# A NOVEL DEMODULATION AND SELECTION PILOT POWER TRADE-OFF FOR CODEBOOK-BASED IRS WITH IMPERFECT CHANNEL ESTIMATES

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# ABSTRACT

The codebook-based scheme for intelligent reflecting surfaces (IRSs) provides flexibility in controlling the training overhead. In it, the reflection pattern with the largest received signal strength is selected from a pre-specified codebook and configured at the IRS. We analyze a training scheme that exploits a novel trade-off between the powers allocated for selection pilots, which are used to select the reflection pattern, and the demodulation pilot, which is used to estimate the channel for demodulation. We develop a novel selection-aware estimator of the beamforming gain of the selected reflection pattern. We derive a tight bound for the achievable rate and an elegant closed-form expression for the beamforming gain. These account for the impact of imperfect channel estimates on the selection of the reflection pattern and the coherent demodulation of the data symbols. The proposed scheme achieves a higher rate than conventional schemes by allocating substantially different powers to the selection and demodulation pilots and data symbols.

*Index Terms*— Intelligent reflecting surface, estimation, training, beamforming, power adaptation.

# 1. INTRODUCTION

An intelligent reflecting surface (IRS) consists of many small, lowcost passive elements, each of which can induce a controlled phase shift to the incident electromagnetic signal. This controllability can be exploited to improve throughput, coverage, and energy efficiency of next-generation wireless communication systems [1,2].

The transmitter or access point (AP) requires the estimates of the cascaded channels from it to the user through each IRS element to determine the optimal reflection pattern at the IRS [1,3,4]. Hence, cascaded channel estimation plays a crucial role in an IRS-aided system and has spurred a lot of research. In the classical on-off estimation scheme [5,6], the IRS turns on one element at a time. However, as many pilots as the IRS elements are needed. The discrete Fourier transform (DFT)-based scheme [3, 7], in which different columns of the DFT matrix are used as reflection patterns for different pilot symbols, achieves a lower mean square error (MSE) than the on-off scheme, but with the same training overhead. For IRSs with tens to hundreds of elements, the training overhead can be large. To reduce this overhead, IRS element grouping [3], active IRS elements [8], and different machine-learning approaches [9–11] have been pursued. However, these works do not consider pilot power adaptation.

An alternative to the above estimation-based schemes is the codebook-based scheme [12–14]. In it, K pre-designed reflection

patterns are used at the IRS, one for each pilot transmission. The pattern that maximizes the received signal strength is selected and configured at the IRS during data transmission. The codebook-based scheme is practically appealing because its pilot overhead is fixed. Its control signaling overhead, which the AP incurs to send the index of the selected reflection pattern, is also significantly lower than that of the estimation-based schemes, which need to communicate the phase shift of each IRS element.

Imperfect channel estimates, which are inevitable in practice because of finite pilot powers, affect the codebook-based scheme in two ways: i) they can cause a sub-optimal reflection pattern to be selected, and ii) this sub-optimal pattern together with imperfect channel estimates affect demodulation and lower the data rate. Only a subset of these effects are studied in the fixed pilot power schemes in the literature on codebook-based training [12, 15–19].

## 1.1. Focus and Contributions

We analyze a codebook-based IRS-assisted communication system. We account for imperfect channel estimates during the selection of the reflection pattern and coherent demodulation. We leverage the intuition that the former is more robust to the imperfect estimates. This leads us to a novel trade-off between the powers allocated for selection and demodulation, which we exploit to improve the rate.

We make the following contributions:

i) We propose a novel training scheme that separates the tasks of selection of the reflection pattern and demodulation. In it, the AP first sends K selection pilots to the user, one for each reflection pattern configured at the IRS. The user selects the reflection pattern with the highest received signal strength and feeds back its index to the AP, which the AP forwards to the IRS. The AP then sends a demodulation pilot to enable the user to estimate the effective channel gain, followed by data. This scheme does not require channel reciprocity.

ii) For the above training scheme, we analyze the achievable rate for orthogonal codebooks, which includes the DFT and Hadamard matrix based codebooks [7]. The analysis accounts for the impact of imperfect channel estimates on the selection of the reflection pattern and demodulation, and the training, control signaling, and feedback overheads. It relies upon two novel theoretical results. First, we derive in closed form the probability density function (PDF) of the effective channel gain corresponding to the selected reflection pattern and its beamforming gain. Second, we derive a novel selectionaware linear minimum mean square error (LMMSE) estimator of the effective channel gain, which is used for coherent demodulation. It exploits the fact that the statistics of the effective channel gain of the selected pattern differ from those of an arbitrary pattern.

iii) Our numerical results show that the proposed scheme achieves a higher rate than the conventional codebook-based and estimation-based schemes in regimes of practical interest. The

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Fig. 1. System model and an illustration of the codebook-based training and data transmission scheme.

reason behind this is its ability to allocate higher power to the demodulation pilot than a selection pilot. This advantage has not been exploited in the literature [12–19].

## 1.2. Outline and Notations

The system model and the proposed transmission scheme are described in Section 2. Section 3 derives an achievable rate expression and optimizes it. Numerical results are presented in Section 4. Our conclusions follow in Section 5.

*Notation:* We denote the probability of an event B by Pr (B) and the expectation of a random variable (RV) X by  $\mathbb{E}[X]$ . The notation  $X \sim C\mathcal{N}(\mu, \sigma^2)$  means that X is a complex normal RV with mean  $\mu$  and variance  $\sigma^2$ .

#### 2. SYSTEM MODEL AND PROPOSED SCHEME

The system model is illustrated in Figure 1. A single antenna AP transmits data to a single antenna user using an IRS with M elements. The direct link between the AP and the user is blocked due to obstacles [1, 14]. Let  $g_m$  and  $h_m$  denote the complex channel gains from the AP to the  $m^{\text{th}}$  IRS element and from  $m^{\text{th}}$  IRS element to the user, respectively. We consider quasi-static flat fading channel model. Let  $\mathbf{g} = [g_1, g_2, \ldots, g_M]^T$  and  $\mathbf{h} = [h_1, h_2, \ldots, h_M]^T$ , where  $(.)^T$  denotes transpose.

For analytical tractability, we focus on the following model: i)  $h_1, h_2, \ldots, h_M$  are independent [4, 6, 12]. ii) A dominant lineof-sight (LoS) link exists between the AP and the IRS [12]. This is justifiable when the AP is deployed on a tower and the IRS on a rooftop. Hence, **g** is a deterministic LoS channel gain vector. iii) The IRS-user link undergo Rayleigh fading with path-loss  $\beta_h$ . Therefore,  $\mathbf{h} \sim \mathcal{CN}(\mathbf{0}, \beta_h \mathbf{I}_M)$ . Let  $\beta_g$  denote the path-loss of the AP-IRS link.

We note that the achievable rate of codebook-based training has not been analyzed even for the above model. While multiple antennas have been considered at the transmitter in [12, 14–18], perfect channel estimates are implicitly assumed for demodulation and in the capacity expressions. Instead, in [3], the focus is on MSE and the impact of imperfect estimates on demodulation is not considered.

When AP transmits symbol x and the reflection pattern  $\phi = [\phi_1, \phi_2, \dots, \phi_M]^T$  is used, the signal y received by the user is [1]

$$y = \left(\sum_{m=1}^{M} h_m \phi_m g_m\right) x + n = \mathbf{r}^T \phi x + n, \tag{1}$$

where  $\mathbf{r} = [r_1, r_2, \dots, r_M]^T = [h_1g_1, h_2g_2, \dots, h_Mg_M]$  is the cascaded reflected channel gain vector and  $n \sim \mathcal{CN}(0, \sigma^2)$  is additive white Gaussian noise.

#### 2.1. Proposed Training and Transmission Scheme

A codebook consisting of  $K \leq M$  orthogonal reflection patterns  $\phi(1), \phi(2), \ldots, \phi(K)$  is used for generating the reflection coefficients at the IRS. Our scheme has following four phases:

*i)* Selection Phase: The AP sequentially transmits K selection pilots, each with power  $P_s$ . When the  $i^{th}$  pilot p = 1 is transmitted, the reflection pattern at the IRS is  $\phi(i)$ . The signal  $y_i$  at the user is

$$y_i = \sqrt{P_s} \mathbf{r}^T \boldsymbol{\phi}(i) p + n_i = \sqrt{P_s} h_{\text{eq}}(i) p + n_i.$$
<sup>(2)</sup>

Here,  $n_1, n_2, \ldots, n_K \sim C\mathcal{N}(0, \sigma^2)$  are independent and identically distributed (i.i.d.) additive white Gaussian noises, and  $h_{eq}(i) = \mathbf{r}^T \phi(i)$  is the effective end-to-end AP-user channel.

The user selects the reflection pattern index denoted by s as

$$s = \arg\max_{i \in \{1, 2, \dots, K\}} \{ |y_i|^2 \},\tag{3}$$

where |.| denotes the absolute value. The index s is fed back to the AP using only  $\lceil \log_2(K) \rceil$  bits, where  $\lceil.\rceil$  denotes the ceiling function. The AP then sends s to the IRS via the control link, again using  $\lceil \log_2(K) \rceil$  bits. Let  $\tau_f$  and  $\tau_c$  denote the feedback and control signaling durations, respectively.

*ii) Estimation Phase:*  $\phi(s)$  is configured as the reflection pattern at the IRS. The AP sends an extra demodulation pilot p = 1 with power  $P_c$  to enable the user to accurately estimate the selected downlink channel gain  $h_{eq}(s)$  for coherent demodulation. The signal  $y_c$  at the user is

$$y_c = \sqrt{P_c} h_{eq}(s) + n_c, \tag{4}$$

where  $n_{\rm c} \sim \mathcal{CN}(0, \sigma^2)$  is additive white Gaussian noise.

*iii) Feedback of Rate:* Based on its estimate of  $h_{eq}(s)$ , the user feeds back to the AP a data rate R that it can receive. We derive the expression for R below.

*iv) Data Transmission Phase:* The AP sends d downlink data symbols to the user with power  $P_d$  and rate R.

Note that  $P_s$ ,  $P_c$ , and  $P_d$  can be different.

## 3. PROBLEM FORMULATION AND ANALYSIS

We first develop a novel selection-aware LMMSE estimate for  $h_{eq}(s)$  and then derive an upper bound on the achievable rate. We then formulate our optimal pilots and data power allocation problem.

## **3.1.** Selection-Aware Estimator for $h_{eq}(s)$

We employ the LMMSE estimator for estimating  $h_{eq}(s)$  from  $y_c$ . The estimate  $\hat{h}_{eq}(s)$  is given by [20, Ch. 12]<sup>1</sup>

$$h_{\rm eq}(s) = ay_c + b,\tag{5}$$

where

$$a = \frac{\operatorname{Cov}\left(y_{c}, h_{eq}(s)\right)}{\operatorname{Var}\left(y_{c}\right)} \text{ and } b = \left(1 - \sqrt{P_{c}}a\right) \mathbb{E}\left[h_{eq}(s)\right], \quad (6)$$

Cov(.,.) denotes covariance, and Var(.) denotes variance. To derive *a* and *b*, we need the statistics of  $h_{eq}(s)$ , which are different

<sup>&</sup>lt;sup>1</sup>The two observations  $y_c$  and  $y_s$  can be used to obtain a more refined channel estimate of  $h_{eq}(s)$ . However, the gains turn out to be marginal.

from that of  $h_{eq}(1), h_{eq}(2), \ldots, h_{eq}(K)$  due to selection. Since  $h_1 \ldots, h_M$  are i.i.d., we can prove that  $h_{eq}(1), h_{eq}(2), \ldots, h_{eq}(K)$  are i.i.d. complex normal RVs with mean 0 and variance  $M\beta_g\beta_h$ . Let  $\sigma_y^2 = P_s M\beta_g\beta_h + \sigma^2$  denote the variance of  $y_i$ . The PDF of  $h_{eq}(s)$  is as follows.

**Theorem 1.** The PDF of  $h_{eq}(s) = e_{rs} + je_{is}$  is given by

$$f_{e_{rs},e_{is}}(x,z) = \frac{K\sigma_y^2}{\pi M\beta_g\beta_h} \sum_{l=0}^{K-1} \binom{K-1}{l} \frac{(-1)^l}{\sigma_y^2 + l\sigma^2} \\ \times \exp\left(\frac{-(l+1)\sigma_y^2}{\left(\sigma_y^2 + l\sigma^2\right)M\beta_g\beta_h} \left(x^2 + z^2\right)\right), \ x, z \in \mathbb{R}.$$
(7)

*Proof.* The proof is given in Appendix 6.1.  $\Box$ 

The symmetric form of the PDF in Theorem 1 implies that  $\mathbb{E}[h_{eq}(s)] = 0$ . Hence, b = 0. Thus, from (5) and (6), the selection-aware estimator is given by

$$\hat{h}_{\rm eq}(s) = \frac{\sqrt{P_{\rm c}}\mathbb{E}\left[|h_{\rm eq}(s)|^2\right]}{P_{\rm c}\mathbb{E}\left[|h_{\rm eq}(s)|^2\right] + \sigma^2}y_{\rm c}.$$
(8)

A selection-unaware approach would have instead led to the following estimator:  $\hat{h}_{eq}(s) = \frac{\sqrt{P_c}M\beta_g\beta_h}{P_cM\beta_g\beta_h + \sigma^2}y_c$ .

Theorem 1 leads to the following novel closed-form expression for the passive beamforming gain  $B_g = \mathbb{E} \left[ |h_{eq}(s)|^2 \right]$ .

**Corollary 1.** The passive beamforming gain  $B_g$  is given by

$$B_g = \frac{M\beta_g \beta_h \left( P_s M\beta_g \beta_h \left[ \psi(K+1) + \gamma \right] + \sigma^2 \right)}{P_s M\beta_g \beta_h + \sigma^2}, \quad (9)$$

where  $\psi(.)$  is the digamma function [21, Ch. 6] and  $\gamma$  is the Euler-Mascheroni constant [21, Ch. 6].

From (9), we can show that  $B_g$  is a concave function of  $P_s$ . It is also a concave function of K. Thus, there is a diminishing increase in the beamforming gain when  $P_s$  or K increase.

#### 3.2. Achievable Rate with Imperfect Channel Estimates

The achievable rate R in the presence of channel estimation errors, after accounting for the training, control signaling, and feedback overheads, is given by  $[22]^2$ 

$$R = \frac{d}{T_{\rm c}} \mathbb{E}\left[\log_2\left(1 + \frac{P_{\rm d} \left|\hat{h}_{\rm eq}(s)\right|^2}{\sigma^2 + P_{\rm d}\sigma_{\hat{h}_{\rm eq}(s)}^2}\right)\right],\tag{10}$$

where  $\sigma_{\hat{h}_{eq}(s)}^2 = \mathbb{E}\left[\left|h_{eq}(s) - \hat{h}_{eq}(s)\right|^2\right]$  is the mean square error (MSE) of the estimate of  $h_{eq}(s)$ , d is the number of data symbols, and  $T_c = d + K + 1 + 2\tau_f + \tau_c$  is the coherence interval duration in symbols. Using the Jensen's inequality, we get the following bound:

$$R \leq \frac{d}{T_{\rm c}} \log_2 \left( 1 + \frac{P_{\rm d} \mathbb{E} \left[ \left| \hat{h}_{\rm eq}(s) \right|^2 \right]}{\sigma^2 + P_{\rm d} \sigma_{\hat{h}_{\rm eq}(s)}^2} \right).$$
(11)

<sup>2</sup>The achievable rate expression in (10) is an approximation because  $h_{eq}(s)$  is not a complex normal RV. However, it is accurate for smaller K and at lower SNRs.

We shall see in Section 4 that the above bound is tight for large M due to channel hardening. From (4) and (8), we can show that

$$\mathbb{E}\left[\left|\hat{h}_{eq}(s)\right|^{2}\right] = \frac{P_{c}B_{g}^{2}}{P_{c}B_{g} + \sigma^{2}}, \ \sigma_{\hat{h}_{eq}(s)}^{2} = \frac{B_{g}\sigma^{2}}{P_{c}B_{g} + \sigma^{2}}.$$
 (12)

Substituting these in (11) yields the following result.

**Theorem 2.** The achievable rate of the codebook-based IRS-aided system in the presence of channel estimation errors is bounded by

$$R \le \frac{d}{T_c} \log_2 \left( 1 + \frac{P_d P_c B_g^2}{\sigma^2 \left[ (P_d + P_c) B_g + \sigma^2 \right]} \right), \tag{13}$$

where  $B_g$  is given in (9).

The total power  $P_{\rm T}$  consumed for the pilot and data transmissions is equal to  $KP_{\rm s} + P_{\rm c} + dP_{\rm d}$ . Based on (13), the problem of maximizing the upper bound on R subject to the total power constraint at AP can be stated as

$$\max_{P_{\rm s},P_{\rm c},P_{\rm d}}\left\{\frac{P_{\rm d}P_{\rm c}B_{\rm g}^2}{\sigma^2\left[(P_{\rm d}+P_{\rm c})B_{\rm g}+\sigma^2\right]}\right\},\tag{14}$$

s.t. 
$$KP_{\rm s} + P_{\rm c} + dP_{\rm d} = P_{\rm T},$$
 (15)

$$P_{\rm s} \ge 0, P_{\rm c} \ge 0, P_{\rm d} \ge 0.$$
 (16)

Note that the selection pilot power  $P_s$  appears implicitly through  $B_g$ . We solve the above non-convex problem numerically.

# 4. NUMERICAL RESULTS AND DISCUSSIONS

We set M = 128 and consider  $T_c = 400$  symbols. We consider the simplified path loss model [6] with  $\beta_g = -34$  dB and  $\beta_h = -54$  dB. The noise variance is  $\sigma^2 = -113.8$  dB. We set  $\tau_f = \tau_c = 1$ . The results are averaged over 10000 fade realizations. We consider a DFT-based codebook at the IRS.

We benchmark the proposed optimal power allocation-based codebook scheme with the following:

*i)* Equal Power Allocation (EPA): The total power is equally allocated to the pilots and data. Furthermore, the unordered statistics of the beamforming gain are used to estimate  $h_{eq}(s)$ . The training, signaling, and feedback overhead is  $K + 1 + 2\tau_f + \tau_c$ .

*ii) Estimation-based Scheme* [3]: We use the DFT-based estimation scheme since it outperforms the on-off scheme [3]. Its training, signaling, and feedback overhead is equal to  $M + M\tau_c + 1 + \tau_f$ .

*iii) Genie-aided Scheme*: The selection of reflection pattern is taken to be noise-free and  $h_{eq}(s)$  is known perfectly to the AP and the user. There is no training, signaling, and feedback overhead.

*iv)* Random Phase Shift (RPS): A random phase shift is configured at each IRS element. Thus, no selection pilots are needed. The training, signaling, and feedback overhead is  $1 + \tau_f$ .

Figure 2 plots the achievable rate of all the schemes as a function of the average SNR  $P_T M \beta_g \beta_h / (T_c \sigma^2)$ . The proposed scheme achieves a higher rate than EPA and RPS for all SNRs. The gap between it and the genie-aided scheme is small and decreases as the SNR increases. Only at low SNRs, does the estimation-based scheme marginally outperform the proposed scheme. This is because the beamforming gain of the former is proportional to  $M^2$  [4], while that of the latter is proportional to  $M\psi(K + 1)$  (see (9)). At high SNRs, the large training and control overheads cause the estimationbased scheme's rate to be lower than even RPS.

Figure 3(a) plots the ratio of the optimal selection pilots' power to the total power as a function of the average SNR for different values of K. It compares them with EPA, where this ratio is equal to



Fig. 2. Performance Benchmarking: Achievable rate as a function of the average SNR (K = 25).



Fig. 3. Pilot power allocation as a function of the average SNR.

 $K/T_c$ . For the proposed scheme, we see that the ratio decreases for all K as the average SNR increases. At low SNRs or larger K, it allocates relatively more power to the selection pilots to improve the odds that the optimal reflection pattern is selected. At high SNRs, it sets aside more power for the demodulation pilot and the data. Figure 3(b) plots the corresponding ratio of the optimal demodulation pilot power to the total power. Notably, the proposed scheme allocates 13 to 14 dB more power to the demodulation pilot than EPA at all SNRs. Also, we observe that  $P_c \gg P_s$ . For example, at K = 50and SNR of 0 dB,  $P_c/P_s = 15$ .

Figure 4 plots the achievable rate and the upper bound in (13) as a function of K for different  $T_c$ . We observe that the upper bound is tight, which is due to channel hardening. For small K, the achievable rate increases as K increases because the beamforming gain increases. However, for larger K, the rate decreases because of the increase in the training overhead. Only for large coherence interval durations does the estimation-based scheme have a higher rate.

## 5. CONCLUSIONS

For the orthogonal codebook-based IRS system, we proposed a novel training scheme that could allocate more power to the demodulation pilot than the selection pilots. We derived a tight upper bound for its achievable rate and a closed-form expression for the average beamforming gain. The analysis accounted for imperfect



Fig. 4. Achievable rate as a function of the number of reflection patterns (K). The maximum rate is shown by the marker 'x'.

channel estimates, which affected both selection and coherent demodulation. The proposed scheme achieved a higher rate than the estimation-based scheme except at low SNRs or when the number of IRS elements was comparable to the coherence interval duration. Future research directions include considering multiple antennas, and considering correlation between the cascaded channels. With the direct link, an interesting problem is determining the pilot powers as a function of the link's strength relative to the cascaded links.

## 6. APPENDIX

## 6.1. Outline of Proof of Theorem 1

Due to space constraints, we only show the key steps. Let  $h_{eq}(m) = e_{rm} + je_{im}$ , for  $m \in \{1, 2, ..., K\}$ . The cumulative distribution function (CDF) of  $h_{eq}(s)$  can be written as  $F_{e_{rs},e_{is}}(x,z) = \Pr(e_{rs} \le x, e_{is} \le z)$ . It is given by

$$F_{e_{rs},e_{is}}(x,z) = \sum_{i=1}^{K} \Pr\left(e_{rs} \le x, e_{is} \le z, s=i\right), \quad (17)$$

$$= K \Pr(e_{r1} \le x, e_{i1} \le z, s = 1).$$
 (18)

Let  $\mathcal{K}_1 = \{2, 3, \dots, K\}$ . Using (3), conditioning on  $e_{r1}, e_{i1}$  and taking expectation over them, the probability in (18) can be written as

$$\mathbb{E}\left[\mathbb{1}_{\{e_{r1}\leq x, e_{i1}\leq z,\}}\Pr\left(|y_1|^2 > |y_i|^2, \forall i \in \mathcal{K}_1 \middle| e_{r1}, e_{i1}\right)\right], \quad (19)$$

where  $\mathbb{1}_{\{.\}}$  denotes the indicator function. Conditioned on  $e_{r1}, e_{i1}$ , we can show that  $|y_1|^2$  is a non-central chi-square RV with PDF  $e^{-(w+P_se^2)/\sigma^2}I_0\left(\sqrt{4P_se^2w/\sigma^4}\right)/\sigma^2$ , for  $w \ge 0$ , where  $e = \sqrt{e_{r1}^2 + e_{i1}^2}$  and  $I_0(.)$  is the zeroth-order modified Bessel function of the first kind [23, Ch. 5]. Using this, we can then show that

$$\Pr\left(|y_{1}|^{2} > |y_{i}|^{2}, \forall i \in \mathcal{K}_{1}|e_{r1}, e_{i1}\right) = \frac{1}{\sigma^{2}} \int_{0}^{\infty} e^{-\frac{w+P_{s}e^{2}}{\sigma^{2}}} I_{0}\left(\sqrt{\frac{4P_{s}e^{2}w}{\sigma^{4}}}\right) \left(1 - e^{-\frac{w}{\sigma^{2}}}\right)^{K-1} dw. \quad (20)$$

Expanding the term  $(1 - \exp(-w/\sigma_y^2))^{K-1}$  and using the identities in [24, (2) and (9)] and [25, (2.33)] simplifies (20). Substituting it in (19) and averaging over  $e_{r1}$  and  $e_{i1}$  yields the expression for  $F_{e_{rs},e_{is}}(x,z)$ . Differentiating this with respect to x and z gives us the expression for PDF in (7).

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