A Novel Constrained Estimator for Selective Feedback in OFDM and its Implications

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***Abstract*—Reduced feedback schemes facilitate scheduling and rate adaptation in orthogonal frequency division multiplexing systems while reducing the number of subchannels (SCs) for which channel state information (CSI) is fed back. In the popular selective feedback scheme, each user feeds back CSI about only its strongest SC to the base station (BS). We focus on the less studied, but practically important problem of transmitting reliably even on SCs that are fed back by no user or few users. We derive a constrained, non-linear minimum mean square error estimator that enables the BS to estimate the power gains of all the unreported SCs of a user. The novelty of the estimator lies in its exploitation of the structure of feedback generated by the selective feedback scheme. It improves the average cell throughput compared to several conventional approaches – without any additional feedback overhead. The improvements occur for both uncorrelated and correlated SCs, and for the channel-aware greedy and fair round-robin schedulers.**

I. Introduction

Techniques such as channel-aware scheduling and rate adaptation have enabled current and next generation orthogonal frequency division modulation (OFDM) systems to offer high data rates to many users [1]. In these systems, several contiguous subcarriers are grouped into subchannels (SCs), with the bandwidth of an SC typically being less than the coherence bandwidth of the channel. For example, in Long Term Evolution (LTE), twelve subcarriers are grouped together into a physical resource block of bandwidth of 180 kHz [2].

In order to implement scheduling and rate adaptation, considerable channel state information (CSI) is required by the base station (BS) as it needs to know the channel gains of all the users for all the SCs. This information needs to be fed back by the users to the BS in frequency-division duplexing systems since the uplink and downlink channels are not reciprocal. Feedback is also necessary in time-division duplexing systems when the uplink and downlink interferences or the transmit and receive radio frequency chains are not symmetric. Such a large feedback overhead lowers the overall spectral efficiency of the system and can overwhelm the uplink feedback channels.

To reduce the feedback overhead, several schemes have been proposed in the OFDM literature. These include threshold-based feedback [3], [4], best-m feedback [5]–[8], and subcarrier clustering [9]. In the best-m scheme, each user sorts its SC gains in descending order. It then reports to the BS, the channel power gains and indices of only the SCs with the m largest power gains. The selective feedback scheme corresponds to $m = 1$, which ensures the lowest feedback overhead.

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While selective feedback significantly reduces the feedback overhead, it has its downsides. It leads to scenarios in which some of the SCs are not reported by any of the users. In [5], [6], [8], [10], when no user reports its gain for an SC, BS does not transmit data on that SC as it does not know the channel gain of that SC for any user. However, this reduces the downlink spectral efficiency. Another issue is that the approaches in [5], [6], [8], [10] only select a user from among those that reported that SC. This limits the ability of the BS to exploit multi-user diversity when few users report an SC.

A. Focus and Contributions

We address the less studied, but important problem of transmitting reliably even on SCs that are fed back by no user or few users. We develop a systematic solution for selective feedback. Our results are practically relevant for fourth generation cellular standards such as LTE, which employ a variant of the best-m scheme [2]. We make the following contributions:

- We propose a novel, constrained minimum mean square error (MMSE) channel estimator for the selective feedback scheme, which enables the BS to assign an SC to a user who might not even have reported it. Its novelty lies in its exploitation of the additional information provided by selective feedback, which is that the estimated SC power gains must be less than that of the reported SC. This makes our problem different from classical OFDM channel estimation, in which the receiver estimates the subcarrier gains using pilots [11]. Here, the MMSE estimate is linear when the complex baseband subcarrier gains are jointly Gaussian. Instead, our constrained MMSE estimate is a non-linear function of the reported SC gains.
- We first derive the MMSE estimate in closed-form when the SC gains are uncorrelated. We then generalize our approach to exponentially and uniformly correlated SC gains by using results from order statistics of correlated random variables (RVs).
- We also study the system-level performance implications. For two different frequency-domain schedulers, namely, the channel-aware greedy scheduler, which maximizes cell throughput, and the round-robin (RR) scheduler, which ensures fairness, we show that the proposed estimator improves downlink cell throughput. Another issue that is addressed is how the rate adaptation algorithm should handle errors in the estimate obtained.
- We show that our approach reduces the root mean square error (RMSE) and increases the average throughput com-

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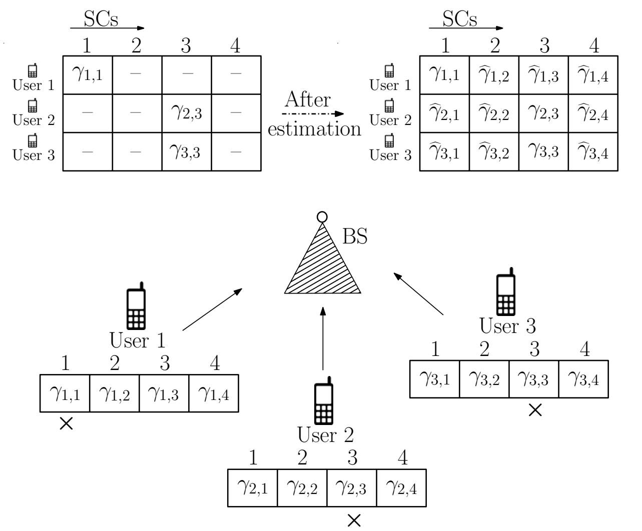

Fig. 1. Illustration of best-m feedback by 3 users and scheduling at the BS ($N = 4$ and $m = 1$). Cross marks ($\times$) denote the SCs that are fed back.

pared to the approaches pursued in [5], [6], [8], [12]. Notably, these gains are achieved without requiring any additional feedback.

B. Organization and Notation

The system model is discussed in Sec. II. The MMSE estimator is presented in Sec. III. The throughput implications are studied in Sec. IV. Our conclusions follow in Sec. V.

The notation $\underline{y} \leq x$ means that all the elements of vector $\underline{y}$ are less than or equal to x. The probability of an event A is denoted by $\mathbf{P}\left[A\right]$. The conditional probability of A given B is denoted by $\mathbf{P}\left[A|B\right]$. The cumulative distribution function (CDF) of an RV X is denoted by $F_X(\cdot)$, the probability density function (PDF) by $f_X(\cdot)$, and the conditional PDF of RV X given $Y = y$ by $f_X(x|Y = y)$. Expectation with respect to RV X is denoted by $\mathbb{E}_X\left[\cdot\right]$. The subscript shall be dropped if it is obvious from context. The expectation conditioned on event A by $\mathbb{E}\left[\cdot|A\right]$. The notation $p\left(X = x, A\right)$ involving RV X and event A is defined as $\lim_{\delta \to 0} \frac{P[x \leq X \leq x+\delta, A]}{\delta}$. Furthermore, $\binom{s}{l_1,\ldots,l_p}$ denotes $\frac{s!}{l_1!\ldots l_p!}$ and $(\cdot)^\star$ denotes complex conjugate.

II. System Model

The system bandwidth is divided into N SCs. There are K users in a cell, each with a single antenna. For SC n, let $H_{k,n}$ denote the downlink SC gain from the BS to user k. We assume Rayleigh fading. Therefore, the SC power gains $\gamma_{k,n} = |H_{k,n}|^2$, for $1 \leq n \leq N$, are exponential RVs each with mean $\mathbb{E}\left[|H_{k,n}|^2\right] = \bar{\gamma}_k$. Note that the mean power gain of all SCs of a user can be shown to be the same as they have the same marginal probability distribution [1]. The SC gains of different users are assumed to be mutually independent, which is justified since the users are spatially separated.

In the selective feedback scheme, each user orders its SCs according to their power gains. For a user k, the ordered SC power gains are denoted as

$$\gamma_{k,1:N} \leq \gamma_{k,2:N} \leq \cdots \leq \gamma_{k,N:N},$$

where $r:N$ denotes the index of the SC with the r^{th} smallest power gain [13]. The users are assumed to know their SC power gains without error [3]–[7], [9]. The user then feeds back its largest SC power gain, $\gamma_{k,N:N}$ along with the corresponding index to the BS. We note that, in practice, the index of the modulation and coding (MCS) is fed back instead of the gain [6]. However, this is beyond the scope of this paper given that the constrained estimator with SC power gain feedback itself has not been explored before.

III. MMSE Estimator for Selective Feedback

Let $\underline{\gamma}_k^{(n)} = [\gamma_{k,1}, \ldots, \gamma_{k,n-1}, \gamma_{k,n+1}, \gamma_{k,N}]$. Given that SC n is the best (i.e., $\underline{\gamma}_k^{(n)} \leq x$) and its power gain, which is reported to the BS, is $\gamma_{k,n} = x$, the MMSE estimate $\widehat{\gamma}_{k,j}$ of the power gain of SC j, for $j \neq n$, can be shown from first principles to be [14]

$$\widehat{\gamma}_{k,j} = \mathbb{E}\left[\gamma_{k,j} | \gamma_{k,n} = x,\ \underline{\gamma}_k^{(n)} \leq x\right]. \tag{1}$$

Using the theorem of expectation and Bayes' theorem, we get

$$\begin{aligned} \widehat{\gamma}_{k,j} &= \int_0^\infty y f_{\gamma_{k,j}}\left(y | \gamma_{k,n} = x, \underline{\gamma}_k^{(n)} \leq x\right) dy, \\ &= \frac{\int_0^\infty y p\left(\gamma_{k,j} = y, \gamma_{k,n} = x, \underline{\gamma}_k^{(n)} \leq x\right) dy}{p(\gamma_n = x, \underline{\gamma}_k^{(n)} \leq x)}. \end{aligned} \tag{2}$$

A. Uncorrelated SC Gains

To build intuition, we first study the scenario in which the SC gains are uncorrelated.

Result 1: For uncorrelated SC gains, the MMSE estimate $\widehat{\gamma}_{k,j}$ of unreported SC j, given that SC $n \neq j$ is the best and its reported power gain is $\gamma_{k,n} = x$, is given by

$$\widehat{\gamma}_{k,j} = \bar{\gamma}_k - \frac{x}{e^{\frac{x}{\bar{\gamma}_k}} - 1}. \tag{3}$$

Proof: Since the SC gains are independent and identically distributed (i.i.d.) exponential RVs, for $x \geq 0$, the denominator in (2) is given by

$$\begin{aligned} p\left(\gamma_{k,n} = x, \underline{\gamma}_k^{(n)} \leq x\right) &= f_{\gamma_{k,n}}(x) \left(\mathbf{P}\left[\gamma_{k,n} \leq x\right]\right)^{N-1}, \\ &= \frac{e^{-\frac{x}{\bar{\gamma}_k}}}{\bar{\gamma}_k}\left(1 - e^{-\frac{x}{\bar{\gamma}_k}}\right)^{N-1}. \end{aligned} \tag{4}$$

Evaluating numerator of (2)*:* Again exploiting the fact that the SC gains are i.i.d. exponential RVs, the probability term in the integrand in the numerator is given by

$$\begin{aligned} &p\left(\gamma_{k,j} = y, \gamma_{k,n} = x, \underline{\gamma}_k^{(n)} \leq x\right) \\ &\quad = \begin{cases} \frac{e^{-\frac{x}{\bar{\gamma}_k}}}{\bar{\gamma}_k} \frac{e^{-\frac{y}{\bar{\gamma}_k}}}{\bar{\gamma}_k}\left(1 - e^{-\frac{x}{\bar{\gamma}_k}}\right)^{N-2}, & 0 \leq y \leq x, \\ 0, & \text{else.} \end{cases} \end{aligned} \tag{5}$$

Using this, the integral in the numerator evaluates to $e^{-\frac{x}{\bar{\gamma}_k}}\left(1 - e^{-\frac{x}{\bar{\gamma}_k}}\right)^{N-2}\left(1 - e^{-\frac{x}{\bar{\gamma}_k}}\left(1 + \frac{x}{\bar{\gamma}_k}\right)\right)$.

Substituting this and (4) in (2) yields the result in (3). ∎

Notice that here, $\widehat{\gamma}_{k,j}(x)$ does not depend on N. Further, it is a non-linear function of x. As shown in Appendix B, the mean square error (MSE) for SC j, given that SC $n \neq j$ is the best, can be written in closed-form as

$$\mathrm{MSE} = \bar{\gamma}_k^2 + \frac{N\bar{\gamma}_k^2}{N-1}\left(\Psi^{(1)}(1) - \Psi^{(1)}(N) + \left[\sum_{u=1}^{N-1}\frac{1}{u}\right]^2\right) - \frac{N\bar{\gamma}_k^2}{N-2}\left(\Psi^{(1)}(1) - \Psi^{(1)}(N-1) + \left[\sum_{u=1}^{N-2}\frac{1}{u}\right]^2\right), \quad (6)$$

where $\Psi^{(1)}(\cdot)$ is the trigamma function [15, Table 6.1]. Notice also that the ratio $\mathrm{MSE}/\bar{\gamma}_k^2$ is independent of $\bar{\gamma}_k$.

B. Uniform and Exponential Correlation Models

In the uniform correlation model, the coefficient of correlation between any two SC gains of a user is the same: $\frac{\mathbb{E}[H_n H_l^\star]}{\bar{\gamma}_k} = \rho$, for $\rho \geq 0$ and $n \neq l, 1 \leq n, l \leq N$.

Result 2: For uniformly correlated SC gains, the MMSE estimate $\widehat{\gamma}_{k,j}$ of SC j, given that SC n is the best and its reported power gain is $\gamma_{k,n} = x$, is $\widehat{\gamma}_{k,j} = \frac{A_j(x,n)}{B(x)}$, where

$$A_j(x,n) = \sum_{s=0}^{\infty}\left(\frac{\rho}{1+(N-1)\rho}\right)^s \sum_{\substack{l_1,\ldots,l_N \geq 0 \\ l_1+\cdots+l_N = s}} \binom{s}{l_1,\ldots,l_N} \times \zeta_{l_n+1}\left(x, \bar{\gamma}_k(1-\rho)\right)(l_j+1)\bar{\gamma}_k(1-\rho) \times L\left(\frac{x}{\bar{\gamma}_k(1-\rho)}, l_j+2\right)\prod_{\substack{i=1 \\ i\neq j,n}}^{N} L\left(\frac{x}{\bar{\gamma}_k(1-\rho)}, l_i+1\right), \quad (7)$$

$$B(x) = \frac{1}{N}\sum_{s=0}^{\infty}\left(\frac{\rho}{1+(N-1)\rho}\right)^s \sum_{\substack{l_1,\ldots,l_N \geq 0 \\ l_1+\cdots+l_N = s}} \binom{s}{l_1,\ldots,l_N} \times \sum_{p=1}^{N}\zeta_{l_p+1}\left(x, \bar{\gamma}_k(1-\rho)\right)\prod_{\substack{i=1 \\ i\neq p}}^{N} L\left(\frac{x}{\bar{\gamma}_k(1-\rho)}, l_i+1\right), \quad (8)$$

$\zeta_l(v,c) = \frac{e^{-\frac{v}{c}}v^{l-1}}{(l-1)!c^l}$, and $L(x,l) = \frac{1}{(l-1)!}\int_0^x e^{-t}t^{l-1}\,dt$ is the lower incomplete Gamma function [15].

Proof: The proof is given in Appendix A. ∎

Note: The infinite series in (7) and (8) are typical in several problems involving correlated gains [16]–[18]. For our problem, we have found that 25 terms for $\rho = 0.5$ and 65 terms for $\rho = 0.8$ are sufficient to ensure numerical accuracy. The number of terms increases with ρ because of the presence of the $1-\rho$ denominator term in $L\left(\frac{x}{\bar{\gamma}_k(1-\rho)}, l\right)$ in (7) and (8).

In the exponential correlation model, for a user k, the coefficient of correlation between SCs n and l is given by $\mathbb{E}\left[H_{k,n}H_{k,l}^\star\right]/\bar{\gamma}_k = \rho^{|n-l|}$, for $n \neq l, 1 \leq n, l \leq 1, \ldots, N$.

Result 3: For exponentially correlated SC gains, the MMSE estimate $\widehat{\gamma}_{k,j}$ of SC j, given that SC n is the best and its reported power gain is $\gamma_{k,n} = x$, is given by $\widehat{\gamma}_{k,j} = \frac{A_j(x,n)}{B(x)}$. Here,

$$A_N(x,1) = (1-\rho^2)\sum_{s=0}^{\infty}\rho^{2s}\sum_{\substack{l_1,\ldots,l_{N-1}\geq 0 \\ l_1+\cdots+l_{N-1}=s}} \zeta_{l_1+1}\left(x, \bar{\gamma}_k(1-\rho^2)\right) \times \Psi(x, l_{N-1})\prod_{i=2}^{N-1}\Omega_2(x, l_{i-1}, l_i). \quad (9)$$

For $1 < j < N$,

$$A_j(x,1) = (1-\rho^2)\sum_{s=0}^{\infty}\rho^{2s}\sum_{\substack{l_1,\ldots,l_{N-1}\geq 0 \\ l_1+\cdots+l_{N-1}=s}} \zeta_{l_1+1}\left(x, (1-\rho^2)\bar{\gamma}_k\right) \times L\left(\frac{x}{\bar{\gamma}_k(1-\rho^2)}, l_{N-1}+1\right)\Omega_1(x, l_{j-1}, l_j) \times \prod_{\substack{i=2 \\ i\neq j}}^{N-1}\Omega_2(x, l_{i-1}, l_i). \quad (10)$$

For $1 < n < N$,

$$A_N(x,n) = (1-\rho^2)\sum_{s=0}^{\infty}\rho^{2s}\sum_{\substack{l_1,\ldots,l_{N-1}\geq 0 \\ l_1+\cdots+l_{N-1}=s}} L\left(\frac{x}{(1-\rho^2)\bar{\gamma}_k}, l_1+1\right) \times \Psi(x, l_{N-1})\Phi(x, l_{n-1}, l_n)\prod_{\substack{i=2 \\ i\neq n}}^{N-1}\Omega_2(x, l_{i-1}, l_i), \quad (11)$$

$$A_1(x,n) = (1-\rho^2)\sum_{s=0}^{\infty}\rho^{2s}\sum_{\substack{l_1,\ldots,l_{N-1}\geq 0 \\ l_1+\cdots+l_{N-1}=s}} L\left(\frac{x}{(1-\rho^2)\bar{\gamma}_k}, l_{N-1}+1\right) \times \Psi(x, l_1)\Phi(x, l_{n-1}, l_n)\prod_{\substack{i=2 \\ i\neq n}}^{N-1}\Omega_2(x, l_{i-1}, l_i). \quad (12)$$

For $1 < j < N$ and $j \neq n$,

$$A_j(x,n) = (1-\rho^2)\sum_{s=0}^{\infty}\rho^{2s}\sum_{\substack{l_1,\ldots,l_{N-1}\geq 0 \\ l_1+\cdots+l_{N-1}=s}} L\left(\frac{x}{(1-\rho^2)\bar{\gamma}_k}, l_1+1\right) \times L\left(\frac{x}{\bar{\gamma}_k(1-\rho^2)}, l_{N-1}+1\right)\Phi(x, l_{n-1}, l_n) \times \Omega_1(x, l_{j-1}, l_j)\prod_{\substack{i=2 \\ i\neq j,n}}^{N-1}\Omega_2(x, l_{i-1}, l_i). \quad (13)$$

Further, $A_1(x,N) = A_N(x,1)$. Lastly, for $1 < j < N$,

$$A_j(x,N) = (1-\rho^2)\sum_{s=0}^{\infty}\rho^{2s}\sum_{\substack{l_1,\ldots,l_{N-1}\geq 0 \\ l_1+\cdots+l_{N-1}=s}} L\left(\frac{x}{\bar{\gamma}_k(1-\rho^2)}, l_1+1\right) \times \zeta_{l_{N-1}+1}\left(x, (1-\rho^2)\bar{\gamma}_k\right)\Omega_1(x, l_{j-1}, l_j) \times \prod_{\substack{i=2 \\ i\neq j}}^{N-1}\Omega_2(x, l_{i-1}, l_i). \quad (14)$$

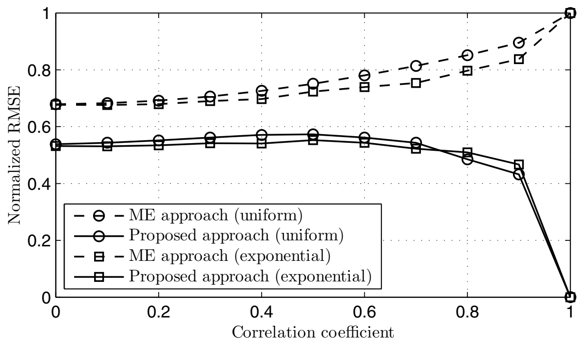

Fig. 2. Normalized RMSE (RMSE/$\bar{\gamma}$) as a function of correlation coefficient ρ for ME and proposed approaches ($N = 4$).

In the above expressions, we have:

$\Omega_1(x,l_1,l_2) = \frac{\binom{l_1+l_2}{l_1} L\left(\frac{x(1+\rho^2)}{(1-\rho^2)\bar{\gamma}_k}, l_1+l_2+2\right)(l_1+l_2+1)(1-\rho^2)\bar{\gamma}_k}{(1+\rho^2)^{l_1+l_2+2}}$,

$\Omega_2(x,l_1,l_2) = \frac{\binom{l_1+l_2}{l_1} L\left(\frac{x(1+\rho^2)}{(1-\rho^2)\bar{\gamma}_k}, l_1+l_2+1\right)}{(1+\rho^2)^{l_1+l_2+1}}$,

$\Phi(x,l_1,l_2) = \binom{l_1+l_2}{l_1} \frac{\zeta_{l_1+l_2+1}\left(x, \frac{(1-\rho^2)\bar{\gamma}_k}{(1+\rho^2)}\right)}{(1+\rho^2)^{l_1+l_2+1}}$, and

$\Psi(x,l) = (l+1)\bar{\gamma}_k(1-\rho^2) L\left(\frac{x}{(1-\rho^2)\bar{\gamma}_k}, l+2\right)$.

The denominator $B(x)$ is given by

$$\begin{aligned} B(x) = {} & \frac{(1-\rho^2)}{N} \sum_{s=0}^{\infty} \rho^{2s} \sum_{\substack{l_1,\ldots,l_{N-1}\geq 0 \\ l_1+\cdots+l_{N-1}=s}} \left[\prod_{i=2}^{N-1} \Omega_2(x,l_{i-1},l_i)\right] \\ & \times \left[\zeta_{l_1+1}\left(x,(1-\rho^2)\bar{\gamma}_k\right) L\left(\frac{x}{(1-\rho^2)\bar{\gamma}_k}, l_{N-1}+1\right) \right. \\ & \left. + L\left(\frac{x}{(1-\rho^2)\bar{\gamma}_k}, l_1+1\right) \zeta_{l_{N-1}+1}\left(x,(1-\rho^2)\bar{\gamma}_k\right)\right] \\ & + L\left(\frac{x}{(1-\rho^2)\bar{\gamma}_k}, l_1+1\right) L\left(\frac{x}{(1-\rho^2)\bar{\gamma}_k}, l_{N-1}+1\right) \\ & \times \sum_{j=2}^{N-1} \Phi(x,l_{j-1},l_j) \left[\prod_{\substack{i=2 \\ i\neq j}}^{N-1} \Omega_2(x,l_{i-1},l_i)\right], \end{aligned} \quad (15)$$

Proof: The proof is omitted due to space constraints. ■

C. Simulation Results

Figure 2 plots the RMSE normalized by $\bar{\gamma}_k$ of an unreported SC as a function of ρ for the proposed estimator. Results are shown for both uniform and exponential correlation models. To get a better understanding, also shown is the normalized RMSE for the mean-as-estimate (ME) approach, which ignores the information fed back by the selective feedback scheme and, thus, sets the estimate as $\bar{\gamma}_k$.[1] The normalization with respect to $\bar{\gamma}_k$ is done because it can be shown that, in general, the normalized RMSE is not a function of $\bar{\gamma}_k$ (c.f. (6)). Notice that this behavior is different from conventional OFDM channel estimation in which the RMSE, in fact, decreases as $\bar{\gamma}_k$ increases. Further, note that it is assumed that $\bar{\gamma}_k$ and ρ are known at the BS. In practice, the users can feed back these values, which can be obtained from the power delay profile.

Even when the SC gains are uncorrelated ($\rho = 0$), we observe that the proposed estimator yields a 20.5% lower RMSE. As expected, for $\rho = 0$, the RMSE is the same for the uniform and exponential correlation models. The RMSE is relatively high even though the proposed estimator is optimal in the MSE sense because $N-1$ SC gains are being estimated from just one reported SC gain. The RMSE of the proposed estimator decreases as ρ increases and tends to zero. This behavior is different from the RMSE for the ME approach, which increases to unity as ρ increases. This demonstrates the benefits of taking into account the fed back SC power gain.

[1]The conventional outage approach can be interpreted as follows. It sets the estimated channel gain as zero, which is why it never transmits on an unreported SC. This corresponds to a larger normalized RMSE of unity.

IV. System Impact Evaluation

We now evaluate the impact of using the proposed estimator on the average downlink cell throughput. Now, the BS can use not just the reported SC's power gain but also the estimate of the power gains of all the unreported SCs for each user. For user k, let $\tilde{\gamma}_{k,l}$ denote the power gain of SC n that is used by the BS for scheduling and rate adaptation as follows:

$$\tilde{\gamma}_{k,l} = \begin{cases} \gamma_{k,l}, & \text{if SC } l \text{ is reported by user } k, \\ \widehat{\gamma}_{k,l}, & \text{if SC } l \text{ is not reported by user } k. \end{cases} \quad (16)$$

A. Scheduling and Rate Adaptation

The user $i_n^\star$ assigned to SC n depends on the scheduler. We present results for the following two schedulers:

- *Greedy:* Among all users, the one with the largest power gain for SC n is chosen to transmit on that SC: $i_n^\star = \operatorname{argmax}_{1\leq i\leq K}\ \tilde{\gamma}_{i,n}$.
- *RR:* Users are assigned to an SC in a predetermined order.

We study discrete rate adaptation, since it is always used in practice [2]. The BS has available to it M MCSs indexed $1,2,\ldots,M$ with rates $0 = R_1 < R_2 < \cdots < R_M$. The range $[0,\infty)$ of SC power gains is divided into M disjoint intervals, one for each MCS, by means of $M+1$ thresholds $0 = T_1 < T_2 < \cdots < T_{M+1} = \infty$. For the selected user $i_n^\star$, if $\tilde{\gamma}_{i_n^\star,n} \in [T_r, T_{r+1})$, an MCS r with rate R_r is assigned.

Distinguishing between estimated and reported SC gains: The rate adaptation algorithm needs to handle one subtle difference between how estimation errors affect MSE and throughput. If a user is scheduled on an SC that it has reported, then the rate assigned to that SC will be reliably decoded since $\tilde{\gamma}_{k,n} = \gamma_{k,n}$. However, for an unreported SC, $\widehat{\gamma}_{k,n}$ need not equal $\gamma_{k,n}$. If $\tilde{\gamma}_{k,n} > T_r > \gamma_{k,n}$, then due to the waterfall nature of the error rate curves, an outage occurs as the transmitted packet cannot be decoded by the BS. Instead, if $\tilde{\gamma}_{k,n} < \gamma_{k,n}$, then no such outage occurs as the rate chosen can be reliably decoded. On the other hand, the MMSE criterion treats undershoots and overshoots equally.

We address this issue as follows. When a user is scheduled on an SC it has not reported and $\tilde{\gamma}_{k,n} \in [T_r, T_{r+1})$, then a one-level lower MCS $r-1$ with rate R_{r-1} is assigned to it.

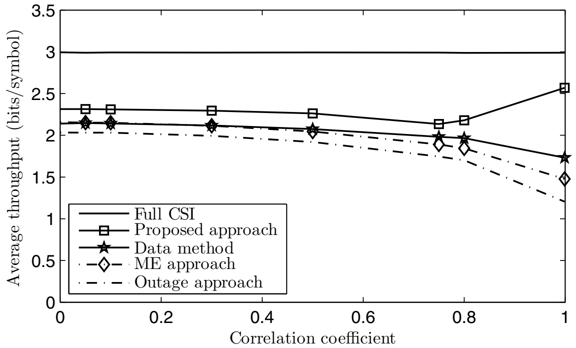

Fig. 3. Greedy scheduler: Throughput as a function of correlation coefficient ρ for different approaches ($K = 10$, $N = 10$, and $\bar{\gamma}_k = 9$ dB, $1 \leq k \leq K$).

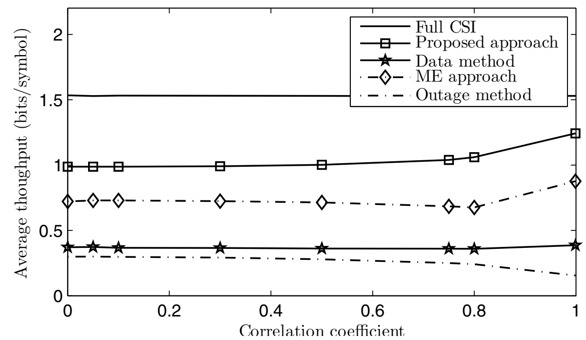

Fig. 4. RR scheduler: Throughput as a function of correlation coefficient ρ for different estimators ($K = 10$, $N = 10$, and $\bar{\gamma}_k = 9$ dB, $1 \leq k \leq K$).

B. Simulation Results

We now present Monte Carlo simulation results that are averaged over 5000 samples. The $M = 16$ rates are as specified in LTE [2, Table 10.1]. These range from $R_2 = 0.15$ bits/symbol to $R_{16} = 5.55$ bits/symbol. The thresholds for discrete rate adaptation are calculated using the formula [19]: $R_r = \log_2(1 + \zeta T_r)$, where $\zeta = 0.398$ accounts for the coding loss of a practical code [19]. The number of users K is 10 and number of SCs N is 10. We focus on the single cell scenario [5]–[9], and set $\bar{\gamma}_1 = \bar{\gamma}_2 = \cdots = \bar{\gamma}_K = 9$ dB.

We benchmark the average cell throughput of the proposed approach against the following approaches:

- *Conventional outage approach [5], [6], [8], [9]:* The BS does not transmit any data on an SC that was not reported by any user, i.e., it declares an outage.
- *ME approach:* The BS sets $\tilde{\gamma}_{k,i} = \bar{\gamma}_k$ if SC n is not reported by user k.
- *Data method [12]:* An SC that is not reported by any user is assigned to the user selected for an adjacent reported SC. Its MCS is one level lower than that assigned to the adjacent SC. In case no adjacent SC has been reported by any user, the BS declares an outage on this SC.
- *Ideal full CSI:* The BS is assumed to know the power gains of all SCs for all users.

Figure 3 plots the average cell throughputs of the various approaches as a function of ρ for the greedy scheduler. For $\rho = 0$, the proposed approach improves average throughput by 7.4% compared to the Data method and the ME approach and 13.8% compared to the outage approach. For $\rho = 0.8$, the corresponding gains increase to 10.7%, 17.9%, and 27.6%. Notice that as ρ increases from 0 to 0.75, the average throughput of the proposed approach marginally decreases. This is due to a reduction in frequency diversity as ρ increases. However, this is compensated by the more accurate estimates for larger ρ. The results for the exponential correlation model are not shown due to space constraints. Given that only one gain is reported among 10 SCs, the average throughput with full CSI of all 10 SCs is a loose upper bound for all the schemes.

Figure 4 plots the corresponding results for the RR scheduler. As expected, it is less than that with the greedy scheduler. In this case, the relative gains of the proposed estimator are even greater. For example, for $\rho = 0$, the gains over ME approach, Data method, and outage approach are 38.8%, 170.2%, and 244.8%, respectively. Another point to note is that the proposed and ME approaches outperform the Data method. This is because the ad hoc approach in the latter makes it transmit only on those SCs that either are reported or are adjacent to the reported SC.

V. Conclusions

We proposed a novel approach for selective feedback in which the BS uses the reported highest SC gain to estimate the gains of the unreported SCs for each user. The BS then utilizes these constrained, non-linear MMSE estimates for scheduling and rate adaptation. We saw that the system throughput is driven not just by the feedback scheme but also the scheme used to estimate the unreported SC gains. Unlike the ME approach, whose RMSE increased as ρ increased, the RMSE of the proposed estimator decreased all the way to 0. We saw that using the proposed approach improved the average cell throughput for greedy and RR schedulers and for both uncorrelated and correlated SCs.

Appendix

A. Proof of Result 2

Evaluating denominator of (2): Using the fact that the SC gains are statistically identical, we get

$$p\left(\gamma_{k,n} = x, \underline{\gamma}_k^{(n)} \leq x\right) = \frac{f_{\gamma_{k,N:N}}(x)}{N}. \tag{17}$$

To evaluate $f_{\gamma_{k,N:N}}(x)$, we start with the joint PDF $f_{\underline{\gamma}}(\underline{y})$ of $\underline{\gamma} = [\gamma_{k,1}, \ldots, \gamma_{k,N}]$, which is given by [18, (91)]

$$f_{\underline{\gamma}}(\underline{y}) = \frac{1-\rho}{1+(N-1)\rho} \sum_{s=0}^{\infty} \left(\frac{\rho}{1+(N-1)\rho}\right)^s \times \sum_{\substack{l_1,\ldots,l_N \geq 0 \\ l_1+\cdots+l_N = s}} \binom{s}{l_1,\ldots,l_N} \prod_{i=1}^{N} \zeta_{l_i+1}\left(y_i, \bar{\gamma}_k(1-\rho)\right). \tag{18}$$

By definition, $F_{\gamma_{k,N:N}}(x) = \int_0^x \ldots \int_0^x f_{\underline{\gamma}}(\underline{y})\, dy_1 \ldots dy_N$. Integrating (18) and using the identity [15] $L\left(\frac{x}{\bar{\gamma}_k(1-\rho)}, l_i+1\right) = \int_0^x \zeta_{l_i+1}\left(y_i, \bar{\gamma}_k(1-\rho)\right) dy_i$, we get

$$F_{\gamma_{k,N:N}}(x) = \frac{1-\rho}{1+(N-1)\rho} \sum_{s=0}^{\infty} \left(\frac{\rho}{1+(N-1)\rho}\right)^s \times \sum_{\substack{l_1,\ldots,l_N \geq 0 \\ l_1+\cdots+l_N=s}} \binom{s}{l_1,\ldots,l_N} \prod_{i=1}^{N} L\left(\frac{x}{\bar{\gamma}_k(1-\rho)}, l_i+1\right).$$

Differentiating this with respect to x and substituting in (17) yields

$$p\left(\gamma_{k,n}=x,\ \underline{\gamma}_k^{(n)} \leq x\right) = B(x) = \frac{1-\rho}{N(1+(N-1)\rho)} \times \sum_{s=0}^{\infty} \left(\frac{\rho}{1+(N-1)\rho}\right)^s \sum_{\substack{l_1,\ldots,l_N \geq 0 \\ l_1+\cdots+l_N=s}} \binom{s}{l_1,\ldots,l_N} \times \sum_{p=1}^{N} \zeta_{l_p+1}\left(x, \bar{\gamma}_k(1-\rho)\right) \prod_{\substack{i=1 \\ i\neq p}}^{N} L\left(\frac{x}{\bar{\gamma}_k(1-\rho)}, l_i+1\right). \quad (19)$$

Evaluating numerator of (2)*:* Without loss of generality, let $j \leq n$. Let the N-length vector $\underline{z}$ be defined as

$$\underline{z} = [z_1,\ldots,z_{j-1}, y, z_{j+1},\ldots,z_{n-1}, x, z_{n+1},\ldots,z_N].$$

The numerator is written as the following $(N-1)$-fold integral over the variables z_k, for $k \in \{1,2,\ldots,N\} \setminus \{j,n\}$, and y:

$$\int_0^x \!\!\cdots\! \int_0^x y f_{\underline{\gamma}}(\underline{z})\, dz_1 \ldots dz_{j-1} dy dz_{j+1} \ldots dz_{n-1} dz_{n+1} \ldots dz_N.$$

Substituting the expression for the joint PDF $f_{\underline{\gamma}}(\cdot)$ of $\underline{\gamma}$ from (18), it can be shown that the integral simplifies to

$$\frac{1-\rho}{1+(N-1)\rho} \sum_{s=0}^{\infty} \left(\frac{\rho}{1+(N-1)\rho}\right)^s \times \sum_{\substack{l_1,\ldots,l_N \geq 0 \\ l_1+\cdots+l_N=s}} \binom{s}{l_1,\ldots,l_N} \zeta_{l_n+1}\left(x, \bar{\gamma}_k(1-\rho)\right) \times \prod_{\substack{i=1 \\ i\neq j,n}}^{N} L\left(\frac{x}{\bar{\gamma}_k(1-\rho)}, l_i+1\right) \int_0^x y \zeta_{l_j+1}\left(y, \bar{\gamma}_k(1-\rho)\right) dy. \quad (20)$$

From the definition of $\zeta_{l_j+1}\left(y, \bar{\gamma}_k(1-\rho)\right)$, we can show that $\int_0^x y\zeta_{l_j+1}\left(y, \bar{\gamma}_k(1-\rho)\right) dy = (l_j+1)(\bar{\gamma}_k(1-\rho))L\left(\frac{x}{\bar{\gamma}_k(1-\rho)}, l_j+2\right)$. Substituting this in (20) yields the expression for $A_j(x,n)$ in (7).

B. *Brief Derivation of* (6)

By definition, MSE is given by

$$\mathrm{MSE} = \mathbb{E}_{\gamma_{k,j},\gamma_{k,N:N},N:N}\left[(\gamma_{k,j} - \widehat{\gamma}_{k,j})^2 | N:N \neq j\right]. \quad (21)$$

Using (3), this can be shown to simplify to

$$\mathrm{MSE} = \int_0^{\infty} \int_0^x \left(y - \bar{\gamma}_k + \frac{x}{e^{\frac{x}{\bar{\gamma}_k}} - 1}\right)^2 \times \frac{p(\gamma_{k,j}=y, \gamma_{k,n}=x, N:N=n)}{\mathbf{P}\left[N:N \neq j\right]}\, dy dx. \quad (22)$$

Since the SC gains are i.i.d., it can be shown that $\mathbf{P}\left[N:N \neq j\right] = 1 - (1/N)$ and

$$p(\gamma_{k,j}=y, \gamma_{k,n}=x, N:N=n) = \frac{1}{\bar{\gamma}_k^2} e^{-\frac{x}{\bar{\gamma}_k}} e^{-\frac{y}{\bar{\gamma}_k}} \times \left(1 - e^{-\frac{x}{\bar{\gamma}_k}}\right)^{N-2}. \quad (23)$$

Substituting these in (22) and simplifying further yields (6).

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