Impact of Imperfect Power Control on Splitting and Capture-Based Fast Distributed Selection

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***Abstract*—Opportunistic selection aims to select a node that improves the overall system performance the most. Selection is challenging as the nodes are geographically distributed and have only local knowledge. Yet, selection needs to be fast in order to allocate more time to the data transmission phase that exploits the selected node's services. In this paper, we analyze the impact of imperfect power control on a fast, distributed, splitting-based selection scheme that exploits the capture effect by allowing the transmitting nodes to have different target receive powers and uses information about the total received power to speed up selection. The scheme owes its speed to the use of different target powers that facilitate capture. However, imperfect power control makes the received power deviate from the target and, hence, affects performance. Our analysis quantifies how it changes the selection speed, leads to the selection of wrong node, or no node getting selected. We also quantify the effect of imperfect power control on the net system throughput and the extent of error beyond which power control is not useful.**

I. Introduction

Opportunistic selection finds applications in many wireless systems. For example, in the downlink of a cellular system, selection of the mobile station with the highest channel power gain improves throughput [1]. In cooperative communications, relay selection exploits spatial diversity [2], [3]. In wireless sensor networks, sensor selection reduces energy consumption and increases network lifetime [4].

In the above systems, determining the best node requires the nodes to be ordered according to their ability to improve the system performance. Formally, this ability is captured in terms of a real-valued *metric* that quantifies how useful the node will be if selected. For example, in amplify-and-forward relaying, the metric is the harmonic mean of the source-to-relay and relay-to-destination channel gains [5]. In the proportional-fair scheduler, the ratio of the demanded to the average rate is the metric [1]. Since the nodes are geographically distributed, each node knows only its metric and not that of the others. Hence, distributed selection schemes are needed [5]–[10].

An important example of a distributed selection scheme is the splitting-based scheme [10]. In it, the nodes whose metrics lie between two thresholds transmit in a slot and the other nodes do not transmit. At the end of every slot, a coordinating node called sink broadcasts an idle, success, or collision outcome to all the nodes based on whether zero, one, or multiple nodes transmitted in that slot. Accordingly, the thresholds are adjusted for the next slot. The algorithm continues until the best node is selected.

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The splitting scheme was speeded up significantly by exploiting capture by the Variable Power Multiple Access Selection Power-Based Splitting scheme (VPMAS-PS) [11]. In VPMAS-PS also, a node transmits in a slot if and only if its metric lies between two thresholds. Furthermore, a node that transmits also adjusts its transmit power so that its received power is one of L possible target power levels. The different target powers make selection possible even in the presence of multiple simultaneous transmissions, which otherwise would have resulted in a collision and a wasted slot.

For example, consider a case with two target powers, namely, P_H and P_L, where $P_L = \gamma\sigma^2, P_H = \gamma(\eta P_L + \sigma^2)$, γ is the minimum signal-to-interference-plus-noise power ratio (SINR) at which the sink can decode, and σ^2 is the noise power. With this, even when $\nu \leq \eta$ nodes with target power P_L transmit, a single node with target power P_H has SINR $= \frac{P_H}{\nu P_L + \sigma^2} \geq \gamma$, and, hence, gets decoded. Sink selects the node that it decodes first. We shall refer to η as the *adversary order*. Furthermore, VPMAS-PS uses the total received power to estimate the best node's target power to speed up selection even more.

However, in practice, power control errors at the transmitter make its receive power deviate from its target. These errors arise due to channel estimation errors, quantization errors, and outdated channel information. This can affect the performance of VPMAS-PS and other splitting-based and, in general, capture-based medium access control (MAC) protocols that exploit power control [12]–[14]. While imperfect power control has been studied in the literature extensively, the focus has been on the MAC protocols. For example, in [15], power control error was shown to increase the throughput of a slotted ALOHA system. In [16], [17], the outage probability and capacity of code-division multiple-access (CDMA) systems was analyzed as a function of this error. In [18], the impact of power control error on the use of direct sequence (DS)-CDMA in low earth orbiting satellite links was studied.

Our focus on the impact of imperfect power control on selection schemes distinguishes our work from the literature on this topic. To gain a quantitative understanding, we focus on VPMAS-PS, as it is the fastest known distributed, scalable selection scheme that is designed to exploit power control.

We quantify how imperfect power control affects the selection by (i) changing the time required to select the best node, (ii) leading to a wrong selection, and (iii) causing selection outage, in which no node gets selected. We analytically characterize the time taken to select the best node for a system

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with $L = 2$ target power levels.[1] As this is a random variable (RV), which depends on the realizations of the metrics and the power control errors of the various nodes, we carefully characterize the selection probability of the best node in a slot. From this, measures such as mean, standard deviation, and higher order moments of the selection time can be computed. We also characterize the impact of imperfect power control on the net system throughput, which accounts for the time overhead of selection and its imperfections.

The paper is organized as follows. Section II presents the system model. In Section III, we analyze the performance of VPMAS-PS with imperfect power control. Results and conclusions follow in Section IV and Section V, respectively.

II. Model

Consider a wireless network with N nodes and a sink. Each node has a real-valued metric that is uniformly distributed in $[0, 1)$.[2] Node $[i]$ denotes the node with the i^{th} largest metric, $\mu_{[i]}$. The sink needs to select the best node, i.e., node [1]. We consider a time-slotted system in which a transmission lasts for one slot. In every slot k, each node maintains two thresholds $\mu_{\max}(k)$ and $\mu_{\min}(k)$. The interval $\Delta(k) = [\mu_{\min}(k), \mu_{\max}(k))$ is referred to as the transmission interval in slot k, and $|\Delta(k)|$ represents its length. The node $[i]$ transmits in slot k if and only if $\mu_{[i]} \in \Delta(k)$. If it transmits in slot k, its target receive power, $P_{[i]}(k)$, is either P_H or P_L, and depends on its metric $\mu_{[i]}$ as follows:

$$P_{[i]}(k) = \begin{cases} P_H, & \mu_{[i]} \in \mathcal{H}\{\Delta(k)\} \\ P_L, & \mu_{[i]} \in \mathcal{L}\{\Delta(k)\} \end{cases}, \tag{1}$$

where $\mathcal{H}\{\Delta(k)\}$and $\mathcal{L}\{\Delta(k)\}$ represent the upper and lower halves of $\Delta(k)$, respectively. Thus, the nodes whose metrics lie in $\mathcal{H}\{\Delta(k)\}$ have higher target power, which improves the chances of their transmissions getting captured. Further, when the best node transmits, its target power is never less than any other transmitting node's target power. For example, in Fig. 1, the nodes [1] and [2] have target power P_H and P_L, in slot k, as $\mu_{[1]}$ and $\mu_{[2]}$ are in $\mathcal{H}\{\Delta(k)\}$ and $\mathcal{L}\{\Delta(k)\}$, respectively.

When M nodes, [1], [2],. . .,$[M]$, transmit in slot k, the SINR of a node $[i]$ in slot k, $\text{SINR}_{[i]}(k)$, is given by

$$\text{SINR}_{[i]}(k) = \frac{P_{[i]}(k)}{\sum\limits_{j \in \{1,\ldots,M\}\setminus\{i\}} P_{[j]}(k) + \sigma^2}. \tag{2}$$

The node [1] gets selected if $\text{SINR}_{[1]}(k) \geq \gamma$. The threshold γ depends on the modulation and coding scheme and is of the order of 8-10 dB.[3]

In order to set its transmit power, a node must know the channel gain from itself to the sink. This knowledge can be acquired by periodic feedback from the sink, but can be imperfect. We assume that in slot k, the sink can measure the total received power $P^{\text{tot}}(k)$ and can sense a transmission. This capability already exists today in the form of the receive signal strength indicator (RSSI).

Notation: The number of nodes that have target power P_H and P_L in slot k are denoted by $n_{\text{H}}(k)$ and $n_{\text{L}}(k)$, respectively. The total number of nodes that transmit in slot k is denoted by $n_{\text{T}}(k) = n_{\text{H}}(k) + n_{\text{L}}(k)$. Further, $X \sim \mathcal{N}(\mu, \sigma^2)$ implies that X is a Gaussian RV with mean μ and variance σ^2. The multinomial $\binom{N}{i_1, i_2, \ldots, i_p}$ stands for $\frac{N!}{(N - i_1 - i_2 - \cdots - i_p))! i_1! i_2! \cdots i_p!}$. Pr$[A]$ denotes the probability of an event A. Probability of an event A conditioned on the event B is denoted by Pr$[A|B]$.

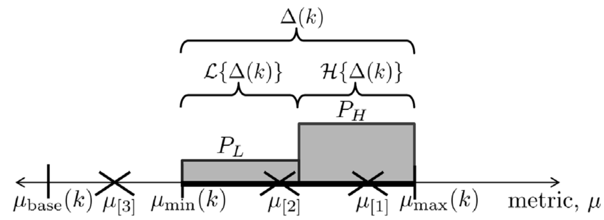

Fig. 1. Target powers and transmissions by different nodes based on the location of their metrics in the interval $[\mu_{\min}(k), \mu_{\max}(k))$.

A. VPMAS-PS Overview

We now give a brief summary of VPMAS-PS. The reader is referred to [11] for a detailed description.

At the beginning of a slot k, each node maintains three thresholds $\mu_{\min}(k)$, $\mu_{\max}(k)$, and $\mu_{\text{base}}(k)$. The threshold $\mu_{\text{base}}(k)$ is set such that $\mu_{[1]}$ lies in $[\mu_{\text{base}}(k), \mu_{\max}(k))$. The scheme is such that in any non-idle slot, the best node always transmits, and is the one that eventually gets selected.

Initialization: In slot 1, metrics of all the nodes lie in $[0, 1)$. Therefore, $\mu_{\text{base}}(1) = 0$ and $\mu_{\max}(1) = 1$. The threshold $\mu_{\min}(1)$ is set so that a fraction z of N nodes transmit in slot 1, which implies that

$$\mu_{\min}(1) = \mu_{\max}(1) - (\mu_{\max}(1) - \mu_{\text{base}}(1))\, z. \tag{3}$$

Thus, $|\Delta(1)| = z$. Appendix A discusses how z is chosen so as to maximize the probability of success in slot 1.

Update of Thresholds: Based on the outcome in slot k that is broadcast by the sink, the thresholds in slot $k+1$ are updated as follows:[4]

Success in slot k, $\mathcal{S}(k)$: The algorithm terminates.

Collision in slot k, $\mathcal{C}(k)$: This implies that at least two nodes have their metrics in $\Delta(k)$. As $\mu_{\min}(k) \leq \mu_{[1]}$, nodes set $\mu_{\text{base}}(k+1) = \mu_{\min}(k)$. The sink then uses $P^{\text{tot}}(k)$ to locate where $\mu_{[1]}$ lies as follows: if $P^{\text{tot}}(k) \leq P_H$, then all the transmitting nodes have target power P_L and their metrics must lie in $\mathcal{L}\{\Delta(k)\}$, which becomes $\Delta(k+1)$. If $P^{\text{tot}}(k) > P_H$, then it is very likely that $\mu_{[1]}$ lies in $\mathcal{H}\{\Delta(k)\}$,

[1]We focus only on two power level scenario in this paper due to space constraints. The scheme easily generalizes to $L \geq 3$ power levels. However, its analysis becomes cumbersome.

[2]In case the metric μ has a non-uniform continuous probability distribution function with CDF F, then $\eta = F(\mu)$ can be defined as a new metric. It is uniformly distributed between $[0, 1)$ and preserves order.

[3]CDMA systems use typically much lower decoding thresholds, but at the expense of extra bandwidth. We do not consider these systems.

[4]Broadcast is assumed to be error free due to its low payload.

which then becomes $\Delta(k+1)$.[5] Note that the length of transmission interval is *halved* and the sink broadcasts an *additional* 1-bit information about whether $P^{\text{tot}}(k)$ exceeds P_H or not when a collision occurs.

Idle in slot k, $\mathcal{I}(k)$: If no collision has occurred in any of the previous slots, then the N nodes' metrics must be uniformly distributed in the interval $[\mu_{\text{base}}(k), \mu_{\min}(k))$. Hence, $\mu_{\max}(k+1) = \mu_{\min}(k)$, $\mu_{\text{base}}(k+1) = \mu_{\text{base}}(k)$, and $\mu_{\min}(k+1) = \mu_{\max}(k+1) - (\mu_{\max}(k+1) - \mu_{\text{base}}(k+1))\, z$. Otherwise, only the following rare event can lead to an idle in slot k. A collision occurs in slot $k-1$ with $P^{\text{tot}}(k-1) > P_H$ but all the transmitting nodes' metrics lie in $\mathcal{L}\{\Delta(k-1)\}$. Hence, $\mathcal{H}\{\Delta(k-1)\}$ becomes $\Delta(k)$ and results in $\mathcal{I}(k)$. In such a case, $\mathcal{L}\{\Delta(k-1)\}$ is made $\Delta(k+1)$ and $\mu_{\text{base}}(k+1) = \mu_{\text{base}}(k)$.

B. Imperfect Power Control

We model the power control error as a lognormal RV, as this model has been used in [15]–[17]. Then, the received power of the transmitting node $[i]$ in slot k is equal to $P_{[i]}(k)e^{l_{[i]}}$, where $e^{l_{[i]}}$ models the power control error and $l_{[i]} \sim \mathcal{N}(0, \sigma_l^2)$. When the M nodes $[1], [2], \ldots, [M]$ transmit, $\text{SINR}_{[i]}(k)$ in (2) now takes the form

$$\text{SINR}_{[i]}(k) = \frac{P_{[i]}(k)e^{l_{[i]}}}{\sum\limits_{j \in \{1,\ldots,M\}\setminus\{i\}} P_{[j]}(k)e^{l_{[j]}} + \sigma^2}. \quad (4)$$

Now a node $[i] \neq [1]$ may get selected in slot k if $\text{SINR}_{[i]}(k) \geq \gamma$. Also note that even if one node transmits in the slot, it may not get selected due to its power control error. Henceforth, with a little abuse of notation, we shall refer to this case also as $\mathcal{C}(k)$. Further, a new case can arise now in which, despite nodes transmitting with target power P_H in slot k, the total received power falls below P_H and, thus, $\Delta(k+1) = \mathcal{L}\{\Delta(k)\}$.[6]

III. Analysis: Impact of Imperfect Power Control

We now analyze the impact of imperfect power control on the time required to select the best node. For this, we evaluate the probability that the best node is selected in each slot.

We first evaluate $\Pr[\mathcal{S}(k)]$ for $k = 1$ and 2 exactly. Then, for $k \geq 3$, we calculate the probability of the four most likely sequences that lead to $\mathcal{S}(k)$. We do so because the number of sequences that lead to $\mathcal{S}(k)$ grows exponentially with k. The perfect power control scenario enables us to identify these sequences. For example, the identified sequences neglect the highly unlikely idle event after a collision. Further, to ensure analytical tractability, we ignore the effect of noise when two or more nodes transmit and the event that a node $[i]$ can be decoded in slot k when more than two nodes have target power at least $P_{[i]}(k)$. The events where more than four nodes transmit simultaneously are also unlikely and are ignored.

[5] Another rare event that can lead to $P^{\text{tot}}(k) > P_H$ is when all the transmitting nodes have a target power P_L. This requires that more than $\frac{P_H - \sigma^2}{P_L} \approx \eta\gamma$ nodes have their metrics in $\mathcal{L}\{\Delta(k)\}$, which is very unlikely.

[6] This issue also triggers a change in VPMAS-PS to cover the case when $n_{\text{T}}(k) = n_{\text{H}}(k)$. In this case, an idle occurs in slot $k+1$, which otherwise is impossible. Thus, $\Delta(k+2) = \mathcal{H}\{\Delta(k)\}$.

A. Probability of Success in Slot 1, $Pr[\mathcal{S}(1)]$

From the law of total probability, we have

$$\Pr[\mathcal{S}(1)] = \Pr[\mathcal{S}(1), n_{\text{H}}(1) > 0] + \Pr[\mathcal{S}(1), n_{\text{H}}(1) = 0]. \quad (5)$$

The term $\Pr[\mathcal{S}(1), n_{\text{H}}(1) > 0]$ is the probability of success in the first slot when at least one node, including the best node, has target power P_H. The total number of ways $n_{\text{H}}(1)$ and $n_{\text{L}}(1)$ nodes can be chosen from the N nodes is $\binom{N}{n_{\text{L}}(1), n_{\text{H}}(1)}$. In slot 1, the probability that a node is silent is $1-z$ and the probability that a node has a target power P_H is $\frac{z}{2}$, which is also the probability that a node has a target power P_L. A success occurs if $\text{SINR}_{[1]}(1) \geq \gamma$. Hence,

$$\Pr[\mathcal{S}(1), n_{\text{H}}(1) > 0] = \sum_{n_{\text{L}}(1)=0}^{N-n_{\text{H}}(1)} \sum_{n_{\text{H}}(1)=1}^{N} \binom{N}{n_{\text{L}}(1), n_{\text{H}}(1)} \times (1-z)^{N-n_T(1)} \left(\frac{z}{2}\right)^{n_T(1)} \Pr\left[\text{SINR}_{[1]}(1) \geq \gamma\right]. \quad (6)$$

When $n_{\text{T}}(1) = n_{\text{H}}(1) = 1$, we have $\Pr\left[\text{SINR}_{[1]}(1) \geq \gamma\right] = \Pr\left[\frac{P_H e^{l_{[1]}}}{\sigma^2} \geq \gamma\right] = Q\left(\ln\left(\frac{\gamma\sigma^2}{P_H}\right)\right)$, where $Q(.)$ is the Gaussian Q-function.

When $n_{\text{T}}(k) > 1$ and $n_{\text{H}}(1) > 0$, neglecting σ^2 yields

$$\Pr\left[\text{SINR}_{[1]}(1) \geq \gamma\right] \approx \Pr\left[\frac{e^{l_{[1]}}}{\sum_{i=2}^{n_{\text{H}}(1)} e^{l_{[i]}} + \frac{P_L}{P_H}\sum_{j=n_{\text{H}}(1)+1}^{n_{\text{H}}(1)+n_{\text{L}}(1)} e^{l_{[j]}}} \geq \gamma\right]. \quad (7)$$

We approximate the denominator $\sum\limits_{i=2}^{n_{\text{H}}(1)} e^{l_{[i]}} + \frac{P_L}{P_H} \sum\limits_{j=n_{\text{H}}(1)+1}^{n_{\text{H}}(1)+n_{\text{L}}(1)} e^{l_{[j]}}$ in (7) by a lognormal RV e^{l_α}, where $l_\alpha \sim \mathcal{N}(\mu_\alpha, \sigma_\alpha^2)$. Here, μ_α and σ_α^2 are given in closed-form by the Fenton-Wilkinson (F-W) method [19]. Hence, (7) simplifies to $\Pr\left[\text{SINR}_{[1]}(1) \geq \gamma\right] \approx \Pr\left[\frac{e^{l_{[1]}}}{e^{l_\alpha}} \geq \gamma\right] = Q\left(\frac{\ln(\gamma)+\mu_\alpha}{\sqrt{\sigma_l^2+\sigma_\alpha^2}}\right)$.

Similarly, the second term in (5) is given by

$$\Pr[\mathcal{S}(1), n_{\text{H}}(1) = 0] = \sum_{n_{\text{L}}(1)=1}^{N} \binom{N}{n_{\text{L}}(1)} \left(\frac{z}{2}\right)^{n_{\text{L}}(1)} \times (1-z)^{N-n_{\text{L}}(1)} \Pr\left[\text{SINR}_{[1]}(1) \geq \gamma\right]. \quad (8)$$

Here also, for $n_{\text{L}}(1) = 1$, we have $\Pr\left[\text{SINR}_{[1]}(1) \geq \gamma\right] = Q\left(\ln(\frac{\gamma\sigma^2}{P_L})\right)$. For $n_{\text{L}}(1) > 1$, upon approximating $\sum_{j=2}^{n_{\text{L}}(1)} e^{l_{[j]}}$ by e^{l_β}, where $l_\beta \sim \mathcal{N}(\mu_\beta, \sigma_\beta^2)$, we get $\Pr\left[\text{SINR}_{[1]}(1) \geq \gamma\right] \approx Q\left(\frac{\ln(\gamma)+\mu_\beta}{\sqrt{\sigma_l^2+\sigma_\beta^2}}\right)$.

B. Probability of Success in Slot 2, $Pr[\mathcal{S}(2)]$

A success in slot 2 can be preceded by $\mathcal{I}(1)$ or $\mathcal{C}(1)$. Hence,

$$\Pr[\mathcal{S}(2)] = \Pr[\mathcal{I}(1)] \Pr[\mathcal{S}(2)|\mathcal{I}(1)] + \Pr[\mathcal{C}(1), \mathcal{S}(2)]. \quad (9)$$

As the algorithm effectively restarts if an idle occurs in slot 1, we have $\Pr[\mathcal{S}(2)|\mathcal{I}(1)] = \Pr[\mathcal{S}(1)]$. This happens when all the N nodes are silent in slot 1. Hence, $\Pr[\mathcal{I}(1)] = (1-z)^N$.

The event $\{\mathcal{C}(1), \mathcal{S}(2)\}$ above can occur only if:

E1) In slot 1, no node gets selected, i.e., $\mathrm{SINR}_{[i]}(1) < \gamma$, for $i = 1, \ldots, n_{\mathrm{T}}(1)$.

E2) For a success in slot 2, $\mu_{[1]} \in \Delta(2)$. This happens if $P^{\mathrm{tot}}(1) > P_H$ when $P_{[1]}(1) = P_H$, and $P^{\mathrm{tot}}(1) \leq P_H$ when $P_{[1]}(1) = P_L$.

E3) In slot 2, the best node is selected, i.e., $\mathrm{SINR}_{[1]}(2) \geq \gamma$.

From the law of total probability, the second term in (9) is

$$\Pr[\mathcal{C}(1), \mathcal{S}(2)] = \Pr[n_{\mathrm{H}}(1) = 0, \mathcal{C}(1), \mathcal{S}(2)] + \Pr[n_{\mathrm{H}}(1) > 0, \mathcal{C}(1), \mathcal{S}(2)]. \quad (10)$$

In (10), the event $\{n_{\mathrm{H}}(1) = 0, \mathcal{C}(1), \mathcal{S}(2)\}$ occurs if slot 1 results in a collision and all the $n_{\mathrm{T}}(1)$ nodes, including the best node, have target power P_L, and is followed by a success in slot 2. As all the $n_{\mathrm{T}}(1)$ nodes' metrics are in $\mathcal{L}\{\Delta(1)\}$, E2 requires $\mathcal{L}\{\Delta(1)\}$ to be chosen as $\Delta(2)$. All the $n_{\mathrm{T}}(1)$ nodes then transmit again in second slot. Thus, $n_{\mathrm{T}}(1) = n_{\mathrm{L}}(1) = n_{\mathrm{T}}(2) = n_{\mathrm{H}}(2) + n_{\mathrm{L}}(2)$. Among these $n_{\mathrm{T}}(2)$ nodes, at least one node should have target power P_H in slot 2, as otherwise a collision will occur again in slot 2. This, therefore, implies that $P_{[1]}(2) = P_H$.

As the length of the transmission interval is halved with each collision, the probability that a node has target power P_H in slot 2 is $\frac{z}{4}$, which is also the probability that a node has target power P_L in slot 2. Therefore,

$$\begin{aligned}
&\Pr[n_{\mathrm{H}}(1) = 0, \mathcal{C}(1), \mathcal{S}(2)] = \\
&\sum_{n_{\mathrm{L}}(1)=1}^{N} \sum_{n_{\mathrm{H}}(2)=1}^{n_{\mathrm{L}}(1)} \binom{N}{n_{\mathrm{L}}(2), n_{\mathrm{H}}(2)} \left(\frac{z}{4}\right)^{n_{\mathrm{L}}(1)} (1-z)^{N-n_{\mathrm{L}}(1)} \\
&\quad \times \Pr\left[\mathrm{SINR}_{[1]}(1) < \gamma, \ldots, \mathrm{SINR}_{[n_{\mathrm{L}}(1)]}(1) < \gamma,\right. \\
&\qquad\qquad \left. P^{\mathrm{tot}}(1) \leq P_H, \mathrm{SINR}_{[1]}(2) \geq \gamma\right]. \quad (11)
\end{aligned}$$

To calculate (11), we consider below the following three cases: (i) $n_{\mathrm{L}}(1) = 1$, (ii) $n_{\mathrm{L}}(1) = 2$, and (iii) $n_{\mathrm{L}}(1) \geq 3$.

Case (i): If $n_L(1) = 1$, then E1, E2, and E3 require $\frac{P_L e^{l_{[1]}}}{\sigma^2} < \gamma$, $P_L e^{l_{[1]}} + \sigma^2 \leq P_H$, and $\frac{P_H e^{l_{[1]}}}{\sigma^2} \geq \gamma$, respectively. Hence, $\kappa_1 \leq l_{[1]} < \kappa_2$, where $\kappa_1 = \ln\left(\frac{\sigma^2 \gamma}{P_H}\right)$ and $\kappa_2 = \min\left(\ln\left(\frac{\sigma^2 \gamma}{P_L}\right), \ln\left(\frac{P_H - \sigma^2}{P_L}\right)\right)$. Its probability α_1 is given by

$$\alpha_1 = Q\left(\frac{\kappa_1}{\sigma_l}\right) - Q\left(\frac{\kappa_2}{\sigma_l}\right). \quad (12)$$

Case (ii): If $n_L(1) = 2$, then E1 requires $\frac{e^{l_{[1]}}}{e^{l_{[2]}}} < \gamma$ and $\frac{e^{l_{[2]}}}{e^{l_{[1]}}} < \gamma$, E2 requires $P_L(e^{l_{[1]}} + e^{l_{[2]}}) \leq P_H$, and E3 requires $\frac{P_H e^{l_{[1]}}}{P_L e^{l_{[2]}}} \geq \gamma$. Putting it all together, we get $e^{l_{[2]}} < \left(\frac{P_H}{P_L}\right)$, and $\vartheta_1(l_{[2]}) < l_{[1]} < \vartheta_2(l_{[2]})$, where $\vartheta_1(l_{[2]}) \triangleq \max\left(\ln\left(\frac{e^{l_{[2]}}}{\gamma}\right), \ln\left(\frac{\gamma P_L e^{l_{[2]}}}{P_H}\right)\right)$ and $\vartheta_2(l_{[2]}) \triangleq \min\left(\ln\left(\gamma e^{l_{[2]}}\right), \ln\left(\frac{P_H}{P_L} - e^{l_{[2]}}\right)\right)$. Hence, the probability α_2 that $l_{[1]}$ satisfies the required constraint is

$$\alpha_2 = \frac{1}{\sqrt{2\pi\sigma_l^2}} \int_{-\infty}^{\ln\left(\frac{P_H}{P_L}\right)} e^{-\frac{{l_{[2]}}^2}{2\sigma_l^2}} \left[Q\left(\frac{\vartheta_1(l_{[2]})}{\sigma_l}\right) - Q\left(\frac{\vartheta_2(l_{[2]})}{\sigma_l}\right)\right] \mathrm{d}l_{[2]}.$$

Case (iii): If $n_L(1) \geq 3$, E1 is true by default from the assumptions stated at the beginning of this analysis. E2 requires $P^{\mathrm{tot}}(1) \leq P_H$, where $P^{\mathrm{tot}}(1) = P_L\left(\sum_{i=1}^{n_{\mathrm{L}}(1)} e^{l_{[i]}}\right) = P_L\left(e^{l_{[1]}} + \sum_{i=2}^{n_{\mathrm{H}}(2)} e^{l_{[i]}} + \sum_{j=1}^{n_{\mathrm{L}}(2)} e^{l_{[j]}}\right) \approx P_L\left(e^{l_{[1]}} + e^{l_\eta} + e^{l_\lambda}\right)$, where e^{l_η} and e^{l_λ} approximate the sums $\sum_{i=2}^{n_{\mathrm{H}}(2)} e^{l_{[i]}}$ and $\sum_{j=1}^{n_{\mathrm{L}}(2)} e^{l_{[j]}}$, respectively, with $l_\eta \sim \mathcal{N}(\mu_\eta, \sigma_\eta^2)$ and $l_\lambda \sim \mathcal{N}(\mu_\lambda, \sigma_\lambda^2)$. And, E3 requires the following:

$$\mathrm{SINR}_{[1]}(2) \approx \frac{P_H e^{l_{[1]}}}{P_H e^{l_\eta} + P_L e^{l_\lambda}} \geq \gamma.$$

Altogether, it is required that $e^{l_\lambda} < \frac{P_H}{P_L}, e^{l_\eta} < \frac{P_H}{P_L} - e^{l_\lambda}$, $\vartheta_3(l_\eta, l_\lambda) \leq l_{[1]} \leq \vartheta_4(l_\eta, l_\lambda)$, where $\vartheta_3(l_\eta, l_\lambda) = \ln\left(\gamma e^{l_\eta} + \frac{\gamma P_L}{P_H} e^{l_\lambda}\right)$ and $\vartheta_4(l_\eta, l_\lambda) = \ln\left(\frac{P_H}{P_L} - e^{l_\eta} - e^{l_\lambda}\right)$. For a given $n_{\mathrm{L}}(1)$ and $n_{\mathrm{H}}(2)$, the probability α_3 that $l_{[1]}$ satisfies the constraints is

$$\begin{aligned}
\alpha_3 \approx \frac{1}{2\pi\sigma_\eta\sigma_\lambda} &\int_{-\infty}^{\ln\left(\frac{P_H}{P_L}\right)} \int_{-\infty}^{\ln\left(\frac{P_H}{P_L} - e^{l_\lambda}\right)} e^{-\frac{(l_\eta - \mu_\eta)^2}{2\sigma_\eta^2}} e^{-\frac{(l_\lambda - \mu_\lambda)^2}{2\sigma_\lambda^2}} \\
&\times \left[Q\left(\frac{\vartheta_3(l_\eta, l_\lambda)}{\sigma_l}\right) - Q\left(\frac{\vartheta_4(l_\eta, l_\lambda)}{\sigma_l}\right)\right] \mathrm{d}l_\eta \mathrm{d}l_\lambda.
\end{aligned}$$

In (10), $\Pr[n_{\mathrm{H}}(1) = 0, \mathcal{C}(1), \mathcal{S}(2)]$ is lower bounded by the sum of probabilities of the above three cases. Similarly, $\Pr[n_{\mathrm{H}}(1) > 0, \mathcal{C}(1), \mathcal{S}(2)]$ in (10) is calculated.

C. Probability of Success in Slot k, $Pr[\mathcal{S}(k)], k > 2$

From the law of total probability, we have

$$\begin{aligned}
\Pr[\mathcal{S}(k)] &= \Pr[\mathcal{I}(1), \mathcal{S}(k)] + \Pr[\mathcal{C}(1), \mathcal{S}(k)], \\
&= \Pr[\mathcal{I}(1)] \Pr[\mathcal{S}(k-1)] + \Pr[\mathcal{C}(1), \mathcal{S}(k)]. \quad (13)
\end{aligned}$$

Equation (13) follows as VPMAS-PS starts afresh after $\mathcal{I}(1)$.

To calculate $\Pr[\mathcal{C}(1), \mathcal{S}(k)]$ in (13), we now track the following four sequences, namely, $\mathrm{A}_{1,k}, \mathrm{A}_{2,k}, \mathrm{A}_{3,k}$, and $\mathrm{A}_{4,k}$, which result in the event $\{\mathcal{C}(1), \mathcal{S}(k)\}$.

Sequence $A_{1,k}$: It consists of the following two events: i) only one node transmits in the first $k-1$ slots with target power P_L. Its SINR is below γ in each of these slots and VPMAS-PS correctly splits the transmission interval in each slot, and ii) the node transmits with target power P_H in slot k and gets selected.

The first event requires $\frac{P_L e^{l_{[1]}}}{\sigma^2} < \gamma$ and $P^{\mathrm{tot}}(m) = (P_L e^{l_{[1]}} + \sigma^2) \leq P_H$, for $m \leq k-1$. The second event requires $\frac{P_H e^{l_{[1]}}}{\sigma^2} \geq \gamma$. The probability that $l_{[1]}$ satisfies these conditions is α_1, which is given in (12). As the transmission interval is halved in length with each split, $|\Delta(k)| = \frac{|\Delta(1)|}{2^{k-1}} = \frac{z}{2^{k-1}}$. The probability that a node's metric lies in $\mathcal{H}\{\Delta(k)\}$ is $\frac{z}{2^k}$. Hence,

$$\Pr[A_{1,k}] = \binom{N}{1}(1-z)^{N-1}\left(\frac{z}{2^k}\right)\alpha_1.$$

Sequence $A_{2,k}$: It consists of the following events: i) only two nodes transmit with the same target power and collide for the first $k-1$ slots, ii) the transmission interval splits correctly in each of these $k-1$ slots, and iii) in slot k the nodes transmit with different target powers, and a success occurs.

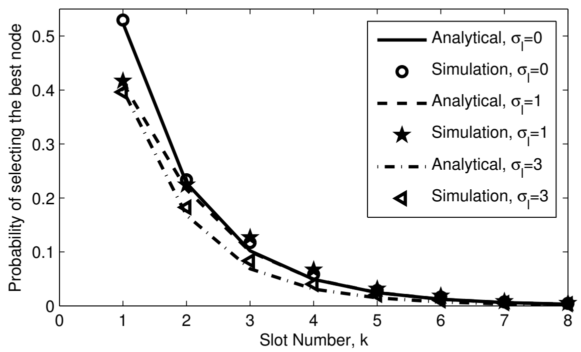

Fig. 2. Probability of selecting the best node in different slots ($\eta = 5$).

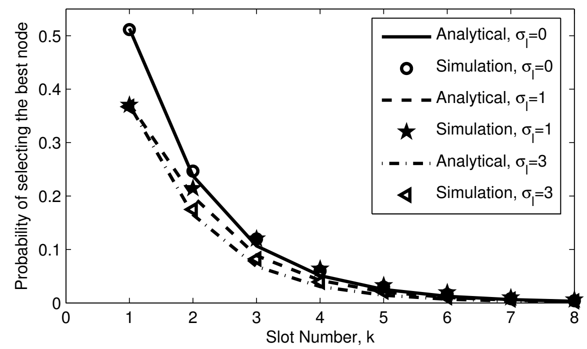

Fig. 3. Probability of selecting the best node in different slots ($\eta = 2$).

Sequence $A_{3,k}$: This is the same as $A_{2,k}$ except that three nodes instead of two have same target power and collide in the first $k-1$ slots. In slot k, one node has target power P_H, other two nodes have target power P_L, and a success occurs.

Sequence $A_{4,k}$: This consists of the following three events: i) three nodes transmit in slot 1, the best node and at least one other node have the same target power and collide in the first $k-1$ slots, ii) VPMAS-PS correctly splits the interval in each of these $k-1$ slots, and iii) in slot k, only two nodes transmit and have different target powers resulting in a success.

Due to space constraints, the derivations of probabilities of $\mathrm{A}_{2,k}$, $\mathrm{A}_{3,k}$, and $\mathrm{A}_{4,k}$ are not shown.

IV. Results

We now verify the analytical results using Monte Carlo simulation results that use 25000 runs. Unless mentioned otherwise, we use $N = 50$, $\gamma = 10$ dB, and $\sigma^2 = -110$ dBm. In each slot, the simulation compares the SINR given in (4) with the threshold γ to determine whether a collision or a success has occurred in slot k.

A. Impact on Selection and Outage Probability

The probability of selecting the best node in slot k, for $k \geq 1$, is shown in Fig. 2 for $\eta = 5$ and in Fig. 3 for $\eta = 2$ for different values of the power control error standard deviation σ_l. We see that the analytical and simulated results match each other well. The slight difference between the two arises due to the lognormal approximation used in the analysis and because some unlikely sequences are not accounted for. As σ_l increases, the probability of selecting the best node in any slot decreases. The changes are more prominent up to the first six slots. This affects the time required to select the best node and the probability that it will eventually get selected. Further, as η decreases, the algorithm becomes more susceptible to power control error, which makes intuitive sense.

The outage probability, measured from simulations, as a function of σ_l is shown in Fig. 4. The analytical expressions for it are not shown due to space constraints. It increases as σ_l increases. Its rate of increase is steepest for $2 \leq \sigma_l \leq 4$. Here also, for smaller η, a relatively small σ_l can easily cause an outage.

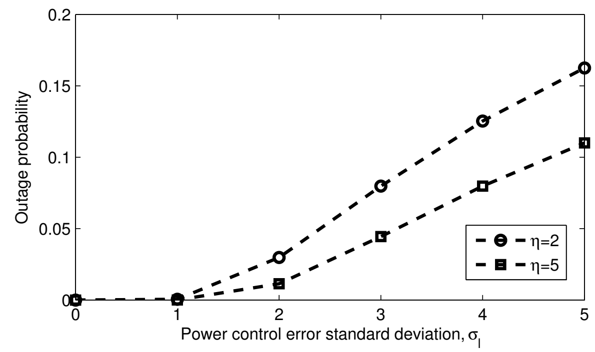

Fig. 4. Outage probability versus power control error standard deviation σ_l.

B. Impact on Net System Throughput

We saw that power control error changes the selection speed, leads to an outage, or selection of a suboptimal node. We now quantify the collective impact of these on the net downlink system throughput of an opportunistic, frequency-division duplex (FDD) wireless system, where the base station (BS) tries to select the node to which it has the highest channel power gain, and transmits data to it. Let g_i be the power gain of the channel from the BS to the node i. We assume that g_i's are independent and identically distributed exponential RVs. The selection phase lasts for T slots and the data transmission phase that follows it lasts for D slots.[7] If node i is selected, the net throughput of the system S, with fading-averaged downlink signal-to-noise-ratio $\mathrm{SNR}_{\mathrm{dl}}$, and with channel state information at the node is

$$S = \frac{D}{T+D} \log_2 \left(1 + g_i \mathrm{SNR}_{\mathrm{dl}}\right) \text{ bits/symbol.} \qquad (14)$$

We aim to explore how imperfect power control and the relative fraction of time spent on selection affect the net throughput. We also compare with the conventional splitting scheme that uses only one target power level. To be fair, we set the target power of the single-power level scheme to $\frac{P_H+P_L}{2} = \gamma\sigma^2 \left(\frac{\eta\gamma+2}{2}\right)$.

[7] Adaptations such as ending the selection phase if a success happens early improve throughput, but are not considered here due to space constraints.

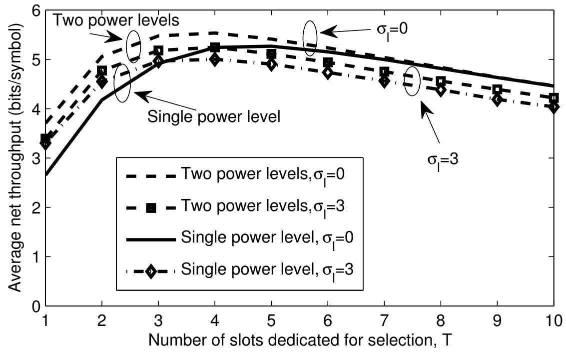

Fig. 5. Average net throughput versus selection duration ($\eta = 5$).

Figure 5 plots the net throughput as a function of T for different σ_l. We set $D = 15, \mathrm{SNR}_{\mathrm{dl}} = 10$ dB, and the mean of g_i as 6 dB. We see that a fundamental trade-off exists between T and the average net throughput. Too small a T often leads to no selection, while too large a T allocates relatively less time for data transmission. Here, $T = 4$ maximizes the average throughput for $\eta = 5$ for the two-power level scheme, and is not more than the optimal T for the single-power level scheme. The two-power level scheme outperforms the single-power level scheme for both $\sigma_l = 0$ and 3.

V. Conclusions

The multiple access-based distributed selection algorithm VPMAS-PS facilitates capture by making each node control its transmit power such that the receive power level takes one out of two values. By doing so, it quickly selects the best node. However, we saw that imperfect power control can cause the receive power to deviate from the targeted value. This increases the time required to select the best node. It can even lead to a suboptimal node being selected or even a selection outage, in which no node ever gets selected.

Assuming a lognormal power control error model, we derived the probability of selecting the best node in each slot. The key challenge that the analysis tackled is the exponential increase in the number of possible trajectories of the algorithm in the presence of imperfect power control. We also investigated the collective impact of this error on the downlink throughput of an opportunistic wireless system. We found that while the performance of VPMAS-PS with two-power levels degrades as the power control error variance increases, it still outperforms the single-power level scheme for $\sigma_l \leq 3$. The effect of power control error can also be ameliorated by increasing the adversary order η, but this requires a larger dynamic range for the transmit power.

Appendix

A. Choosing z

The probability of success S_r when r nodes transmit is

$$S_r = \begin{cases} 1, & \text{if } r = 1 \\ \frac{r}{2^r}, & \text{if } 1 < r \leq \eta + 1 \\ 0, & \text{otherwise} \end{cases}. \tag{15}$$

This follows because $r = 1$ ensures a success. If $1 < r \leq \eta+1$, a success occurs only if one node has target power P_H and the remaining $r-1$ nodes have target power P_L. The total number of ways this can happen is r. As the probability that a transmitting node has a target power P_H is $\frac{1}{2}$, which is also the probability that a transmitting node has a target power P_L, S_r in this case is $\frac{r}{2^r}$. For $r > n+1$, a collision is inevitable. Hence, $S_r = 0$.

If ϵ is the transmission probability of a node in slot 1, then $\Pr[\mathcal{S}(1)] = \sum_{r=1}^{N} S_r \binom{N}{r} \epsilon^r (1-\epsilon)^{N-r}$. Then, z is given by

$$z = \underset{0 \leq \epsilon \leq 1}{\arg\max} \sum_{r=1}^{N} S_r \binom{N}{r} \epsilon^r (1-\epsilon)^{N-r}. \tag{16}$$

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