This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE Globecom 2010 proceedings.

Performance Analysis of User Selected Subband Channel Quality Indicator Feedback Scheme of LTE

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***Abstract*—Frequency-domain scheduling and rate adaptation have helped next generation orthogonal frequency division multiple access (OFDMA) based wireless cellular systems such as Long Term Evolution (LTE) achieve significantly higher spectral efficiencies. To overcome the severe uplink feedback bandwidth constraints, LTE uses several techniques to reduce the feedback required by a frequency-domain scheduler about the channel state information of all subcarriers of all users. In this paper, we analyze the throughput achieved by the User Selected Subband feedback scheme of LTE. In it, a user feeds back only the indices of the best M subbands and a single 4-bit estimate of the average rate achievable over all selected M subbands. In addition, we compare the performance with the subband-level feedback scheme of LTE, and highlight the role of the scheduler by comparing the performances of the unfair greedy scheduler and the proportional fair (PF) scheduler. Our analysis sheds several insights into the working of the feedback reduction techniques used in LTE.**

I. INTRODUCTION

Frequency-domain scheduling is one of the key enablers of high data rates in fourth generation wireless cellular standards such as Long Term Evolution (LTE) [1], [2] and the IEEE 802.16m Advanced WiMAX [3], which employ orthogonal frequency division multiple access (OFDMA) in the physical layer. In OFDMA, frequency-domain scheduling is performed by dividing the bandwidth into several hundred subcarriers. These subcarriers are assigned to users with higher channel gains to improve overall system throughput.

In order to perform frequency-domain scheduling, the base station (BS), which is also called the eNodeB in LTE parlance, ideally needs to know the instantaneous channel state information (CSI) for all subcarriers for all users (UEs) in the cell. Since the uplink and downlink channels are not reciprocal in the popular frequency division duplex (FDD) mode of operation in LTE, this channel information needs to be fed back to the BS by each UE. Such extensive subcarrier-level feedback is practically infeasible as it consumes an extremely large amount of uplink resources. Hence, a balance needs to be struck between gains due to multiuser diversity and the amount of feedback required. The extent of the balance also depends on the scheduler used by the BS. For example, the use of a proportional fair (PF) scheduler ensures fairness among UEs while still exploiting multi-user diversity [4] [5, Sec. 6.7.1], [6], [7]. A PF scheduler takes into account not only the channel state of the subcarriers of each UE but also the average rate experienced by the UE.

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This work was partially supported by a project funded by the Ministry of Information Technology, India.

Several feedback reduction techniques have been studied in the literature. In [8], each UE sends CSI for a subcarrier only if the subcarrier's channel gain is above a certain threshold. In [4], each UE only indicates a pre-specified number of subcarriers that have the best gains, and what their gains are. In [9], a one bit feedback scheme, in which each UE sends a feedback bit only if its channel gain exceeds a pre-defined threshold, is shown to be asymptotically optimal in terms of sum capacity. Thresholding combined with subcarrier grouping, in which a UE feeds back the minimum channel gain of a group of subcarriers, was considered in [10].

A practical system such as LTE employs a pragmatic mixture of several of the above techniques in order to achieve a significant reduction in the feedback overhead. In LTE, the CSI is quantized into a 4-bit value called channel quality indicator (CQI). Moreover, only the average of the CSI of a possibly large group of subcarriers is reported in the CQI. Therefore, the frequency resolution of the CQI fed back is quite meager.

In this paper, we develop an analysis for the performance of the UE selected subband feedback mechanism used in LTE downlinks. Such an analysis is relevant because most of the LTE-specific literature that deals with either scheduling algorithms or limited feedback is simulation based due to the analytical complexity of the problem [6], [7], [11]. We develop expressions for the average net throughput achieved by a PF scheduler with UE selected subband feedback. These results are also contrasted with the greedy scheduler, which sacrifices fairness to maximize sum throughput. We show that unless there are very few UEs, the throughput of UE selected subband feedback scheme is quite close to the subband-level feedback scheme of LTE, which requires more feedback. Also, for the above CQI feedback schemes, we show that PF scheduler improves fairness significantly with a marginal reduction in throughput.

The paper is organized as follows. We first provide a brief overview of the LTE frame structure and its feedback reporting schemes in Sec. II. This motivates the system model developed in Sec. III, and its analysis in Sec. IV. Numerical results and conclusions follow in Sec. V and Sec. VI, respectively.

II. LTE FRAME STRUCTURE AND CQI FEEDBACK

In LTE, each downlink *frame* is 10 ms long and consists of ten subframes, each of duration 1 ms. A subframe consists of two 0.5 ms slots, with each slot consisting of seven OFDM symbols. In the frequency domain, the system bandwidth, B, is divided into several subcarriers, each of bandwidth of 15 kHz. The number of subcarriers available depends upon B, which ranges from 1.25 MHz to 20 MHz. A set of twelve consecutive subcarriers for a duration of one slot is called a *Physical Resource Block (PRB)*.

Feedback: The feedback information sent by the (UE) is called the Channel Quality Indicator (CQI), and consists of 4 bits. The CQI value indicates an estimate of the modulation and coding scheme (MCS) that the UE can receive reliably from the BS. It is typically based on the measured received signal quality, which can be estimated, for example, using the pilots sent by the BS on the downlink. The $2^4 = 16$ MCSs and their rates are tabulated in [12, Tbl. 7.2.3-1].

The BS controls how often and when the UE feeds back CQI. The finest possible frequency resolution for CQI reporting is a subband, which consists of q contiguous PRBs. Depending on the system bandwidth and the feedback scheme, q ranges from 2 to 8.

The UE can report the CQI using one of three different feedback schemes. Specifically, in *Wideband feedback*, the UE reports one wideband CQI value for the whole system bandwidth. In *Subband-level feedback*, the UE reports the CQI for each subband. In *UE selected subband feedback,* the UE reports the position of the M subbands that have the highest CQIs and only a single CQI value that indicates the channel quality when averaged over all these M subbands.[1]

PRB Allocation and Signaling: Based on the CQI reports from all the UEs, the BS decides which PRB to allocate to which UE. It then uses one of three *Resource Allocation Types* to signal to each UE on the downlink control channel the specific PRBs that are allocated to it. The three allocation types trade off the control signaling overheads in slightly different ways. *A key point to note is that the PRB is the smallest block of frequency that can be allocated to a UE. However, this allocation is based on a coarser subband frequency resolution of CQI fed back.*

III. System Model

In this paper, we will focus on the UE selected subband CQI reporting scheme. The wideband CQI scheme is not of interest to us since it does not enable frequency-domain scheduling. The subband-level feedback scheme has been partially analyzed in [13].

Consider a BS that serves K UEs. Let N denote the total number of PRBs available. The total number of subbands is $S = \lceil N/q \rceil$, where $\lceil . \rceil$ denotes the ceil function. The channel of each UE is assumed to undergo block Rayleigh fading, and is assumed to be constant over a 1 ms subframe. For a given UE, all the subcarriers within a PRB have the same channel gain, and the channel gains across different PRBs are assumed to be independent and identically distributed (i.i.d.). This is a valid and common assumption [14], [15] in LTE because the 180 kHz bandwidth of a PRB is close to the coherence bandwidth of the channel for a typical delay spread of 4-5 μs [2, Sec. 5.3.2]. While all the PRBs assigned to a UE use the same MCS in LTE, we assume for analytical tractability that different PRBs assigned to a UE can use different MCSs. The simulations in [16] show that the difference in throughput between the two is less than 5%.

Notation: The received SNR in a subframe for UE k in the n^{th} PRB is $\gamma_{n,k}$. It depends on the fading in PRB n, shadowing, and the distance of the UE from the BS. To keep the notation simple, the subband containing the n^{th} PRB is denoted by $s(n)$. Let r_i denote the rate in bits/symbol achieved by using the MCS corresponding to the i^{th} CQI value. Let C_k denote the single CQI value reported by UE k. It can take one of $L = 16$ possible values. For ease of explanation, we shall no longer distinguish between r_i and its 4-bit index i (where $1 \leq i \leq L$), and shall just say that a UE reports a CQI value of $C_k = r_i$. We shall denote the expectation of a RV X by $E[X]$ and the probability of an event A by $\Pr(A)$. Similarly, $E[X|A]$ denotes the conditional expection given A and $\Pr(B|A)$ denotes the conditional probability of B given A. For a set $\mathcal{I}$, $|\mathcal{I}|$ shall denote its cardinality.

A. UE Selected Subband CQI Feedback

To enable analytical tractability and gain insights into the performance of the CQI feedback scheme of LTE, we assume that the CQI value reported by a UE is the MCS that corresponds to the arithmetic mean SNR of the SNRs of the PRBs of the best M subbands [17]. Alternate mathematical models, such as effective exponential SNR, for determining the CQI value also exist [18]. However, these are analytically intractable and are beyond the scope of this paper.

The subband SNR, $\gamma_{s,k}^{\text{sub}}$, of the k^{th} UE for subband s is

$$\gamma_{s,k}^{\text{sub}} = \frac{1}{q} \sum_{n \in \mathcal{PRB}(s)} \gamma_{n,k}, \tag{1}$$

where $\mathcal{PRB}(s)$ denotes the set of PRBs in subband s. Since $\gamma_{n,k}$ are i.i.d., $\gamma_{s,k}^{\text{sub}}$ is a chi-squared (χ^2) RV with $2q$ degrees of freedom and mean σ_k^2.

In the UE selected subband feedback scheme, UE k orders the subband SNRs of its S subbands as $\gamma_{(1),k}^{\text{sub}} \geq \cdots \geq \gamma_{(M),k}^{\text{sub}} \geq \cdots \geq \gamma_{(S),k}^{\text{sub}}$, where (i) is the index of the subband with the i^{th} largest CQI. It then reports the set $\mathcal{I}(k) = \{(1), \ldots, (M)\}$, which consists of the M subbands with the highest CQIs.

As mentioned, UE k also reports a single CQI, C_k, that depends on the SNR, γ_k^{rep}, obtained by averaging over its M selected subbands as follows:

$$\gamma_k^{\text{rep}} = \frac{1}{M} \sum_{i=1}^{M} \gamma_{(i),k}^{\text{sub}}. \tag{2}$$

Based on γ_k^{rep}, the CQI reported, C_k, equals r_i if $\gamma_k^{\text{rep}} \in [T_{i-1}, T_i)$. Here, $T_0, \ldots, T_L$ are the set of link adaptation thresholds that ensure that a target block error rate of 10%

[1]LTE further reduces the CQI overhead in both the subband-level and UE selected subband feedback schemes as follows. A UE reports a 2-bit differential CQI value for each subband and a wideband CQI value for the whole system bandwidth. We shall ignore the minor performance degradation due to this differential feedback.

is met should the BS transmit over the entire subband [2, Fig. 10.1]. Note that γ_k^{rep} is not fed back, only C_k is.

B. BS scheduling

The BS uses $\mathcal{I}(k)$ and C_k reported by all the K UEs to determine for each PRB which UE to transmit to. This determination also depends on the scheduler used by the BS. We shall consider two schedulers: (i) the greedy scheduler, which maximizes system throughput without considering fairness, and (ii) the proportional fair scheduler, which also attempts to achieve fairness across UEs by accounting for the average rate experienced by each UE.

Specifically, the schedulers are defined as follows. Let $\mathcal{Z}_{s(n)}$ denote the subset of UEs that have selected the subband $s(n)$.

- *Greedy scheduler:* The n^{th} PRB gets assigned to the UE that reported the highest CQI value among the UEs that selected subband $s(n)$.
- *PF scheduler:* The n^{th} PRB gets assigned to UE $k^*(n)$ if [4] $k^*(n) = \arg\max_{k \in \mathcal{Z}_{s(n)}} \frac{C_k}{E[C_k]}$. Thus, a PRB gets assigned to the UE whose CQI exceeds its mean rate the most. This ensures fairness across UEs with different mean rates. Since the channel gains of the PRBs are statistically identical, the above model is also equivalent to that of a PF scheduler that considers the mean rate experienced by a UE over the entire system bandwidth.

Both schedulers can be succinctly defined as follows in terms of a UE-specific metric $M_k(C_k)$. PRB n is assigned to UE $k^*(n)$ if

$$k^*(n) = \arg \max_{k \in \mathcal{Z}_{s(n)}} \{M_k(C_k)\}, \tag{3}$$

where $M_k(C_k) \triangleq C_k$ for the greedy scheduler and $M_k(C_k) \triangleq \frac{C_k}{E[C_k]}$ for the PF scheduler. The BS then transmits data on PRB n to UE $k^*(n)$ at a rate $C_{k^*(n)}$. If multiple UEs have the same highest value of $M_k(C_k)$, then one of them is chosen with uniform probability.

Outage: Since the CQI value corresponds to the average SNR for the best M subbands, the actual SNR for the n^{th} PRB may be less than the lower threshold of the MCS being used. In such a case, we say that an *outage* has occurred, and the throughput is 0 in that subframe.[2] Outage for PRB n also occurs if $\mathcal{Z}_{s(n)}$ is a null set, since the PRB is not allocated to anyone.

IV. Analysis

A. UE Selected Subband Scheme

A common stochastic model for $\gamma_{n,k}$ is that it is an exponential random variable (RV) with mean σ_k^2 [9]. This corresponds to a Rayleigh fading channel model where the BS and the UEs are each equipped with one antenna and where co-channel interference, if any, is modeled as noise. We now derive the expression for the throughput of the general scheduler of (3). The following three lemmas shall lead us to the final expression for the throughput in (9).

Lemma *1:* Let UE k be selected (sel.) for the n^{th} PRB and let r_i be the CQI value that it reports. Then, the conditional probability that $\gamma_{n,k}$ is less than T_{i-1} is

$$\begin{aligned} &\Pr\left(\gamma_{n,k} < T_{i-1} | C_k = r_i, k \text{ is sel. for } n^{\text{th}} \text{ PRB}\right) \\ &\quad = (Mq-1)! \sum_{l=0}^{Mq-2} \frac{(-1)^l T_{i-1}^{l+1}}{(l+1)!(Mq-2-l)!(\sigma_k^2)^{l+1}} \\ &\times \frac{\left[\gamma\left(Mq-1-l, \frac{MqT_i}{\sigma_k^2}\right) - \gamma\left(Mq-1-l, \frac{MqT_{i-1}}{\sigma_k^2}\right)\right]}{\gamma\left(Mq, \frac{MqT_i}{\sigma_k^2}\right) - \gamma\left(Mq, \frac{MqT_{i-1}}{\sigma_k^2}\right)}, \end{aligned} \tag{4}$$

where $\gamma(k, x)$ is the Incomplete Gamma function [19, Chp. 6].

Proof: The proof is relegated to Appendix A. ■

Lemma *2:* The probability that a UE k reports a CQI of r_i is $\Pr(C_k = r_i) = \Pr(C_k \le r_i) - \Pr(C_k \le r_{i-1})$, where

$$\begin{aligned} &\Pr(C_k \le r_i) \\ &\approx \frac{1}{\beta} \int_0^{T_i} \frac{(Mq)^{Mq} z^{Mq-1} e^{-\frac{Mqz}{\sigma_k^2}}}{(Mq-1)!(\sigma_k^2)^{Mq}} \left(\frac{\gamma\left(q, \frac{qz}{\sigma_k^2}\right)}{(q-1)!}\right)^{(S-M)} dz, \end{aligned} \tag{5}$$

for $1 \le i \le L$, and

$$\beta \approx \sum_{i=1}^{U} w_i \frac{\alpha_i^{Mq-1}}{(Mq-1)!} \left(\frac{\gamma\left(q, \frac{\alpha_i}{M}\right)}{(q-1)!}\right)^{(S-M)}. \tag{6}$$

Here, w_i and α_i are the weights and abscissa of Gauss-Laguerre quadrature, respectively, and are tabulated in [19, Tbl. 25.9].

Proof: The proof is relegated to Appendix B. ■

In Fig. 1, we plot, for a UE (say UE 1) the cumulative density function (cdf) of the CQI of C_1 using the approximation developed in the above lemma (with $U = 6$), and compare it with the empirical cdf generated using 50,000 samples. The set of rates $\{r_i\}$ for the 16 different MCSs used in LTE are strictly as per [2, Tbl. 10.1], and lie in the range 0 to 5.6 bits/symbol. We can see that the approximation is reasonably accurate. Notice that the probability that UE 1 reports low rate MCSs ($r_i \le 2.7$ bits/symbol) or very high rate MCSs ($r_i \ge 3.9$ bits/symbol) is negligible.

Lemma *3:* For the PF scheduler, the probability that UE k is selected for PRB n given that $k \in \mathcal{Z}_{s(n)}$ and $C_k = r_i$ is

$$\begin{aligned} &\Pr\left(k \text{ is sel. for } n^{\text{th}} \text{ PRB} | k \in \mathcal{Z}_{s(n)}, C_k = r_i\right) \\ &\qquad\qquad = \prod_{\substack{l \ne k \\ l \in \mathcal{Z}_{s(n)}}} \Pr(C_l \le a_{l,i}), \end{aligned} \tag{7}$$

where $a_{l,i}$ is the largest rate that is strictly less than $\frac{E[C_l]}{E[C_k]} r_i$ and $\Pr(C_l \le a_{l,i})$ is given by Lemma 2.

[2] In practice, when the SNR of the PRB is below the MCS threshold, the data might still be received correctly, albeit with a higher error probability. Therefore, the outage model provides a lower bound on the throughput. However, it is accurate when the block error rate declines sharply with SNR.

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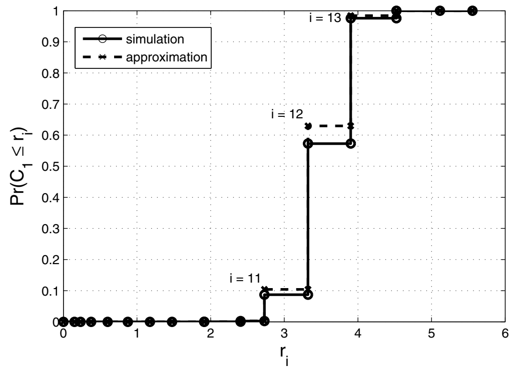


Fig. 1. Plot of the approximate cdf of C_1 and the cdf generated from simulations ($S = 4$, $M = 2$, $\sigma_1^2 = 13$ dB).

Proof: The result follows because UE k, which reports a CQI of r_i, is selected only if $C_l < \frac{E[C_l]}{E[C_k]} r_i$, for all UEs $l \neq k$.[3] ■

Similarly, for the greedy scheduler, we have:

Lemma *4:* For the greedy scheduler, the probability that UE k is selected for PRB n given that $k \in \mathcal{Z}_{s(n)}$ and $C_k = r_i$ is

$$\begin{aligned}\Pr&\left(k \text{ is sel. for } n^{\text{th}} \text{ PRB} | k \in \mathcal{Z}_{s(n)}, C_k = r_i\right) \\ &= \sum_{\mathcal{A} \subseteq \mathcal{Z}_{s(n)} \setminus \{k\}} \frac{1}{|\mathcal{A}|+1} \left[\prod_{t_1 \in \mathcal{A}} \Pr\left(C_{t_1} = r_i\right)\right] \\ &\quad \times \left[\prod_{t_2 \in \mathcal{Z}_{s(n)} \setminus \{\mathcal{A} \cup \{k\}\}} \Pr\left(C_{t_2} \leq r_{i-1}\right)\right], \quad (8)\end{aligned}$$

where $\sum_{\mathcal{A} \subseteq \mathcal{Z}_{s(n)} \setminus \{k\}}$ denotes the summation over all subsets $\mathcal{A}$ of $\mathcal{Z}_{s(n)} \setminus \{k\}$.

Proof: The proof is similar to that for Lemma 3, and is omitted. It is given in [20]. ■

Result *1:* The average throughput, R_n, for PRB n is

$$\begin{aligned}R_n = &\sum_{\mathcal{Z}_{s(n)}} \Pr\left(\mathcal{Z}_{s(n)}\right) \sum_{k \in \mathcal{Z}_{s(n)}} \sum_{i=1}^{L} r_i \Pr\left(C_k = r_i\right) \\ &\times \left(1 - \Pr\left(\gamma_{n,k} < T_{i-1} | C_k = r_i, k \text{ is sel. for } n^{\text{th}} \text{ PRB}\right)\right) \\ &\quad \times \Pr\left(k \text{ is sel. for } n^{\text{th}} \text{ PRB} | k \in \mathcal{Z}_{s(n)}, C_k = r_i\right). \quad (9)\end{aligned}$$

Here, $\Pr\left(\mathcal{Z}_{s(n)}\right) = \left(\frac{M}{S}\right)^{|\mathcal{Z}_{s(n)}|}\left(1 - \frac{M}{S}\right)^{K - |\mathcal{Z}_{s(n)}|}$. The expression for $\Pr\left(\gamma_{n,k} < T_{i-1} | C_k = r_i, k \text{ is sel. for } n^{\text{th}} \text{ PRB}\right)$ is given in Lemma 1, and $\Pr\left(C_k = r_i\right)$ is given in Lemma 2. For the PF and greedy schedulers, $\Pr\left(k \text{ is sel. for } n^{\text{th}} \text{ PRB} | k \in \mathcal{Z}_{s(n)}, C_k = r_i\right)$ is given by Lemmas 3 and 4, respectively.

Proof: Since the SNRs of different PRBs of a UE are i.i.d., the probability that a UE selects the subband $s(n)$ is M/S. Hence, $\Pr\left(\mathcal{Z}_{s(n)}\right) = \left(\frac{M}{S}\right)^{|\mathcal{Z}_{s(n)}|}\left(1 - \frac{M}{S}\right)^{(K - |\mathcal{Z}_{s(n)}|)}$. A rate r_i is achieved if the UE selected for the PRB fed back a CQI value equal to r_i and there was no outage. This yields the expression for the average throughput in (9). ■

[3]The equality case $C_l = \frac{E[C_l]}{E[C_k]} r_i$ occurs with probability zero since the event $E[C_l] = E[C_k]$, for the two UEs l and k, occurs with probability zero in a random deployment of UEs in a cell.

V. Simulation Results And Comparisons

We now verify the analytical results using Monte Carlo simulations that average over 50,000 samples. The set of 16 rates is the same as specified for LTE [12, Tbl. 7.2.3-1]. A rate r_i is considered achievable in a PRB (without outage) if the SNR of the PRB exceeds the threshold T_{i-1}. For the purpose of illustration, the relationship between the rates and thresholds is modeled as [21], [22]

$$r_i = \log_2\left(1 + \alpha T_{i-1}\right), \quad (10)$$

where α is the coding gain loss. We use $\alpha = 0.398$, which corresponds to a deviation of 4 dB from the Shannon limit [21]. Note that the thresholds can be determined from simulation studies, as well. A subband consists of $q = 4$ PRBs. The number of subbands is $S = 4$, out of which each UE selects $M = 2$ best subbands. The fading across PRBs is i.i.d., as per the system model. To model the effect of different locations of UEs in a cell, we set the mean for the K UEs as $\sigma_1^2 = \lambda$, $\sigma_2^2 = \lambda/a, \ldots, \sigma_K^2 = \lambda/a^{K-1}$. Thus, the mean SNR decreases by a factor of a from the first UE to the second one, and so on. The closer a is to 1, the more statistically identical the UEs' channels are. We shall illustrate the results using $\lambda = 13$ dB, $a = 1.2$, and $N = 6$ UEs.

In Fig. 2, we plot the average throughput as a function of the number of UEs for the subband-level and UE selected subband feedback schemes for the PF scheduler. The expression for average throughput for the subband-level feedback scheme is not presented in this paper to conserve space; it is found by combining the approaches used here and in [13]. We observe that the analytical results are quite close to the simulation ones. The minor difference is due to the approximation used in Lemma 2. As the number of UEs increases beyond 5, the throughput decreases marginally for the UE selected subband feedback scheme. This is because the PF scheduler ensures fairness among the UEs even though the mean SNR of the additional UEs considered decreases.

In Fig. 3, we plot the simulation and analysis results for the average throughput for the subband-level and UE selected subband feedback schemes for the greedy scheduler. We use the same parameters as that of the PF scheduler. The results for subband-level feedback scheme are derived in [13]. Note that the analytical results are again close to the simulation results. We can see that, unlike the PF scheduler, as the number of UEs in a cell increases, the average throughput per PRB increases for the greedy scheduler. Also, as the number of UEs increases, the throughput of the UE selected subband scheme is quite close to that of the subband-level feedback scheme.

Figure 4 shows the individual throughputs achieved by each UE for the greedy and PF schedulers. Unlike the greedy scheduler, which allocates significantly higher rates to UEs

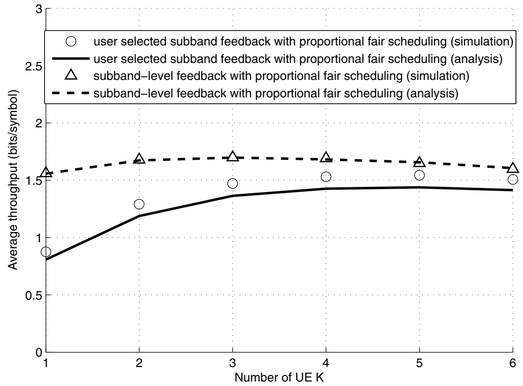


Fig. 2. Average throughput vs. number of UEs for the PF scheduler.

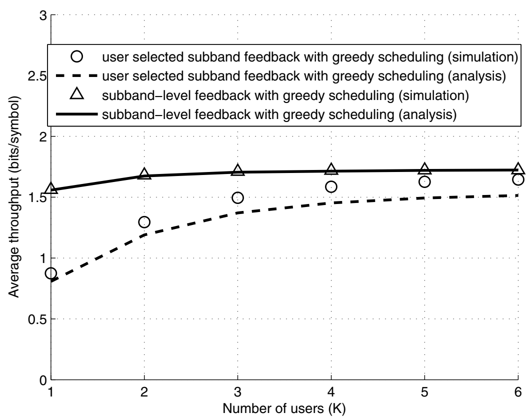
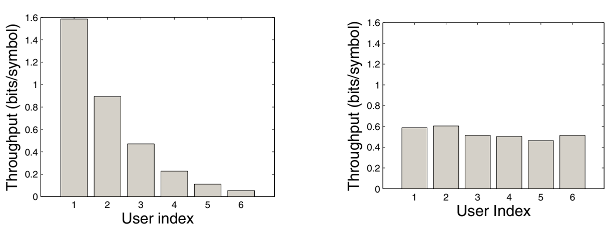


Fig. 3. Average throughput vs. number of UEs for the greedy scheduler.

with higher mean channel gains, we observe that the PF scheduler is more fair.



(a) Greedy scheduler

(b) PF scheduler

Fig. 4. Comparison of the throughputs of different UEs for the PF and greedy schedulers.

VI. Conclusions

In an OFDMA-based system such as LTE, the frequency-domain scheduler exploits multi-user diversity and frequency diversity by assigning different PRBs to different UEs based on the channel quality estimates reported by the UEs. However, the CQI report, on the basis of which the assignment is done, has a coarse frequency granularity in the UE selected subband feedback scheme. We developed an analysis for the throughput of the UE selected subband feedback scheme for both the proportional fair and greedy schedulers. We found that the UE selected subband feedback scheme performs almost as well as the subband-level feedback scheme, despite using lesser feedback. We also showed that proportional fair scheduler provides fairness at the expense of a marginal reduction in throughput. The analysis also led to a new approximation for the probability distribution function (pdf) of a sum of a subset of *ordered* χ^2 RVs. An interesting avenue for future work is extending the analysis to also handle multiple antennas at the LTE transmitter and receiver, and analyze the effect of high correlation, if any, across PRBs.

Appendix

A. Proof of Lemma 1

Given that UE k is selected for the n^{th} PRB and $C_k = r_i$

$$\begin{aligned}&\Pr\left(\gamma_{n,k} < T_{i-1} | C_k = r_i, k \text{ is sel. for } n^{\text{th}} \text{ PRB}\right)\\ &\stackrel{(a)}{=} \Pr\left(\gamma_{n,k} < T_{i-1} | C_k = r_i\right),\\ &\stackrel{(b)}{=} \frac{\Pr\left(T_{i-1} \le \frac{1}{M}\sum_{r\in\mathcal{I}(k)} \gamma_{r,k}^{\text{sub}} < T_i, \gamma_{n,k} < T_{i-1}\right)}{\Pr\left(T_{i-1} \le \frac{1}{M}\sum_{r\in\mathcal{I}(k)} \gamma_{r,k}^{\text{sub}} < T_i\right)}.\end{aligned} \tag{11}$$

Here, (a) follows because the probability that $\gamma_{n,k}$ is less than T_{i-1}, given that $C_k = r_i$, does not depend on whether UE k is selected for the n^{th} PRB or not. And, (b) follows because the event $C_k = r_i$ is same as the event $T_{i-1} \le \frac{1}{M}\sum_{r\in\mathcal{I}(k)} \gamma_{r,k}^{\text{sub}} < T_i$.

Note that $\gamma_{r,k}^{\text{sub}}$ and $\gamma_{n,k}$ are not independent since the subband SNR is the arithmetic mean of the SNRs of its constituent PRBs. Consequently, the numerator in (11) is evaluated as follows.

$$\begin{aligned}&\Pr\left(T_{i-1} \le \frac{1}{M}\sum_{r\in\mathcal{I}(k)} \gamma_{r,k}^{\text{sub}} < T_i, \gamma_{n,k} < T_{i-1}\right)\\ &= \int_0^{T_{i-1}} f_{\gamma_{n,k}}(y) \Pr\left(T_{i-1} \le \frac{1}{M}\sum_{r\in\mathcal{I}(k)} \gamma_{r,k}^{\text{sub}} < T_i | \gamma_{n,k} = y\right) dy,\end{aligned}$$

where $f_{\gamma_{n,k}}(y) = \frac{e^{-\frac{y}{\sigma_k^2}}}{\sigma_k^2}$, $y \ge 0$, is the pdf of $\gamma_{n,k}$. Since $\gamma_{r,k}^{\text{sub}}$ is the average SNR of q PRBs, given that $\gamma_{n,k} = y$, we have $\gamma_{r,k}^{\text{sub}} = \frac{\sigma_k^2}{2qM} B + \frac{y}{qM}$, where, B is a χ^2 RV with $2(Mq-1)$ degrees of freedom and mean $2(Mq-1)$. Thus,

$$\begin{aligned}&\Pr\left(T_{i-1} \le \frac{1}{M}\sum_{r\in\mathcal{I}(k)} \gamma_{r,k}^{\text{sub}} < T_i, \gamma_{n,k} < T_{i-1}\right)\\ &= \int_0^{T_{i-1}} \frac{e^{-\frac{y}{\sigma_k^2}}}{\sigma_k^2} \int_{T_{i-1}-\frac{y}{Mq}}^{T_i - \frac{y}{Mq}} \frac{(Mq)^{Mq-1} x^{Mq-2} e^{-\frac{Mqx}{\sigma_k^2}}}{(Mq-2)!(\sigma_k^2)^{Mq-1}}\, dx\, dy.\end{aligned} \tag{12}$$

The denominator of (11) is simply

$$\Pr\left(T_{i-1} \le \frac{1}{M}\sum_{r\in\mathcal{I}(k)} \gamma_{r,k}^{\text{sub}} < T_i\right) = \int_{T_{i-1}}^{T_i} \frac{(Mq)^{Mq} x^{Mq-1} e^{-\frac{Mqx}{\sigma_k^2}}}{(Mq-1)!(\sigma_k^2)^{Mq}}\, dx. \quad (13)$$

Evaluating the integrals in (12) and (13) in closed-form and substituting in (11) yields the desired result.

B. Proof of Lemma 2

From Sec. III-A, $\Pr(C_k \le r_i)$ is given by

$$\Pr(C_k \le r_i) = \Pr\left(0 \le \frac{1}{M}\sum_{i=1}^{M} \gamma_{(i),k}^{\text{sub}} < T_i\right). \quad (14)$$

To evaluate the above expression, we need the pdf of the sum of M ordered χ^2 RVs, which is analytically intractable [23]. A new approximate expression below that involves only a single integral circumvents this problem as follows.

$$\begin{aligned}
&\Pr\left(0 \le \frac{1}{M}\sum_{i=1}^{M} \gamma_{(i),k}^{\text{sub}} < T_i\right)\\
&\overset{(a)}{=} \sum_{i_1,\ldots,i_M} \Pr\left(0 \le \frac{1}{M}\sum_{i\in\mathcal{I}_k} \gamma_{i,k}^{\text{sub}} < T_i, \mathcal{I}_k = \{i_1,\ldots,i_M\}\right),\\
&\overset{(b)}{=} \binom{S}{M}\Pr\left(0 \le \frac{1}{M}\sum_{i=1}^{M} \gamma_{i,k}^{\text{sub}} < T_i, \mathcal{I}_k = \{1,\ldots,M\}\right)
\end{aligned} \quad (15)$$

Here, (a) follows from the law of total probability. We get (b) because all possible sets of M selected subbands are statistically identical since the S subband SNRs are i.i.d.

Without loss of generality, let Λ be the event that subbands $\mathcal{I}_k = \{1,\ldots,M\}$ are selected by UE k. Then, $\Pr(\Lambda) = \Pr\left(\gamma_{M+1,k}^{\text{sub}},\ldots,\gamma_{S,k}^{\text{sub}} \le \min\left(\gamma_{1,k}^{\text{sub}},\ldots\gamma_{M,k}^{\text{sub}}\right)\right)$. Clearly, since the min is less than the arithmetic mean, we have $\Pr(\Lambda) \le \Pr(\widetilde{\Lambda})$, where $\widetilde{\Lambda}$ is the event that $\gamma_{M+1,k}^{\text{sub}},\ldots,\gamma_{S,k}^{\text{sub}} \le \frac{1}{M}\sum_{i=1}^{M}\gamma_{i,k}^{\text{sub}}$. Thus, from (14) and (15),

$$\begin{aligned}
\Pr(C_k \le r_i) &\le \binom{S}{M}\Pr\left(0 \le \frac{1}{M}\sum_{i=1}^{M} \gamma_{i,k}^{\text{sub}} < T_i, \widetilde{\Lambda}\right),\\
&= \binom{S}{M}\int_0^{T_i} \Pr\left(\frac{1}{M}\sum_{i=1}^{M}\gamma_{i,k}^{\text{sub}} = z\right)\\
&\qquad \times \Pr\left(\gamma_{M+1,k}^{\text{sub}} \le z\right)^{(S-M)} dz. \quad (16)
\end{aligned}$$

At the same time, we know that $\Pr(C_k \le r_L) = 1$. This motivates the following approximation in which the upper bound in (16) is divided by a factor $\binom{S}{M}\beta$, where

$$\beta = \int_0^{T_L} \frac{(Mq)^{Mq} z^{Mq-1} e^{-\frac{Mqz}{\sigma_k^2}}}{(Mq-1)!(\sigma_k^2)^{Mq}} \left(\frac{\gamma\left(q, \frac{qz}{\sigma_k^2}\right)}{(q-1)!}\right)^{(S-M)} dz. \quad (17)$$

This expression is obtained by using (13) and the fact that $\gamma_{i,k}^{\text{sub}}$ is a χ^2 RV with $2q$ degrees of freedom and mean σ_k^2. This makes the approximation exact for $\Pr(C_k \le r_L)$. The expression in (6) is a Gauss-Laguerre approximation of β.

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