Performance Analysis of Fixed Gain Amplify-and-Forward Relaying with Time-Efficient Cascaded Channel Estimation

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***Abstract*—In a cooperative system with an amplify-and-forward relay, the cascaded channel training protocol enables the destination to estimate the source-destination channel gain and the product of the source-relay (SR) and relay-destination (RD) channel gains using only two pilot transmissions from the source. Notably, the destination does not require a separate estimate of the SR channel. We develop a new expression for the symbol error probability (SEP) of AF relaying when imperfect channel state information (CSI) is acquired using the above training protocol. A tight SEP upper bound is also derived; it shows that full diversity is achieved, albeit at a high signal-to-noise ratio (SNR). Our analysis uses fewer simplifying assumptions, and leads to expressions that are accurate even at low SNRs and are different from those in the literature. For instance, it does not approximate the estimate of the product of SR and RD channel gains by the product of the estimates of the SR and RD channel gains. We show that cascaded channel estimation often outperforms a channel estimation protocol that incurs a greater training overhead by forwarding a quantized estimate of the SR channel gain to the destination. The extent of pilot power boosting, if allowed, that is required to improve performance is also quantified.**

I. Introduction

Relay-based cooperation is an appealing technology because it exploits the independent path provided by a relay between a source and a destination to overcome the shortcomings of point-to-point communication. Several cooperation protocols have been proposed in the literature, *e.g.*, [1]–[6]. Among these, the amplify-and-forward (AF) protocol, in which the relay simply amplifies and forwards its received signal to the destination, is considered to be easy to implement as the relay does not need to decode its received signal.

The performance analysis of the AF protocol is an active and challenging area of research. One reason that makes such an analysis challenging is that the received signal component contains a product (or a cascade) of the SR and RD channel gains. Accurate expressions for symbol error probability (SEP) and bounds have been derived in [2]–[6] assuming perfect channel state information (CSI) at the destination. However, in practical situations, the receiver has to first acquire CSI, which might be imperfect. In [7], [8], the SR, relay-destination (RD), and source-destination (SD) channel estimates are all assumed to be available at the destination. Further, in [8], the channel estimation error is modeled as additive Gaussian noise with a fixed variance that does not depend on the received signal-to-noise-ratio (SNR). While [7] derived an expression for SEP of MPSK and MQAM over Rayleigh channels in the form of a double integral, [8] derived the SEP over a Rician channel in the asymptotic regime of large SNR. In [9], an SEP expression was derived, but only for BPSK. In [10], the diversity order was determined for orthogonal and non-orthogonal AF protocols. An outage probability analysis with imperfect CSI was instead developed in [11].

Another category of papers deals with training protocols for AF [12]–[14]. In [13], it was pointed out that it is difficult for the destination to acquire separate estimates of the SR and RD channel gains because the relay then has to estimate the SR channel gain. Doing so also increases the training overhead since the relay must quantize and forward its estimate to the destination. An estimation method, in which the relay precodes its received signal, was also proposed in [13]. In [14], the relay instead forwards the noisy training signal it receives from the destination to the source. The above papers primarily focus on estimation, and use simulations to compute the SEP.

In this paper, we focus on an analytical performance characterization of the cascaded channel estimation protocol [12] and fixed gain AF relaying [1], [13]. In it, the source broadcasts the pilot symbol in the first phase and the relay forwards to the destination the amplified version of the signal it receives in the second phase. At the end of second phase, the destination estimates the SD channel gain and the product of SR and RD channel gains. It uses these two estimates for coherently demodulating the data symbols. However, it does not have an estimate of the SR channel. The cascaded training protocol is time-efficient and relevant as it spans only two symbol durations.

Instead of postulating a model for the estimation error, we develop an accurate model for the channel estimation error for the cascaded channel estimation protocol. We develop an expression for the SEP of MPSK modulation in the form of single integral. A key goal of the analysis is to use as few simplifying approximations as possible. The analysis leads to a different analysis method and new expressions for the SEP, which are accurate even at low SNRs. For instance, unlike [9], [10], we do not assume that the estimate of the product of SR and RD channel gains is the same as the product of the estimates of SR and RD channel gains, which is accurate only under high SNR and with many pilot symbols [10]. In [11], the above estimate was even replaced by its perfect CSI

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equivalent to enable analysis. We also develop a new closed-form tight upper bound for the SEP, derive the asymptotic SEP expressions for large SNR, and characterize the diversity order. We also quantify the effectiveness of pilot power boosting by the source and relay, if allowed, in reducing the SEP. Thus, the paper develops a rigorous understanding of the practically relevant cascaded channel estimation method.

The paper is organized as follows. Section II develops the system model. The SEP is analyzed in Sec. III. The results and conclusions follow in Sec. IV and Sec. V, respectively. Mathematical derivations are relegated to the Appendix.

II. Model

Consider a system with a source node, S, that transmits data to a destination node, D, over a frequency-flat Rayleigh fading channel. An AF relay, R, forwards the signal from the S to D. All the nodes are equipped with a single transmit and receive antenna, and operate in the half-duplex mode, i.e., they cannot transmit and receive simultaneously. Let h_{sd}, h_{sr}, and h_{rd} denote the SD, SR, and RD channels, respectively. These are modeled as circularly symmetric complex Gaussian random variables (RVs) with variances σ^2_{hsd}, σ^2_{hsr}, and σ^2_{hrd}, respectively, and are mutually independent. We assume that h_{sr}, h_{rd}, and h_{sd} remain constant during the training and data communication phases, which are described below.

A. Two Phase Data Communication

In the first phase, the source transmits a data symbol that is drawn with equal probability from the MPSK constellation of size M. In this first phase, the received signals at the destination, y_{sd}, and the relay, y_{sr}, are given by

$$y_{sd} = \sqrt{E_s} h_{sd} x + n_{sd}, \tag{1}$$

$$y_{sr} = \sqrt{E_s} h_{sr} x + n_{sr}, \tag{2}$$

where E_s is the symbol energy transmitted by the source, $|x|^2 = 1$, and n_{sd} and n_{sr} are additive circularly symmetric complex additive white Gaussian noise (CAWGN) at the destination and the relay, respectively, with variance σ^2_n.

In the second phase, the relay amplifies y_{sr} by a factor α_d and forwards it to the destination. Therefore, the signal, y_{rd}, received by the destination in the second phase is

$$y_{rd} = \sqrt{E_s} \alpha_d g x + \alpha_d h_{rd} n_{sr} + n_{rd}, \tag{3}$$

where $g = h_{sr} h_{rd}$ is called the *cascaded channel gain* and n_{rd} is CAWGN with variance σ^2_n. Let $\bar{\gamma}_{sd} = \frac{\sigma^2_{sd} E_s}{\sigma^2_n}$, $\bar{\gamma}_{sr} = \frac{\sigma^2_{sr} E_s}{\sigma^2_n}$, and $\bar{\gamma}_{rd} = \frac{\sigma^2_{rd} E_r}{\sigma^2_n}$, denote the SD, SR, and RD fading-averaged channel SNRs.

In a fixed gain AF relay, the relay gain is set as $\alpha_d = \sqrt{\frac{E_r}{\sigma^2_{sr} E_s + \sigma^2_n}}$ to ensure an average relay transmit energy of E_r.

B. Two Slot Cascaded Channel Estimation Protocol

Training precedes data communication; it is done only once in every coherence interval. In the first slot, the source transmits a pilot symbol p such that $|p|^2 = 1$. In the second slot, the relay amplifies it by a fixed gain α_t and forwards to the destination. The relay does not estimate any channel gain nor does it feed-forward any channel estimate to the destination. Using (1) and (3), the signals r_{sd} and r_{rd}, received by the destination in the first and second slots, respectively, are given by

$$r_{sd} = \sqrt{\varepsilon E_s} h_{sd} p + w_{sd}, \tag{4}$$

$$r_{rd} = \sqrt{\varepsilon E_s} \alpha_t g p + \alpha_t h_{rd} w_{sr} + w_{rd}, \tag{5}$$

where $\alpha_t = \sqrt{\frac{\varepsilon E_r}{\varepsilon \sigma^2_{sr} E_s + \sigma^2_n}}$ and ε is called the pilot power boosting factor. Note that the relay also boosts its transmit power to εE_r in the training phase. Here, w_{sd}, w_{rd}, and w_{sr} are CAWGN; each has variance of σ^2_n. They are also mutually independent of n_{sd}, n_{sr}, and n_{rd}. Note also that $\alpha_t \neq \alpha_d$.

Using the two signals r_{sd} and r_{rd}, the destination comes up with an estimate of the SD channel, $\hat{h}_{sd}$, and of the cascaded channel, $\hat{g}$, using an linear minimum mean square (LMMSE) estimator. We do not use the minimum mean square error (MMSE) estimator because it is analytically intractable for AF systems. This is because neither the overall channel g nor the effective noise $\alpha_t h_{rd} w_{sr} + w_{rd}$ in (5) are Gaussian. For the same reason even simulating the MMSE estimator is challenging.

The LMMSE channel estimates of h_{sd} and g are given by [15]

$$\hat{h}_{sd} = L_{sd} \frac{r_{sd}}{p} \quad \text{and} \quad \hat{g} = L_g \frac{r_{rd}}{p}, \tag{6}$$

where

$$L_{sd} = \frac{\sigma^2_{sd} \sqrt{\varepsilon E_s}}{\sigma^2_{sd} \varepsilon E_s + \sigma^2_n}, \; L_g = \frac{\sigma^2_{sr} \sigma^2_{rd} \alpha_t \sqrt{\varepsilon E_s}}{\sigma^2_{sr} \sigma^2_{rd} \alpha^2_t \varepsilon E_s + \sigma^2_{w_{srd}}}, \tag{7}$$

and $\sigma^2_{w_{srd}} = \left(\alpha^2_t \sigma^2_{rd} + 1\right) \sigma^2_n$.

Thus, the SD channel estimate, $\hat{h}_{sd}$, is perturbed by the noise term w_{sd}. Similarly, the noise term $w_{srd} \triangleq \alpha_t h_{rd} w_{sr} + w_{rd}$ perturbs the cascaded channel estimate, $\hat{g}$, which is not a Gaussian RV. This is due to the presence of products of two Gaussian RVs in (5). To enable analytical tractability, we will assume that the noise term w_{srd} is CAWGN with variance $\sigma^2_{w_{srd}} = \left(\alpha^2_t \sigma^2_{rd} + 1\right) \sigma^2_n$. Such an approximation has been used to good effect in [9], [13] and in double differential modulation performance analysis [16]. It corresponds to a worst case noise model [17]. Similarly, $n_{srd} \triangleq \alpha_d h_{rd} n_{sr} + n_{rd}$ is assumed to be CAWGN with variance $\sigma^2_{srd} = \left(\alpha^2_d \sigma^2_{rd} + 1\right) \sigma^2_n$. We shall verify the accuracy of these assumptions in Sec. IV.

III. Symbol Error Probability Analysis

We now analyze the SEP with imperfect CSI for MPSK. We shall use the following notation. The probability of an event A and the conditional probability of A given B are denoted by $\Pr(A)$ and $\Pr(A|B)$, respectively. For an RV Z, its probability density function (PDF) is denoted by $p_Z(z)$, and $\mathbf{E}[Z]$ and $\mathbf{var}[Z]$ denote its expectation and variance, respectively. Similarly, $p_{Z|A}(z)$, $\mathbf{E}[Z|A]$, and $\mathbf{var}[Z|A]$ denote the conditional PDF, expectation, and variance, respectively, of Z given A. Finally, $\tau_1 = O(\tau_2)$ means that $|\tau_1| < \kappa |\tau_2|$ for some constant κ for a small enough τ_2.

A. Decision Variable and its Statistics

The maximum likelihood (ML) decision variable, D, for detecting data symbol x is based on the observables y_{sd}, y_{rd}, $\hat{h}_{sd}$, and $\hat{g}$. Since w_{sd}, n_{sd}, w_{srd}, and n_{srd} are independent, based on maximal ratio combining (MRC), D is given by[1]

$$D = \frac{\left(\mathbf{E}\left[y_{sd}|\hat{h}_{sd},\hat{g},x\right]\right)^{*} y_{sd}}{\mathbf{var}\left[y_{sd}|\hat{h}_{sd},\hat{g},x\right]} + \frac{\left(\mathbf{E}\left[y_{rd}|\hat{h}_{sd},\hat{g},x\right]\right)^{*} y_{rd}}{\mathbf{var}\left[y_{rd}|\hat{h}_{sd},\hat{g},x\right]}. \tag{8}$$

Conditioned on the estimates, $\hat{h}_{sd}$ and $\hat{g}$, and x, it follows from (1) that y_{sd} is a complex Gaussian RV. As shown in Appendix A, $\mathbf{E}\left[y_{sd}|\hat{h}_{sd},\hat{g},x\right] = \sqrt{E_s}\hat{h}_{sd}x$ and $\mathbf{var}\left[y_{sd}|\hat{h}_{sd},\hat{g},x\right] = \left(\frac{E_s\sigma_{sd}^2}{\varepsilon E_s\sigma_{sd}^2+\sigma_n^2}+1\right)\sigma_n^2$.

Conditioned on the estimates, $\hat{h}_{sd}$ and $\hat{g}$, and x, it follows from (3) that y_{rd} is not a complex Gaussian RV. However, for the purpose of evaluating its conditional moments, we will assume that g and $\hat{g}$ are jointly Gaussian RVs, and shall verify its accuracy in Sec. IV. This is necessary for analytical tractability because both g and $\hat{g}$ are products of two complex Gaussian RVs. As shown in Appendix B, we then get

$$\mathbf{E}\left[y_{rd}|\hat{h}_{sd},\hat{g},x\right] = \alpha_d\sqrt{E_s}\hat{g}x,$$
$$\mathbf{var}\left[y_{rd}|\hat{h}_{sd},\hat{g},x\right] = \frac{\alpha_d^2\sigma_{sr}^2\sigma_{rd}^2\sigma_{w_{srd}}^2E_s}{\alpha_t^2\sigma_{sr}^2\sigma_{rd}^2\varepsilon E_s+\sigma_{w_{srd}}^2} + \sigma_{srd}^2. \tag{9}$$

From the above results, the moments of D are given as follows.

Lemma *1:*

$$\mathbf{var}\left[D|\hat{h}_{sd},\hat{g},x\right] = \mathbf{E}\left[D|\hat{h}_{sd},\hat{g},x\right]$$
$$= \frac{E_s\left|\hat{h}_{sd}\right|^2}{\left(\frac{E_s\sigma_{sd}^2}{\varepsilon E_s\sigma_{sd}^2+\sigma_n^2}+1\right)\sigma_n^2} + \frac{\alpha_d^2E_s|\hat{g}|^2}{\frac{\alpha_d^2\sigma_{sr}^2\sigma_{rd}^2E_s\sigma_{w_{srd}}^2}{\alpha_t^2\sigma_{sr}^2\sigma_{rd}^2\varepsilon E_s+\sigma_{w_{srd}}^2}+\sigma_{srd}^2}.$$

Proof: The proof is similar to that in Appendix B, and is omitted. ■

B. SEP Expression

Theorem *1:* With noisy channel estimates obtained from cascaded channel estimation, the SEP of MPSK is given by

$$P_{\text{MPSK}} = \frac{\psi\left(\varepsilon,\bar{\gamma}_{sr},\bar{\gamma}_{rd}\right)}{\pi\sin^2\left(\frac{\pi}{M}\right)}\int_0^{\frac{M-1}{M}\pi}\frac{\sin^2\theta}{1+\frac{\varepsilon\sin^2\left(\frac{\pi}{M}\right)\bar{\gamma}_{sd}^2}{(1+(1+\varepsilon)\bar{\gamma}_{sd})\sin^2\theta}}$$
$$\times U\left(1,1;\frac{1+\varepsilon\left(\bar{\gamma}_{sr}+\bar{\gamma}_{rd}\right)}{\varepsilon^2\bar{\gamma}_{sr}\bar{\gamma}_{rd}}+\psi\left(\varepsilon,\bar{\gamma}_{sr},\bar{\gamma}_{rd}\right)\frac{\sin^2\theta}{\sin^2\left(\frac{\pi}{M}\right)}\right)d\theta, \tag{10}$$

where

$$\psi\left(\varepsilon,\bar{\gamma}_{sr},\bar{\gamma}_{rd}\right) = \frac{1}{\varepsilon^4\bar{\gamma}_{sr}^3\bar{\gamma}_{rd}^3}\left(1+\varepsilon\left(\bar{\gamma}_{sr}+\bar{\gamma}_{rd}\right)\right)$$
$$\times\left(1+\varepsilon\left(\bar{\gamma}_{sr}+\bar{\gamma}_{rd}\right)+\varepsilon^2\bar{\gamma}_{sr}\bar{\gamma}_{rd}\right)$$
$$\times\left(\bar{\gamma}_{sr}\bar{\gamma}_{rd}+\left(1+\bar{\gamma}_{sr}+\bar{\gamma}_{rd}\right)\left[1+\frac{\varepsilon^2\bar{\gamma}_{sr}\bar{\gamma}_{rd}}{1+\varepsilon\left(\bar{\gamma}_{sr}+\bar{\gamma}_{rd}\right)}\right]\right),$$

and $U(a,b;z)$ is the confluent hypergeometric function of the second kind [18, (9.210)].

Proof: The proof is relegated to Appendix C. ■

The expression in (10) is in the form of a single integral and is different from those in [3], [7], [8]. A closed-form upper bound on the SEP is then as follows.

Corollary *1:*

$$P_{\text{MPSK}} \le \frac{1}{\pi\sin^2\left(\frac{\pi}{M}\right)}\psi\left(\varepsilon,\bar{\gamma}_{sr},\bar{\gamma}_{rd}\right)f(M,\beta)$$
$$\times U\left(1,1;\frac{1+\varepsilon\left(\bar{\gamma}_{sr}+\bar{\gamma}_{rd}\right)}{\varepsilon^2\bar{\gamma}_{sr}\bar{\gamma}_{rd}}+\psi\left(\varepsilon,\bar{\gamma}_{sr},\bar{\gamma}_{rd}\right)\frac{1}{\sin^2\left(\frac{\pi}{M}\right)}\right), \tag{11}$$

where $\beta = \frac{\varepsilon\sin^2\left(\frac{\pi}{M}\right)\bar{\gamma}_{sd}^2}{1+(1+\varepsilon)\bar{\gamma}_{sd}}$ and $f(M,\beta) = \pi\left(1-\frac{1}{M}\right) - \frac{\pi}{2}\sqrt{\frac{\beta}{\beta+1}} - \sqrt{\frac{\beta}{\beta+1}}\arctan\left(\sqrt{\frac{\beta}{\beta+1}}\cot\left(\frac{\pi}{M}\right)\right)$.

Proof: The proof is relegated to Appendix D. ■

We obtain further insights into the SEP by analyzing the asymptotic regime in which $\bar{\gamma}_{sr}, \bar{\gamma}_{rd}$, and $\bar{\gamma}_{sd}$ are large.

Theorem *2:* In the asymptotic regime of $\bar{\gamma}_{sr}$, $\bar{\gamma}_{rd}$, and $\bar{\gamma}_{sd}\to\infty$, the SEP expression simplifies to

$$P_{\text{MPSK}} = \frac{(1+\varepsilon)^2}{4\epsilon^2\sin^4\left(\frac{\pi}{M}\right)}\left(\frac{1}{\bar{\gamma}_{sr}}+\frac{1}{\bar{\gamma}_{rd}}\right)\frac{1}{\bar{\gamma}_{sd}}\log\left(\vartheta^{-1}\right), \tag{12}$$

where $\vartheta = \frac{1}{\varepsilon}\left(\frac{1}{\bar{\gamma}_{sr}}+\frac{1}{\bar{\gamma}_{rd}}\right)\left(1+\frac{(1+\varepsilon)}{\sin^2\left(\frac{\pi}{M}\right)}\right)$.

Proof: The proof is relegated to Appendix E. ■

The expression in (12) differs from that in [3], [8] as it has an extra $\log\left(\vartheta^{-1}\right)$ term that depends on ε, $\bar{\gamma}_{sr}$, and $\bar{\gamma}_{rd}$. It can be easily shown from (12) that a full diversity order of two is achieved. However, the full diversity order is achievable only at high SNRs. Note that this is not an artifact of the cascaded channel estimation protocol. For example, even with perfect CSI, a similar result occurs [19].

IV. Simulations

We now present Monte Carlo simulation results that use 10^5 samples . We assume $\bar{\gamma}_{sr} = \bar{\gamma}_{rd} = \bar{\gamma}_{sd} = \gamma_0$. Figure 1 plots the SEP for 8PSK and 16PSK as a function of γ_0. Notice that the analytical and simulation results are in good agreement even at SNRs as low as 1 dB. This is because, unlike [9], [10], we do not assume that $\hat{g} = \hat{h}_{sr}\hat{h}_{rd}$, which is true only if the SNR is large or many pilot symbols are used. The close agreement clearly shows that the approximations made to compute (9) and the assumption that w_{srd} and n_{srd} are Gaussian are accurate. Unlike [8], no error floor occurs. Furthermore, the upper bound is tight and is within 0.2 dB of the exact SEP. Compared to a receiver with perfect CSI, imperfect CSI leads to a 3.2 dB loss in SNR when SEP $= 10^{-2}$. This gap can be reduced by boosting the pilot power, as shown in Fig. 2. Increasing the pilot energy improves SEP because the channel estimation error decreases. For example, $\varepsilon = 5$ gives a 2 dB performance gain for the same data SNR.

In Fig. 3, we compare cascaded channel estimation with the disintegrated channel estimation (D-CE) scheme of [12], in

[1]This assumes that conditioned on $\hat{g}$, w_{srd} is Gaussian.

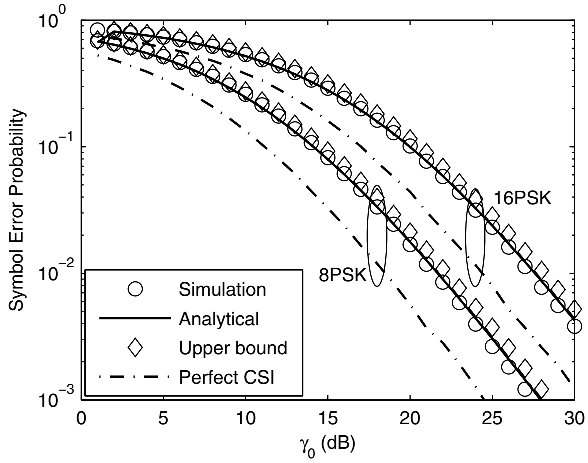

Fig. 1. SEP of cascaded channel estimation for 8PSK and 16PSK constellations ($\bar{\gamma}_{sr} = \bar{\gamma}_{rd} = \bar{\gamma}_{sd} = \gamma_0$ and $\varepsilon = 1$)

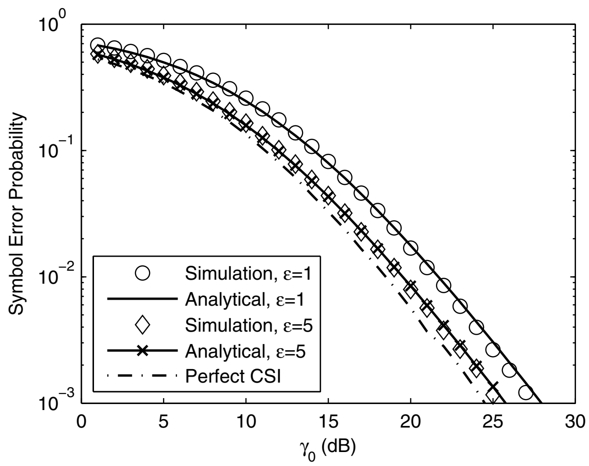

Fig. 2. SEP of cascaded channel estimation for different pilot power boosting factors, ε (8PSK and $\bar{\gamma}_{sr} = \bar{\gamma}_{rd} = \bar{\gamma}_{sd} = \gamma_0$)

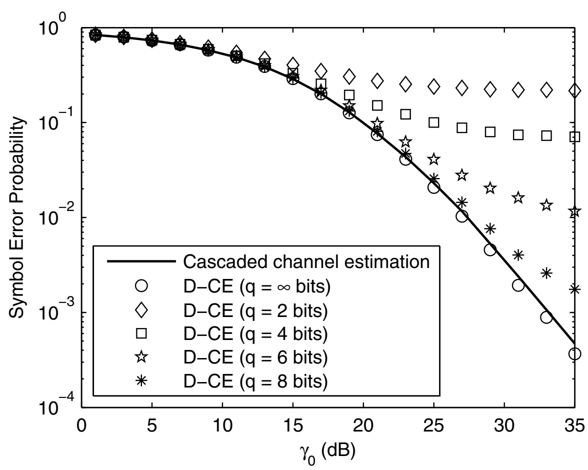

Fig. 3. Comparison of SEP of cascaded channel estimation and disintegrated channel estimation (D-CE) (16PSK, $\bar{\gamma}_{sr} = \bar{\gamma}_{rd} = \bar{\gamma}_{sd} = \gamma_0$, and $\varepsilon = 1$)

which the relay estimates h_{sr} and forwards a q-bit quantized estimate of it to the destination. As in [12], the forwarding is assumed to be error-free and instantaneous. It can be seen that despite incurring a larger training overhead, D-CE underperforms cascaded channel estimation for $q \leq 8$ bits. Even with $q = \infty$ bits, the gap in performance is less than 0.5 dB at an SEP of 10^{-3}.

V. Conclusions

In this paper, we analyzed the performance of a fixed gain single relay AF system that employs a time-efficient cascading channel estimation protocol. In this protocol, the destination estimates the cascaded channel gains of the SR and RD channels and does not acquire the SR channel estimate. We derived an expression for the SEP in the form of a single integral and also a tight upper bound that was within 0.2 dB of the exact answer. Unlike the approaches pursued in the literature, our analysis uses fewer simplifying assumptions. As a result, both the bound and the exact SEP expression are accurate even at low SNRs. We saw that a diversity order of 2 is achieved with cascaded channel estimation, albeit at high SNRs. We showed that pilot power boosting, if allowed, improves SEP. Thus, with pilot power boosting and cascaded channel estimation, performance close to perfect CSI is achievable with a short training phase.

Appendix

A. Derivation of Conditional Mean and Variance of y_{sd}

From (1), it follows that

$$\mathbf{var}\left[y_{sd}|\hat{h}_{sd}, x\right] = E_s \mathbf{var}\left[h_{sd}|\hat{h}_{sd}\right] + \sigma_n^2. \quad (13)$$

Using standard conditional Gaussian results and properties of LMMSE [15] to evaluate $\mathbf{var}\left[h_{sd}|\hat{h}_{sd}\right]$ yields the desired expression for the conditional variance.

The expression for the conditional mean also directly follows from standard conditional Gaussian results since $\hat{h}_{sd}$ is the LMMSE estimate of h_{sd} and n_{sd} is independent of $\hat{h}_{sd}$.

B. Derivation of Conditional Mean and Variance of y_{rd}

From (3), it follows that

$$\mathbf{var}\left[y_{rd}|\hat{g}, x\right] = \alpha_d^2 E_s \mathbf{var}\left[g|\hat{g}, x\right] + \sigma_{srd}^2. \quad (14)$$

Proceeding along the same lines as in Appendix A yields the desired result.

C. Proof of Theorem 1

Using Lemma 1 and [20, (40)], the SEP expression for MPSK given that D is a Gaussian RV is

$$P(\text{Err}) = \frac{1}{\pi}\int_0^{\frac{M-1}{M}\pi} \mathbf{E}\left[\exp\left(-\frac{E_s\left|\hat{h}_{sd}\right|^2}{\left(\frac{E_s\sigma_{sd}^2}{\varepsilon E_s\sigma_{sd}^2+\sigma_n^2}+1\right)\sigma_n^2}\right)\frac{\sin^2\left(\frac{\pi}{M}\right)}{\sin^2\theta}\right] \times \mathbf{E}\left[\exp\left(-\eta\left|\hat{g}\right|^2\right)\right] d\theta, \quad (15)$$

where $\eta = \frac{\alpha_d^2 E_s}{\frac{\alpha_d^2 \sigma_{sr}^2 \sigma_{rd}^2 E_s \sigma_{w_{srd}}^2}{\alpha_t^2 \sigma_{sr}^2 \sigma_{rd}^2 \varepsilon E_s + \sigma_{w_{srd}}^2} + \sigma_{srd}^2} \frac{\sin^2\left(\frac{\pi}{M}\right)}{\sin^2\theta}$. From (7) and (4), it can be shown that $\hat{h}_{sd}$ is a zero-mean Gaussian RV with variance $\hat{\sigma}_{sd}^2 = L_{sd}^2 \left(\varepsilon E_s \sigma_{sd}^2 + \sigma_n^2\right)$. Therefore, the first expectation in (15) evaluates to $\left(1 + \frac{\varepsilon \sin^2\left(\frac{\pi}{M}\right)\bar{\gamma}_{sd}^2}{(1+(1+\varepsilon)\bar{\gamma}_{sd})\sin^2\theta}\right)^{-1}$. Using (5) and (7), we have

$$|\hat{g}|^2 = L_g^2 \left(\varepsilon E_s \alpha_t^2 |g|^2 + |w_{srd}|^2 + 2\sqrt{\varepsilon E_s}\alpha_t |g| |w_{srd}| \cos\phi\right),$$

where ϕ is uniformly distributed in $[-\pi,\pi]$. Using the mutual independence of the RVs g, w_{srd}, and ϕ, we can write

$$\mathbf{E}\left[\exp\left(-\eta|\hat{g}|^2\right)\right] = \mathbf{E}_{|g|,|w_{srd}|}\left[\exp\left(-\eta L_g^2 \varepsilon E_s \alpha_t^2 |g|^2\right) \times \exp\left(-\eta L_g^2 |w_{srd}|^2\right) \mathbf{E}_\phi\left[\Delta(\phi)\right]\right], \quad (16)$$

where $\Delta(\phi) = \exp\left(-2\eta L_g^2 \alpha_t \sqrt{\varepsilon E_s} |g| |w_{srd}| \cos\phi\right)$. Using [18, (3.339)], it can be shown that

$$\mathbf{E}_\phi\left[\Delta(\phi)\right] = I_0\left(2\eta L_g^2 \alpha_t \sqrt{\varepsilon E_s} |g| |w_{srd}|\right), \quad (17)$$

where I_0 is the modified Bessel function of the first kind of order 0 [18, (8.406.1)].

The RV $|w_{srd}|$ is Rayleigh with variance $\sigma_{w_{srd}}^2$. It can be shown that the PDF of $|g|^2 = |h_{sr}|^2 |h_{rd}|^2$ is

$$p_{|g|^2}(z) = \frac{2}{\sigma_{rd}^2 \sigma_{sr}^2} K_0\left(\frac{2\sqrt{z}}{\sigma_{sr}\sigma_{rd}}\right), \quad z \geq 0, \quad (18)$$

where K_0 is the modified Bessel function of second kind of order 0 [18, (8.407.1)]. Substituting (17), (18), and [18, (6.631.4),(6.631.3)] in (16) and simplifying yields (10).

D. Brief Proof of Corollary 1

The second term, $\mathbf{E}\left[\exp\left(-\eta|\hat{g}|^2\right)\right]$, in the integrand in (15) can be upper bounded by replacing θ with $\pi/2$. Simplifying further along the lines of Appendix C, we get

$$P_{\text{MPSK}} \leq \frac{1}{\pi\sin^2\left(\frac{\pi}{M}\right)} \psi\left(\varepsilon, \bar{\gamma}_{sr}, \bar{\gamma}_{rd}\right) \times U\left(1,1; \frac{1+\varepsilon\left(\bar{\gamma}_{sr}+\bar{\gamma}_{rd}\right)}{\varepsilon^2 \bar{\gamma}_{sr}\bar{\gamma}_{rd}} + \psi\left(\varepsilon, \bar{\gamma}_{sr}, \bar{\gamma}_{rd}\right) \frac{1}{\sin^2\left(\frac{\pi}{M}\right)}\right) \times \int_0^{\frac{M-1}{M}\pi} \frac{\sin^2\theta}{\sin^2\theta + \frac{\varepsilon\sin^2\left(\frac{\pi}{M}\right)\bar{\gamma}_{sd}^2}{1+(1+\varepsilon)\bar{\gamma}_{sd}}} d\theta. \quad (19)$$

Substituting $\cot\theta = x$ in (19) and evaluating the integral in closed-form using partial fractions leads to the desired result.

E. Proof of Theorem 2

For large SNRs, it can be shown that [21, (13.5.9)]

$$U\left(1,1; \frac{1+\varepsilon\left(\bar{\gamma}_{sr}+\bar{\gamma}_{rd}\right)}{\varepsilon^2 \bar{\gamma}_{sr}\bar{\gamma}_{rd}} + \psi\left(\varepsilon, \bar{\gamma}_{sr}, \bar{\gamma}_{rd}\right) \frac{1}{\sin^2\left(\frac{\pi}{M}\right)}\right) = -\zeta - \log\left(\frac{1+\varepsilon\left(\bar{\gamma}_{sr}+\bar{\gamma}_{rd}\right)}{\varepsilon^2 \bar{\gamma}_{sr}\bar{\gamma}_{rd}} + \psi\left(\varepsilon, \bar{\gamma}_{sr}, \bar{\gamma}_{rd}\right) \frac{1}{\sin^2\left(\frac{\pi}{M}\right)}\right) + O\left(\frac{\bar{\gamma}_{sr}+\bar{\gamma}_{rd}}{\bar{\gamma}_{sr}\bar{\gamma}_{rd}}\right), \quad (20)$$

where ζ is the Euler-Mascheroni constant [18, (9.73)]. Similarly, for large β, it can be shown that

$$f\left(M, \beta\right) = \frac{\pi}{4\beta} + O\left(\frac{1}{\beta^2}\right). \quad (21)$$

Substituting this result and (20) in Corollary 1 and simplifying yields (12).

References

[1] R. U. Nabar, H. Bolcskei, and F. W. Kneubuhler, "Fading relay channels: performance limits and space-time signal design," *IEEE J. Sel. Areas. Commun.*, vol. 22, pp. 1099–1109, Aug. 2004.

[2] P. A. Anghel and M. Kaveh, "Exact symbol error probability of a cooperative network in a Rayleigh-fading environment," *IEEE Trans. Wireless Commun.*, vol. 3, pp. 1416–1421, Sep. 2004.

[3] A. Ribeiro, X. Cai, and G. B. Giannakis, "Symbol error probabilities for general cooperative links," *IEEE Trans. Wireless Commun.*, vol. 4, pp. 1264–1273, May 2005.

[4] M. O. Hasna and M.-S. Alouini, "A performance study of dual-hop transmissions with fixed gain relays," *IEEE Trans. Wireless Commun.*, vol. 3, pp. 1963–1968, Nov. 2004.

[5] Y. Li and S. Kishore, "Asymptotic analysis of amplify-and-forward relaying in Nakagami-fading environments," *IEEE Trans. Wireless Commun.*, vol. 6, pp. 4256–4262, Dec. 2007.

[6] B. Medepally and N. B. Mehta, "Voluntary energy harvesting relays and selection in cooperative wireless networks," *IEEE Trans. Wireless Commun.*, vol. 9, pp. 3543–3553, Nov. 2010.

[7] Y. Wu and M. Patzold, "Performance analysis of cooperative communication systems with imperfect channel estimation," in *Proc. ICC*, Jun. 2009.

[8] S. S. Ikki, M. Feteiha, and M. Uysal, "Performance analysis of cooperative diversity networks with imperfect channel estimation over Rician fading channels," in *Proc. Intl. Conf. Telecommun. (ICT)*, pp. 160–165, Apr. 2010.

[9] C. S. Patel and G. L. Stuber, "Channel estimation for amplify and forward relay based cooperation diversity systems," *IEEE Trans. Wireless Commun.*, vol. 6, pp. 2348–2356, Jun. 2007.

[10] B. Gedik and M. Uysal, "Impact of imperfect channel estimation on the performance of amplify-and-forward relaying," *IEEE Trans. Wireless Commun.*, vol. 8, pp. 1468–1479, Mar. 2009.

[11] F. S. Tabataba, P. Sadeghi, and M. R. Pakravan, "Outage probability and power allocation of amplify and forward relaying with channel estimation errors," *IEEE Trans. Wireless Commun.*, vol. 10, pp. 124–134, Jan. 2011.

[12] B. Gedik and M. Uysal, "Two channel estimation methods for amplify-and-forward relay networks," in *Proc. Canadian Conf. on Electrical and Computer Eng. (CCECE)*, pp. 615–618, May 2008.

[13] F. Gao, T. Cui, and A. Nallanathan, "On channel estimation and optimal training design for amplify and forward relay networks," *IEEE Trans. Wireless Commun.*, vol. 7, pp. 1907–1916, May 2008.

[14] H. Yomo and E. de Carvalho, "A CSI estimation method for wireless relay network," *IEEE Commun. Lett.*, vol. 11, pp. 480–482, Jun. 2007.

[15] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. Prentice Hall Signal Processing Series, 1993.

[16] M. R. Bhatnagar and A. Hjorungnes, "SER expressions for double differential modulation," in *Proc. Inf. Theory for Wireless Networks (ITWN)*, Jul. 2007.

[17] Y. Jia and A. Vosoughi, "Transmission resource allocation for training based amplify-and-forward relay systems," *IEEE Trans. Wireless Commun.*, vol. 10, pp. 450–455, Feb. 2011.

[18] L. S. Gradshteyn and L. M. Ryzhik, *Tables of Integrals, Series and Products*. Academic Press, 2007.

[19] M. Uysal, "Diversity analysis of space-time coding in cascaded Rayleigh fading channels," in *Proc. WCNC*, pp. 1471–1474, Apr. 2006.

[20] M.-S. Alouini and A. J. Goldsmith, "A unified approach for calculating error rates of linearly modulated signals over generalized fading channels," *IEEE Trans. Commun.*, vol. 47, pp. 1324–1334, Sep. 1999.

[21] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*. Dover Publications, Inc., 1972.