Allocating Multiple D2D Users to Subchannels With Partial CSI in Multi-Cell Scenarios

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***Abstract*—We address the problem of allocating multiple device-to-device (D2D) pairs per subchannel in a multi-cell scenario with multiple subchannels and unknown inter-D2D and inter-cell interference. We propose a scheme to feedback q bits about the signal-to-interference-plus-noise ratio of a D2D pair for each subchannel that ensures that the D2D rates can be achieved with a pre-specified probability of outage. Along with this, the base station (BS) has only statistical information about the inter-cell interference, and has to provide a quality-of-service guarantee to the scheduled cellular users. We formulate the subchannel allocation problem as a generalized assignment problem, and propose a low-complexity locally greedy algorithm (LGA) to solve it. LGA provably achieves a D2D sum rate that is at least $1/2$ and $1/3$ of the maximum achievable D2D sum rate for $q = 1$ and $q \geq 2$, respectively. We then propose a rate upgradation (RU) step that enhances the D2D rate by exploiting an inherent asymmetry in the channel state information (CSI) at the BS and D2D pairs. LGA with RU achieves a spectral efficiency that is markedly better than conventional approaches that assign one D2D pair per subchannel, and close to that of a system with full CSI of the intra-cell links even for small q.**

I. Introduction

Device-to-device (D2D) communication enables users to communicate directly with each other without routing their data through the base station (BS). In the 3GPP standard specification [1], D2D users can operate either in the dedicated mode, in which dedicated subchannels are assigned to D2D users, or in the underlay mode, in which the D2D users reuse the uplink spectrum used by the cellular users (CUs). In the underlay mode, which we focus on, assigning more D2D pairs to a subchannel can improve the overall spectral efficiency, but it can also be counter-productive because it increases inter-D2D interference and the interference caused to the CU scheduled on that subchannel.

Interference-aware allocation of subchannels to the D2D pairs is important to provide a minimum quality-of-service (QoS) to the CUs, while achieving a high spectral efficiency. While the initial works on it focused on the single-cell scenario, recent works address the multi-cell scenario. We summarize the most pertinent ones below.

Single-cell Scenario: A model in which only one D2D pair is assigned to a subchannel is considered in [2]. Assignment of multiple D2D pairs to a subchannel to improve the spectral efficiency is considered in [3]–[8]. The approaches in [3], [4] are tailored for the idealized case in which the BS has full channel state information (CSI) about the CU to BS, CU to D2D receiver (DRx), D2D transmitter (DTx) to BS, and DTx to DRx links. A statistical CSI model is considered in [5], [6], in which the BS only knows the probability density functions of the CU to DRx and DTx to DRx channel power gains. In [5], a dynamic programming approach and a simpler heuristic rule are proposed for allocating subchannels to the CUs and multiple D2D pairs. Game-theoretic approaches are employed in [7], [8] to allocate subchannels to the D2D pairs.

Multi-cell Scenario: In [9], a game-theoretic approach is considered to allocate bandwidth to a D2D pair that lies in the overlapping area of two cells. In [10], a power control algorithm is proposed to determine the powers of one CU and one D2D pair, which are subject to a constraint on the inter-cell interference they cause to the BSs. In [11], using the statistics of inter-cell interference, an admission control method is proposed to admit D2D pairs to a single subchannel.

A. Focus and Contributions

We address the problem of assigning multiple D2D pairs per subchannel in a multi-cell scenario with multiple subchannels. Our goal is to maximize the D2D sum rate while providing a QoS guarantee for the CUs assigned to the subchannels.

Our model and problem formulation address the following challenges that arise in this less investigated and practically relevant scenario. First, the CSI available at the BS, which makes the resource allocation decisions, is inherently partial. The BS has CSI of the CU to BS and DTx to BS links, but not that of the DTx to DRx and CU to DRx links, as it is neither a transmitter nor a receiver in these links. Therefore, feedback from the D2D pair to the BS is needed.

Second, when feeding back CSI, D2D pair does not have CSI of inter-D2D interference links from the DTxs of other D2D pairs to its DRx, as it does not yet know which D2D pairs will share a subchannel with it. Third, neither the BS nor a D2D user knows the inter-cell interference it experiences from neighboring cells as this requires it to have a priori information about the users scheduled in the neighboring cells and the channel gain from those users to it. In light of the above uncertainties, ensuring that the rates determined by the BS for the CUs and D2D pairs are reliably decodable is challenging.

We make the following contributions:

- *Feedback with Reliability Guarantees for D2D Users:* We propose a feedback scheme in which a D2D user

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feeds back only q bits to the BS about its signal-to-interference-plus-noise ratio (SINR) for each subchannel. The feedback, which exploits the statistics of the inter-cell and inter-D2D interference, is designed to guarantee a probability of outage less than a pre-specified value ϵ_D. It provides the BS with a conservative estimate of the rate that the D2D pair can communicate on that subchannel.

- *Novel Low-Complexity Algorithm with Theoretical Performance Guarantee:* The subchannel allocation problem with the above model turns out to be an instance of the generalized assignment problem (GAP), which is known to be NP-hard [12]. We propose the locally greedy algorithm (LGA), which combines a greedy algorithm for the knapsack problem [13] and the Goundan-Schultz algorithm [12]. It allocates multiple D2D pairs to subchannels, while ensuring that each CU transmits at a pre-specified rate with a probability of outage less than a pre-specified value ϵ_C. An appealing feature of LGA is the theoretical guarantee about its performance and its low polynomial-time complexity. Specifically, we prove that the D2D sum rate it achieves is at least $1/2$ and $1/3$ of the maximum achievable D2D sum rate for $q = 1$ and $q \geq 2$ bits, respectively.
- *Rate Upgradation to Exploit CSI Asymmetry:* We propose a novel rate upgradation (RU) step that exploits an asymmetry that is inherent in the CSI at the BS and D2D pairs. Our performance benchmarking shows that LGA with RU achieves a D2D sum rate that is within 19% of that with full intra-cell CSI at the BS even for small values of q that are of practical interest. It also markedly outperforms the semi-orthogonal sharing algorithm (SSA) of [2].

Comments: The above feedback scheme that ensures reliable communication even with partial CSI and random inter-cell and inter-D2D interference, is different from the idealized full CSI models in [2]–[4]. Our model is also different from the statistical CSI models considered in [5], [6], which do not guarantee that the outage constraint is satisfied for all users and do not exploit the instantaneous CSI that is often available at the D2D users to improve their rates. Our approach also differs from the game-theoretic models of [7]–[9], which require time-consuming multiple interactions between users. To the best of our knowledge, the GAP formulation of the subchannel allocation problem is not available in the literature on assigning multiple D2D pairs per subchannels. It is different from the multi-cell scenario models in [9]–[11], which consider only one subchannel.

B. Outline

Section II discusses the system model. Section III presents LGA and RU. Section IV presents numerical results. Our conclusions follow in Section V.

II. System Model

We consider a reference cell in which there are N orthogonal uplink subchannels, indexed $1, 2, \ldots, N$, and M D2D pairs, indexed $1, 2, \ldots, M$. Let $\mathcal{S} = \{1, 2, \ldots, N\}$ and $\mathcal{D} = \{1, 2, \ldots, M\}$. We assume that each subchannel has been assigned to a CU already by the scheduler at the BS. Therefore, without loss of generality, let CU i be assigned to subchannel i. The reference cell is surrounded by neighboring cells, which reuse the same subchannels.

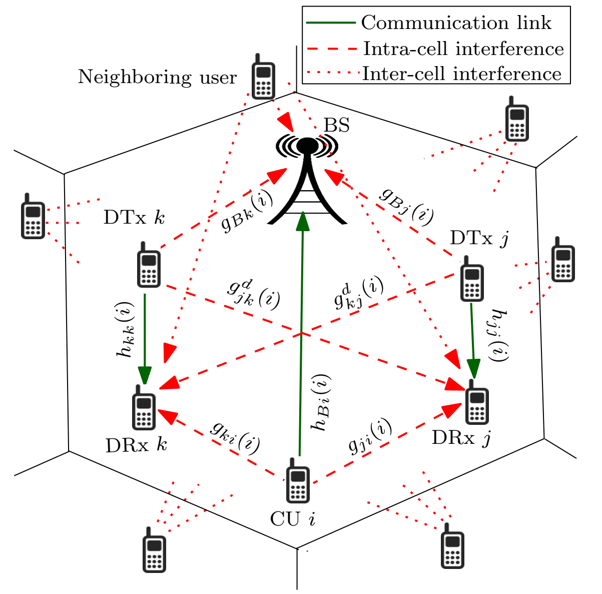

Fig. 1. Multi-cell system model that illustrates the links from DTx to DRx, CU to BS, CU to DRx, and DTx to BS, and the inter-cell interference from neighboring CUs to BS and DRxs.

The transmit power of the CU is P^c and that of the DTx of a D2D pair is P^d. Let $g_{ji}(i)$ be the channel power gain from CU i to the DRx of D2D pair j on subchannel i. The uplink channel power gain from CU i to the BS on subchannel i is $h_{Bi}(i)$. The channel power gain of the DTx to DRx link of D2D pair j on subchannel i is $h_{jj}(i)$, and from the DTx of D2D pair j to the BS is $g_{Bj}(i)$. The channel power gain from the DTx of D2D pair k to the DRx of D2D pair j on subchannel i is $g^d_{jk}(i)$. The system model is shown in Fig. 1.

A. CSI Model

The local CSI available at the BS and the D2D pairs in the reference cell is inherently different and is as follows:

- *At BS:* The BS knows $h_{Bi}(i)$ and $g_{Bj}(i), \forall i \in \mathcal{S}, j \in \mathcal{D}$. The BS can estimate these from the reference signals transmitted by the users.
- *At D2D Pairs:* The DRx of D2D pair j knows $h_{jj}(i)$ and $g_{ji}(i), \forall i \in \mathcal{S}$. It can estimate them using, for example, sounding reference signals transmitted by the users [1].

Inter-D2D Interference: The D2D pair j does not know $g^d_{jk}(i)$, $\forall k \in \mathcal{D}, j \neq k, i \in \mathcal{S}$. Therefore, the inter-D2D interference $I_{jk} = P^d g^d_{jk}(i)$ from the DTx of D2D pair k to the DRx of D2D pair j is considered as a random variable (RV); only its statistics are known to the D2D pair.

Inter-cell Interference: The BS and D2D pair j, $\forall j \in \mathcal{D}$, in the reference cell experience inter-cell interferences I_B and

I_j^D, respectively, from users in the neighboring cells. These are RVs that are not known a priori to the BS and D2D pair j; only their statistics are known to them.

SINR of CU: Let x_{ij} be an indicator variable that is 1 if subchannel i is assigned to D2D pair j, and is 0 otherwise. Then, given x_{ij}, $\forall j \in \mathcal{D}$, the SINR $\xi_i^c(i)$ of CU i on its allocated subchannel i is given by

$$\xi_i^C(i) = \frac{P^c h_{Bi}(i)}{\sum_{j=1}^{M} x_{ij} P^d g_{Bj}(i) + I_B + \sigma^2}, \tag{1}$$

where σ^2 is the Gaussian noise power.

SINR of D2D pair: The SINR $\xi_j^D(i)$ of D2D pair j on subchannel i is

$$\xi_j^D(i) = \frac{P^d h_{jj}(i)}{P^c g_{ji}(i) + I_j + \sigma^2}, \tag{2}$$

where I_j is the sum of inter-D2D and inter-cell interferences:

$$I_j = \sum_{k=1, k\neq j}^{M} x_{ik} I_{jk} + I_j^D. \tag{3}$$

B. Limited Feedback Model and Implications

Unlike the D2D pair j, the BS does not know $h_{jj}(i)$ and $g_{ji}(i)$. Therefore, the DRx sends a q-bit feedback δ_{ij} to the BS about its SINR. However, even the D2D pair does not know its SINR exactly since it does not know the inter-cell and inter-D2D interferences. Even so, it can guarantee that its SINR on subchannel i exceeds a value $T_{ij}(\epsilon_D)$ with a probability $1-\epsilon_D$, where $T_{ij}(\epsilon_D)$ is computed as follows:

$$\Pr\left\{\frac{P^d h_{jj}(i)}{P^c g_{ji}(i) + I_j + \sigma^2} \geq T_{ij}(\epsilon_D)\right\} = 1 - \epsilon_D. \tag{4}$$

Rearranging and writing in terms of the cumulative distribution function (CDF) $F_j(\cdot)$ of I_j, we get

$$F_j\left(\frac{P^d h_{jj}(i)}{T_{ij}(\epsilon_D)} - P^c g_{ji}(i) - \sigma^2\right) = 1 - \epsilon_D. \tag{5}$$

Rearranging terms again, we get

$$T_{ij}(\epsilon_D) = \frac{P^d h_{jj}(i)}{P^c g_{ji}(i) + {F_j}^{-1}(1-\epsilon_D) + \sigma^2}, \tag{6}$$

where ${F_j}^{-1}(\cdot)$ is the inverse of the CDF of I_j.

We note that at the time of generating feedback, a D2D pair does not know which D2D pairs will interfere with it. Therefore, to evaluate ${F_j}^{-1}(\cdot)$, we conservatively assume that all the other $(M-1)$ D2D pairs interfere with it. Hence, in the expression for I_j in (3), we replace $\sum_{k=1,k\neq j}^{M} x_{ik} I_{jk}$ with $\sum_{k=1,k\neq j}^{M} I_{jk}$. We shall see in Section IV that significant gains accrue even with this conservative approach.

The DRx sends a q-bit feedback δ_{ij} to the BS by quantizing $T_{ij}(\epsilon_D)$. Let $0 = \Psi_0 < \Psi_1 < \cdots < \Psi_{L-1} < \infty$ be the $L = 2^q$ quantization thresholds. The feedback δ_{ij} is given by

$$\delta_{ij} = k \quad \text{if } \Psi_k \leq T_{ij}(\epsilon_D) < \Psi_{k+1}. \tag{7}$$

Implications: Given δ_{ij}, the BS determines the rate C_{ij} of D2D pair j on subchannel i as

$$C_{ij} = (1-\epsilon_D)\log_2(1+\Psi_{\delta_{ij}}). \tag{8}$$

We shall refer to C_{ij} as the *conservative rate*; this is the only information the BS has about the rates of the D2D pairs.

C. Minimum Rate Guarantee for CUs with Outage Constraint

We require that CU i, whose SINR is affected by the unknown inter-cell interference I_B, must be able to transmit at a minimum rate $R_{\min}^{(i)}$ with a probability of outage that is at most ϵ_C, which is a system parameter. Therefore,

$$\Pr\left\{\log_2\left(1+\xi_i^C(i)\right) \geq R_{\min}^{(i)}\right\} \geq \ 1-\epsilon_C. \tag{9}$$

Substituting (1) and rearranging terms, we get

$$\sum_{j=1}^{M} x_{ij} w_{ij} \leq b_i, \tag{10}$$

where $w_{ij} = P^d g_{Bj}(i)$, $b_i = \frac{P^c h_{Bi}(i)}{2^{R_{\min}^{(i)}}-1} - \sigma^2 - F_B^{-1}(1-\epsilon_C)$, and $F_B^{-1}(\cdot)$ is the inverse of the CDF of I_B. Thus, the sum of interferences at the BS from the DTxs of the D2D pairs assigned to subchannel i should not exceed b_i.

Comment: The above formulation is general because it applies to any CDF of I_j and I_B. Note that I_j can be D2D pair specific since the inter-cell and inter-D2D interferences at different locations in a cell are different. For example, with Rayleigh fading and lognormal shadowing, I_j and I_B are sums of composite Rayleigh-lognormal RVs. Therefore, they can be well approximated as lognormal RVs [14, Ch. 3]. In this case, let the dB-mean and dB-standard deviation of I_j be μ_j and σ_j, respectively, and those of I_B be μ_B and σ_B. Then, the CDF of I_j can be shown to be $F_j(x) = 1 - Q(\frac{10\log_{10}(x)-\mu_j}{\sigma_j})$, where $Q(\cdot)$ is the Q-function [14, Ch. 3]. Therefore, in this case, ${F_j}^{-1}(x) = 10^{0.1(\mu_j + \sigma_j Q^{-1}(1-x))}$, for $x \geq 0$, where $Q^{-1}(\cdot)$ is the inverse Q-function. Similarly, the inverse of the CDF of I_B is $F_B^{-1}(x) = 10^{0.1(\mu_B + \sigma_B Q^{-1}(1-x))}$, for $x \geq 0$.

D. Subchannel Allocation Problem Formulation

Our problem of allocating multiple D2D pairs to subchannels to maximize the sum of conservative rates is as follows:

$$\mathcal{P}: \max_{x_{ij}, \forall i \in \mathcal{S}, j \in \mathcal{D}} \left\{\sum_{i=1}^{N}\sum_{j=1}^{M} x_{ij} C_{ij}\right\}, \tag{11}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} x_{ij} \leq 1, \quad \forall j \in \mathcal{D}, \tag{12}$$

$$\sum_{j=1}^{M} x_{ij} w_{ij} \leq b_i, \quad \forall i \in \mathcal{S}, \tag{13}$$

$$x_{ij} \in \{0,1\}, \quad \forall i \in \mathcal{S}, j \in \mathcal{D}. \tag{14}$$

Constraint (12) mandates that at most one subchannel can be assigned to a D2D pair, and Constraint (13) ensures the minimum rate for CUs with a given outage constraint. The problem $\mathcal{P}$ is an instance of GAP, which is NP-hard [12].

III. Proposed Algorithm and Rate Upgradation

A. Locally Greedy Algorithm (LGA)

To describe LGA, we first define the key terminology. A *feasible* D2D set for subchannel i is a set of D2D pairs whose cumulative interference at the BS on that subchannel is not more than b_i (cf. (13)). The tuple (i, s_i) consists of subchannel i and its associated feasible D2D set s_i. Let $\mathcal{B}_i = \{(1, s_1), (2, s_2), \ldots, (i, s_i)\}$, where $s_1, s_2, \ldots, s_i$ need not be mutually exclusive. Define a set function f as

$$f(\mathcal{B}_i) \triangleq \sum_{j=1}^{M} \max_{l \in \{1,2,\ldots,i\}} \{C_{lj} : \exists\, (l, s_l) \in \mathcal{B}_i, j \in s_l\}. \quad (15)$$

Here, $f(\mathcal{B}_i)$ is the D2D sum rate achieved by the set $\mathcal{B}_i$. It is the sum of conservative rates of the D2D pairs present in $s_1, s_2, \ldots, s_i$, such that if a D2D pair appears in the feasible D2D set of multiple subchannels then its conservative rate is the maximum among those subchannels.

It can be shown that $f(\cdot)$ is a non-decreasing set function i.e., $f(\mathcal{X}) \le f(\mathcal{Z})$ if $\mathcal{X} \subseteq \mathcal{Z}$. It can also be shown to be a submodular function that satisfies [15]

$$\rho_e(\mathcal{X}) \ge \rho_e(\mathcal{Z}),\ \forall \mathcal{X} \subseteq \mathcal{Z},\ e \notin \mathcal{Z}, \quad (16)$$

where

$$\rho_e(\mathcal{A}) \triangleq f(\mathcal{A} \cup \{e\}) - f(\mathcal{A}), \quad (17)$$

represents the incremental gain in f when e is included in $\mathcal{A}$. Let p_{ij} denote the *incremental gain* obtained by adding the tuple $(i, \{j\})$ to $\mathcal{B}_{i-1}$. Then,

$$p_{ij} = \rho_{(i,\{j\})}(\mathcal{B}_{i-1}). \quad (18)$$

Using LGA we find $\mathcal{B}_N$ as follows.

Algorithm Description: We set $\mathcal{B}_0 = \emptyset$, where $\emptyset$ is the null set, $f(\emptyset) = 0$, and start with subchannel $i = 1$. Given $\mathcal{B}_{i-1}$, the feasible D2D set s_i is selected for subchannel i as follows: Compute p_{ij} for those D2D pairs j whose interference w_{ij} at the BS is not more than b_i. These D2D pairs are arranged in the non-increasing order of $\frac{p_{ij}}{w_{ij}}$, which is the ratio of incremental gain to the interference at the BS. Using order statistics notation, let $[l]$ represent the D2D pair with l^{th} largest ratio of $\frac{p_{ij}}{w_{ij}}$. Thus,

$$\frac{p_{i[1]}}{w_{i[1]}} \ge \frac{p_{i[2]}}{w_{i[2]}} \ge \cdots \ge \frac{p_{i[M]}}{w_{i[M]}}. \quad (19)$$

We find the D2D pair $[d]$ such that the set of D2D pairs $\{[1], [2], \ldots, [d-1]\}$ is feasible while $\{[1], \ldots, [d-1], [d]\}$ is not. If the incremental gain $\sum_{j=1}^{d-1} p_{i[j]}$ of the tuple $(i, \{[1], \ldots, [d-1]\})$ is greater than the incremental gain $p_{i[d]}$ of the tuple $(i, \{[d]\})$, then $s_i = \{[1], \ldots, [d-1]\}$, otherwise $s_i = \{[d]\}$.[1] Then, $\mathcal{B}_i = \mathcal{B}_{i-1} \cup \{(i, s_i)\}$. This procedure is repeated until $i = N$. From the definition in (17), we get

$$\rho_{(i,s_i)}(\mathcal{B}_{i-1}) = f(\mathcal{B}_i) - f(\mathcal{B}_{i-1}). \quad (20)$$

[1]For any subchannel i, if no feasible D2D set is possible then we consider $s_i = \emptyset$. On the other hand, if the entire D2D set $\{[1], [2], \ldots, [M]\}$ is feasible, then $s_i = \{[1], [2], \ldots, [M]\}$.

Once the set $\mathcal{B}_N$ is obtained, the D2D pairs in s_i are allocated to subchannel i. However, if a D2D pair is present in the feasible D2D set of multiple subchannels, then it is allocated only to the subchannel for which it has the maximum conservative rate. The final D2D sum rate $f(\mathcal{B}_N)$ of LGA is

$$f(\mathcal{B}_N) = \sum_{i=1}^{N} \rho_{(i,s_i)}(\mathcal{B}_{i-1}). \quad (21)$$

The pseudocode of LGA is given in Algorithm 1.

Algorithm 1 Locally Greedy Algorithm

1: **Initialization:** Set $\mathcal{B}_0 = \emptyset$
2: **for** subchannel $i = 1$ to N **do**
- Given $\mathcal{B}_{i-1}$, compute p_{ij} of all D2D pairs for which $w_{ij} \le b_i$.
- Arrange the D2D pairs in the non-increasing order of $\frac{p_{ij}}{w_{ij}}$: $\frac{p_{i[1]}}{w_{i[1]}} \ge \frac{p_{i[2]}}{w_{i[2]}} \ge \cdots \ge \frac{p_{i[M]}}{w_{i[M]}}$.
- Find $[d]$ such that $\sum_{j=1}^{d-1} w_{i[j]} \le b_i$ and $\sum_{j=1}^{d} w_{i[j]} > b_i$.
- Set $s_i = \{[1], \ldots, [d-1]\}$ if $\sum_{j=1}^{d-1} p_{i[j]} > p_{i[d]}$, else $s_i = \{[d]\}$.
- $\mathcal{B}_i \leftarrow \mathcal{B}_{i-1} \cup \{(i, s_i)\}$.

3: **end for**
4: Allocate D2D pairs to subchannels to maximize conservative rate.

LGA provides the following desirable theoretical guarantee about its performance.

Result *1:* The D2D sum rate achieved by LGA is at least $1/2$ of the optimal D2D sum rate for $q = 1$ and is at least $1/3$ of the optimal D2D sum rate for $q \ge 2$.

Proof: The proof is given in Appendix A. ■

Computational Complexity: Finding a feasible D2D set out of the M D2D pairs for a subchannel entails a complexity of $\mathcal{O}(M \log M)$. Therefore, for N subchannels, the complexity of LGA, which runs at the BS, is $\mathcal{O}(NM \log M)$.

B. Rate Upgradation (RU) at D2D user

There is an inherent asymmetry in the CSI at the BS and D2D pairs. While the BS only knows the conservative rate C_{ij} from the q-bit feedback, the D2D pair knows $T_{ij}(\epsilon_D)$ and the corresponding rate $(1-\epsilon_D)\log_2(1+T_{ij}(\epsilon_D))$, which is greater than or equal to C_{ij}. Therefore, after subchannel allocation, D2D pair j can increase its transmission rate from C_{ij} to $(1-\epsilon_D)\log_2(1+T_{ij}(\epsilon_D))$ while ensuring that its outage probability does not exceed ϵ_D. We refer to this as the RU step.

IV. Numerical Results

We now present Monte Carlo simulation results for the multi-cell scenario. In the reference cell of radius $R = 500$ m, we drop the N CUs and the DRxs of the M D2D pairs uniformly. The DTx lies with uniform probability within a circle of radius 50 m around the DRx. We illustrate the results for Rayleigh fading and lognormal shadowing with a dB-standard deviation of 6. The pathloss, as a function of the distance d between nodes, is given by $0.01 d^{-\eta}$, where $\eta = 3.5$. Also, we set $\sigma^2 = -120$ dBm, $P^c = 10$ dBm, $P^d = -10$ dBm, $\epsilon_C = 0.1$, and $R_{\min}^{(i)} = 1$ bps/Hz, $\forall i \in \mathcal{S}$.

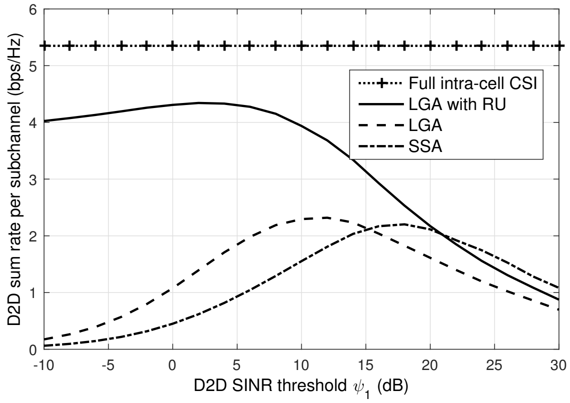

Fig. 2. Performance benchmarking: Comparison of LGA and LGA with RU with SSA scheme ($q = 1$, $N = 8$, $M = 12$, and $\epsilon_D = 0.1$).

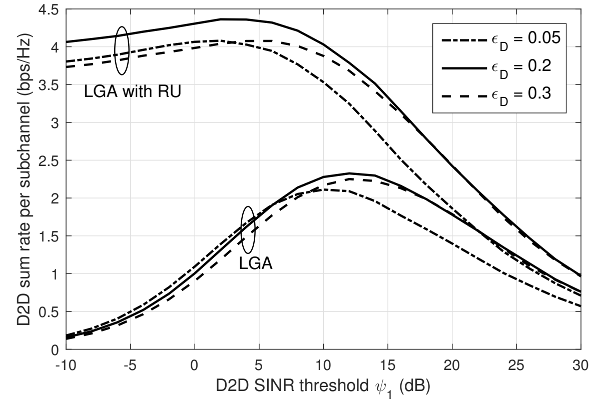

Fig. 3. Effect of ϵ_D: Comparison of D2D sum rates of LGA and LGA with RU for different values of ϵ_D ($q = 1$, $N = 8$, and $M = 12$).

To capture the statistics of the inter-cell interference terms I_B and I_j^D, we proceed as follows. From 10,000 realizations of shadowing, small-scale fading, and locations of cellular users in the neighboring cells that are assigned to a subchannel, the interference on that subchannel is measured at the BS and the DRx of D2D pair j. Interference from D2D pairs in the neighboring cells is not modeled since it is small. From these, the CDFs of I_B and I_j^D are determined. The statistics of the inter-D2D interference I_{jk} is obtained by averaging over shadowing and small-scale fading, given the locations of the D2D pairs j and k. From this, the statistics of $\sum_{k=1,k\neq j}^{M} I_{jk} + I_j^D$ (cf. Section II-B) are determined.

Benchmarking: We compare LGA with the following:

- *Semi-orthogonal Sharing Algorithm (SSA)* [2]: In this, at most one D2D pair is assigned to a subchannel. It has the advantage of no inter-D2D interference (cf. (2)). For fair comparison, the CSI model considered is the same as for LGA. Here, the optimal subchannel allocation can be found using the Kuhn-Munkres algorithm [16, Ch. 3].
- *Full Intra-Cell CSI:* In this, the BS has CSI of CU to BS, DTx to DRx (of the same D2D pair), DTx to BS, and CU to DRx links in the cell it serves. However, it only knows the statistics for the inter-cell and inter-D2D interferences. Note that exhaustive search has a prohibitively large computational complexity of $\mathcal{O}(2^{MN})$. Therefore, we use LGA to determine the subchannel allocation.

Due to fundamental differences in the CSI and feedback models, and QoS guarantees, a comparison with the schemes proposed in [3]–[8], which aim to allocate multiple D2D pairs to a subchannel is not possible. For example, [3], [4] assume that the BS has full CSI of all the links in the system. In [5], [6], no feedback is considered and the algorithm does not guarantee that the outage probability constraint is satisfied for all users. The game-theoretic models in [7], [8] assume a multi-step interaction between users over time and only consider the single-cell scenario.

1) $q = 1$: Fig. 2 compares the D2D sum rate per subchannel as a function of the SINR threshold Ψ_1. As Ψ_1 increases, the D2D sum rate of LGA increases, reaches a maximum value, and then decreases. This is because, for small values of Ψ_1, the number of D2D pairs whose SINR exceeds Ψ_1 is large while the conservative rate per D2D pair is small. On the other hand, for large Ψ_1, the number of D2D pairs whose SINRs exceed Ψ_1 decreases, even though the conservative rate per D2D pair increases. The optimal threshold for LGA is 12 dB and for LGA with RU is 2 dB. RU increases the maximum D2D sum rate of LGA by 87%, and makes it within 19% of that of full intra-cell CSI. It is 97% more than that of SSA.

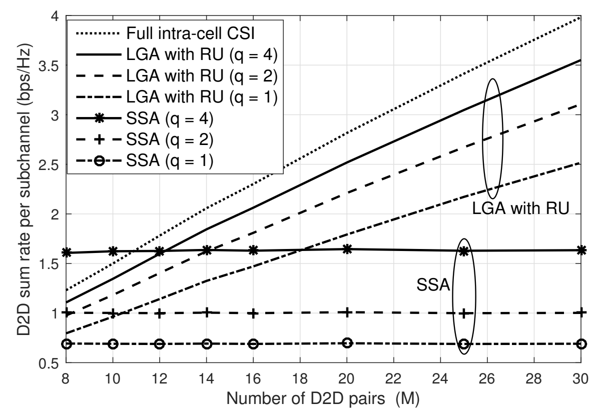

Fig. 4. Performance benchmarking and effect of feedback resolution: Comparison of D2D sum rates for different q ($N = 8$ and $\epsilon_D = 0.1$).

Fig. 3 investigates the impact of ϵ_D. The D2D sum rate initially increases as ϵ_D increases from 0.05 to 0.2. This is because in $C_{ij} = (1 - \epsilon_D)\log_2(1 + \Psi_{\delta_{ij}})$, the term $\Psi_{\delta_{ij}}$ increases as the outage requirement is weakened. However, the D2D sum rate decreases when ϵ_D increases from 0.2 to 0.3 because the pre-multiplication factor $(1 - \epsilon_D)$ decreases.

2) $q \geq 2$: Fig. 4 plots the D2D sum rate per subchannel as a function of M for different q. For $q \geq 2$, it becomes computationally intractable to optimize the $L = 2^q$ quantization thresholds. We, therefore, use the percentile threshold-based quantized feedback scheme, which is described in [17]. The

D2D sum rate increases as M increases because of multi-user diversity. It increases as q increases due to better feedback resolution. We see that LGA with RU outperforms SSA for large M. The performance of SSA is insensitive to M.

V. Conclusions

We considered the multi-cell scenario, in which the BS had partial CSI. We proposed a q-bit feedback model that ensured that the rate assigned to the D2D pairs was achievable within a pre-specified outage probability. We proposed a polynomial-time algorithm LGA that enabled multiple D2D pairs to share a subchannel with a scheduled CU. LGA ensured a minimum rate for the CUs for a pre-specified outage probability. Theoretically, LGA achieved at least $1/2$ and $1/3$ of the optimal D2D sum rate for $q = 1$ and $q \geq 2$, respectively. Practically, with RU, its performance was close to that with full intra-cell CSI and markedly better than conventional approaches. Future work involves modeling imperfect CSI and allowing a D2D pair to simultaneously transmit on multiple subchannels.

Appendix

A. Proof of Result 1

Let $x^*_{ij}, \forall i \in \mathcal{S}, j \in \mathcal{D}$, be the optimal solution for $\mathcal{P}$. Let $t^*_i = \{j : x^*_{ij} = 1, j \in \mathcal{D}\}$ be the feasible D2D set associated with subchannel i in the optimal solution. Define the set of tuples $\mathcal{T}^* = \{(i, t^*_i) \mid 1 \leq i \leq N\}$. The optimal D2D sum rate for $\mathcal{P}$ is then $f(\mathcal{T}^*) = \sum_{i=1}^{N} \sum_{j=1}^{M} x^*_{ij} C_{ij}$. The solution obtained by LGA is $\mathcal{B}_N = \{(i, s_i) \mid 1 \leq i \leq N\}$. Let $x_{ij}, \forall i \in \mathcal{S}, j \in \mathcal{D}$, be the allocation derived from $\mathcal{B}_N$. The D2D sum rate of LGA is $f(\mathcal{B}_N) = \sum_{i=1}^{N} \sum_{j=1}^{M} x_{ij} C_{ij}$.

Since f is non-decreasing and submodular, it satisfies the following inequality [15, Prop. 2.2 (iv′)]:

$$f(\mathcal{T}^*) \leq f(\mathcal{B}_N) + \sum_{i=1}^{N} \rho_{\left(i, t^*_i\right)} \left(\mathcal{B}_N\right) 1_{\left\{t^*_i \neq s_i\right\}}, \tag{22}$$

$$\leq f(\mathcal{B}_N) + \sum_{i=1}^{N} \rho_{\left(i, t^*_i\right)} \left(\mathcal{B}_N\right). \tag{23}$$

Since $\mathcal{B}_{i-1} \subseteq \mathcal{B}_N$, invoking the submodular property, we get

$$\sum_{i=1}^{N} \rho_{\left(i, t^*_i\right)} \left(\mathcal{B}_N\right) \leq \sum_{i=1}^{N} \rho_{\left(i, t^*_i\right)} \left(\mathcal{B}_{i-1}\right). \tag{24}$$

We now evaluate $\sum_{i=1}^{N} \rho_{\left(i, t^*_i\right)} \left(\mathcal{B}_{i-1}\right)$ for $q = 1$ and $q \geq 2$.

1) *$q = 1$:* The conservative rate of a D2D pair is either 0 or $\log_2(1+\Psi_1)$. Hence, from (18), p_{ij} is either 0 or $\log_2(1+\Psi_1)$. In Step 2 (cf. Algorithm 1), if $p_{ij} = 0$, then D2D pair j will not be included in s_i. Therefore, for the other D2D pairs, the set s_i is formed by taking the D2D pairs in the increasing order of their interference w_{ij} until s_i is no longer feasible. Therefore, Step 2 ensures that the sum of the incremental gains of the D2D pairs in s_i is the largest among all the feasible sets. Hence, $\rho_{\left(i, t^*_i\right)} \left(\mathcal{B}_{i-1}\right) \leq \rho_{(i, s_i)} \left(\mathcal{B}_{i-1}\right)$. Summing over all the subchannels, we get

$$\sum_{i=1}^{N} \rho_{\left(i, t^*_i\right)} \left(\mathcal{B}_{i-1}\right) \leq \sum_{i=1}^{N} \rho_{(i, s_i)} \left(\mathcal{B}_{i-1}\right) = f(\mathcal{B}_N). \tag{25}$$

The above equality follows from (21). Substituting (24) and (25) in (23), we get $f(\mathcal{T}^*) \leq 2f(\mathcal{B}_N)$.

2) *$q \geq 2$:* It can be shown using the result in [13, Ch. 2] about the greedy algorithm for the knapsack problem that Step 2 of LGA selects a feasible D2D set s_i for subchannel i such that the incremental gain of adding (i, s_i) to $\mathcal{B}_{i-1}$ is at least half of the optimal incremental gain. The proof is involved and is not repeated here. Therefore, $\rho_{\left(i, t^*_i\right)} \left(\mathcal{B}_{i-1}\right) \leq 2\rho_{(i, s_i)} \left(\mathcal{B}_{i-1}\right)$. Summing over all the subchannels, we get

$$\sum_{i=1}^{N} \rho_{\left(i, t^*_i\right)} \left(\mathcal{B}_{i-1}\right) \leq 2 \sum_{i=1}^{N} \rho_{(i, s_i)} \left(\mathcal{B}_{i-1}\right) = 2f(\mathcal{B}_N). \tag{26}$$

Substituting (24) and (26) in (23), we get $f(\mathcal{T}^*) \leq 3f(\mathcal{B}_N)$.

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