An Accurate Model For Interference From Spatially Distributed Shadowed Users in CDMA Uplinks

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***Abstract*—A detailed characterization of interference power statistics in CDMA systems is of considerable practical and theoretical interest. Such a characterization for uplink inter-cell interference has been difficult because of transmit power control, randomness in the number of interfering mobile stations, and randomness in their locations. We develop a new method to model the uplink inter-cell interference power as a lognormal distribution, and show that it is an order of magnitude more accurate than the conventional Gaussian approximation even when the average number of mobile stations per cell is relatively large and even outperforms the moment-matched lognormal approximation considered in the literature. The proposed method determines the lognormal parameters by matching its moment generating function with a new approximation of the moment generating function for the inter-cell interference. The method is tractable and exploits the elegant spatial Poisson process theory. Using several numerical examples, the accuracy of the proposed method in modeling the probability distribution of inter-cell interference is verified for both small and large values of interference.**

I. INTRODUCTION

CDMA (Code Division Multiple Access) cellular communications systems are by their very design interference limited. The data signals are spread at the transmitter by pseudo-random spreading codes that are *approximately* orthogonal to help separate different users. However, the suppression of other-user interference is not perfect, and the residual interference is the determining factor for the performance (capacity, transmission quality) of the system. Since third-generation cellular systems (3GPP, cdma2000) based on CDMA serve millions of users, clearly, a detailed investigation of interference power statistics in CDMA systems is of great practical as well as theoretical interest.

The current paper deals with *inter-cell* interference in the *uplink*. Generally, interference statistics are different between the *uplink* and *downlink* of the system. Also, the properties of *inter-cell* and *intra-cell* interference are different. Intra-cell interference is a well understood phenomenon for both uplink and downlink. In the downlink, even though the spreading codes used within one cell are truly orthogonal to each other at the transmitter, delay dispersion of the channel can destroy this orthogonality. Therefore, the intra-cell interference is the transmit power multiplied by the pathloss and the so-called orthogonality factor [1]. In the uplink, in which the spreading codes are not designed to be perfectly orthogonal, power control ensures that the strength of all the (spread) signals arriving at the receiver is the same. (The despreading operation just suppresses the interference by a constant factor.) Thus, the interference level is simply a constant times the number of active users [2].

Inter-cell interference in the downlink is the non-coherent sum of a small number of interfering signals equal to the number of base stations in the neighborhood of the considered cell. Each of those signals undergoes a different pathloss, shadowing, and small-scale fading. The total interference power is then the sum of a deterministic number of lognormal or Suzuki-distributed variables, whose statistics can be approximated using the methods described in [3], for example.

Uplink inter-cell interference power is a more complicated, and less investigated, topic. Compared to the uplink intra-cell interference, we find that the interfering signals undergo (uncontrolled) shadowing and fading in their respective wireless channels, and the path loss depends on the user location. Compared to downlink inter-cell interference, complications arise from the fact that the transmit power of the interfering mobile station (MS) is power-controlled *by a different cell*. Furthermore, the number of MSs is itself random. These factors prevent a closed-form analytical characterization of the inter-cell interference probability distribution, and approximations need to be found. A widely used approximation for the uplink inter-cell interference is the Gaussian approximation, which is motivated by the central limit theorem related to the large number of interferers [4]. An additional justification that has been given is that the use of power control and cell selection in CDMA systems reduces the relative variations in the received powers [5, Chp. 4].

However, it has been shown recently [6], [7] that the Gaussian approximation usually does not give satisfactory accuracy even in the presence of power control. Even refinements of the Gaussian approach based on Edgeworth approximations [4] leave considerable scope for improvement. In this paper, we introduce a much better approximation based on a lognormal distribution whose parameters are obtained by matching its moment generating function (MGF) with that developed for

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the uplink inter-cell interference at a sufficient number of points. The method is analytically tractable and, as we show, accurate even when the average user density per cell is high.

The fact that a lognormal distribution is a better approximation for inter-cell interference power is well understood for the downlink case since it is known that the sum of lognormal RVs is well approximated by a lognormal RV. Different methods have been developed in [3], [8]–[11]. In other words, while the distribution of the sum does eventually become a Gaussian RV, the rate of convergence of the sum's distribution to the Gaussian distribution is slow due to the skewed nature of the lognormal distribution [12]. Approximations other than the lognormal such as the modified power-lognormal approximation [13], Type IV Pearson distribution [14], [15], and compound distributions [16], [17] have also been developed.

Modeling the uplink inter-cell interference is a different problem because of additional dimensions such as the use of power control, the random number of interfering users served by neighboring base stations (BSs), and their random locations, which all affect the power transmitted by the MSs.[1] It makes the MGF or characteristic function matching methods developed in [3], [11] inapplicable and necessitates a new approach.

The new method developed in this paper first requires a tractable approximation for the MGF of the uplink inter-cell interference power. For this, we employ the elegant theory of spatial Poisson processes [18] to model the spatial randomness of the interfering MSs. The theory provides an analytically tractable and reasonable model for the spatial randomness observed in a CDMA uplink, and has been used effectively in several wireless system design problems [4], [19]. A similar spatial model was also used in [20], which also considered the problem of modeling uplink inter-cell interference with power control and cell selection. However, [20] only computed the cumulants of the interference, and required multi-dimensional Monte Carlo integration to do this. The MGF matching method developed in this paper is inherently well suited to leverage spatial Poisson process theory to approximate the probability distribution of the uplink inter-cell interence. Interference from both first-tier and second-tier interference cells is accounted for.

We also compare our method with the alternate moment-matching based lognormal approximation approach developed in [6], [7] for modeling inter-cell uplink interference. As was observed in the papers, while the moment matching approach worked better than the Gaussian approximation, considerable scope for improvement remained in modeling the distribution for small values of the interference. The proposed approach overcomes these problems and leads to a significantly more accurate match with the actual distribution. Furthermore, the same method can now be used to accurately approximate the distribution of small and large values of interference.

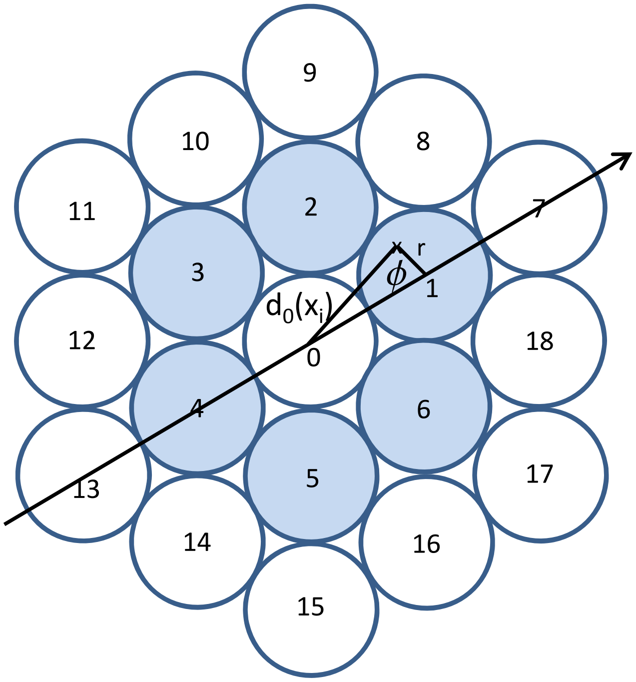

Fig. 1. Cellular layout showing reference cell and first- and second-tier cells.

The paper is organized as follows. The uplink system model is developed in Sec. II. The alternate lognormal approximation method for modeling interference is developed in Sec. III. Simulation results are presented in Sec. IV, and are followed by our conclusions in Sec. V.

II. SYSTEM MODEL

Figure 1 shows the cellular layout consisting of a cell 0, which we henceforth refer to as the *reference cell*, and adjacent first-tier and second-tier interfering cells. In order to enable analytical tractability, the cells are taken to be circular and not hexagonal in shape. A cell k is served by BS k, which is located at the cell's center. If first-tier interferers are considered, k takes a value between 1 to 6. For second-tier interferers, k takes a value between 7 to 18. Let D_k denote the distance between BS k and the reference BS 0. This case also includes the scenario where Rayleigh or Ricean fading is also present because a composite Rayleigh-lognormal or a Rice-lognormal random variable's distribution can be well approximated by a lognormal [21].[2]

Consider an MS i inside cell k, and, therefore, served by BS k. Let $\mathbf{x}_i$ denote the absolute or global position vector of MS i with respect to a universal center. Let $d_k(\mathbf{x}_i)$ denote the corresponding distance of the MS from BS k.

When the MS i transmits a signal with power P_i, the short-term fading-averaged receive signal power at BS k is

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[1] Cell selection, *i.e.*, the fact that the MS connects to the BS to which it has the strongest link, even if it is not the geographically closest, introduces further randomness. It can be taken into account in our framework, but it is omitted due to space restrictions.

[2] Inverting a Suzuki channel will make the uplink inter-cell interference moments unbounded [22]. In practice, the peak transmit power constraint or threshold based power control prevents this, and makes the approximation of a Rayleigh-lognormal by a lognormal work. This modeling also directly applies when diversity techniques mitigate fading or when slow power control that only reacts to shadowing is used.

proportional to $P_i\left(\frac{d_0}{d_k(\mathbf{x}_i)}\right)^{\epsilon} s_i^{(k)}$, where d_0 is a reference distance and ϵ is the path loss coefficient, which typically takes values between 2 and 4 [22]. The variable $s_i^{(k)}$ denotes the shadowing of the uplink channel from MS i to BS k. As mentioned, shadowing is well characterized by a lognormal random variable (RV), and can be written as

$$s_i^{(k)} = 10^{0.1 y_i(k)} = e^{\beta y_i(k)}, \tag{1}$$

where $y_i(k)$ is a Gaussian RV with zero mean and variance $\sigma_i^2(k)$ and $\beta = 0.1 \log_e(10)$. Following terminology used in the literature, we shall refer to $\sigma_i^2(k)$ as the dB variance of the lognormal RV $s_i^{(k)}$. Typically, $\sigma_i(k)$ takes values between 4 and 12. For analytical simplicity, we shall assume that $\sigma_i(k) = \sigma$ for all i and k. We assume $y_i(k)$ to be independent and identically distributed for different values of i and k.

If k is the serving cell for a user at location $\mathbf{x}_i$, power control ensures that its transmit power is set so that

$$P_{\text{tx}} \frac{s_i^{(k)}}{d_k(i)^{\epsilon}} = \gamma. \tag{2}$$

Hence, the interference generated by a user i at the reference cell 0 equals

$$I(\mathbf{x}_i, k) = P_{\text{tx}} \frac{s_i^{(0)}}{d_0(\mathbf{x}_i)^{\epsilon}} = \gamma e^{\beta(y_i(0) - y_i(k))} \left(\frac{d_k(\mathbf{x}_i)}{d_0(\mathbf{x}_i)}\right)^{\epsilon}. \tag{3}$$

Therefore, the total interference at the reference cell from the N_k users served by cell k equals

$$\sum_{i=1}^{N_k} I(\mathbf{x}_i, k) = \gamma \sum_{i=1}^{N_k} e^{\beta(y_i(0) - y_i(k))} \left(\frac{d_k(\mathbf{x}_i)}{d_0(\mathbf{x}_i)}\right)^{\epsilon}. \tag{4}$$

A. Spatial Poisson Process Model for Users

We now model the statistics of N_k and the user locations within the cell. For this, we use a spatial Poisson process. While we limit our attention in this paper to homogeneous Poisson point processes, the analysis can be extended to handle non-homogeneous processes as well.

Definition: Let $\#\mathcal{S}$ denote the number of elements in a set $\mathcal{S}$. Then, a random countable subset $\Pi \subset \mathbb{R}^d$ is called a homogeneous spatial Poisson process with intensity λ if the RV $N(\mathcal{B}) = \#(\Pi \cap \mathcal{B})$, $\mathcal{B} \subset \mathbb{R}^d$, satisfies [18]:

1) For all $n \geq 1$ and disjoint $\mathcal{B}_1, \ldots, \mathcal{B}_n \subset \mathbb{R}^d$, the RVs $N(\mathcal{B}_1), \ldots, N(\mathcal{B}_n)$ are independent,
2) $N(\mathcal{B})$ is Poisson distributed with mean $\lambda |\mathcal{B}|$, where $|\mathcal{B}|$ denotes the volume (Lebesgue measure) of $\mathcal{B}$.

When applied to a two-dimensional ($d = 2$) cell of area A, we get the following interpretation. The probability that N_k users occur within the cell k is Poisson distributed with mean λA, and is given by

$$\Pr(N_k = l) = \frac{(\lambda A)^l}{l!} \exp(-\lambda A). \tag{5}$$

Furthermore, conditioned on N_k, the geographical locations of the N_k MSs are uniformly distributed over the cell area.

Comment: This model implies that with power control, the probability distribution of the *intra-cell interference power* in a cell of area A is simply a Poisson RV with mean $\gamma \lambda A$ [4] (See also the discussion in Sec. I.). It is for this reason that we focus primarily on the inter-cell interference in this paper.

III. LOGNORMAL APPROXIMATION METHOD

We now develop the MGF matching based lognormal approximation method. A key result that enables the proposed method that is developed next is captured in the following theorem.

Theorem *1:* Let $\psi_k(s)$ denote the MGF of the interference from an arbitrary user in cell k. Then, the MGF, $\Psi_k(s)$, of the total uplink interference from cell k equals

$$\Psi_k(s) = e^{\lambda A(\psi_k(s) - 1)}. \tag{6}$$

Proof: When N_k users are present in cell k, the total interference from cell k equals $\sum_{i=1}^{N_k} I(\mathbf{x}_i, k)$. Therefore, $\Psi_k(s) = \mathbf{E}_{N_k, \mathbf{x}_i}\left[e^{-s \sum_{i=1}^{N_k} I(\mathbf{x}_i, k)}\right]$, where $\mathbf{E}_X[.]$ denotes the expectation with respect to the random variable X. Since $I(\mathbf{x}_i, k)$ is independent for different users, using (5) to uncondition over N_k results in

$$\begin{aligned}
\Psi_k(s) &= \mathbf{E}_{N_k}\left[\prod_{i=1}^{N_k} \mathbf{E}_{\mathbf{x}_i}\left[e^{-s I(\mathbf{x}_i, k)}\right]\right], \\
&= \sum_{N_k=0}^{\infty} \frac{(\lambda A)^{N_k}}{N_k!} e^{-\lambda A} \left(\psi_k(s)\right)^{N_k}, \\
&= e^{\lambda A(\psi_k(s) - 1)}.
\end{aligned}$$

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We now evaluate $\psi_i(s)$. From (3), we have

$$\psi_i(s) = \mathbf{E}_{\mathbf{x}_i}\left[\frac{1}{2\sigma\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-\frac{t^2}{4\sigma^2}} \exp\left(-s\gamma e^{\beta t} \left(\frac{d_k(\mathbf{x}_i)}{d_0(\mathbf{x}_i)}\right)^{\epsilon}\right) dt\right]. \tag{7}$$

Using Gauss-Hermite quadrature, we get

$$\psi_i(s) \approx \frac{1}{\sqrt{\pi}} \sum_{n=1}^{W_H} w_n \mathbf{E}_{\mathbf{x}_i}\left[\exp\left(-s\gamma e^{2\beta\sigma a_n} \left(\frac{d_k(\mathbf{x}_i)}{d_0(\mathbf{x}_i)}\right)^{\epsilon}\right)\right]. \tag{8}$$

Averaging over $\mathbf{x}_i$, we get

$$\psi_i(s) \approx \frac{2}{\pi^{3/2} R^2} \sum_{n=1}^{W_H} w_n \times \int_0^R \int_0^{\pi} r \exp\left(-s\gamma e^{2\beta\sigma a_n}\left(1 + \frac{D_k^2}{r^2} - 2\frac{D_k}{r}\cos\phi\right)^{-\frac{\epsilon}{2}}\right) d\phi dr,$$

where (r, ϕ) are polar coordinates of $\mathbf{x}_i$ with respect to BS k. Using the variable substitution $\cos\phi = z$ we get

$$\psi_i(s) \approx \frac{2}{\pi^{3/2} R^2} \sum_{n=1}^{W_H} w_n \times \int_0^R \int_{-1}^{1} \frac{r}{\sqrt{1 - z^2}} \exp\left(-s\gamma e^{2\beta\sigma a_n}\left(1 + \frac{D_k^2}{r^2} - 2z\frac{D_k}{r}\right)^{-\frac{\epsilon}{2}}\right) dz dr.$$

Applying Gauss-Chebyshev quadrature [23], we get

$$\psi_i(s) \approx \frac{2}{\sqrt{\pi} R^2 W_C} \sum_{n=1}^{W_H} w_n \sum_{n'=1}^{W_C}$$
$$\int_0^R r \exp\left(-s\gamma e^{2\beta\sigma a_n} \left(1 + \frac{D_k^2}{r^2} - 2\frac{D_k}{r} a_{n'} \right)^{-\frac{\epsilon}{2}} \right) dr, \quad (9)$$

where $a_{n'}$, $1 \leq n' \leq W_C$, are the abscissa of Gauss-Chebyshev quadrature and are readily tabulated in [23]. A key point to note is that even a small value for W_H and W_C such as 6 suffices to accurately match the observed interference distribution for the system parameters typical of cellular uplinks.

Therefore, the MGF of the uplink inter-cell interference from a cell k is approximately given by $\Psi_k(s) \approx \widetilde{\Psi}_k(s)$, where $\widetilde{\Psi}_k(s)$ is defined in (10). Any further simplification seems difficult. Therefore, the single integral above needs to be evaluated numerically.

A. The Proposed MGF Matching Method

The MGF of the approximating lognormal RV with dB mean and dB variance μ_{eq} and σ_{eq}, respectively, is approximately given by [3] $\sum_{n=1}^{W_H} \frac{w_n}{\pi} \exp\left[-s \exp\left(\frac{\sqrt{2} a_n \sigma_{\text{eq}} + \mu_{\text{eq}}}{\xi} \right) \right]$, where a_n and w_n, as defined earlier, are the W_H Gauss-Hermite quadrature abscissa and weights, respectively. Consequently, the parameters of the approximating lognormal are obtained by solving the following system of two non-linear equations that match the MGF at $i = 1$ and $i = 2$:

$$\widetilde{\Psi}_k(s_i) = \sum_{n=1}^{W_H} \frac{w_n}{\sqrt{\pi}} \exp\left[-s_i \exp\left(\frac{\sqrt{2} a_n \sigma_{\text{eq}} + \mu_{\text{eq}}}{\xi} \right) \right]. \quad (11)$$

For convenience, s_1 and s_2 are taken to be positive real numbers.

Since the uplink interference from a first- or second-tier cell is independent of that from other cells, the MGF of the total interference is simply $\prod_k \Psi_k(s)$, where k indexes all the first- and second-tier cells. Therefore, to determine the parameters of the lognormal that approximates the total interference, one just replaces $\widetilde{\Psi}_k(s_i)$ with $\prod_k \widetilde{\Psi}_k(s_i)$ in (11).

The choice of s_1 and s_2 depends on the portion of the distribution needs that to be modeled accurately. In general, the weighting function interpretation of the MGF in [3] indicates that larger values of s_1 and s_2 emphasize accuracy in matching the distribution for small values of the interference power. Similarly, choosing smaller values for s_1 and s_2 will emphasize accuracy in matching the distribution for larger interference values.

IV. SIMULATION RESULTS AND MODEL VERIFICATION

We now compare the accuracy achieved in modeling the observed uplink inter-cell interference. We compare the lognormal approximation with the conventional Gaussian approximation and the moment-matching based lognormal approximation method of [6]. The mean and variance of the Gaussian distribution are obtained by equating them with the mean and variance of the total interference, along lines similar to that in Sec. III.

We also plot the CDF and CCDF obtained from Monte Carlo simulations that uses 3×10^5 sample points. In each sample point, the users locations and the number of users per cell constitute a realization of the spatial Poisson process described in Sec. II-A. And, the shadowing of the different users to different BSs' channels are generated as independent lognormal random variables. Since only power control is being considered, a user's serving BS is the one nearest to it. Each user sets its transmit power so as to compensate for the shadowing and path loss to its serving BS as per (2).

The comparison is done on the basis of the cumulative distribution function (CDF) and the complementary CDF (CCDF) of the inter-cell interference power, as is often done in the literature [3], [10], [11]. Comparing the CDF is instructive because small values of the CDF reveal the accuracy in tracking the head portion (small interference values) of the probability distribution. Similarly, small values of the CCDF reveal the accuracy in tracking the tail portion (large interference values) of the interference probability distribution.

The system parameters used in the simulations are: path loss exponent $\epsilon = 4$, power control threshold $\gamma = 8$ dB, lognormal dB standard deviation $\sigma = 6$, cell radius $R = 400$ m, and $D = 800$ m. We first study the accuracy of the proposed approach in modeling the inter-cell interference from a first-tier cell. Thereafter, we evaluate the ability of the proposed approach to model the total interference from all the first-tier and second-tier cells. For the proposed MGF matching method, $s_1 = 0.01$ and $s_2 = 0.02$ were used for matching the CDF, and $s_1 = 5 * 10^{-7}$ and $s_2 = 10^{-4}$ were used for matching the CCDF. These values were determined numerically, and were chosen to ensure that an accurate approximation is obtained for a wide range of user densities.

Figure 2 compares the CDFs of the interference power from a first-tier cell generated by the various methods when the average number of users per cell is relatively small ($\lambda A = 10$). Figure 3 does the same when the average number of users per cell is tripled to $\lambda A = 30$. In both cases, it can be seen that the lognormal approximation, be it from the moment-matching method or the proposed method, is an order of magnitude more accurate. In fact, the Gaussian CDF saturates for small values of the interference [6]. This occurs due to the positivity constraint on the interference power. The Gaussian CDF, say with mean μ_G and variance σ_G^2, saturates for small interference values at $Q(\mu_G/\sigma_G)$. On the other hand, this saturation does not happen for the lognormal CDF when the interference, say I, from a cell tends to 0, because it equals $Q\left(\frac{|\log_e(I)|}{\sigma_{\text{eq}}}\right)$ for small I.

Furthermore, the proposed approach is considerably more accurate than the moment-matched lognormal approximation for both the user densities. One reason follows from the analysis developed in [6], which shows that the moment-matching method captures the randomness in the number of users only through its first two moments.

$$\widetilde{\Psi}_k(s) \triangleq e^{-\lambda A} \prod_{n=1}^{W_H} \prod_{n'=1}^{W_C} \exp\left(\frac{2\lambda A w_n}{\sqrt{\pi} R^2 W_C} \int_0^R r \exp\left(-s\gamma e^{2\beta\sigma a_n} \left(1 + \frac{D_k^2}{r^2} - 2\frac{D_k}{r} a_{n'}\right)^{-\frac{\epsilon}{2}} \right) \right) dr. \tag{10}$$

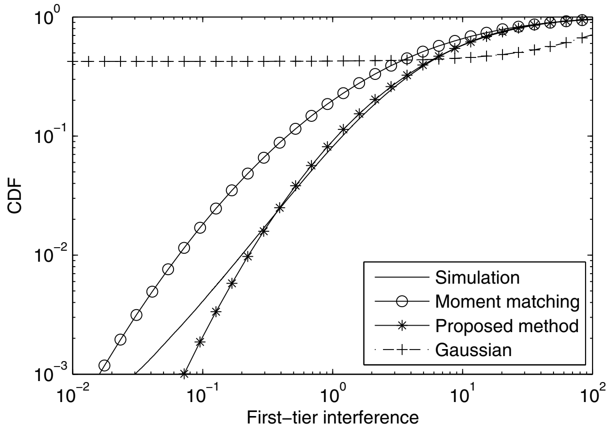

Fig. 2. Lower user density case (10 users/cell on average): Comparison of CDFs of interference from a first-tier cell using proposed method, moment-matching method, Gaussian approximation, and simulations.

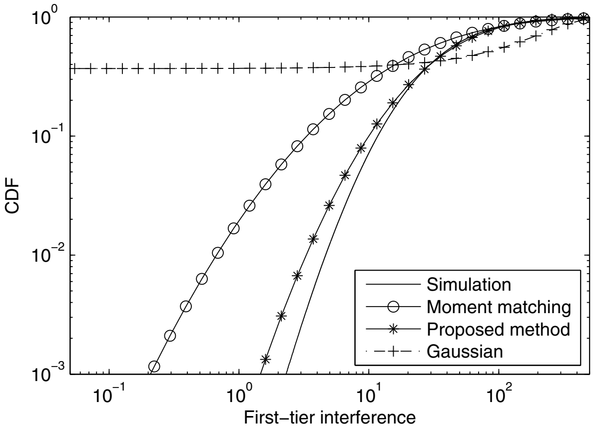

Fig. 3. Higher user density case (30 users/cell on average): Comparison of CDFs of interference from a first-tier cell using proposed method, moment-matching method, Gaussian approximation, and simulations.

Figure 4 now compares the CCDF of the various methods when $\lambda A = 30$. (The figure for $\lambda A = 10$ is qualitatively similar, and is not shown here.) The proposed MGF-matched lognormal approximation and the moment-matched lognormal approximation overlap with each other, and are better in tracking the CCDF than the conventional Gaussian approximation. This is similar to the case of a fixed number of lognormal interferers considered in [3], [10], in which the moment-matched Fenton-Wilkinson method was accurate in approximating very small values of the CCDF.

The CDF and CCDF of the total uplink inter-cell interference from all the first- and second-tier cells is plotted in Fig. 5. It can be seen that due to the many more interferers that contribute to the total interference, the accuracy of the Gaussian approximation improves. However, the lognormal approximation obtained using the proposed MGF-matching method is still significantly better.

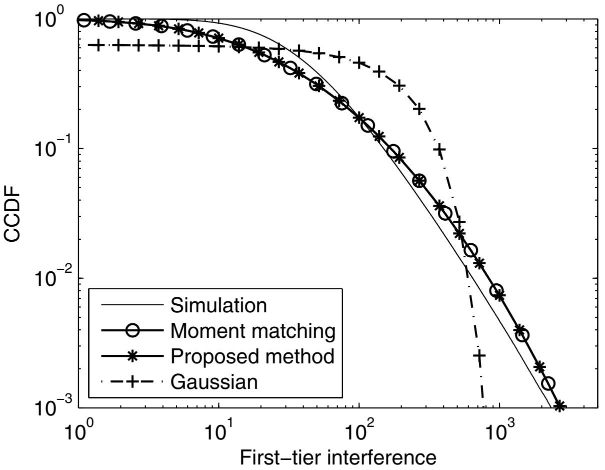

Fig. 4. Comparison of CCDFs of interference from a first-tier cell using proposed method, moment-matching method, Gaussian approximation, and simulations for an average of 30 users per cell.

V. CONCLUSIONS

We argued that while the uplink and downlink intra-cell interference powers and downlink inter-cell interference power are well modeled, considerable work remains in characterizing the distribution of the uplink inter-cell interference power in CDMA systems. This is because of the presence of additional sources of randomness in the uplink such as power control of an interfering user by its serving cell, the random number of interfering users whose signals undergo path loss and lognormal shadowing, and their random locations within a cell.

We developed a new expression for the MGF of the uplink inter-cell power. By matching it with corresponding MGF of the approximating lognormal, we determined the approximating lognormal's parameters. We showed that doing so leads to a model that is an order of magnitude more accurate than the conventional Gaussian approach, which is motivated by the law of large numbers. Furthermore, the proposed model is also significantly more accurate than the moment-matched lognormal approximation approach in modeling the distribution of small values of interference. Future research involves developing a finer time trace model for the inter-cell interference to analyze its impact on a call session or data session.

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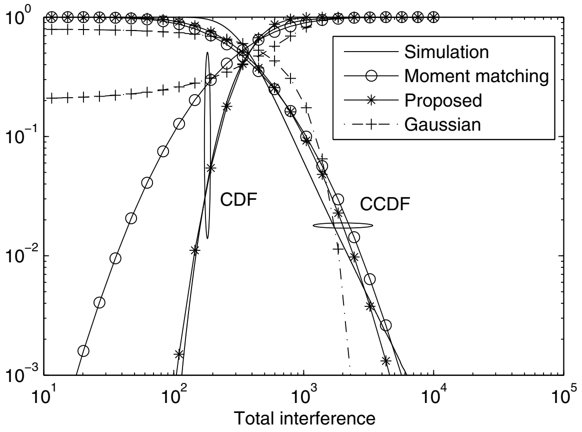

Fig. 5. Comparison of CDF and CCDF of the total inter-cell interference as generated by proposed method, moment-matched lognormal approximation, Gaussian approximation, and simulations.

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