Optimal Energy-Efficient Antenna Selection and Power Adaptation for Underlay Spectrum Sharing

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***Abstract*—We propose a novel joint antenna selection and power adaptation rule that maximizes the energy-efficiency (EE) of an underlay spectrum sharing system. In an underlay system, a secondary user shares the spectrum with a higher priority primary user, but is subject to tight constraints on the interference it generates. Our approach exploits the spatial diversity benefits of multiple antennas to improve the secondary user's performance, but with a hardware complexity and cost comparable to a single antenna system. We prove that the proposed rule is EE-optimal for a secondary transmitter that is subject to the interference-outage constraint, which generalizes the widely used peak interference constraint, and the peak transmit power constraint. It has a novel form different from the conventional rules considered in the literature. We present an insightful geometrical characterization that brings out the dependence of the optimal power on the channel gains within the secondary system and between the secondary and primary systems. The proposed rule achieves a markedly higher EE compared to conventional rules.**

I. Introduction

Next generation wireless communication systems are expected to achieve higher data rates and support a variety of new applications despite severe constraints on the spectrum available and energy. For example, 5G systems are expected to achieve 10 times higher spectral efficiency and 100 times higher energy-efficiency (EE) [1]. The severe shortage of spectrum has led to a rejuvenation of interest in spectrum sharing by the regulators [2], [3]. Spectrum sharing makes multiple systems share the same spectrum to improve its utilization. In its underlay mode, secondary users (SUs) share the spectrum used by higher priority primary users (PUs) [4], [5]. However, to protect the PU, the SU is subject to tight constraints on the interference it causes.

Multiple antenna techniques such as transmit antenna selection (TAS) have attracted attention because of their ability to improve the SU's performance but with a low hardware complexity [5]. In TAS, the secondary transmitter (STx) dynamically selects one antenna and transmits to the secondary receiver (SRx). By requiring only one radio-frequency (RF) chain, TAS cuts down on components such as digital-to-analog converter, mixer, filter, and amplifiers that contribute the most to a multi-antenna system's cost. For this reason, TAS has been adopted in commercial standards such as 4G Long Term Evolution [6].

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EE maximization for underlay spectrum sharing has been studied in [7]–[9]. In [7], energy-efficient power adaptation when the STx is subject to the average interference power constraint and different combinations of peak and average power constraints is studied. In [8], EE-maximizing power adaptation when the STx is subject to both peak power and average power constraints and the interference-outage constraint, is studied. In [9], energy-efficient power adaptation is studied for various combinations of peak and average power constraints, and peak and average interference constraints.

TAS for underlay spectrum sharing has been explored from different angles in the literature. For example, symbol error probability-minimizing TAS rules for an STx that is subject to interference-outage and transmit power constraints are considered in [5], [10], and signal-to-interference-plus-noise ratio (SINR)-maximizing TAS rules for an STx that is subject to the peak interference constraint are studied in [4]. Rate-maximizing TAS rules for an STx that is subject to peak interference and transmit power constraints are studied in [11].

A. Contributions

From the above discussion, we see that the design of an optimal TAS rule that maximizes the EE of an underlay spectrum sharing system is an open problem. We propose a novel EE-optimal joint antenna selection and power adaptation (EE-ASPA) rule for a secondary system that is subject to the interference-outage constraint and the peak power constraint. The interference-outage constraint limits the probability that the interference power at the primary receiver (PRx) exceeds a threshold. It generalizes the widely used peak interference constraint [5], [10]. The peak power constraint limits the transmit power of the STx, and arises due to power amplifier limitations [5], [7].

EE-ASPA selects the antenna and sets its power to maximize an instantaneous reward function that is parametrized by a power penalty η and an interference penalty λ. Its mathematical form is very different from the antenna selection and power adaptation (ASPA) rules considered in the literature [5], [12]–[14]. The form brings out how the interference constraint makes the antenna selected and its power depend not only on the STx-SRx channel power gain, but also on the interference link gain from the STx to the PRx, and how the consideration of EE affects the choice of the antenna and its power. We propose an iterative algorithm to determine η and λ.

We then present an insightful and novel geometric characterization of the optimal power as a function of the STx-SRx

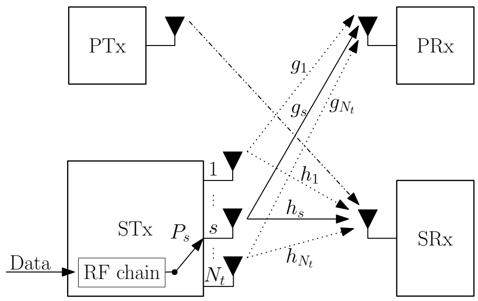


Fig. 1. System model showing an STx with N_t antennas and one RF chain that transmits to an SRx using antenna s in the presence of a PRx.

and STx-PRx channel power gains of the selected antenna. We also study the small peak power regime in which the optimal transmit power adaptation simplifies to a simpler binary power control. We present explicit expressions for the EE and the interference-outage probability in this regime.

Our numerical results show that EE-ASPA achieves an EE that is larger by up to 170% compared to the other rules in the literature. The results also bring out how different rules trade off between the spectral efficiency and the EE, and how EE-ASPA optimally balances the two. On account of its optimality, EE-ASPA provides a fundamental new benchmark for the EE of TAS for underlay spectrum sharing.

B. Outline and Notations

Section II describes the system model. Section III presents EE-ASPA and proves its optimality. Numerical results are presented in Section IV. Our conclusions follow in Section V.

Notations: We show scalar variables in normal font, vector variables in lowercase bold font, and sets in calligraphic font. $\Pr(X)$ and $\mathbb{E}[X]$ denote the probability and the expectation, respectively, of X. The indicator function is denoted by $\mathbb{1}_{\{a\}}$, which equals one if a is true and is zero else. We denote $\max\{a,0\}$ by $(a)^+$.

II. System Model and Problem Statement

The system model is illustrated in Figure 1. It consists of a primary system and a secondary system that share the same spectrum of bandwidth B. The secondary system consists of an STx with N_t transmit antennas and an SRx with a single receive antenna.[1] The channel power gains from the i^{th} antenna of the STx to the SRx and the PRx are denoted by h_i and g_i, respectively. Let $\mathbf{h} \triangleq [h_1,\ldots,h_{N_\mathrm{t}}]$ and $\mathbf{g} \triangleq [g_1,\ldots,g_{N_\mathrm{t}}]$. The STx chooses an antenna $s \in \{1,\ldots,N_\mathrm{t}\}$ and transmits with a power P_s.

The STx knows $\mathbf{h}$ and $\mathbf{g}$, as has been widely assumed in the spectrum sharing literature [5], [7], [12]–[14]. In the time division duplex mode, the STx can estimate $\mathbf{h}$ and $\mathbf{g}$ using channel reciprocity by transmitting or receiving from the antennas one by one. In the frequency division duplex mode, the STx can estimate $\mathbf{h}$ via feedback from the SRx, and $\mathbf{g}$ using techniques such as hidden power feedback loop [15]. The SRx performs coherent demodulation. For this, it only needs to know the complex baseband channel gain from selected STx antenna s to itself. The SRx can estimate it from the pilot symbols transmitted along with the data [5].

EE Definition: The EE η_EE is defined as the ratio of average rate to average power consumed at the STx. From Shannon's formula, the instantaneous rate is given by $\log\left(1+P_s h_s/\sigma^2\right)$, and $\sigma^2 = \sigma_\mathrm{p}^2 + \sigma_\mathrm{n}^2$ is the sum of interference power σ_p^2 at the SRx from the primary transmitter (PTx) and the noise power σ_n^2 at the SRx.[2] σ^2 is assumed to be known to the STx. The instantaneous power consumed is equal to $\xi P_s + P_\mathrm{c}$, where ξ is the inverse of the power amplifier efficiency of the STx and P_c is the constant power drain due to STx components such as mixer, filter, and digital-to-analog converter [16]. Therefore,

$$\eta_\mathrm{EE} = \frac{B\mathbb{E}\left[\log\left(1+\frac{P_s h_s}{\sigma^2}\right)\right]}{\mathbb{E}\left[\xi P_s + P_\mathrm{c}\right]} \text{ bits/J.} \quad (1)$$

The expectations in (1) are taken over both $\mathbf{h}$ and $\mathbf{g}$.

Constraints: The STx is subject to the following constraints:

1) *Peak Power Constraint :* The secondary transmit power cannot exceed a peak transmit power P_max.
2) *Interference-Outage Constraint :* An interference-outage occurs when the interference power $P_s g_s$ at the PRx exceeds a threshold τ, which is a system parameter. The constraint requires that an interference-outage should occur with a probability at most O_max:

$$\Pr\left(P_s g_s > \tau\right) \leq O_\mathrm{max}. \quad (2)$$

Problem Statement: An ASPA rule $\phi : (\mathbb{R}^+)^{N_\mathrm{t}} \times (\mathbb{R}^+)^{N_\mathrm{t}} \to \{1,\ldots,N_\mathrm{t}\} \times [0,P_\mathrm{max}]$ determines for each realization of $\mathbf{h}$ and $\mathbf{g}$ the transmit antenna $s \in \{1,\ldots,N_\mathrm{t}\}$ and its transmit power $P_s \in [0,P_\mathrm{max}]$. Let $\mathcal{D}$ be the set of all ASPA rules. Our aim is to find an optimal ASPA rule ϕ^* that maximizes the EE and satisfies the above constraints. Since B is a constant, our stochastic, constrained optimization problem is as follows:

$$\mathcal{P}_0: \quad \max_{\phi\in\mathcal{D}} \quad \left\{\frac{\mathbb{E}\left[\log\left(1+\frac{P_s h_s}{\sigma^2}\right)\right]}{\mathbb{E}\left[\xi P_s + P_\mathrm{c}\right]}\right\}, \quad (3)$$

$$\text{s.t.} \quad \Pr\left(P_s g_s > \tau\right) \leq O_\mathrm{max}, \quad (4)$$

$$0 \leq P_s \leq P_\mathrm{max}, \quad (5)$$

$$(s, P_s) = \phi(\mathbf{h},\mathbf{g}). \quad (6)$$

III. EE-ASPA

We define the EE-ASPA rule $\phi^*(\mathbf{h},\mathbf{g})$ as

$$(s^*, P_{s^*}^*) = \underset{P_i\in[0,P_\mathrm{max}],\ i\in\{1,\ldots,N_\mathrm{t}\}}{\operatorname{argmax}} \left\{\Omega_i\left(P_i;\eta,\lambda\right)\right\}, \quad (7)$$

where $\Omega_i\left(P_i;\eta,\lambda\right)$ is the instantaneous reward function of antenna i when it transmits with power P_i. It is defined as

$$\Omega_i\left(P_i;\eta,\lambda\right) \triangleq \log\left(1+\frac{P_i h_i}{\sigma^2}\right) - \eta\xi P_i - \lambda \mathbb{1}_{\{P_i g_i > \tau\}}, \quad (8)$$

[1]The results can be easily generalized for the scenario with multiple receive antennas.

[2]The interference from the PTx to the SRx is assumed to be Gaussian as has been widely assumed in the literature due to its tractability [5].

where $\eta \geq 0$ and $\lambda \geq 0$ are two constants. We do not explicitly show the dependence of $\Omega_i(P_i; \eta, \lambda)$ on h_i and g_i to keep the notation simple.

Consider first the constrained regime in which the interference-outage constraint is active. For this regime, we now show that EE-ASPA is the optimal solution for the problem $\mathcal{P}_0$ with the help of three lemmas.

Lemma *1:* For a given $\eta \geq 0$, let λ be chosen such that $\Pr(P_{s^*}^* g_{s^*} > \tau) = O_{\max}$, where $(s^*, P_{s^*}^*) = \phi^*(\mathbf{h}, \mathbf{g})$. Then, ϕ^* solves the optimization problem $\mathcal{P}_1(\eta)$, where

$$\mathcal{P}_1(\eta): \ \max_{\phi \in \mathcal{D}} \left\{ \mathbb{E}\left[\log\left(1 + \frac{P_s h_s}{\sigma^2}\right)\right] - \eta \mathbb{E}\left[\xi P_s + P_{\mathrm{c}}\right] \right\}, \tag{9}$$

s.t. (4), (5), (6).

Proof: The proof is given in Appendix A. ■

Lemma *2:* Let η^* be the value of the power penalty η at which the maximum value of the objective function of $\mathcal{P}_1(\eta)$ is 0. For this value of η^*, let λ^* be chosen such that $\Pr(P_{s^*}^* g_{s^*} > \tau) = O_{\max}$, where $(s^*, P_{s^*}^*) = \phi^*(\mathbf{h}, \mathbf{g})$. Then, η^* is the optimal EE of $\mathcal{P}_0$ and ϕ^* solves $\mathcal{P}_0$.

Proof: The proof is given in Appendix B. ■

Lemma 2 implies that ϕ^* is the optimal ASPA rule. However, the rule requires η^* to be known. We determine it iteratively. In the k^{th} iteration, for a given $\eta = \eta^{(k)}$, let $\lambda^{(k)}$ denote the corresponding interference penalty at which $\Pr(P_{s^{(k)}} h_{s^{(k)}} > \tau) = O_{\max}$. Here, $s^{(k)}$ is the optimal antenna and $P_{s^{(k)}}$ is its optimal power in the k^{th} iteration. Recall that $s^{(k)}$ and $P_{s^{(k)}}$ are functions of $\mathbf{h}$ and $\mathbf{g}$. Then,

$$\eta^{(k+1)} = \frac{\mathbb{E}\left[\log\left(1 + \frac{P_{s^{(k)}} h_{s^{(k)}}}{\sigma^2}\right)\right]}{\mathbb{E}\left[\xi P_{s^{(k)}} + P_{\mathrm{c}}\right]}. \tag{10}$$

We set $\eta^{(0)}$ as zero to initialize this iteration.

Lemma *3:* The sequence $\eta^{(0)}, \eta^{(1)}, \ldots, \eta^{(k)}, \ldots$ is a strictly monotonically increasing sequence that converges to η^*.

Proof: Following the method in [17], we can show that the sequence is strictly monotonically increasing. Also, we can prove that the optimal value of $\mathcal{P}_1(\eta)$ monotonically decreases and becomes zero only when $\eta = \eta^*$. ■

We now present an explicit characterization of the optimal transmit power and the optimal antenna.

Result *1:* The optimal antenna s^* is given by

$$s^* = \operatorname*{argmax}_{i \in \{1, \ldots, N_{\mathrm{t}}\}} \left\{\Omega_i(P_i^*; \eta^*, \lambda^*)\right\}, \tag{11}$$

where P_i^* is the optimal power of antenna i if it is selected. It is given by

$$P_i^* = \begin{cases} \min\left\{\widetilde{P}(h_i), P_{\max}\right\}, & \text{for } \mathcal{R}_1, \ (12a) \\ \widetilde{P}(h_i), & \text{for } \mathcal{R}_2, \ (12b) \\ \operatorname*{argmax}_{P_i \in \left\{\widetilde{P}(h_i), \tau/g_i\right\}} \left\{\Omega_i(P_i; \eta^*, \lambda^*)\right\}, & \text{for } \mathcal{R}_3, \ (12c) \\ \operatorname*{argmax}_{P_i \in \{P_{\max}, \tau/g_i\}} \left\{\Omega_i(P_i; \eta^*, \lambda^*)\right\}, & \text{for } \mathcal{R}_4, \ (12d) \end{cases}$$

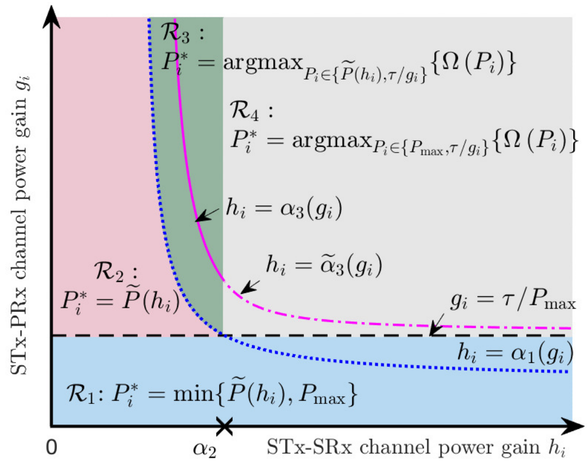


Fig. 2. Illustration of the four decision regions for antenna i depending on the values of h_i, g_i, α_2, and $\tau/P_{\max}$ with the optimal power in each region.

where

$$\widetilde{P}(h_i) = \sigma^2 \left(\frac{1}{\alpha_0} - \frac{1}{h_i}\right)^+, \tag{13}$$

and $\alpha_0 = \eta^* \xi \sigma^2$. The regions $\mathcal{R}_1$, $\mathcal{R}_2$, $\mathcal{R}_3$, and $\mathcal{R}_4$ are:

$$\mathcal{R}_1 = \left\{(h_i, g_i) : g_i \leq \frac{\tau}{P_{\max}}\right\}, \tag{14a}$$

$$\mathcal{R}_2 = \left\{(h_i, g_i) : h_i \leq \alpha_1(g_i), \ g_i > \frac{\tau}{P_{\max}}\right\}, \tag{14b}$$

$$\mathcal{R}_3 = \left\{(h_i, g_i) : \alpha_1(g_i) < h_i \leq \alpha_2, \ g_i > \frac{\tau}{P_{\max}}\right\}, \tag{14c}$$

$$\mathcal{R}_4 = \left\{(h_i, g_i) : h_i > \alpha_2, \ g_i > \frac{\tau}{P_{\max}}\right\}, \tag{14d}$$

where

$$\alpha_1(g_i) = \frac{\alpha_0}{\left(1 - \frac{\eta^* \xi \tau}{g_i}\right)^+} \text{ and } \alpha_2 = \frac{\alpha_0}{(1 - \eta^* \xi P_{\max})^+}. \tag{15}$$

Proof: The proof is given in Appendix C. ■

To understand the four regions, we first define five key values of h_i, namely, α_0, $\alpha_1(g_i)$, α_2, $\alpha_3(g_i)$, and $\widetilde{\alpha}_3(g_i)$, at which the behaviour of the reward function in (8) can change. The point at which $\widetilde{P}(h_i)$ first becomes non-zero is α_0. The point at which $\widetilde{P}(h_i) = \tau/g_i$ is $\alpha_1(g_i)$. The point at which $\widetilde{P}(h_i) = P_{\max}$ is α_2.

The point at which $\Omega_i(\tau/g_i; \eta^*, \lambda^*) = \Omega_i\left(\widetilde{P}(h_i); \eta^*, \lambda^*\right)$ is $\alpha_3(g_i)$. Rearranging terms in the equation yields

$$\alpha_3(g_i) = \frac{\alpha_0}{\left(\omega_{\lambda^*} - \frac{\eta^* \xi \tau}{g_i}\right)^+}, \tag{16}$$

where $\omega_{\lambda^*} = -W_0\left(-e^{-1-\lambda^*}\right)$ and $W_0(.)$ denotes the LambertW function on its principal branch [18].

Lastly, the point at which $\Omega_i(\tau/g_i;\eta^*,\lambda^*) = \Omega_i(P_{\max};\eta^*,\lambda^*)$ is $\widetilde{\alpha}_3(g_i)$. Rearranging terms in the equation yields

$$\widetilde{\alpha}_3(g_i) = \frac{\sigma^2 - \sigma^2 e^{-\lambda^*+\eta^*\xi\left(\frac{\tau}{g_i}-P_{\max}\right)}}{\left(e^{-\lambda^*+\eta^*\xi\left(\frac{\tau}{g_i}-P_{\max}\right)}P_{\max} - \frac{\tau}{g_i}\right)^+}. \tag{17}$$

With the help of Result 1, we understand how the transmit power changes as a function of the STx-SRx and STx-PRx channel gains. In the following, an antenna i is *outage-inducing* if $P_i^* g_i > \tau$, and is *non-outage-inducing* otherwise.

- In $\mathcal{R}_1$, from (12a), $P_i^* = 0$, for $0 \le h_i \le \alpha_0$, $P_i^* = \widetilde{P}(h_i) > 0$, for $\alpha_0 < h_i \le \alpha_2$, and $P_i^* = P_{\max}$, for $h_i > \alpha_2$. Since $P_{\max} \le \tau/g_i$, antenna i is non-outage-inducing in the entire region.
- In $\mathcal{R}_2$, from (12b), we have $P_i^* = \widetilde{P}(h_i) \le \tau/g_i$ and antenna i is non-outage-inducing in this region.
- In $\mathcal{R}_3$, from (12c), it follows that $P_i^* = \tau/g_i$, for $\alpha_1(g_i) < h_i \le \alpha_3(g_i)$, and $P_i^* = \widetilde{P}(h_i)$, otherwise. Also, from (14c), we can show that $\tau/g_i < \widetilde{P}(h_i) \le P_{\max}$ here. Therefore, antenna i is outage-inducing when $h_i > \alpha_3(g_i)$ in $\mathcal{R}_3$.
- In $\mathcal{R}_4$, from (12d), it follows that $P_i^* = \tau/g_i$, for $\alpha_2 < h_i \le \widetilde{\alpha}_3(g_i)$, and $P_i^* = P_{\max}$, otherwise. Also, from (14d), we can show that $\tau/g_i < P_{\max} < \widetilde{P}(h_i)$ here. Hence, antenna i is outage-inducing when $h_i > \widetilde{\alpha}_3(g_i)$.

Determining λ^*: Recall that in EE-ASPA, for a given η^*, we need to find λ^* such that the interference-outage constraint in (4) is satisfied with equality. We find λ^* iteratively using the sub-gradient algorithm [19, Ch. 3]. In the ℓ^{th} iteration of the algorithm, λ is updated as $\lambda^{(\ell)} = \left[\lambda^{(\ell-1)} - t\left(O_{\max} - P_{\text{out}}^{(\ell-1)}\right)\right]^+$, where t is the step size and $P_{\text{out}}^{(\ell-1)}$ is the outage probability after the $(\ell-1)^{\text{th}}$ iteration. The algorithm stops when either $P_{\text{out}}^{(\ell)} = O_{\max}$ or $\lambda^{(\ell)} = 0$. The subgradient is bounded by $O_{\max}$, which guarantees the convergence of the algorithm to the neighbourhood of λ^*. When the interference-outage constraint is inactive, $\lambda^* = 0$.

Note that finding η^* and the corresponding λ^* needs to be done only once. Given these two parameters, the proposed rule requires $\mathcal{O}(N_{\text{t}})$ comparisons every coherence interval.

1) Rate-Optimal Rule: It can be shown that setting $\eta^* = 0$ leads to the following novel rate-optimal rule.

Corollary *1:* The optimal ASPA rule that maximizes the average rate subject to the interference-outage and peak power constraints is as follows:

$$P_i = \begin{cases} \frac{\tau}{g_i}, & \text{if } P_{\max} \in \left[\frac{\tau}{g_i}, \frac{\sigma^2}{h_i}e^{\lambda}\left(1+\frac{\tau h_i}{\sigma^2 g_i}\right) - \frac{\sigma^2}{h_i}\right), \\ P_{\max}, & \text{else}, \end{cases}$$
$$s = \underset{i\in\{1,\ldots,N_{\text{t}}\}}{\operatorname{argmax}} \left\{\log\left(1+\frac{P_i h_i}{\sigma^2}\right) - \lambda \mathbb{1}_{\{P_i g_i > \tau\}}\right\}, \tag{18}$$

where λ is chosen such that $\Pr(P_s g_s > \tau) = O_{\max}$.

A. Insights: Small P_{max} Regime

In this regime, the interference-outage constraint is inactive and $\lambda^* = 0$ because the transmit power is small. Here, we can show that the antenna with the largest STx-SRx channel power gain is optimal. From (14), it is easy to see that (h_i, g_i) for antenna i falls in region $\mathcal{R}_1$ with high probability in this regime. Also, from the expressions for α_0 and α_2 in Result 1, it follows that $\alpha_2 \to \alpha_0$ as $P_{\max} \to 0$. Hence, $P_i^* = P_{\max}$, for $h_i > \alpha_0$. Thus, from (12a), EE-ASPA simplifies to

$$s^* = \underset{i\in\{1,\ldots,N_{\text{t}}\}}{\operatorname{argmax}} \{h_i\} \text{ and } P_i^* = \begin{cases} 0, & \text{if } h_i \le \alpha_0, \\ P_{\max}, & \text{else.} \end{cases} \tag{19}$$

Therefore, binary power control is optimal for small $P_{\max}$.

For this policy, the outage probability P_{out} and the EE can be analyzed. For Rayleigh fading, $h_1, \ldots, h_{N_{\text{t}}}$ are independent and identically distributed exponential random variables with unit power, and so are $g_1, \ldots, g_{N_{\text{t}}}$.

Result *2:* In the low $P_{\max}$ regime, P_{out} is given in closed-form by

$$P_{\text{out}} = N_{\text{t}} e^{-\frac{\tau}{P_{\max}}} \sum_{k=0}^{N_{\text{t}}-1} \binom{N_{\text{t}}-1}{k} \frac{(-1)^k}{k+1} e^{-(k+1)\alpha_0}, \tag{20}$$

and η^* is the solution of the following equation:

$$\eta = N_{\text{t}}\left[\xi P_{\max}\left(1 - \left[1 - e^{-\eta\xi\sigma^2}\right]^{N_{\text{t}}}\right) + P_{\text{c}}\right]^{-1}$$
$$\times \sum_{k=0}^{N_{\text{t}}-1} \binom{N_{\text{t}}-1}{k} \frac{(-1)^k}{k+1} \left[\log\left(1+\eta\xi P_{\max}\right) e^{-(k+1)\eta\xi\sigma^2}\right.$$
$$\left. + e^{\frac{(k+1)\sigma^2}{P_{\max}}} E_1\left((k+1)\sigma^2\left[\frac{1}{P_{\max}} + \eta\xi\right]\right)\right], \tag{21}$$

where $E_1(x) = \int_x^\infty e^{-t}/t dt$ is the exponential integral [20, Ch. 5.1].

Proof: The proof is skipped to conserve space. ■

The expression in (20) brings out how P_{out} decreases as $P_{\max}$ decreases. Computing η^* from (21) is much simpler than computing it from the iterative approach discussed earlier.

IV. Benchmarking and Numerical Results

We compare the EE, average rate, and average power consumption of EE-ASPA with the rate-optimal rule, minimum interference rule [12], maximum ratio rule [13], and maximum signal power rule [14]. As originally proposed, the last three rules set the power as $P_i = \min\{P_{\max}, \tau/g_i\}$ and have a zero interference-outage probability. In order to enable the secondary system to exploit the non-zero outage probability $O_{\max}$ that is permitted and ensure a fair comparison, we modify the power adaptation for these rules as follows: $P_i = P_{\max}$, if $g_i \le \tau/P_{\max}$, else

$$P_i = \begin{cases} P_{\max}, & \text{with probability } q, \\ \frac{\tau}{g_i}, & \text{with probability } 1-q, \end{cases} \tag{22}$$

where q is set numerically such that $\Pr(P_s g_s > \tau) = O_{\max}$. The minimum interference rule selects antenna $s =$

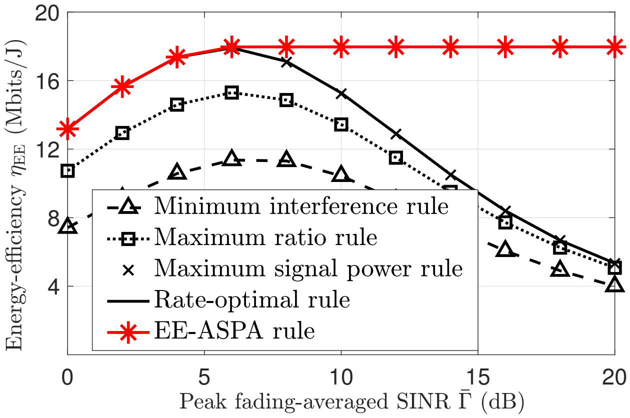
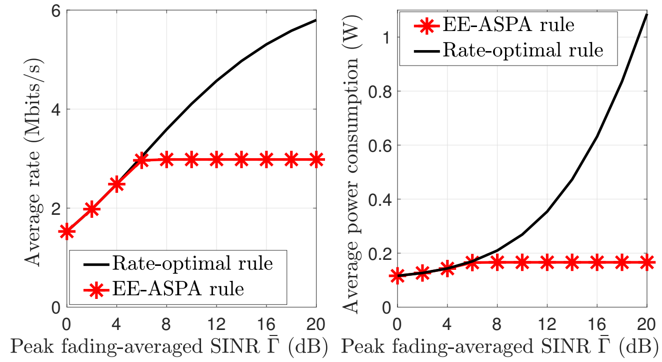


Fig. 3. EE comparison of various ASPA rules ($N_{\rm t} = 4$, $\tau/\sigma^2 = 3$ dB, and $O_{\max} = 0.05$).



(a) Average rate. (b) Average power consumption.

Fig. 4. Trade-off between the average rate and the average power consumption ($N_{\rm t} = 4$, $\tau/\sigma^2 = 3$ dB, and $O_{\max} = 0.05$).

$\text{argmin}_{i\in\{1,\ldots,N_{\rm t}\}}\{g_i\}$. The maximum ratio rule selects antenna $s = \text{argmax}_{i\in\{1,\ldots,N_{\rm t}\}}\{h_i/g_i\}$. The maximum signal power rule selects antenna $s = \text{argmax}_{i\in\{1,\ldots,N_{\rm t}\}}\{h_i P_i\}$.

In the Monte Carlo simulations, we average over 10^5 realizations of Rayleigh fading channels. The parameters are set such that the peak fading-averaged SINR $\bar{\Gamma} = P_{\max}\mu/\sigma^2$ is 7 dB. For example, this is achieved when $P_{\max} = 15$ dBm, the path-loss for the STx-SRx link μ is -114 dB, the path-loss for the STx-PRx link is -121 dB, σ^2 is -106 dBm, and the bandwidth is 1 MHz. Here, $P_{\rm c} = 98$ mW and $\xi = 2.86$ [16]. We have found that atmost 4 iterations of η and atmost 1000 iterations of λ for a given η are required for the algorithm to converge.

Figure 3 plots the EE in Mbits/J as a function of $\bar{\Gamma}$ for the aforementioned rules. For small $P_{\max}$, the peak power constraint limits the EE. Increasing $P_{\max}$, therefore, leads to an increase in the EE. However, for large $P_{\max}$, the interference-outage constraint limits the EE, which then saturates for EE-ASPA. However, for the other rules, the EE decreases for larger values of $P_{\max}$. The rate-optimal rule and EE-ASPA have the same EE for a $\bar{\Gamma}$ up to 6 dB. This is because for small values of $P_{\max}$, the average power consumption is dominated by $P_{\rm c}$, and maximizing EE and average rate are equivalent.

To better understand the trade-off between rate and power consumed, Figure 4(a) plots the average rates of the EE-ASPA and rate-optimal rules, and Figure 4(b) plots the average power consumed by them. The average rate and average power of EE-ASPA both increase as $\bar{\Gamma}$ increase, and they eventually saturate. EE-ASPA avoids larger rates because of the much larger transmit powers they require. On the other hand, as $\bar{\Gamma}$ increases, the average rate of the rate-optimal rule increases at a logarithmic rate but its average power consumed increases exponentially. Thus, its EE eventually decreases.

V. Conclusion

We proposed a novel and optimal ASPA rule that maximized the EE of an underlay spectrum sharing system that was subject to the interference-outage and the peak power constraints. The selected antenna and its power maximized a reward function that consisted of three terms and was parametrized by the penalty constants η^* and λ^*. These constants were determined iteratively. We saw that the optimal power depended on which of the four regions the STx-SRx and STx-PRx channel gains lay in. The optimal rule achieved a much higher EE than the ASPA rules proposed in the literature. Unlike these rules, its average rate and power consumed both saturated as the peak power increased. An interesting avenue for future work is to consider imperfect channel state information at the STx.

Appendix

A. Proof of Lemma 1

We say that an ASPA rule is feasible when it satisfies both constraints in (4) and (5). Consider any feasible ASPA rule $(s, P_s) = \phi(\mathbf{h}, \mathbf{g})$. From the definition of ϕ^* in (7),

$$\Omega_{s^*}\left(P_{s^*}^*; \eta, \lambda\right) \geq \Omega_s\left(P_s; \eta, \lambda\right). \qquad (23)$$

Averaging (23) over $\mathbf{h}$ and $\mathbf{g}$, and rearranging terms that constitute $\Omega_i\left(P_i; \eta, \lambda\right)$ yields

$$\mathbb{E}\left[\log\left(1 + \frac{P_{s^*}^* h_{s^*}}{\sigma^2}\right) - \eta\xi P_{s^*}^*\right] \geq \mathbb{E}\left[\log\left(1 + \frac{P_s h_s}{\sigma^2}\right)\right] - \eta\xi\mathbb{E}\left[P_s\right] - \lambda\left(\mathbb{E}\left[\mathbb{1}_{\{P_s g_s > \tau\}}\right] - \mathbb{E}\left[\mathbb{1}_{\left\{P_{s^*}^* g_{s^*} > \tau\right\}}\right]\right). \qquad (24)$$

We know that $\mathbb{E}\left[\mathbb{1}_{\{P_s g_s > \tau\}}\right] = \Pr\left(P_s g_s > \tau\right) \leq O_{\max}$ since ϕ is a feasible ASPA rule. Furthermore, from the choice of λ, $\mathbb{E}\left[\mathbb{1}_{\left\{P_{s^*}^* g_{s^*} > \tau\right\}}\right] = O_{\max}$. Thus,

$$\mathbb{E}\left[\mathbb{1}_{\{P_s g_s > \tau\}}\right] - \mathbb{E}\left[\mathbb{1}_{\left\{P_{s^*}^* g_s^* > \tau\right\}}\right] = \Pr\left(P_s g_s > \tau\right) - O_{\max} \leq 0.$$

Hence, $\mathbb{E}\left[\log\left(1 + \frac{P_{s^*}^* h_{s^*}}{\sigma^2}\right) - \eta\xi P_{s^*}^*\right] \geq \mathbb{E}\left[\log\left(1 + \frac{P_s h_s}{\sigma^2}\right)\right] - \eta\xi\mathbb{E}\left[P_s\right]$. Thus, EE-ASPA solves the problem $\mathcal{P}_1(\eta)$.

B. Proof of Lemma 2

We are given that the maximum value of the objective function of $\mathcal{P}_1(\eta^*)$ is 0. From Lemma 1, we know that ϕ^* in (7) maximizes the objective function of $\mathcal{P}_1(\eta^*)$ when

$\eta = \eta^*$. Let $(s, P_s) = \phi(\mathbf{h}, \mathbf{g})$ be any feasible ASPA rule. Hence,

$$\mathbb{E}\left[\log\left(1+\frac{P_s h_s}{\sigma^2}\right)\right] - \eta^* \mathbb{E}\left[\xi P_s + P_{\mathrm{c}}\right] \le 0, \text{ for } \phi,$$
$$\mathbb{E}\left[\log\left(1+\frac{P_{s^*}^* h_{s^*}}{\sigma^2}\right)\right] - \eta^* \mathbb{E}\left[\xi P_{s^*}^* + P_{\mathrm{c}}\right] = 0, \text{ for } \phi^*. \quad (25)$$

Rearranging terms, we get

$$\eta^* = \frac{\mathbb{E}\left[\log\left(1+\frac{P_{s^*}^* h_{s^*}}{\sigma^2}\right)\right]}{\mathbb{E}\left[\xi P_{s^*}^* + P_{\mathrm{c}}\right]} \ge \frac{\mathbb{E}\left[\log\left(1+\frac{P_s h_s}{\sigma^2}\right)\right]}{\mathbb{E}\left[\xi P_s + P_{\mathrm{c}}\right]}. \quad (26)$$

Hence, the result follows.[3]

C. Brief Proof of Result 1

We consider the cases $P_{\max} \le \tau/g_i$ and $P_{\max} > \tau/g_i$ separately below.

1) $P_{max} \le \tau/g_i$: This corresponds to the region $\mathcal{R}_1$ in (14a). In this case, for $P_i \le \tau/g_i$, we get

$$\Omega_i\left(P_i; \eta^*, \lambda^*\right) = \log\left(1+\frac{P_i h_i}{\sigma^2}\right) - \eta^* \xi P_i. \quad (27)$$

Thus, $\Omega_i\left(P_i; \eta^*, \lambda^*\right)$ is concave in P_i. From the first order conditions, we can show that $\widetilde{P}\left(h_i\right)$ in (13) maximizes $\Omega_i\left(P_i; \eta^*, \lambda^*\right)$. Combining this with the peak power constraint, we get $P_i^* = \min\left\{\widetilde{P}\left(h_i\right), P_{\max}\right\}$.

2) $P_{max} > \tau/g_i$: In this case, for $P_i > \tau/g_i$, we get

$$\Omega_i\left(P_i; \eta^*, \lambda^*\right) = \log\left(1+\frac{P_i h_i}{\sigma^2}\right) - \eta^* \xi P_i - \lambda^*. \quad (28)$$

The following three sub-cases arise depending on where $\widetilde{P}\left(h_i\right)$ lies with respect to τ/g_i and $P_{\max}$:

a) $\widetilde{P}\left(h_i\right) \le \tau/g_i < P_{max}$: Rearranging the terms in $\widetilde{P}\left(h_i\right) \le \tau/g_i$ yields $h_i \le \alpha_1(g_i)$, where $\alpha_1(g_i)$ is given in (15). This corresponds to the region $\mathcal{R}_2$ in (14b). The reward function is given by (27), for $P_i \in (0, \tau/g_i]$, and by (28), otherwise. Since $\widetilde{P}\left(h_i\right) \le \tau/g_i$, from the above discussion, it maximizes $\Omega_i\left(P_i; \eta^*, \lambda^*\right)$. Also, $\widetilde{P}\left(h_i\right) < P_{\max}$ in $\mathcal{R}_2$. Therefore, $P_i^* = \widetilde{P}\left(h_i\right)$. This yields (12b).

b) $\tau/g_i < \widetilde{P}\left(h_i\right) \le P_{max}$: From the definition of $\widetilde{P}\left(h_i\right)$ in (13), it is easy to see that the condition $\tau/g_i < \widetilde{P}\left(h_i\right) \le P_{\max}$ is equivalent to $\alpha_1(g_i) < h_i \le \alpha_2$, where α_2 is given in (15). This corresponds to region $\mathcal{R}_3$ in (14c). Here, $\Omega_i\left(P_i; \eta^*, \lambda^*\right)$ monotonically increases for $P_i \in [0, \tau/g_i]$. At $P_i = \tau/g_i$, $\Omega_i\left(P_i; \eta^*, \lambda^*\right)$ drops by λ^*. It, then monotonically increases for $P_i \in (\tau/g_i, \widetilde{P}\left(h_i\right)]$, and monotonically decreases for $P_i \in (\widetilde{P}\left(h_i\right), P_{\max}]$. Hence, the reward function is maximized at either τ/g_i or $\widetilde{P}\left(h_i\right)$. This yields (12c).

c) $\tau/g_i < P_{max} < \widetilde{P}\left(h_i\right)$: It can be shown that the condition $P_{\max} < \widetilde{P}\left(h_i\right)$ is equivalent to $h_i > \alpha_2$. This corresponds to the region $\mathcal{R}_4$ in (14d). Here, $\Omega_i\left(P_i; \eta^*, \lambda^*\right)$ monotonically increases for $P_i \in [0, \tau/g_i]$. Then, as above, it drops by λ^* at $P_i = \tau/g_i$. It then monotonically increases for $P_i \in (\tau/g_i, P_{\max}]$. Hence, the reward function is maximized at either τ/g_i or $P_{\max}$. This yields (12d).

Substituting P_i^* in (7) yields the optimal antenna in (11).

[3]The proof uses ideas from [17]. Our problem is a stochastic fractional programming problem which differs from that in [17].

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