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Voluntary Cooperative Energy Harvesting Relay Nodes: Analysis and Benefits

Bhargav Medepally Neelesh B. Mehta, *Senior Member, IEEE*

***Abstract*—The use of energy harvesting (EH) nodes as cooperative relays is an attractive solution that harnesses the spatial diversity of a multi-relay network and also addresses the vexing problem of a relay's batteries getting drained when it forwards information to the destination. For a general class of stationary and ergodic energy harvesting processes, we analytically characterize the performance of a cooperative system in which the EH nodes are amplify-and-forward relays, which volunteer to relay if and only if they have sufficient energy for transmission. We show that when such systems employ relay selection, the energy usage at any relay and, consequently, its availability for relaying depend not only on its energy harvesting process but also the total number of relays and the relay selection policy. Further insight is gained by a two-fold asymptotic analysis that considers the cases where the signal-to-noise ratio (SNR) or the number of relays is large. The optimal static transmit power setting at the EH relays is also determined. Altogether, our results show that EH relays are beneficial and different from conventional cooperative relays.**

I. Introduction

Cooperative communication using relays promises significant improvements in throughput and reliability [1]–[5]. When multiple nodes are present, selecting the most suitable relay based on current channel states harnesses the spatial diversity and, yet, avoids the tight synchronization required among multiple transmitting relays [5]–[7].

In forwarding to the destination a representation of the signal it has received from the source, a relay expends its own energy. Since running power cables to supply energy is often impractical or cumbersome in several scenarios, this energy is typically supplied by a pre-charged battery. The more often a relay is selected, the sooner its battery drains out. When the battery drains out, the relay can no longer assist in transmission. While fairness mechanisms [8] and network lifetime maximization techniques [9] mitigate this problem, they only delay the inevitable – all the relays still do eventually run out of energy.

An attractive alternate solution is offered by using energy harvesting (EH) nodes as relays. These nodes harvest energy from the environment to carry out their communication tasks [10], [11]. Energy is harvested using solar, vibration, thermoelectric effects, and other physical phenomena [12], [13]. EH relays promise perpetual network operation without periodic battery replacements. This is because an EH node that drains out its battery can harvest energy later and again become available for relaying.

B. Medepally is with ST-Ericsson, Bangalore, India. He was at the Indian Institute of Science (IISc), Bangalore, India, during the course of this work. N. B. Mehta is with the Dept. of Electrical Communication Eng. at IISc.
Emails: bhargavmvpk@gmail.com, nbmehta@ece.iisc.ernet.in

A key aspect that affects an EH relay's performance is its *energy profile*, which mathematically models the random harvesting of energy with time at an EH node. In general, the energy profile depends on the device physics [10], [14], [15]. Depending on the energy profile, the harvested energy is often available in random and small amounts and only sporadically. Storing the excess harvested energy in a battery or a super-capacitor for later use helps mitigate this randomness, but only partially. The energy profile places fundamental limitations on how effective an EH relay can be. For example, increasing the transmit energy improves the communication reliability. However, it also uses up an EH relay's harvested energy, and decreases the probability that it will volunteer as a relay. Fewer relays to choose from, in turn, reduces the spatial diversity in the system. Thus, while energy harvesting has benefits, it comes at the cost of a more involved link and network design [14], [16].

Understanding the performance benefits of using EH relays and optimizing their usage is, thus, an important and challenging problem, and is the focus of this paper. We analyze a system that uses EH cooperative amplify-and-forward (AF) relays. We focus on AF relaying because it has been considered to be suitable for wireless sensor networks that require low complexity nodes, and has been well investigated in the literature [3]–[8], [17]. *In our system, the EH nodes volunteer to relay if and only if they have enough energy to transmit the data.* Our analysis only requires that the energy harvesting process be stationary and ergodic, and thus applies to several energy profiles. To gain further insights, we also develop a two-fold asymptotic analysis for the cases where the number of EH relays is large, but the total energy harvested by all of them is fixed, and when the end-to-end signal-to-noise ratio (SNR) is large. The analysis also provides a general characterization of the conditions under which the nodes operate in an *energy unconstrained* regime, where the randomness in energy harvesting no longer affects an EH relay's availability.

The sum total of our results shows that systems that use voluntary EH relays differ from conventional cooperative systems and provide an attractive solution for improving performance. Interestingly, we show that *random relay selection*, in which any one of the nodes with sufficient energy is randomly selected, is a useful benchmark for our system. This also highlights how the energy usage at relays – and, consequently, its availability – is a function of the selection policy.

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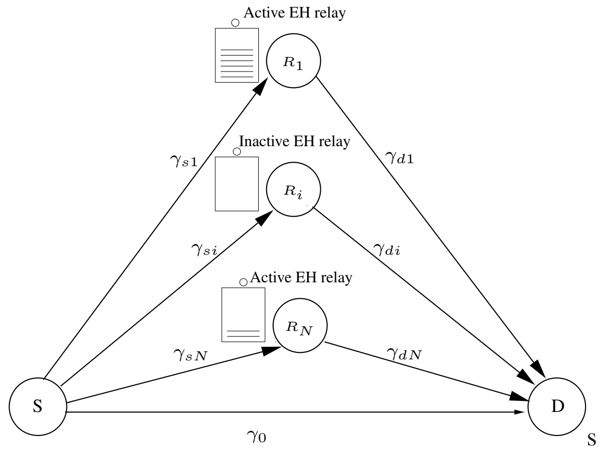

Fig. 1. Voluntary cooperative energy harvesting relays with battery storage capability assisting communication between a source and a destination

The paper is organized as follows. Section II sets up the energy harvesting and relay cooperation model. The analysis is developed in Section III. Simulations are presented in Section IV, and are followed by our conclusions in V. Brief mathematical proofs are given in the Appendix.

II. Energy Harvesting and Cooperation Model

As shown in Figure 1, the scenario we consider consists of a source (S), a destination (D), and N energy harvesting AF relays $R_1, \ldots, R_N$. The source and destination nodes are conventional non-EH nodes that want to communicate with each other. The various source-relay channels are assumed to be independent and identically distributed (i.i.d.), and so are the various relay-destination channels. The instantaneous channel gain amplitudes of the S–R_i and R_i–D channels are denoted by h_{si} and h_{di}, respectively. The direct S–D channel gain amplitude is denoted by h_0, and is independent of the S–R_i and R_i–D channel gain amplitudes. The analysis can be generalized to handle the general case in which the various channels are not statistically identical. However, due to space constraints, this general analysis is not presented in this paper.

The cooperative transmission of a symbol, x, takes place over two time phases, each of duration T sec [1], [3], [7]: In the first time phase, S transmits x with power P_s, which is received by the destination and relays. In the second time phase, one of the relays amplifies and forwards the signal it has received to the destination. It is possible that due to the energy harvesting nature of the relays, no relay is available in the second phase. At the end of the second phase, the destination optimally combines the signals it has received in the two phases. All channels are assumed to be Rayleigh, frequency-flat, block-fading channels that remain constant over at least the two phases required for cooperative transmission.

A. Energy Harvesting and Storage Model

The energy harvested by a Relay i over time is assumed to be a stationary and ergodic process, with mean E^{av} J/sec. No other limiting assumptions about it are made in our analysis. Thus, this general model encompasses several energy profiles assumed in the literature. For example, in [14], [15], an energy E_s is harvested with probability ρ in every time slot. A Markov model was used in [18]. The model also encompasses multiple- or continuous-valued harvested energy and time correlations, if any.

The EH relay stores its harvested energy in its battery, and draws energy from it to transmit. The battery helps the EH relay partially overcome the randomness in the energy harvested. The battery capacity is assumed to be unbounded in order to enable analytical tractability.

B. Harvested Energy Usage and Active Relays

When a Relay i transmits, it does so at a constant power P, as has been used in [1]–[3], [5], [8]. Let $\alpha_i(t)$ denote the signal amplification by Relay i if it were to transmit. Then,

$$\alpha_i^2 = \frac{P}{P_s \left|h_{si}\right|^2 + N_0}, \tag{1}$$

where N_0 is the additive white Gaussian noise (AWGN) power. Thus, a relay inverts the effect of its source-relay link. The SNR at the destination, γ_D, then equals [17]:

$$\gamma_D = \gamma_0 + \frac{\gamma_{si}\gamma_{di}}{\gamma_{si} + \gamma_{di} + 1}, \tag{2}$$

where $\gamma_0 = \left|h_0\right|^2 \frac{P_s T}{N_0}$, $\gamma_{si} = \left|h_{si}\right|^2 \frac{P_s T}{N_0}$, and $\gamma_{di} = \left|h_{di}\right|^2 \frac{P_i T}{N_0}$. Let $\mathbf{E}\left[\gamma_0\right] = \bar{\gamma}_0$, $\mathbf{E}\left[\gamma_{si}\right] = \bar{\gamma}_s$, and $\mathbf{E}\left[\gamma_{di}\right] = \bar{\gamma}_d$, where $\mathbf{E}\left[.\right]$ denotes expectation.

In forwarding a symbol, a relay consumes PT energy, where T is the symbol transmission duration. A relay amplifies and forwards its received signal only if at least PT energy is stored in its battery. Otherwise, it does not even receive. A relay that has sufficient energy to transmit shall be called an *active* relay.

C. Energy Neutrality and Energy Constrained Relays

The energy availability at a relay is subject to the fundamental *energy neutrality* constraint, which mandates that the energy used by the relay thus far should not exceed the energy harvested by it. A key consequence of this is that a relay might not be active for some time instants.

Let ξ_i denote the steady state probability that Relay i is active. Depending on ξ_i, we define the following two terms; these shall play a pivotal role in the analysis.

- *Energy unconstrained relay:* A Relay i is said to be energy unconstrained when $\xi_i = 1$.
- *Energy constrained relay:* A Relay i is said to be energy constrained when $\xi_i < 1$.

An energy unconstrained relay is, thus, *always* active.

D. Relay Selection

As mentioned, when multiple active relays are available, *one* of them is selected to forward the symbol.[1] The relay that maximizes the end-to-end SNR of (2) is selected as this minimizes the symbol error rate (SER). We call this the *best relay selection* rule.

[1] We consider symbol-by-symbol forwarding per phase in this paper primarily for notational simplicity. The analysis easily generalizes to the practical case where a relay forwards a block of D symbols instead.

III. SER Analysis

It follows from symmetry that all relays have the same probability of being active, *i.e.*, $\xi_i = \xi$. We now determine the symbol error rate (SER) when MPSK constellation is used. The analysis generalizes to MQAM constellations, as well.

The energy stored in the battery of a relay clearly depends on how often it has been selected in the past and the energy it has harvested thus far. Consequently, the probability that the relay is active, ξ, depends on the number of relays, the rate at which energy is harvested by each relay, and the energy used per transmission by a relay, as quantified below.

Proposition 1: Let $\rho = \frac{2E^{\text{av}}}{P}$. The relays are energy constrained when $\rho < \frac{1}{N}$, in which case

$$\xi = 1 - (1 - N\rho)^{\frac{1}{N}}. \tag{3}$$

All the relays are energy unconstrained ($\xi = 1$) when $\rho \geq \frac{1}{N}$.

Proof: The proof is relegated to Appendix A. A key enabler in the proof is a *decoupling approximation*, which assumes that the probability that a relay node is active is independent of the probability of other relay nodes being active. It shall turn out to be very accurate for all ρ. ■

The factor of 2 in ρ occurs because a node will harvest an average energy of $2E^{\text{av}}T$ in the two phases of the amplify-and-forward cooperation protocol. The equation shows the dependency of ξ on N and ρ. As ρ and N increase, expectedly ξ increases. The above result also shows a key property of a voluntary EH relay system – *whether a relay is energy constrained depends on the other EH relays.*

The SER at any time depends on the subset of relays that are active. Due to symmetry, the SER conditioned on any one of the $\binom{N}{k}$ possible subsets of k active relays is the same. Hence, let SER_k denote the SER given that any k out of the total N relays are active. Let $\Lambda_i = \frac{\gamma_{si}\gamma_{di}}{\gamma_{si}+\gamma_{di}+1} \approx \frac{\gamma_{si}\gamma_{di}}{\gamma_{si}+\gamma_{di}}$, and let $f_\Lambda(.)$ and $F_\Lambda(.)$ denote the probability density function (PDF) and cumulative distribution function (CDF), respectively, of Λ_i for an arbitrary relay i.[2]

Proposition 2: SER_k can be written as

$$\text{SER}_k = k \int_0^\infty \psi(x) f_\Lambda(x) F_\Lambda^{k-1}(x)\, dx. \tag{4}$$

For BPSK ($M = 2$),

$$\psi(x) = \frac{1}{2}\text{erfc}\left(\sqrt{x}\right) - \frac{1}{2}\exp\left(\frac{x}{\bar{\gamma}_0}\right)\frac{\text{erfc}\left(\sqrt{x\left(1+\frac{1}{\bar{\gamma}_0}\right)}\right)}{\sqrt{1+\frac{1}{\bar{\gamma}_0}}}, \tag{5}$$

where erfc(.) is the complementary error function [19]. For $M > 2$, $\psi(x) = \psi_1(x) - \psi_2(x)$, where $\psi_1(x) = \text{erfc}\left(\sqrt{x/\beta}\right) - \exp\left(\frac{x}{\bar{\gamma}_0}\right)\frac{\text{erfc}\left(\sqrt{x(\beta+1/\bar{\gamma}_0)}\right)}{\sqrt{1+\beta/\bar{\gamma}_0}}$, $\beta = \csc^2(\pi/M)$, and $\psi_2(x)$ is given by (6) at the top of the next page.

Proof: The proof is given in Appendix B. ■

We can now derive an accurate expression for SER_k.

Proposition 3: The SER given $k > 0$ relays are active is

$$\begin{aligned}\text{SER}_k \approx\;& k\sum_{n=0}^{W} w_n \left(\frac{4a_n\nu}{\mu^2}K_0\left(\frac{2a_n\sqrt{\nu}}{\mu}\right) + \frac{2a_n\sqrt{\nu}}{\mu}K_1\left(\frac{2a_n\sqrt{\nu}}{\mu}\right)\right)\\ &\times \psi\left(\frac{\nu}{\mu}a_n\right)\left(1 - \frac{2a_n\sqrt{\nu}}{\mu}K_1\left(\frac{2a_n\sqrt{\nu}}{\mu}\right)\exp\left(-a_n\right)\right)^{k-1},\end{aligned} \tag{7}$$

where $\nu = \bar{\gamma}_s\bar{\gamma}_d$, $\mu = \bar{\gamma}_s + \bar{\gamma}_d$, a_n and w_n, $1 \leq n \leq W$, are the W abscissa and weights, respectively, of Gauss-Laguerre quadrature, and K_l is the modified Bessel function of the second kind of order l [20]. When $k = 0$ relays are active,

$$\begin{aligned}\text{SER}_0 =\;& \frac{1}{2}\left(1 - \frac{1}{\sqrt{1+\beta/\bar{\gamma}_0}}\right) - \\ &\frac{1}{\pi}\left(\frac{\arctan\left(\sqrt{\beta-1}/\sqrt{1+\frac{\beta}{\bar{\gamma}_0}}\right)}{\sqrt{1+\frac{\beta}{\bar{\gamma}_0}}} - \arctan\left(\sqrt{\beta-1}\right)\right).\end{aligned} \tag{8}$$

Proof: The proof is in Appendix C. ■

Note that W required for numerical accuracy is small.

From Propositions 1, 2, and 3, the final SER expression for a system with N EH relays can be shown to be:

Result *1:*

$$\begin{aligned}\text{SER} \approx\;& (1-\xi)^N \text{SER}_0 + \\ & N\xi\sum_{n=0}^{W} w_n \left(\frac{4a_n\nu}{\mu^2}K_0\left(\frac{2a_n\sqrt{\nu}}{\mu}\right) + \frac{2a_n\sqrt{\nu}}{\mu}K_1\left(\frac{2a_n\sqrt{\nu}}{\mu}\right)\right)\\ &\times \psi\left(\frac{\nu}{\mu}a_n\right)\left(1 - \frac{2a_n\xi\sqrt{\nu}}{\mu}K_1\left(\frac{2a_n\sqrt{\nu}}{\mu}\right)\exp\left(-a_n\right)\right)^{N-1},\end{aligned} \tag{9}$$

where SER_0 is given by (8).

Energy unconstrained regime as a special case: The SER is now the same as that of a conventional cooperative system with N available AF relays. This has been analyzed in [4], [5], [7]. However, the expressions derived therein use other approximations such as a truncated Taylor series expansion of $K_l(x)$, and are accurate for large SNR. Our results turn out be as accurate over a wider range of SNR values, which will be useful when we optimize the transmit power settings.

A. Two Asymptotic Regimes

Further insights can be gained about the impact of using EH relays by considering the following asymptotic regimes:

1) Asymptotically large N with $N\rho = \kappa$ fixed: This case corresponds to keeping the total energy harvested by all the relays constant even as the number of relays increases. Only the energy constrained case of $\kappa < 1$ is relevant here.[3] In this case, the following key simplification occurs.

[2] The effect of noise is ignored to make the analysis tractable, see, for example, [3]. However, the simulations in Sec. IV do use the exact expression.

[3] Otherwise, in the energy unconstrained regime, an infinite number of relays will be active, and the diversity order will be infinite.

$$\psi_2(x) = \sum_{r=0}^{\infty} \frac{A_r}{\beta^{r+\frac{1}{2}}} \left(x^{r+\frac{1}{2}} \Gamma\left(-\frac{1}{2} - r\right) + e^{-x} \frac{\Gamma(r+\frac{1}{2})}{\Gamma(r+\frac{3}{2})} {}_1F_1\left(1; \frac{1}{2} - r; x\right) \right) - \sum_{r=0}^{\infty} \frac{A_r a^r}{\beta^{r+\frac{1}{2}}} \exp\left(-\left(1 + \frac{1}{\bar{\gamma}_0}\right) x + \frac{1}{\bar{\gamma}_0} \right)$$
$$\times \left(x^{r+\frac{1}{2}} e^{\left(1+\frac{1}{\bar{\gamma}_0}\right)x} \Gamma\left(-\frac{1}{2} - r\right) + \left(1 + \frac{1}{\bar{\gamma}_0}\right)^{-r-\frac{1}{2}} \frac{\Gamma(r+\frac{1}{2})}{\Gamma(r+\frac{3}{2})} {}_1F_1\left(1; \frac{1}{2} - r; \left(1 + \frac{1}{\bar{\gamma}_0}\right) x\right) \right), \quad (6)$$

where $A_r = \frac{1}{2\pi} \frac{1}{2^r r!} \prod_{j=1}^{r} (2j-1)$, $a = 1 + \beta/\bar{\gamma}_0$, ${}_1F_1(.;.;.)$ is the confluent hyper-geometric function of a single variable, and $\Gamma(.)$ is the Gamma function [19].

Proposition 4: The number of active relays follows a Poisson distribution with mean $\lambda = -\log_e(1-\kappa)$.

Proof: The proof follows from the Poisson approximation of a binomial distribution for large N and fixed ρ. ∎

Therefore, the average SER in (9) simplifies to

$$\text{SER} = e^{-\lambda}\text{SER}_0 + \sum_{n=0}^{W} w_n \psi\left(\frac{\nu}{\mu} a_n\right)$$
$$\times \left(\frac{4a_n\nu}{\mu^2} K_0\left(\frac{2a_n\sqrt{\nu}}{\mu}\right) + \frac{2a_n\sqrt{\nu}}{\mu} K_1\left(\frac{2a_n\sqrt{\nu}}{\mu}\right) \right)$$
$$\times \exp\left(-\frac{2a_n\lambda\sqrt{\nu}}{\mu} K_1\left(\frac{2a_n\sqrt{\nu}}{\mu}\right) \exp(-a_n) \right), \quad (10)$$

where $\psi(x)$ is given in Prop. 2 and SER_0 is given in (8).

2) Asymptotic mean channel gains ($\bar{\gamma}_0, \bar{\gamma}_{si}, \bar{\gamma}_{di} \to \infty$): The SER_0 term, which occurs due to the direct S–D path, dominates in (9). Therefore,

$$\text{SER} = (1-\xi)^N \text{SER}_0 = \frac{(1-\xi)^N}{\bar{\gamma}_0} \left(\frac{\pi - \beta \arctan(\sqrt{\beta - 1})}{2\pi} \right).$$

However, in the energy unconstrained regime, $\text{SER} = \text{SER}_N$. Therefore, the diversity order becomes $N+1$, as in a conventional cooperative system [4], [7].

Implications of the asymptotic analysis:

- *Diversity and coding gain*: While diversity order is relevant primarily at high SNRs, it is useful to study it to better understand the system's performance. In the energy constrained regime, we see that the diversity order is 1. The EH relays, thus, provide a *coding gain* of $-10N\log_{10}(1-\xi)$ over direct S-D transmission.
- *Combined case when $N \to \infty$ (with $N\rho = \kappa$ fixed) and $\bar{\gamma}_0, \bar{\gamma}_{si}, \bar{\gamma}_{di} \to \infty$:* When $\kappa < 1$ (energy constrained regime), it follows that the coding gain is now 23.02λ dB.

IV. Simulations and Discussion

We now study the analytical results graphically and also verify the accuracy of our approximations through Monte Carlo simulations that use up to 10^6 samples. Unless mentioned otherwise, the number of relays is $N = 4$ and best relay selection is used. We set $\mathbf{E}\left[|h_{si}|^2\right] = 1$, $\mathbf{E}\left[|h_{di}|^2\right] = 1$, and $N_0 = 1$. The S–D link is weaker with $\mathbf{E}\left[|h_0|^2\right] = 1/4$. Therefore, $\bar{\gamma}_{si} = \bar{\gamma}_{di} = PT$ and $\bar{\gamma}_0 = \frac{P_sT}{4}$.

Figure 2 plots the SER as a function of the normalized average energy harvesting rate, $2E^{\text{av}}T$, when $P_sT = PT = 10$ dB. As expected, the SER increases as the MPSK constellation

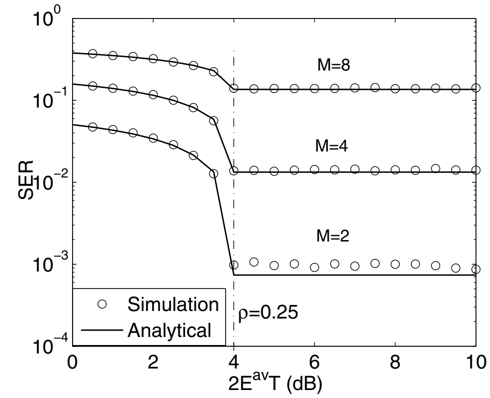

Fig. 2. SER *vs.* $2E^{\text{av}}T$, for different values of constellation size M, $PT = P_sT = 10$ dB

size, M, increases. As E^{av} increases, the SER decreases. It eventually saturates once $2E^{\text{av}}$ exceeds PT/N, where the relays become energy unconstrained; the SER is now entirely determined by the mean channel gains and N.

Figure 3 plots the SER as a function of relay transmission energy, PT, for four different values of ρ. When $\rho < \frac{1}{N} = 0.25$, the relays are energy constrained, and the diversity of the system is 1. Therefore, the SER curves for all $\rho \leq 0.2475$ eventually become parallel to each other for large PT. For $\rho \geq 0.25$, all the relays are energy unconstrained, which makes the diversity order increase to $N+1 = 5$.

The optimal transmit power setting of an EH relay to minimize the SER is investigated in Fig. 4. It plots the SER as a function of PT for two fixed $2E^{\text{av}}T$ values. We observe the following: (i) For small PT, all the relays are energy unconstrained. Therefore, initially, as PT increases, the SER decreases. (ii) Once PT exceeds $2NE^{\text{av}}T$, the relays become energy constrained. Now, the SER is sensitive to ρ, and increases as PT increases. (iii) For large PT (small ρ), the relays expend all their harvested energy and are inactive for most of the time. Therefore, the SER is now determined primarily by the direct S–D channel, and no longer depends on PT. *Thus, the optimum transmit power setting ensures that the relays have just entered the energy unconstrained regime.*

The figure also plots the SER of random relay selection, which can be analyzed along lines similar to that for best relay selection. In it, an active relay is selected randomly without considering its channel gains. Interestingly, the SER of random

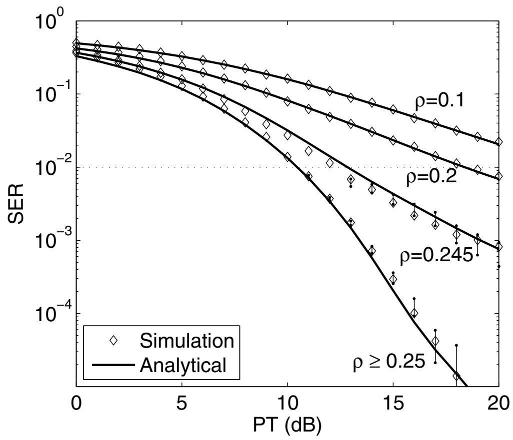

Fig. 3. SER *vs.* PT, as a function of ρ when $P_s = P$, $M = 4$, and $N = 4$ relays. For SER below 10^{-2}, 99% confidence intervals are shown for simulation results.

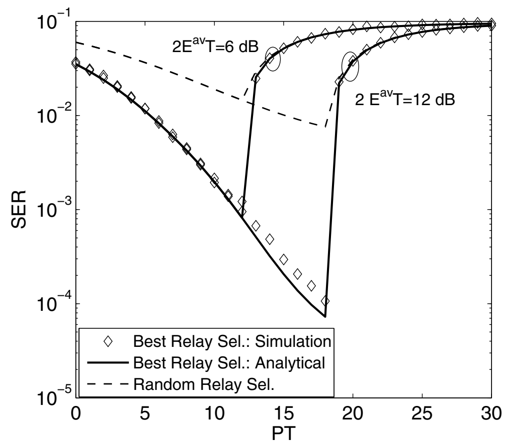

Fig. 4. SER $vs. PT$ for two different normalized E^{av} values when $P_s T =$ 10 dB, $M = 4$ and $N = 4$ Relays

relay selection is comparable to that of best relay selection in a portion the energy constrained regime.

Figure 5 plots the SER as a function of PT, for $N = 4$ and $N \to \infty$. The asymptotic results turn out to be accurate even when N is as small as 4. This figure also shows that random relay selection works almost as well as best relay selection when $\rho = 0.5/N$. However, when ρ is close to $1/N$, best relay selection performs better.

V. Conclusions

We investigated the idea of using energy harvesting nodes as amplify-and-forward cooperative relays that assist conventional source and destination nodes to communicate with each other. For a general class of energy profiles, we derived closed-form expressions for the SER of the system. A key contribution of the analysis is the introduction of an accurate decoupling approximation about the energy availability at the various relays. The analysis showed that the energy profile

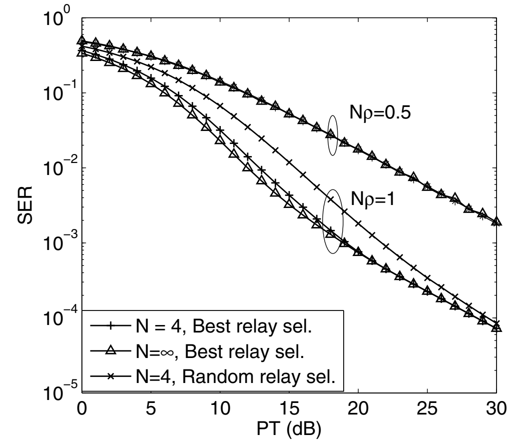

Fig. 5. SER *vs.* PT: Comparison of random relay selection and best relay selection.

influences system performance through a single parameter, namely, the average rate at which energy is harvested by each node. We saw that whether the relays are energy constrained or not depends on the average rate at which they harvest energy, their transmit power, and the number of relays in the system. We also showed that random relay selection can be a useful benchmark to compare against, especially when the relays are energy constrained. Altogether, an EH relay system differs from the conventional cooperative system in several respects, and offer a promising way of improving performance.

Appendix

A. Proof of Proposition 1

First consider the case where the relays are energy constrained. Let $\Pr(R_i \text{ selected})$ denote the probability that R_i is selected. From the energy neutrality constraint and the ergodicity and stationarity of the energy profile, we have

$$\rho = \frac{2E^{\text{av}}T}{PT} = \Pr(R_i \text{ selected}). \quad (11)$$

Since a relay is selected only if it is active, we have

$$\begin{aligned}
&\Pr(R_i \text{ selected}) = \Pr(R_i \text{ active}, R_i \text{ selected}) \\
&= \sum_{r=0}^{N-1} \Pr(R_i \text{ selected} | R_i \text{ is active, } r \text{ other active relays}) \\
&\qquad \times \Pr(R_i \text{ is active, } r \text{ other active relays}). \quad (12)
\end{aligned}$$

From the decoupling approximation,

$$\begin{aligned}
&Pr(R_i \text{ active}, r \text{ other active relays}) \approx \\
&\qquad \Pr(R_i \text{ active}) \binom{N-1}{r} \xi^r (1-\xi)^{N-1-r}. \quad (13)
\end{aligned}$$

From symmetry, we can observe for any r that $\Pr(R_i \text{ selected} | R_i \text{ active}, r \text{ other active relays}) = 1/(r+1)$. Hence,

$$\rho = \xi \sum_{k=0}^{N-1} \frac{1}{r+1} \binom{N-1}{r} \xi^r (1-\xi)^{N-r-1} = \frac{1-(1-\xi)^N}{N}.$$

The desired equation in (1) follows by rearranging terms. This also shows that $\rho N < 1$ when $\xi < 1$. When $\xi = 1$, the system harvests more energy than it can use. Therefore, $\rho N \geq 1$.

B. *Brief Proof of Proposition 2*

From (2), $\gamma_D = \gamma_0 + \Lambda_{[1]}$, where $\Lambda_{[1]} \triangleq \max_i \Lambda_i$. Recall that $\Lambda_i = \frac{\gamma_{si}\gamma_{di}}{\gamma_{si}+\gamma_{di}}$. Using Craig's formula [21]

$$\text{SER}_k = \frac{1}{\pi}\int_0^{\frac{M-1}{M}\pi} \mathcal{M}_{\gamma_D}\left(-\frac{\sin^2(\pi/M)}{\sin^2\phi}\right) d\phi, \tag{14}$$

where the MGF of γ_D, $\mathcal{M}_{\gamma_D}(s) = \mathcal{M}_{\gamma_0}(s)\mathcal{M}_{\Lambda_{[1]}}(s)$. Since γ_0 is exponentially distributed with mean $\bar{\gamma}_0$, $\mathcal{M}_{\gamma_0}(s) = \frac{1}{1+\bar{\gamma}_0 s}$. Furthermore, the PDF of $\Lambda_{[1]}$ is $f_{\Lambda_{[1]}} = k f_\Lambda(x) F_\Lambda^{k-1}(x)$. Therefore, $\mathcal{M}_{\Lambda_{[1]}}(s) = \int_0^\infty k f_\Lambda(x) F_\Lambda(x)^{k-1} e^{-sx} dx$. Thus,

$$\text{SER}_k = \int_0^\infty \psi(x) k f_\Lambda(x) F_\Lambda^{k-1}(x) dx, \tag{15}$$

where $\psi(x) = \frac{1}{\pi}\int_0^{\frac{M-1}{M}\pi} \frac{\exp\left(-\frac{x}{\beta\sin^2\phi}\right)}{1+\left(\frac{\bar{\gamma}_0}{\beta\sin^2\phi}\right)} d\phi$.

1) For BPSK ($M = 2$): We now have the relation, $\psi(x) = \frac{1}{\pi}\int_0^{\pi/2} \frac{\exp\left(-\frac{x}{\sin^2\phi}\right)}{1+\left(\frac{\bar{\gamma}_0}{\sin^2\phi}\right)} d\phi$. Using the variable substitution $\csc^2(\phi) = t^2 + 1$ and the identity in [19, (3.466.1)] yields (5).

2) For MPSK ($M > 2$): Substituting $\csc^2(\phi) = y$ gives

$$\psi(x) = \frac{1}{\pi}\int_1^\infty \frac{e^{-y\left(\frac{x}{\beta}\right)}}{y\sqrt{y-1}(1+y\frac{\bar{\gamma}_0}{\beta})} dy - \frac{1}{2\pi}\int_\beta^\infty \frac{e^{-y\left(\frac{x}{\beta}\right)}}{y\sqrt{y-1}(1+y\frac{\bar{\gamma}_0}{\beta})} dy. \tag{16}$$

Evaluation of the first integral in (16), $\psi_1(x)$, is similar to that for BPSK, which is done above. To evaluate the second integral in (16), $\psi_2(x)$, we take a partial fraction expansion of its integrand to get

$$\psi_2(x) = \frac{1}{2\pi}\int_\beta^\infty \frac{e^{-y\frac{x}{\beta}}}{y\sqrt{y-1}} dy - \frac{1}{2\pi}\int_\beta^\infty \frac{e^{-y\frac{x}{\beta}}}{\sqrt{y-1}(y+\beta/\bar{\gamma}_0)} dy, \tag{17}$$

where $\beta = \csc^2(\pi/M) > 1$ since $M > 2$. Taking the Taylor series expansion of $(1-1/y)^{-1/2}$ we can show that the first term of (17) evaluates to $\sum_{r=0}^\infty A_r \int_\beta^\infty \frac{1}{y^{r+3/2}} e^{-y\frac{x}{\beta}} dy$. The integral in the summation can then be further simplified using the identity in [19, (3.381.6)], The second term of (17) can be simplified similarly to obtain (6).

C. *Brief Proof of Proposition 3*

From [3], the CDF and PDF of Λ_i are, respectively,

$$F_{\Lambda_i}(x) = 1 - \frac{2x}{\sqrt{\nu}} K_1\left(\frac{2x}{\sqrt{\nu}}\right)\exp\left(-\frac{\mu}{\nu}x\right),$$

$$f_{\Lambda_i}(x) = \left(\frac{4x}{\nu}K_0\left(\frac{2x}{\sqrt{\nu}}\right) + \frac{2x}{\sqrt{v_i}}\frac{\mu}{\nu}K_1\left(\frac{2x}{\sqrt{\nu}}\right)\right)\exp\left(-\frac{\mu}{\nu}x\right).$$

Substituting this in (15) and using Gauss-Laguerre quadrature yields (7).

When $k = 0$, we have to only consider the direct link between the source and the destination. Therefore,

$$\text{SER}_0 = \frac{1}{\pi}\int_0^{((M-1)/M)\pi} \frac{1}{1+\bar{\gamma}_0\left(-\frac{\sin^2(\pi/M)}{\sin^2\phi}\right)} d\phi. \tag{18}$$

This upon simplifying yields (8).

References

[1] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, pp. 3062–3080, 2004.

[2] M. O. Hasna and M.-S. Alouini, "End-to-end performance of transmission systems with relays over Rayleigh-fading channels," *IEEE Trans. Wireless Commun.*, vol. 2, pp. 1126–1131, Nov. 2003.

[3] P. A. Anghel and M. Kaveh, "Exact symbol error probability of a cooperative network in a Rayleigh-fading environment," *IEEE Trans. Wireless Commun.*, vol. 3, pp. 1416–1421, Sept. 2004.

[4] S. S. Ikki and M. H. Ahmed, "Performance analysis of generalized selection combining for amplify-and-forward cooperative-diversity networks," in *Proc. ICC*, 2009.

[5] Y. Zhao, R. Adve, and T. J. Lim, "Symbol error rate of selection amplify-and-forward relay systems," *IEEE Commun. Lett.*, vol. 10, pp. 757–759, Nov. 2006.

[6] A. Bletsas, A. Khisti, D. P. Reed, and A. Lippman, "A simple cooperative diversity method based on network path selection," *IEEE J. Sel. Areas Commun.*, vol. 24, pp. 659–672, Mar. 2006.

[7] D. S. Michalopoulos and G. K. Karagiannidis, "Performance analysis of single relay selection in Rayleigh fading," *IEEE Trans. Wireless Commun.*, vol. 7, pp. 3718–3724, Oct. 2008.

[8] D. S. Michalopoulos and G. K. Karagiannidis, "PHY-layer fairness in amplify and forward cooperative diversity systems," *IEEE Trans. Wireless Commun.*, vol. 7, pp. 1073–1083, Mar. 2008.

[9] T. Himsoon, W. P. Siriwongpairat, Z. Han, and K. J. R. Liu, "Lifetime maximization via cooperative nodes and relay deployment in wireless networks," *IEEE J. Sel. Areas Commun.*, vol. 25, pp. 306–317, Feb. 2007.

[10] A. Kansal, J. Hsu, S. Zahedi, and M. B. Srivastava, "Power management in energy harvesting sensor networks," *ACM Trans. Embedded Comput. Syst.*, vol. 7, pp. 1–38, Sept. 2007.

[11] C. R. Murthy, "Power management and data rate maximization in wireless energy harvesting sensors," *International Journal of Wireless Information Networks*, Jul. 2009. DOI: 10.1007/s10776-009-0104-2.

[12] J. A. Paradiso and T. Starner, "Energy scavenging for mobile and wireless electronics," *IEEE Trans. Pervasive Comput.*, pp. 18–27, Jan.–Mar. 2005.

[13] V. Raghunathan, S. Ganeriwal, and M. Srivastava, "Emerging techniques for long lived wireless sensor networks," *IEEE Commun. Mag.*, pp. 108–114, Apr. 2006.

[14] B. Medepally, N. B. Mehta, and C. R. Murthy, "Implications of energy profile and storage on energy harvesting sensor link performance," in *Proc. Globecom*, 2009.

[15] J. Lei, R. Yates, and L. Greenstein, "A generic model for optimizing single-hop transmission policy of replenishable sensors," *IEEE Trans. Wireless Commun.*, vol. 8, pp. 547–551, Feb. 2009.

[16] M. Tacca, P. Monti, and A. Fumagalli, "Cooperative and reliable ARQ protocols for energy harvesting wireless sensor nodes," *IEEE Trans. Wireless Commun.*, vol. 6, pp. 2519–2529, July. 2007.

[17] A. Ribeiro, X. Cai, and G. B. Giannakis, "Symbol error probabilities for general cooperative links," *IEEE Trans. Wireless Commun.*, vol. 4, pp. 1264–1273, May 2005.

[18] D. Niyato, E. Hossain, and A. Fallahi, "Sleep and wakup strategies in solar-powered wireless sensor/mesh networks: Performance analysis and optimization," *IEEE Trans. Mobile Comput.*, vol. 6, pp. 221–236, Feb. 2007.

[19] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*. Academic Press, 4^{th} ed., 1980.

[20] M. Abramowitz and I. Stegun, *Handbook of mathematical functions with formulas, graphs, and mathematical tables*. Dover, 9 ed., 1972.

[21] M. K. Simon and D. Divsalar, "Some new twists to problems involving the Gaussian probability integral," *IEEE Trans. Commun.*, vol. 46, pp. 200–210, Feb. 1998.