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Power and Discrete Rate Adaptation for Energy Harvesting Wireless Nodes

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***Abstract*—Energy harvesting (EH) nodes, which harvest energy from the environment in order to communicate over a wireless link, promise perpetual lifetimes for wireless networks. In this paper, we address the throughput optimization problem for a rate-adaptive EH node that chooses its rate from a set of discrete rates and adjusts its power depending on its channel gain and battery state. First, we show that the optimal throughput of an EH node is upper bounded by the throughput achievable by a node that is subject only to an average power constraint. We then propose a simple transmission scheme for an EH node that achieves an average throughput close to the upper bound. The scheme's parameters can be made to account for energy overheads such as battery non-idealities and the energy required for sensing and processing. The effect of these overheads on the average throughput is also analytically characterized.**

I. Introduction

Improving the network life time is an acute problem in wireless networks that consist of battery-powered nodes [1], [2]. Energy harvesting (EH) nodes circumvent this difficult problem by harvesting energy from the environment using solar, vibration, thermoelectric effects, and other physical phenomena [3], [4]. An EH node that drains out its battery can harvest energy later and again become available. Energy harvesting, thus, offers the tantalizing possibility of a maintenance-free perpetual wireless network.

The operation of an EH node is fundamentally governed by the *energy neutrality constraint*, which demands that, at any point of time, the total amount of energy utilized must be less than or equal to the sum of the initial energy in the battery and the total amount of energy harvested thus far [5].[1] The energy neutrality constraint motivates a significant redesign of the physical and multiple access layers of communication of EH nodes. Energy conservation ceases to be the dominant goal in the design of the protocols. Instead, the problem becomes one of handling the randomness in the energy harvested and ensuring that the energy required for communication can be met. Given its promise, several recent papers in the literature address the communication design aspects of EH nodes, see, for example, [5]–[9] and references therein.

In time-varying channels, adapting the transmission rate and power to the channel fading enables efficient transmission [10]. With recent advances in hardware, rate adaptation has become a promising option even for energy constrained wireless networks [11].

In this paper, we consider the problem of maximizing the throughput of a rate-adaptive EH node that transmits over a fading channel and meets an instantaneous bit error rate (BER) constraint. We focus on the practically relevant case of *discrete rate adaption*, in which the transmitter can only switch between a pre-specified set of rates [12]. This problem differs from the well-studied version in which a node is subject only to an average power constraint [10]. This is because, in our problem, the transmission is affected not only by the channel gain, but also by the availability of energy in the battery. We shall henceforth refer to a node subject to the average power constraint as a *non-EH node*.

We first show that the energy neutrality constraint is tighter than the average power constraint. Motivated by the optimal transmission algorithm for the average power constraint, we then propose a simple EH transmission algorithm and show that its throughput approaches the upper bound arbitrarily closely. We also find that the *greedy scheme*, in which a node tries to transmit with the highest constellation size that its current battery state can support, is significantly sub-optimal. Further, we show how the effect of energy overheads, such as battery storage inefficiencies and internal leakage currents and energy required for sensing and processing, can be incorporated in the algorithm. We also analytically characterize the throughput with and without these overheads.

Rate and power adaptation in EH nodes was also considered in [8], but randomness in energy harvesting was not modeled. While [6] proposed optimal rate and power management policies for EH nodes, the primary focus was on non-fading channels. Further, both [8] and [6] assumed continuous rate adaptation as against discrete rate adaptation, which is the focus of this paper. Reference [9] also addressed a similar problem, but it assumed that the channel gain, energy harvested, and transmit power are all quantized.

The paper is organized as follows. The system model is specified in Sec. II. Section III develops the throughput-optimal adaptation scheme and, thereafter, studies the effect of energy overheads. Simulation results are presented in Sec. IV, and are followed by our conclusions in Sec. V.

II. EH Node Model

Consider an EH node that has data to send to a destination over a frequency-flat, block fading channel of bandwidth B Hz and coherence time T_c s. In order to determine the maximum throughput possible, we assume that the node always has data

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[1]The energy neutrality constraint is same as the law of conservation of energy. We use the former terminology as it is employed in the literature [5].

to send. The received signal in baseband is

$$y = \sqrt{h} e^{j\phi} x + n,$$

where x is the transmitted symbol, h is the channel power gain, ϕ is phase of the channel response, and n is circular symmetric complex additive white Gaussian noise (AWGN) with power spectral density $N_0/2$. We shall call the time interval $[kT_c, (k+1)T_c)$, in which the channel does not change, as slot k. Thus, $h(k)$s are independent and identically distributed (i.i.d.) random variables with probability density function (PDF) and cumulative distribution function (CDF) denoted by $f_H(h)$ and $F_H(h)$, respectively. The EH node and the receiver are assumed to know the channel gain [8], [9].

A. Rate and Power Adaptation

In any time slot k, the constellation size $M(k)$, transmit power $P_{\text{tx}}(k)$, and the channel power gain $h(k)$ are inter-related as follows [10]. The BER is $c_1 \exp\left(\frac{-c_2 h(k) P_{\text{tx}}(k)}{N_0 B (M^{c_3}(k) - c_4)}\right)$, where $c_1, \dots, c_4$ are modulation specific real constants. For example, for M-QAM constellations, $c_1 = 2$, $c_2 = 1.5$, $c_3 = 1$, and $c_4 = 1$. The constellation size is chosen from a set $\mathcal{M} = \{m_1, m_2, \dots, m_M\}$ with $m_1 = 1$ corresponding to no transmission and $m_1 < m_2 < \cdots < m_M$. The choice of $\mathcal{M}$ depends on the hardware complexity of the system. Given the channel power gain, an EH node adapts its constellation size and transmit power to meet an instantaneous BER of P_b. If the chosen constellation size is $M(k) = m_j$, then the throughput is $r_j = \log_2 m_j$ and the required transmit power $P_{\textit{req}}(h(k), m_j)$ is

$$P_{\textit{req}}(h(k), m_j) = \frac{d_j}{h(k)}, \tag{1}$$

where $d_j = -\frac{N_0 B}{c_2}\left(m_j^{c_3} - c_4\right) \log_e\left(\frac{P_b}{c_1}\right)$, for $1 \le j \le M$.

B. Energy Harvesting, Storage, and Consumption

For the EH node, the energy harvesting process is assumed to be stationary and ergodic with mean $\bar{P}_{\text{EH}}$ J/s.[2] We make no other limiting assumptions. This model covers several harvesting models considered in literature, *e.g.*, [13], [14]. It also allows for time correlations, if any, in the harvesting process.

The node stores its harvested energy in a buffer such as a rechargeable battery or a supercapacitor, both of which shall be referred to as a battery henceforth. Initially, we assume that the battery is ideal, *i.e.*, it has infinite capacity and has 100% storage efficiency. Deviations from these assumptions are addressed later in Sec. III-C and Sec. IV.

Let $B(n)$ denote the energy available in the battery at the beginning of the n^{th} slot. Let $D(n)$ and $U(n)$ denote the harvested and utilized power, respectively, during the n^{th} slot, with $U(n)T_c \le B(n)$. Thus,

$$B(n) = T_c \sum_{i=0}^{n-1} (D(i) - U(i)) + B(0). \tag{2}$$

Note that, during time slot i, an EH node can transmit a constellation of size $M(i)$ only if it has sufficient energy stored in its battery, i.e., $B(i) \ge T_c P_{\textit{req}}(h(i), M(i))$.

III. Transmission Rule for an EH Node

Depending on the current battery and channel states, the node must decide which constellation to transmit and at what power. From (1), it is clear that the choice of the constellation fixes the transmit power and vice versa. Let $R(n)$ denote the throughput in time slot n. Our goal is to maximize

$$\bar{R} = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} R(i), \tag{3}$$

subject to

$$B(n) \ge 0, \quad \forall n \in \mathbb{N}. \tag{4}$$

In general, $R(n)$ depends on both $h(n)$ and $B(n)$, which, in turn, depend on the sequences $\{D(i), i<n\}$ and $\{U(i), i<n\}$.

Let $\bar{R}^*_{\text{EH}}(\bar{P}_{\text{EH}})$ and $\bar{R}^*_{\textit{nonEH}}(\bar{P}_{\text{EH}})$ respectively denote the maximum average throughput achieved by an EH node that harvests energy at a rate $\bar{P}_{\text{EH}}$ and a non-EH node with an average power constraint of $\bar{P}_{\text{EH}}$. Then, $\bar{R}^*_{\text{EH}}(\bar{P}_{\text{EH}})$ is upper bounded as follows.

Lemma *1:*

$$\bar{R}^*_{\text{EH}}(\bar{P}_{\text{EH}}) \le \bar{R}^*_{\textit{nonEH}}(\bar{P}_{\text{EH}}). \tag{5}$$

Proof: The proof is relegated to Appendix A. ∎

A key point about the upper bound is that it depends only on the mean $\bar{P}_{\text{EH}}$ and not on $\{D(i), i < n\}$ or its higher moments.

A. Non-EH Case: Achieving $\bar{R}^*_{\textit{nonEH}}(\bar{P}_{EH})$

The optimal transmission policy for a non-EH node subject to an average power constraint of $\bar{P}_{\text{EH}}$ is as follows [10]. A non-EH node transmits with constellation size m_j, if $h \in [\Gamma_j, \Gamma_{j+1})$, where $\Gamma_j = \eta k_j, 1 \le j \le M+1$, and

$$k_j = \begin{cases} 0, & j = 1 \\ -\frac{1}{c_2} \log_e\left(\frac{P_b}{c_1}\right) \frac{m_2^{c_3} - c_4}{\log_2(m_2)}, & j = 2 \\ -\frac{1}{c_2} \log_e\left(\frac{P_b}{c_1}\right) \frac{m_j^{c_3} - m_{j-1}^{c_3}}{\log_2\left(\frac{m_j}{m_{j-1}}\right)}, & 3 \le j \le M \\ \infty, & j = M+1 \end{cases}. \tag{6}$$

The constant η is a solution of

$$\sum_{j=2}^{M} \int_{\eta k_j}^{\eta k_{j+1}} P_{\textit{req}}(h, m_j) f_H(h)\, dh = \bar{P}_{\text{EH}}. \tag{7}$$

The optimum non-EH throughput is given by

$$\bar{R}^*_{\textit{nonEH}}(\bar{P}_{\text{EH}}) = \sum_{j=2}^{M} \log_2(m_j) \int_{\eta k_j}^{\eta k_{j+1}} f_H(h)\, dh. \tag{8}$$

[2] When the statistics of the energy harvesting model itself varies slowly (*e.g.*, solar), the harvesting process can be approximated by a piece-wise stationary process. Our results can then be applied to each piece.

Upon rearranging the terms of the summation, we get

$$\bar{R}^*_{nonEH}(\bar{P}_{\text{EH}}) = \log_2(m_M) - \sum_{j=2}^{M} F_H(\eta k_j)\log_2\left(\frac{m_j}{m_{j-1}}\right).$$

B. EH Transmission Rule and Its Optimality

Motivated by Lemma 1, we propose the following transmission rule for an EH node. We shall use the following notation. For a given $z \in \mathbb{R}$ and a sequence $\{s(n), n \geq 0\}$ of random variables, we will write $\lim_{n\to\infty} s(n) \overset{as}{>} z$, to mean greater than in the almost sure sense, *i.e.*, $\Pr(\lim_{n\to\infty} s(n) > z) = 1$. Similarly, $\overset{as}{=}$ and $\overset{as}{<}$ mean equality and less than, respectively, in the almost sure sense.

EH transmission rule: Transmit with a constellation of size m_j, if $\eta k_j \leq h < \eta k_{j+1}$ and $P_{req}(h, m_j)T_c \leq B(n)$. Else, do not transmit. Here, η is chosen to meet an average power constraint of $\bar{P}_{\text{EH}}$ in (7).

Let $R_{\text{EH}}(\bar{P}_{\text{EH}}, k)$ denote the throughput of the above transmission rule in time slot k. Then, its cumulative average until time slot n is $\bar{R}_{\text{EH}}(\bar{P}_{\text{EH}}, n) = \frac{1}{n}\sum_{k=1}^{n} R_{\text{EH}}(\bar{P}_{\text{EH}}, k)$.

Theorem *1:* For any given $\epsilon > 0$, there exists a $\delta > 0$ such that

$$\lim_{n\to\infty}\left[\bar{R}^*_{nonEH}(\bar{P}_{\text{EH}}) - \bar{R}_{\text{EH}}(\bar{P}_{\text{EH}} - \delta, n)\right] \overset{as}{<} \epsilon.$$

Proof: The proof is relegated to Appendix B. ■

The above result states that the average throughput of an EH node can be made arbitrarily close to that of a non-EH node with an average power constraint $\bar{P}_{\text{EH}}$. In effect, the above scheme lets the energy stored in the battery increase with time and ensures that randomness in energy harvesting does not affect the choice of the transmission rate. The optimal policies in [6], [9] also allowed the battery energy to increase with time. However, their system models are different from ours.

C. Effect of Energy Overheads

Let the efficiency of the battery be $\beta_c \leq 1$ and $\beta_d \leq 1$ for charging and discharging, respectively. Hence, only a fraction β_c (or β_d) of the available energy gets stored in (or utilized from) the battery.[3] Let the energy loss in the battery due to an internal leakage current be L per slot. Let the energy required for sensing and processing be E_p per slot. Thus, the battery state evolves as

$$B(n+1) = (B(n) - E_p - L)^+ + \beta_c D(n)T_c - \frac{U(n)T_c}{\beta_d},$$

where $z^+ \triangleq \max\{z, 0\}$. A sufficient condition for energy neutrality constraint to be satisfied can be easily obtained if we assume that $E_p + L$ is deducted even if the battery has low energy, *i.e.*, $(B(n) - E_p - L)^+ = (B(n) - E_p - L)$.

[3]The efficiency of the storage buffer may depend on charging/discharging current and the voltage profile. We ignore this since, for supercapacitors, β_c and β_d are already close to 1 [15]. We also assume that the harvested energy is always stored in the battery first and then utilized as required. An alternate model that has been considered in the literature is that some of the energy harvested is directly used with 100% efficiency while the remaining unused energy is stored in the battery with a lower efficiency [6], [8]. We do not consider this optimization.

Then, the energy neutrality condition can be simplified to, $\frac{T_c}{n\beta_d}\sum_{i=0}^{n-1} U(i) \leq \frac{T_c}{n}\sum_{i=0}^{n-1} \beta_c D(n) - (E_p + L)$. This implies an average power constraint of $\bar{P}'_{\text{EH}} = \bar{P}_{\text{EH}} - \alpha$, where $\alpha = (1 - \beta_c\beta_d)\bar{P}_{\text{EH}} + \beta_d\frac{E_p + L}{T_c}$. From Theorem 1, it follows that the throughput of an EH node with energy overheads can be made arbitrarily close to that of a non-EH node with an average power constraint of $\bar{P}'_{\text{EH}}$.

The following result characterizes the impact of the this energy overhead α, on the throughput.

Lemma *2:* If η and η' correspond to the average power constraints of $\bar{P}_{\text{EH}}$ and $\bar{P}_{\text{EH}} - \alpha$, respectively, and if $\frac{\eta\alpha}{\sum_{j=2}^{M} f_H(k_j\eta)(d_j - d_{j-1})} \ll 1$, then

$$\eta' \approx \eta\left(1 + \frac{\alpha}{\sum_{j=2}^{M} f_H(k_j\eta)(d_j - d_{j-1})}\right). \tag{9}$$

The corresponding throughputs, $R(\eta)$ and $R(\eta')$, are related as

$$R(\eta') \approx R(\eta) - \frac{\eta\alpha}{N_0 B}. \tag{10}$$

Proof: The proof is relegated to Appendix C. ■

The result shows that for a small α, the above approach is close to optimal. The effect of finite battery capacity will be studied below using simulations.

IV. Simulations and Results

We illustrate our results using square M-QAM constellations with the three available constellations being 4-QAM ($m_2 = 4$), 16-QAM ($m_3 = 16$), 64-QAM ($m_4 = 64$), and $m_1 = 0$. We assume Rayleigh fading channels. Thus, $f_H(h) = \exp(-h)$, for $h \geq 0$. Without loss of generality, we set $N_0BT_c = 1$. For the purpose of illustration, we use the following energy harvesting model [13]: Every node harvests an energy E with probability ρ in every slot. Hence, $\bar{P}_{\text{EH}} = \rho E$. Note that our analysis holds for several other EH models as well. We also verify our analysis using Monte Carlo simulations that are run over 10^6 slots. We also assume that $B(0) = 0$.

The throughput as a function of the average energy harvesting rate, $\bar{P}_{\text{EH}}$, is plotted in Fig. 1 for the following three schemes: (i) Proposed EH transmission scheme, (ii) Greedy scheme, in which an EH node transmits data at the highest rate that can be supported by its current battery state, *i.e.*, it transmits with a constellation of size m_j if $P_{req}(h, m_j)T_c \leq B(n) < P_{req}(h, m_{j+1})T_c$, and (iii) Constant bit rate scheme, in which a node transmits with 16-QAM if sufficient energy is available, or does not transmit otherwise. Also plotted is the throughput of a non-EH node with an average power constraint of $\bar{P}_{\text{EH}}$, which serves as an upper bound. We observe that the average throughput of the EH transmission scheme coincides with its upper bound. Further, the greedy and constant bit rate schemes are significantly sub-optimal. It was also found that the performance of these suboptimal schemes is sensitive to ρ, unlike the proposed EH transmission scheme. The figure is not shown due to space constraints.

Figure 2 plots the cumulative average of the throughput as a function of time n (in slots) for the above mentioned four

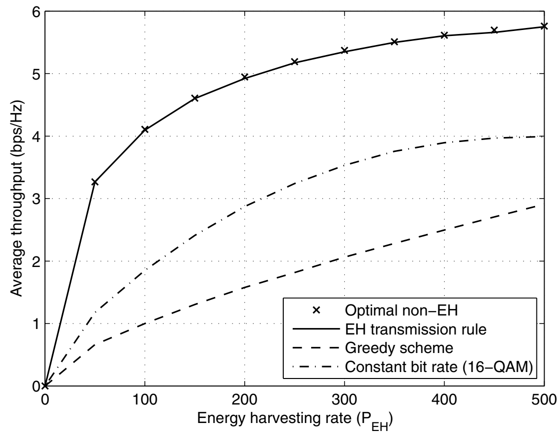

Fig. 1. Average throughput as a function of the average energy harvesting rate ($\rho = 0.01$ and $E = \bar{P}_{\text{EH}}/\rho$).

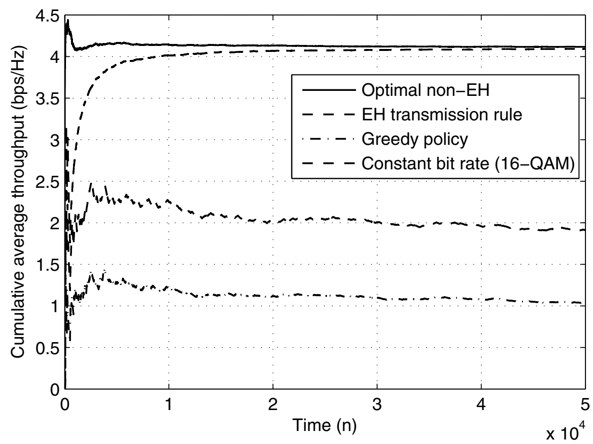

Fig. 2. Cumulative average of throughput as a function of time ($E_s = 10000$, $\rho = 0.01$, and $\bar{P}_{\text{EH}} = 100$).

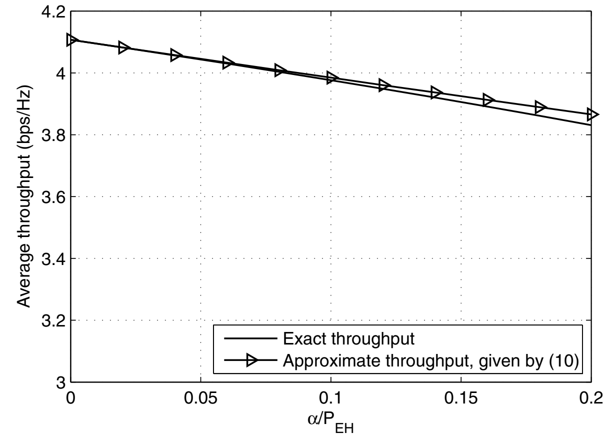

Fig. 3. Zoomed-in view of average throughput as a function of the energy overhead relative to average energy harvesting rate ($\bar{P}_{\text{EH}} = 100$).

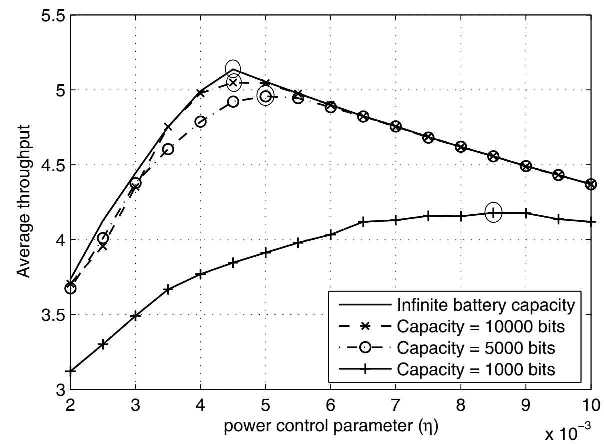

Fig. 4. Zoomed-in view of the average throughput as a function of η ($E_s = 25000$, $\rho = 0.01$, and $\bar{P}_{\text{EH}} = 250$). The peak value of every curve is encircled.

schemes. The plot verifies that the average throughput obtained by the proposed EH transmission rule converges to its upper bound as n increases. In this plot, it is within 5% and 1% of the optimal value for $n \geq 6000$ and for $n \geq 32000$, respectively.

A. Effect of Energy Overheads

Figure 3 plots the average throughput of the EH transmission rule as a function of the energy overhead relative to the average energy harvesting rate, $\alpha/\bar{P}_{\text{EH}}$. Also plotted is the approximate expression for the throughput in (10). We see that the expression is quite accurate. For example, it is off by only 0.7% even when α is 20% of $\bar{P}_{\text{EH}}$.

B. Effect of Finite Battery Capacity

We now consider the practical scenario where the battery capacity is limited. Figure 4 plots the zoomed-in view of the throughput as a function of η for various battery capacities. The battery capacity is characterized in a generic manner in terms of the energy required to transmit a given number bits at the highest constellation size of 64-QAM when the channel power gain is unity. Even with a battery that can store energy enough for transmitting only 5000 bits, the loss in the average throughput is less than 3.5%. Further, the optimal value of η that maximizes the throughput changes marginally as the battery capacity decreases, except for small battery capacities.

V. Conclusions

The energy harvesting functionality motivates a significant redesign of the physical and multiple access layers of wireless networks with battery-powered nodes. We saw that for discrete rate adaptation, the energy neutrality constraint, which governs the operation of EH nodes, is tighter than the average power constraint. We showed that a simple policy in which battery energy is allowed to increase by consuming energy at an average rate that is marginally smaller than the average rate of energy harvesting gives a throughput that is close to the upper bound on the throughput achievable by any scheme. The battery storage inefficiencies and other energy overheads simply entailed a decrease in the average energy harvesting rate. Finally, we saw that the infinite battery capacity model provides a good analytically tractable approximation for typical finite battery capacities.

Our results highlight the importance of throughput-optimal discrete rate and power adaptation in enhancing the performance of a network with EH nodes. Future work includes analyzing a case in which EH node does not always have data to transmit and generalizations to a multi-EH node system.

APPENDIX

A. Proof of Lemma 1

From (2), the constraint in (4) can be written as $B(0) + T_c \sum_{i=0}^{n-1}(D(i) - U(i)) \geq 0$, $\forall n \in \mathbb{N}$. Thus,

$$\lim_{n\to\infty} \frac{1}{n}\sum_{i=0}^{n-1} U(i) \leq \lim_{n\to\infty} \frac{1}{n}\sum_{i=0}^{n-1} D(i) = \bar{P}_{\text{EH}}, \tag{11}$$

which is the average power constraint. Hence, the result.

B. Brief Proof of Theorem 1

We begin our proof with following proposition.

Proposition 2:

$$\lim_{n\to\infty} \frac{1}{n}\sum_{i=0}^{n-1} \left(R_{\textit{nonEH}}(\bar{P}_{\text{EH}} - \delta, i) - R_{\text{EH}}(\bar{P}_{\text{EH}} - \delta, i)\right) \overset{as}{=} 0.$$

Proof: Let $X(n) \triangleq D(n) - U(n)$ and $Y(n) \triangleq D(n) - P_{\textit{req}}(h(n), M(n))$, where $M(n) = m_j$, if $\eta k_j \leq h(n) < \eta k_{j+1}$, and η is chosen to meet an average power constraint of $\bar{P}_{\text{EH}} - \delta$. Thus, $\mathbb{E}[Y(n)] = \delta$. Since a node may not transmit, $X(n) \geq Y(n)$. From (2), we have

$$B(n) = B(0) + T_c \sum_{i=0}^{n-1} X(i) \geq B(0) + nT_c \left(\frac{1}{n}\sum_{i=0}^{n-1} Y(i)\right). \tag{12}$$

Note from (1) that $P_{\textit{req}}(h(n), M(n))$ is finite. Further, since the random process $\{D(n), n \geq 0\}$ is ergodic and $\{P_{\textit{req}}(h(n), M(n)), n \geq 0\}$ is a sequence of i.i.d. random variables, $\{Y(n), n \geq 0\}$ is also an ergodic random process. Hence, $\lim_{n\to\infty} \frac{1}{n}\sum_{i=0}^{n-1} Y(i) \overset{as}{=} \delta$. Since $P_{\textit{req}}(h(n), M(n))$ is finite, it can be shown from (12) that $\lim_{n\to\infty}(B(n) - P_{\textit{req}}(h(n), M(n))) \overset{as}{>} 0$. Therefore, for a large enough n, the battery almost surely has enough energy to support transmission at any rate. Thus,

$$\lim_{n\to\infty} \left(R_{\textit{nonEH}}(\bar{P}_{\text{EH}} - \delta, n) - R_{\text{EH}}(\bar{P}_{\text{EH}} - \delta, n)\right) \overset{as}{=} 0,$$

which leads to the desired result. ■

If $F_H(h)$ is a continuous function of h, then it can be easily proved that $\bar{R}^*_{\textit{nonEH}}(\bar{P}_{\text{EH}})$ is a continuous function of $\bar{P}_{\text{EH}}$. Hence, for every $\epsilon > 0$, there exists a $\delta > 0$ such that

$$\bar{R}^*_{\textit{nonEH}}(\bar{P}_{\text{EH}}) - \bar{R}^*_{\textit{nonEH}}(\bar{P}_{\text{EH}} - \delta) < \epsilon. \tag{13}$$

Invoking the strong law of large numbers for the sequence $\{R_{\textit{nonEH}}(\bar{P}_{\text{EH}} - \delta, n), n \geq 0\}$, which is a sequence of i.i.d. random variables, we get

$$\bar{R}^*_{\textit{nonEH}}(\bar{P}_{\text{EH}} - \delta) \overset{as}{=} \lim_{n\to\infty} \frac{1}{n}\sum_{i=0}^{n-1} R_{\textit{nonEH}}(\bar{P}_{\text{EH}} - \delta, i).$$

The desired result follows by combining the above equation with (13) and Proposition 2.

C. Proof of Lemma 2

For small ϵ, let η and $\eta + \epsilon$ correspond to average power constraints of $\bar{P}_{\text{EH}}$ and $\bar{P}_{\text{EH}} - \alpha$, respectively. Thus, $\alpha \approx -\epsilon \frac{d\bar{P}_{\text{EH}}(\eta)}{d\eta}$. Differentiating (7), we get

$$\alpha = -\epsilon \frac{d\bar{P}_{\text{EH}}(\eta)}{d\eta} = -\frac{\epsilon}{\eta}\sum_{j=2}^{M} d_j \left(f_H(k_{j+1}\eta) - f_H(k_j\eta)\right) dh.$$

Rearranging the terms inside the summation above yields

$$\epsilon = \frac{\alpha\eta}{\sum_{j=2}^{M} f_H(k_j\eta)(d_j - d_{j-1})}. \tag{14}$$

A small change of ϵ in η triggers a change of $\epsilon\frac{dR(\eta)}{d\eta}$ in the throughput. Differentiating (8) with respect to η yeilds $\epsilon\frac{dR(\eta)}{d\eta} = -\epsilon\sum_{j=2}^{M} k_j f_H(k_j\eta)\log_2\left(\frac{m_j}{m_{j-1}}\right)$. Using (14) and $k_j \log_2\left(\frac{m_j}{m_{j-1}}\right) N_0 B T_c = d_j - d_{j-1}$, we get the result. Note that the above results are valid for $\epsilon \ll 1$, which, from (14), is equivalent to $\frac{\eta\alpha}{\sum_{j=2}^{M} d_j[f_H(k_j\eta) - f_H(k_{j+1}\eta)]} \ll 1$.

REFERENCES

[1] G. Anastasi, M. Conti, M. Di Francesco, and A. Passarella, “Energy conservation in wireless sensor networks: A survey,” *Ad Hoc Netw.*, vol. 7, pp. 537–568, May 2009.
[2] I. Akyildiz, W. Su, Y. Sankarasubramaniam, and E. Cayirci, “A survey on sensor networks,” *IEEE Commun. Mag.*, vol. 40, pp. 102–114, Aug. 2002.
[3] J. Paradiso and T. Starner, “Energy scavenging for mobile and wireless electronics,” *IEEE Pervasive Comput.*, vol. 4, pp. 18–27, Jan.–Mar. 2005.
[4] V. Raghunathan, S. Ganeriwal, and M. Srivastava, “Emerging techniques for long lived wireless sensor networks,” *IEEE Commun. Mag.*, vol. 44, pp. 108–114, Apr. 2006.
[5] A. Kansal, J. Hsu, S. Zahedi, and M. B. Srivastava, “Power management in energy harvesting sensor networks,” *ACM Trans. Embedded Comput. Syst.*, vol. 6, Sept. 2007.
[6] V. Sharma, U. Mukherji, V. Joseph, and S. Gupta, “Optimal energy management policies for energy harvesting sensor nodes,” *IEEE Trans. Wireless Commun.*, vol. 9, pp. 1326–1336, Apr. 2010.
[7] D. Niyato, E. Hossain, M. Rashid, and V. Bhargava, “Wireless sensor networks with energy harvesting technologies: A game-theoretic approach to optimal energy management,” *IEEE Wireless Commun.*, vol. 14, pp. 90–96, Aug. 2007.
[8] C. Murthy, “Power management and data rate maximization in wireless energy harvesting sensors,” *Int. J. Wireless Inf. Netw.*, vol. 16, pp. 102–117, Jul. 2009.
[9] M. Gatzianas, L. Georgiadis, and L. Tassiulas, “Control of wireless networks with rechargeable batteries,” *IEEE Trans. Wireless Commun.*, vol. 9, pp. 581–593, Feb. 2010.
[10] A. J. Goldsmith, *Wireless Communications*. Cambridge University Press, 2005.
[11] S. Lanzisera, A. M. Mehta, and K. S. J. Pister, “Reducing average power in wireless sensor networks through data rate adaptation,” in *Proc. ICC*, pp. 480–485, Jun. 2009.
[12] “Wireless LAN medium access control (MAC) and physical layer (PHY) specifications: Amendment 4: Enhancements for higher throughput,” Tech. Rep. 20.5.5.7 (Draft version D2.05), IEEE 802.11 Working Group, July 2007.
[13] B. Medepally, N. B. Mehta, and C. R. Murthy, “Implications of energy profile and storage on energy harvesting sensor link performance,” in *Proc. Globecom*, Dec. 2009.
[14] J. Lei, R. Yates, and L. Greenstein, “A generic model for optimizing single-hop transmission policy of replenishable sensors,” *IEEE Trans. Wireless Commun.*, vol. 8, pp. 547–551, Feb. 2009.
[15] P. Barrade, “Energy storage and applications with supercapacitors,” in *ANAE : Associazione Nazionale Azionamenti Elettrici*, Mar. 2003.