{ or } \frac{1-\rho^2}{\rho^2} \cdot \log\frac{1}{\delta} + O_\epsilon\left(\sqrt{\log\frac{1}{\delta}}\right)$$

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whereas for the simple scheme, the communication is

$$n(\delta, \epsilon) = \frac{1}{1 - h\left(\frac{1-\rho}{2}\right)} \cdot \log \frac{1}{\epsilon} + O_\delta\left(\sqrt{\log \frac{1}{\epsilon}}\right) \text{ or } \frac{2}{\log \frac{1}{1-\rho^2}} \cdot \log \frac{1}{\delta} + O_\epsilon\left(\sqrt{\log \frac{1}{\delta}}\right)$$

Clearly, for all $\rho \notin \{-1, 0, 1\}$,

$$\frac{1}{\rho^2} < \frac{1}{1 - h\left(\frac{1-\rho}{2}\right)} \text{ and } \frac{1-\rho^2}{\rho^2} < \frac{2}{\log \frac{1}{1-\rho^2}}$$

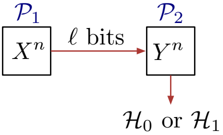
Summary of results

- We present general upper and lower bounds that match for $\texttt{BSS}(\rho)$
- Scheme uses linear correlation as a statistic
- Lower bound uses hypercontractivity to get a measure change bound

Results and Proofs

Minimum one-way communication for independence testing

- Shared randomness between $\mathcal{P}_1$ and $\mathcal{P}_2$, denoted by U
- A distributed test $T = (c, d)$
  - $\mathcal{P}_1$ observes X^n.
  - $\mathcal{P}_1$ sends $B^l = c(X^n, U)$ to $\mathcal{P}_2$.
  - $\mathcal{P}_1$ observes Y^n and receives B^l.
  - $\mathcal{P}_2$ declares $d(Y^n, B^l, U)$.



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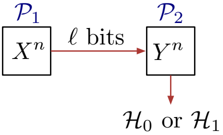

- (c, d) is an (l, δ, ϵ)-test if

$$P_{\mathcal{H}_0}(d(Y^n, B^l, U) = 1) \leq \delta \quad \text{and} \quad P_{\mathcal{H}_1}(d(Y^n, B^l, U) = 0) \leq \epsilon$$

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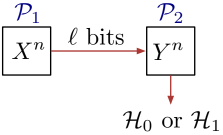

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- Minimum communication:
  $C(\delta, \epsilon)$ is the min l s.t. $\exists$ an (l, δ, ϵ)-test for some n

Proposed scheme for binary sequences

- Based on a scheme for common randomness generation by Guruswamy and Radhakrishnan (2017)

- Reparameterize $\{0, 1\}$ to $\{+1, -1\}$

  i. Let $\mathbb{U}$ be an $(n \times 2^k)$ matrix of $\texttt{Unif}\{-1, +1\}$-valued rvs

  ii. $\mathcal{P}_1$ sends the least j s.t. that $\sum_{i=1}^{n} U_{ij} X_i \geq r\sqrt{n}$

     – if none found, declares $\mathcal{H}_1$

  iii. $\mathcal{P}_2$ declares $\mathcal{H}_0$ if $\sum_{i=1}^{n} U_{ij} Y_i \geq \theta . r\sqrt{n}$

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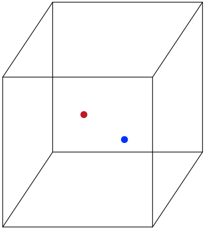

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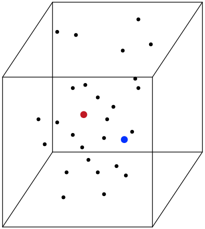

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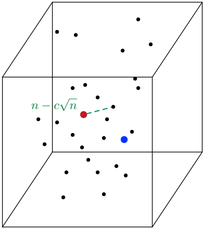

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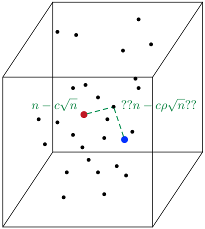

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  Since $\mathbb{E}_{\mathcal{H}_0}[Y|X] = \rho X$, we choose $\theta \approx \rho$

The general scheme

We use the best "linear correlation" we can get from P_{XY}

The maximum correlation of (X, Y) is given by

$$\rho_m(X, Y) = \max_{\substack{f,g \text{ s.t. } \mathbb{E}[f(X)] = \mathbb{E}[g(Y)] = 0 \text{ and} \\ \mathbb{E}[f(X)^2] = \mathbb{E}[g(Y)^2] = 1}} \mathbb{E}[f(X)g(Y)]$$

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Consider (X, Y) with $\rho_m(X, Y) = \rho$

The maximizing f and g satisfy

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We choose $\theta \approx \rho$.

Performance guarantees

Theorem (Upper bound for small ϵ, δ)

For $\delta, \epsilon \in (0, 1/2)$ and P_{XY} with $\rho_m(X, Y) = \rho$,

$$C(\delta, \epsilon) \leq \frac{1}{\rho^2} \cdot \left(\sqrt{\log \frac{1}{\epsilon}} + \sqrt{(1-\rho^2) \log \frac{1}{\delta}} \right)^2 + O\left(\sqrt{\log \frac{1}{\epsilon\delta}} \right)$$

Theorem (Upper bound for small ϵ, large δ)

For $\epsilon \in (0, 1/2)$, $\delta \in (1/2, 1)$ and P_{XY} with $\rho_m(X, Y) = \rho$,

$$C(\delta, \epsilon) \leq \frac{1}{\rho^2} \cdot \left(\sqrt{\log \frac{1}{\epsilon}} - \sqrt{(1-\rho^2) \log \frac{1}{1-\delta}} \right)^2 + O\left(\sqrt{\log \frac{1}{\epsilon(1-\delta)}} \right)$$

Deriving the lower bound

Given a (deterministic) (l, δ, ϵ)-test (c, d)

let $A_i = \{x^n : c(x^n) = i\}$ and $B_i = \{y^n : d(y^n, i) = 0\}$, $L = 2^l$,

Note that $\{A_1, ..., A_L\}$ is a partition of $\mathcal{X}^n$

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The change of measure bound:

$$1 - \delta \leq \sum_{i=1}^{L} P_{X^n Y^n}(A_i \times B_i)$$

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The change of measure bound: Using Cauchy-Schwarz and Jensen's

$$\begin{aligned}
1 - \delta &\leq \sum_{i=1}^{L} P_{X^n Y^n}(A_i \times B_i) \\
&\leq \sum_{i=1}^{L} \sqrt{P_{X^n}(A_i) P_{Y^n}(B_i)} \\
&\leq \sqrt{L \sum_{i=1}^{L} P_{X^n}(A_i) P_{Y^n}(B_i)} \\
&\leq \sqrt{L\epsilon}
\end{aligned}$$

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Replace Cauchy-Schwarz with a hypercontractivity bound

Deriving the lower bound

For $1 \leq q \leq p < \infty$, with $p' = p/(p-1)$,

P_{XY} is (p,q)-hypercontractive iff $\mathbb{E}[|f(X)g(Y)|] \leq \|f(X)\|_{p'} \|g(Y)\|_q$.

For any rectangle $A \times B$: $P_{XY}(A \times B) \leq P_X(A)^{\frac{1}{p'}} P_Y(B)^{\frac{1}{q}}$

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\begin{aligned}
1 - \delta &\leq \sum_{i=1}^{L} P_{X^nY^n}(A_i \times B_i) \\
&\leq \sum_{i=1}^{L} \left(P_{X^n}(A_i) P_{Y^n}(B_i)\right)^{\frac{1}{q}} P_{X^n}(A_i)^{\frac{1}{p'} - \frac{1}{q}} \\
&\leq \left(\sum_{i=1}^{L} P_{X^n}(A_i) P_{Y^n}(B_i) \right)^{\frac{1}{q}} \left(\sum_{i=1}^{L} P_{X^n}(A_i)^{q'\left(\frac{1}{p'} - \frac{1}{q}\right)} \right)^{\frac{1}{q'}} \\
&\leq \epsilon^{\frac{1}{q}} L^{\frac{1}{p}},
\end{aligned}
$$

where we have assumed $1 \leq p' \leq q$

The lower bound

Theorem (Lower bound 1)

Given $\delta, \epsilon \in (0,1)$ *and* (p,q) *such that* $1 \leq p' \leq q \leq p$ *and* (X,Y) *is* (p,q)*-hypercontractive,*

$$C(\delta,\epsilon) \geq \frac{p}{q}\log\frac{1}{\epsilon} - p\log\frac{1}{1-\delta}$$

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Similarly, using reverse hypercontractivity, we can get:

Theorem (Lower bound 2)

Given $\delta, \epsilon \in (0,1)$ *and* (p,q) *such that* $1 \geq q \geq 0 \geq q' \geq p$ *and* (X,Y) *is* (p,q)*-reverse hypercontractive,*

$$C(\delta, \epsilon) \geq \frac{p}{q} \log \frac{1}{1-\epsilon} - p \log \frac{1}{\delta}$$

Evaluation for $\mathtt{BSS}(\rho)$

For $1 \leq q \leq p$, (X, Y) is (p, q)-hypercontractive iff $\frac{q-1}{p-1} \geq \rho^2$

On optimizing the lower bound over this region, we get the desired bound.

Corollary

For a $\mathtt{BSS}(\rho)$*,* $\delta \in (0, 1/2)$ *and* ϵ *s.t.* $\delta + \epsilon^{\frac{1-|\rho|}{1+|\rho|}} \leq 1$

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For a $\mathtt{BSS}(\rho)$, $\epsilon, \delta \in (0, 1/2)$

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Corollary

For a BSS(ρ), $\delta \in (1/2, 1)$ *and* ϵ *s.t.* $\delta + \epsilon^{\frac{1-|\rho|}{1+|\rho|}} \leq 1$,

$$C(\delta, \epsilon) = \frac{1}{\rho^2}\left(\sqrt{\log\frac{1}{\epsilon}} - \sqrt{\left(1-\rho^2\right)\log\frac{1}{1-\delta}}\right)^2 + O\left(\sqrt{\log\frac{1}{\epsilon(1-\delta)}}\right)$$

Remark – Also works for Gaussian symmetric source $\texttt{GSS}(\rho)$:

$$\begin{pmatrix} X \\ Y \end{pmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}\right)$$

Future directions

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  - $BSS(\rho)$ is ρ away from $BSS(0)$
  - Simple scheme uses $O(1/\rho^2)$ bits and is order-optimal
  - Alphabet size $k > 2$?
- Do not have a practical scheme that outperforms the simple scheme