Reduced Feedback, User Scheduling, and Mode Selection in Asymmetric Full-Duplex Systems

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***Abstract*—In a full-duplex (FD) system, the base station (BS) needs to carefully schedule the uplink and downlink users that transmit simultaneously to control the inter-user interference. The efficacy of scheduling is closely tied to the availability of channel state information and the feedback scheme that conveys it to the BS. We propose a novel user-pair scheduling and mode selection algorithm (UPSMA) and a reduced feedback scheme, in which a user feeds back only a limited number of inter-user interferences that are below a pre-specified threshold. We derive expressions for the uplink and downlink rates of UPSMA in the presence of small-scale fading, large-scale shadowing, and pathloss. These lead to novel scaling laws for the threshold and the uplink and downlink rates. Even with limited feedback, UPSMA achieves a higher sum rate compared to a half-duplex system and conventional FD resource allocation algorithms.**

I. Introduction

A full-duplex (FD) enabled base station (BS) can simultaneously transmit and receive signals on the same frequency band. Consequently, FD has the potential to double the spectral efficiency of cellular systems. While several techniques have been developed to reduce the self-interference (SI), several system-design challenges remain. One of them is controlling the inter-user interference from the uplink user to the downlink user. This requires the BS to carefully choose the users or revert to the half-duplex (HD) mode.

User scheduling and mode selection are intimately linked to the availability of channel state information (CSI) of the inter-user channels at the BS, which affects the downlink signal-to-interference-plus-noise ratios (SINRs) in an FD system. This, in turn, depends on the feedback scheme. The literature differs considerably on its assumptions about CSI and feedback, and the criteria used to schedule the users.

In [1]–[3], the BS schedules the uplink and downlink users that are sufficiently spaced apart to reduce the inter-user interference. In [4], the BS schedules users in the FD mode only if the probability that their inter-user interference is less than a threshold exceeds a minimum value. Three scheduling algorithms are proposed in [5] by Alexandropoulos, Kountouris, and Atzeni (AKA). The first algorithm AKA-1 schedules the uplink and downlink users with the largest channel gains. In the second algorithm AKA-2, the BS first selects the uplink user with the largest channel gain. It then selects the downlink user with the largest SINR. The third algorithm AKA-3 is similar to AKA-2 except that the downlink user is selected first and the uplink user next. The algorithm in [6] maximizes the sum rate while ensuring a minimum rate for each user. Given an uplink or downlink user, the algorithm in [7] pairs another user with it to optimize the sum of the uplink and downlink rates or outage probabilities. Scheduling algorithms for an orthogonal frequency division multiple access system are proposed in [8], [9].

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The algorithms in [1]–[4] require the BS to know the locations of the users and do not consider the effect of fading on inter-user interference. In [6]–[8], the BS is assumed to know the interferences of all the user pairs in the cell. This is practically challenging since the BS cannot estimate an inter-user channel gain by itself; it needs feedback. Since there are $\binom{N}{2}$ user-pairs in a cell with N users, the feedback overhead grows as $O\left(N^2\right)$ and can overwhelm the uplink. The algorithms in [5], [9] reduce the number of inter-user interferences that needs to be fed back to N, but assume that this CSI is available to the BS with infinite resolution.

A. Contributions

We propose a novel user-pair scheduling and mode selection algorithm (UPSMA) and a reduced inter-user interference feedback scheme for a cellular system with an FD BS and HD users. This asymmetric model is motivated by the observation that FD is likely to be first implemented in the BS transceiver.

In UPSMA, the BS first schedules a user on either uplink or downlink. The scheduled user feeds back a set containing at most L users whose inter-user interference is below a threshold γ_{th} and b-bit quantized versions of the interferences. Here, L and b are system parameters that control the feedback overhead. This also enables the BS to bound the inter-user interference and conservatively estimate the downlink SINR even before it decides upon the uplink user. The BS schedules the second user from the feedback set. It reverts to the HD mode if it has a higher rate or the feedback set is empty.

We derive expressions for the uplink and downlink rates of UPSMA in a single-cell scenario. The analysis accounts for small-scale fading, shadowing, and pathloss. It leads to insightful and novel asymptotic scaling laws that reveal how to choose γ_{th} to maximize the sum rate, and how the uplink and downlink rates scale with the number of users.

We present extensive simulation results to benchmark the rate gains from UPSMA in noise-limited single-cell and co-channel interference-limited (CCI) multi-cell scenarios with imperfect SI cancellation. These bring out the critical role that feedback plays in the design of an FD cellular system.

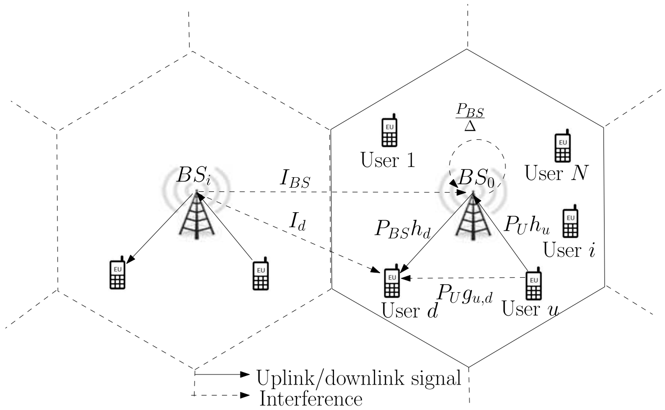


Fig. 1: System model illustrating uplink and downlink channels, inter-user interference, SI, and CCI from the neighboring cells.

B. Outline and Notation

The paper is organized as follows. Section II describes the system model. Section III specifies UPSMA. Its performance is analyzed in Section IV. Simulation results are presented in Section V, and are followed by our conclusions in Section VI.

Notation: $\mathbb{P}(A)$ denotes the probability of an event A and $\mathbb{P}(A|B)$ the conditional probability of A given B. The probability density function (PDF) and cumulative distribution function (CDF) of a random variable (RV) X are denoted by $f_X(.)$ and $F_X(.)$, respectively. The expectation over an RV X is denoted by $\mathbb{E}_X[.]$. The expectation conditioned on an event A is denoted by $\mathbb{E}_X[.|A]$. The subscript is dropped when it is obvious from context. $X \sim \text{Exp}(\mu)$ means that X is an exponential RV with mean μ. The cardinality of a set $\mathcal{A}$ is denoted by $|\mathcal{A}|$. The indicator function is denoted by $1_{\{a\}}$; it is 1 if a is true and is 0 otherwise. We use $O(.)$ and $\Theta(.)$ as per the Bachmann-Landau notation [10, Ch. 2.2].

II. System Model

We consider a cellular system with an FD-capable BS and N HD users per cell. It operates in the time division duplex (TDD) mode [1], [3], [5]. Let $\mathbb{N} = \{1, 2, \ldots, N\}$ denote the set of user indices. The system model is illustrated in Fig. 1. Every user has data to transmit to the BS and receive from it. The channel power gain between user $m \in \mathbb{N}$ and the BS is denoted by h_m. It is given by

$$h_m = \alpha_m \beta_m \mu_m, \tag{1}$$

where $\alpha_m \sim \text{Exp}(1)$ models small-scale fading and β_m is a lognormal RV with dB-mean 0 and dB standard deviation σ_{dB} that models shadowing. And, $\mu_m = K (d_0/d_m)^\eta$ is the pathloss between the BS and user m. Here, d_m is the distance between the BS and user m, d_0 is the critical distance, K is a constant, and η is the pathloss exponent. Similarly, $g_{m,n} = \alpha_{m,n}\beta_{m,n}\mu_{m,n}$ is the channel power gain between users m and n, where $\alpha_{m,n}$ models small-scale fading, $\beta_{m,n}$ models shadowing, and $\mu_{m,n} = K (d_0/d_{m,n})^\eta$. Here, $d_{m,n}$ is the distance between users m and n.

SINR Expressions: The received SINRs at the BS and the users in the FD and HD modes are as follows.

1) FD Mode: When the BS receives a signal from user u and transmits to user d in the FD mode, the FD mode uplink SINR $\gamma_{\text{UL}}^{\text{FD}}(u)$ and downlink SINR $\gamma_{\text{DL}}^{\text{FD}}(u,d)$ are given by

$$\gamma_{\text{UL}}^{\text{FD}}(u) = \frac{P_{\text{U}} h_u}{\frac{P_{\text{BS}}}{\Delta} + I_{\text{BS}} + \sigma_{\text{ul}}^2} \text{ and } \gamma_{\text{DL}}^{\text{FD}}(u,d) = \frac{P_{\text{BS}} h_d}{P_{\text{U}} g_{u,d} + I_d + \sigma_{\text{dl}}^2}, \tag{2}$$

where P_{U} and P_{BS} are the transmit powers of the users and the BS, respectively, I_{BS} and I_d are the CCI powers from the neighboring cells at the BS and at user d, σ_{ul}^2 and σ_{dl}^2 are the noise powers, and Δ is the SI suppression factor that leads to a residual SI power of P_{BS}/Δ.

2) HD Mode: Similarly, in the HD mode, the uplink SINR $\gamma_{\text{UL}}^{\text{HD}}(u)$ and downlink SINR $\gamma_{\text{DL}}^{\text{HD}}(d)$ are given by

$$\gamma_{\text{UL}}^{\text{HD}}(u) = \frac{P_{\text{U}} h_u}{I_{\text{BS}} + \sigma_{\text{ul}}^2} \text{ and } \gamma_{\text{DL}}^{\text{HD}}(d) = \frac{P_{\text{BS}} h_d}{I_d + \sigma_{\text{dl}}^2}. \tag{3}$$

CSI Model: We assume that the BS knows the channel gains $h_1, h_2, \ldots, h_N$. It can estimate them from the uplink pilots in a TDD system. A user can measure the interference from the other users in the cell from their transmissions. Note that the BS cannot measure this on its own. We also assume that a user i feeds back $(I_i + \sigma_{\text{dl}}^2)$ to the BS.

III. UPSMA and Reduced Feedback Scheme

We first present the feedback scheme and then UPSMA.

Let the BS select a user f, which can be an uplink or a downlink user. The criteria for selecting f are described below. User f feeds back a set $\mathcal{B}_f$ that contains at most L users and b-bit quantized versions of the interferences from them. Only users whose inter-user interference lie below a threshold γ_{th} are included in the set. Here, γ_{th} and L are system parameters. If there are more than L such users, then the L users with the lowest interference are fed back.

UPSMA: It schedules users in the following three steps.

1) Scheduling the First User f: From the CSI available to it, it can be seen from (2) that the BS can compute the uplink SINR. However, this is not the case for the downlink SINR because the inter-user interference depends on the uplink user that the BS is yet to choose. Even so, the BS can conservatively estimate $\gamma_{\text{DL}}^{\text{FD}}(u,d)$ for every user d. Since only inter-user interferences that are below γ_{th} are fed back, it follows that

$$\gamma_{\text{DL}}^{\text{FD}}(u,d) \geq \hat{\gamma}_{\text{DL}}^{\text{FD}}(d) \triangleq \frac{P_{\text{BS}} h_i}{\gamma_{\text{th}} + I_d + \sigma_{\text{dl}}^2}, \ \forall u \in \mathcal{B}_f. \tag{4}$$

We shall refer to $\hat{\gamma}_{\text{DL}}^{\text{FD}}(d)$ as the *conservative FD mode downlink SINR*.

Let $\gamma_{\text{UL}}^* = \max_{i \in \mathbb{N}} \left\{\gamma_{\text{UL}}^{\text{FD}}(i)\right\}$ and $\gamma_{\text{DL}}^* = \max_{i \in \mathbb{N}} \left\{\hat{\gamma}_{\text{DL}}^{\text{FD}}(i)\right\}$. If $\gamma_{\text{UL}}^* \geq \gamma_{\text{DL}}^*$, then the BS selects the uplink user $f = \text{argmax}_{i \in \mathbb{N}} \left\{\gamma_{\text{UL}}^{\text{FD}}(i)\right\}$. Else, it selects the downlink user $f = \text{argmax}_{i \in \mathbb{N}} \left\{\hat{\gamma}_{\text{DL}}^{\text{FD}}(i)\right\}$. The user f then feeds back the set $\mathcal{B}_f$ to the BS, as described above.

2) Scheduling the Second User s: If $\mathcal{B}_f = \emptyset$, the BS skips this step. Else, there are two possibilities:

a) If f is an Uplink User: The BS selects the user s for the downlink that maximizes the sum rate as follows:

$$s = \underset{i \in \mathcal{B}_f}{\operatorname{argmax}} \left\{ \log_2 \left(1 + \gamma_{\mathrm{UL}}^{\mathrm{FD}}(f)\right) + \log_2 \left(1 + \gamma_{\mathrm{DL}}^{\mathrm{FD}}(f,i)\right) \right\}. \tag{5}$$

At this stage the BS can compute a more precise value for the downlink SINR $\gamma_{\mathrm{DL}}^{\mathrm{FD}}(f,i)$ for $b \geq 1$ as follows. For $b = 1$, the two interference quantization levels in dB scale are γ_{th} and γ_{th}/Q, where Q is a system parameter. For $b = 2$, the quantization levels are γ_{th}, γ_{th}/Q, γ_{th}/Q^2, and γ_{th}/Q^3, and so on.[1] The BS uses the upper limit of the quantization interval in which inter-user interference lies instead of γ_{th} in (4). However, for *index-only feedback*, i.e., $b = 0$, only user indices are fed back. The BS continues to use the conservative value $\hat{\gamma}_{\mathrm{DL}}^{\mathrm{FD}}(i)$ of (4) as it has no additional information about the inter-user interference. For all b, this also ensures that the downlink rate chosen by the BS has zero error probability.

b) If f is a Downlink User: The BS schedules the uplink user s that maximizes the sum rate as follows:

$$s = \underset{i \in \mathcal{B}_f}{\operatorname{argmax}} \left\{ \log_2 \left(1 + \gamma_{\mathrm{UL}}^{\mathrm{FD}}(i)\right) + \log_2 \left(1 + \gamma_{\mathrm{DL}}^{\mathrm{FD}}(i,f)\right) \right\}. \tag{6}$$

3) Mode Selection: The BS selects the HD mode if $\mathcal{B}_f = \emptyset$ or the FD mode sum rate is less than the HD mode rate of the user f' with the highest uplink or downlink SINR:

$$f' = \underset{i \in \mathbb{N}}{\operatorname{argmax}} \left\{ \gamma_{\mathrm{DL}}^{\mathrm{HD}}(i), \gamma_{\mathrm{UL}}^{\mathrm{HD}}(i) \right\}. \tag{7}$$

Here, f' can be different from f because $\gamma_{\mathrm{DL}}^{\mathrm{HD}}(i)$ in (3) is different from $\hat{\gamma}_{\mathrm{DL}}^{\mathrm{FD}}(i)$ in (4).

Note that UPSMA maximizes sum rate. It is a relevant metric in a TDD FD system because of the simultaneous uplink and downlink transmissions that can occur.

IV. Rate Analysis of UPSMA

We now analyze uplink, downlink, and sum rates of UPSMA. To gain analytical insights, we focus on index-only feedback for the single-cell scenario ($I_{\mathrm{BS}} = I_d = 0$) with zero SI ($\Delta = \infty$). For this, the FD mode uplink and conservative downlink SINRs for user i in (2) and (4) simplify to

$$\gamma_{\mathrm{UL}}^{\mathrm{FD}}(i) = P_{\mathrm{U}} h_i / \sigma_{\mathrm{ul}}^2 \text{ and } \hat{\gamma}_{\mathrm{DL}}^{\mathrm{FD}}(i) = P_{\mathrm{BS}} h_i / \left(\gamma_{\mathrm{th}} + \sigma_{\mathrm{dl}}^2\right). \tag{8}$$

We first evaluate the probability $\mathbb{P}(\mathcal{B}_f)$ that the set $\mathcal{B}_f$ is fed back when user $f \in \mathbb{N}$ is scheduled first. The following two scenarios arise depending on $|\mathcal{B}_f|$:

i) When $|\mathcal{B}_f| < L$: In this case, $i \notin \mathcal{B}_f$ if and only if $P_{\mathrm{U}} g_{f,i} > \gamma_{\mathrm{th}}$. Hence,

$$\begin{aligned} \mathbb{P}(\mathcal{B}_f) &= \mathbb{P}\left(P_{\mathrm{U}} g_{f,i} \leq \gamma_{\mathrm{th}}, \forall i \in \mathcal{B}_f; P_{\mathrm{U}} g_{f,i} > \gamma_{\mathrm{th}}, \forall i \notin \mathcal{B}_f \cup \{f\}\right), \\ &= \left[\prod_{k \in \mathcal{B}_f} F_{g_{f,k}}\left(\frac{\gamma_{\mathrm{th}}}{P_{\mathrm{U}}}\right)\right] \left[\prod_{i \notin \mathcal{B}_f \cup \{f\}} \left(1 - F_{g_{f,i}}\left(\frac{\gamma_{\mathrm{th}}}{P_{\mathrm{U}}}\right)\right)\right]. \end{aligned} \tag{9}$$

ii) When $|\mathcal{B}_f| = L$: In this case, $\mathcal{B}_f$ contains the L users with the lowest interferences. Among them, let user k cause the largest interference. This means that: (i) $g_{f,i} \leq g_{f,k}, \forall i \in \mathcal{B}_f \setminus \{k\}$, (ii) $g_{f,k} \leq g_{f,i}$, $\forall i \notin \mathcal{B}_f \cup \{f\}$, and (iii) $P_{\mathrm{U}} g_{f,k} \leq \gamma_{\mathrm{th}}$. Considering all the possibilities for k, we get

$$\begin{aligned} \mathbb{P}(\mathcal{B}_f) = \sum_{k \in \mathcal{B}_f} \mathbb{P}\big(&g_{f,i} \leq g_{f,k}, \forall i \in \mathcal{B}_f \setminus \{k\}; \\ &g_{f,k} \leq g_{f,i}, \forall i \notin \mathcal{B}_f \cup \{f\}; P_{\mathrm{U}} g_{f,k} \leq \gamma_{\mathrm{th}}\big). \end{aligned} \tag{10}$$

Conditioning on $g_{f,k}$ and then averaging over it, we get

$$\begin{aligned} \mathbb{P}(\mathcal{B}_f) = \sum_{k \in \mathcal{B}_f} \int_0^{\frac{\gamma_{\mathrm{th}}}{P_{\mathrm{U}}}} f_{g_{f,k}}(x) \left[\prod_{i \in \mathcal{B}_f \setminus \{k\}} F_{g_{f,i}}(x)\right] \\ \times \left[\prod_{i \notin \mathcal{B}_f \cup \{f\}} \left(1 - F_{g_{f,i}}(x)\right)\right] dx. \end{aligned} \tag{11}$$

Rate Analysis: From (8), if $P_{\mathrm{U}}/\sigma_{\mathrm{ul}}^2 > P_{\mathrm{BS}}/\left(\gamma_{\mathrm{th}} + \sigma_{\mathrm{dl}}^2\right)$, then for any user, its FD mode uplink SINR is greater than its conservative FD mode downlink SINR. Therefore, f will be an uplink user. We shall refer to this as the *uplink first case*. Else, f is a downlink user, which we shall refer to as the *downlink first case*. The following result gives the uplink rate C_{UL} and downlink rate C_{DL} of UPSMA. For tractability, these are derived assuming that the third step in which the HD mode is selected occurs only if $\mathcal{B}_f = \emptyset$. In effect, these expressions provide lower bounds on the rates.

Result *1:* In the uplink first case,

$$C_{\mathrm{UL}} = \sum_{i \in \mathbb{N}} \int_0^{\infty} \log_2(1+\gamma) f_{\gamma_{\mathrm{UL}}^{\mathrm{FD}}(i)}(\gamma) \left[\prod_{j=1, j \neq i}^{N} F_{\gamma_{\mathrm{UL}}^{\mathrm{FD}}(j)}(\gamma)\right] d\gamma, \tag{12}$$

and

$$\begin{aligned} C_{\mathrm{DL}} &= \sum_{f \in \mathbb{N}} \sum_{\mathcal{B}_f \neq \emptyset} \mathbb{P}(\mathcal{B}_f) \\ &\times \sum_{s \in \mathcal{B}_f} \int_0^{\infty} \log_2(1+\gamma) f_{\hat{\gamma}_{\mathrm{DL}}^{\mathrm{FD}}(s)}(\gamma) \left[\prod_{i \in \mathcal{B}_f \setminus \{s\}} F_{\hat{\gamma}_{\mathrm{DL}}^{\mathrm{FD}}(i)}(\gamma)\right] \\ &\times \int_{\gamma}^{\infty} f_{\hat{\gamma}_{\mathrm{DL}}^{\mathrm{FD}}(f)}(x) \left[\prod_{i \notin \mathcal{B}_f \cup \{f\}} F_{\hat{\gamma}_{\mathrm{DL}}^{\mathrm{FD}}(i)}(x)\right] dx\, d\gamma. \end{aligned} \tag{13}$$

In the downlink first case, the expressions of C_{UL} and C_{DL} are similar to the above expressions for C_{DL} and C_{UL}, respectively. We skip them to conserve space.

Proof: The proof is given in Appendix A. ■

The sum rate is $C_{\mathrm{UL}} + C_{\mathrm{DL}}$. The above expressions are useful because they apply to the general scenario in which the users are at different distances from the BS and also from each other. However, they are involved and require the integrals to be computed numerically. We now use them to gain insights about how to choose γ_{th} and how the uplink and downlink rates scale with the number of users N. We study the asymptotic regime in which N is large, L is fixed, and $\mu_1 = \cdots = \mu_N = \mu'$, and $\mu_{m,n} = \mu''$, for $m \neq n$ [5].

[1] Optimizing the quantization levels, as done, for example, in [11], is beyond the scope of this paper.

A. Asymptotic Insights

The channel gains $h_1, h_2, \ldots, h_N$ and $g_{m,n}$, for $m, n \in \mathbb{N}$ and $m \neq n$, are Suzuki RVs. A Suzuki RV can be well approximated by a lognormal RV since the shadowing dominates the small-scale fading [12].[2] Let $\tilde{\mu}'$ be the dB-mean of the lognormal RV that approximates the channel gain between the BS and a user, and $\tilde{\mu}''$ be the dB-mean for an inter-user channel gain. And, let $\tilde{\sigma}_{\text{dB}}$ be their dB-standard deviation. Then, the scaling law for γ_{th} is as follows.

Lemma *1:* Let $L \leq \left((N-1)\left[1-Q\left(\zeta\sqrt{2\log(N)}\right)\right]\right)^{\kappa}$, for any $0 < \kappa < 1$ and an arbitrary constant $\zeta > 0$, and γ_{th} be set as a function of N as

$$\gamma_{\text{th}} = P_{\text{U}} \exp\left(\frac{1}{\xi}\left(\tilde{\mu}'' - \zeta\tilde{\sigma}_{\text{dB}}\sqrt{2\log(N)}\right)\right), \text{ for } \zeta > 0, \tag{14}$$

where $\xi = 10/\log(10)$. Then, as $N \to \infty$, $\mathbb{P}(|\mathcal{B}_f| < L) \to 0$ for $0 < \zeta < 1$, and $\mathbb{P}(|\mathcal{B}_f| = 0) \to 1$ for $\zeta > 1$.

Proof: The proof follows from (9) and is skipped. ■

We see that if γ_{th} decreases at a rate faster than $\exp\left(-\tilde{\sigma}_{\text{dB}}\sqrt{2\log(N)}/\xi\right)$, then $|\mathcal{B}_f| \to 0$. This is sub-optimal because then the HD mode will be chosen by the BS. Otherwise, $|\mathcal{B}_f| = L$ with probability 1, which enables the BS to exploit multi-user diversity.

Result *2:* Let γ_{th} and L satisfy the conditions of Lemma 1 with $0 < \zeta, \kappa < 1$. Then, in the uplink first case,

$$C_{\text{UL}} = \frac{\tilde{\sigma}_{\text{dB}}\log_2(e)}{\xi}\sqrt{2\log(N)} + \log_2\left(\frac{P_{\text{U}}}{\sigma_{\text{ul}}^2}e^{\frac{\tilde{\mu}'}{\xi}}\right) + O(\log(\log(N))), \tag{15}$$

and

$$\log_2\left(1+\frac{P_{\text{BS}}}{\sigma_{\text{dl}}^2}e^{\frac{\tilde{\mu}'}{\xi}}\right) \leq C_{\text{DL}} \leq L\log_2\left(1+\frac{P_{\text{BS}}}{\sigma_{\text{dl}}^2}e^{\left(\frac{\tilde{\mu}'}{\xi}+\frac{\tilde{\sigma}_{\text{dB}}^2}{2\xi^2}\right)}\right). \tag{16}$$

In the downlink first case, the scaling laws of C_{UL} and C_{DL} are similar to those in (15) and (16), respectively. Therefore, we skip them to conserve space.

Proof: The proof is given in Appendix B. ■

Notice that the $\Theta(\sqrt{\log(N)})$ scaling of C_{UL} in (15) is the same as that of an HD system. This can be proved using an analysis similar to ours. On the other hand, C_{DL} is a constant due to the feedback set size being capped to L.

V. Simulation Results and Benchmarking

We now present Monte Carlo simulation results for both single-cell and multi-cell scenarios that are averaged over 10^6 channel realizations. The simulation parameters are set as follows: cell radius $R = 100$ m, bandwidth is 1 MHz, $\eta = 3.7$, $K = -40$ dB, $d_0 = 1$ m, noise figure is 10 dB, noise power spectral density is -174 dBm/Hz, $\sigma_{\text{dB}} = 8$, and $Q = 2$ dB. In order to generate statistically non-identical channels, we place user i at a distance of iR/N from the BS and at an azimuth angle $2\pi i/N$. We set $P_{\text{U}} = P_{\text{BS}}$ such that the average cell-edge signal-to-noise ratio (SNR) is 0 dB.

We also benchmark with the following algorithms:

- *HD System:* In it, the BS either transmits to a user or receives from it, but does not do both simultaneously. The user with the highest SINR – be it on the uplink or the downlink – is scheduled.
- *Exhaustive Search:* In it, the uplink and downlink user pair with the largest sum rate is determined and its rate is compared with the HD system rate. The mode that achieves the highest sum rate is used.
- *AKA-1, 2, and 3:* In AKA-1, the users with the largest and second largest channel gains are scheduled in the uplink and downlink, respectively. AKA-2 and 3 are as described in Section I. To ensure a fair comparison, in AKA-2 and AKA-3, the BS knows only the L lowest interferences to the first scheduled user and the users that cause them. It knows these with infinite resolution.

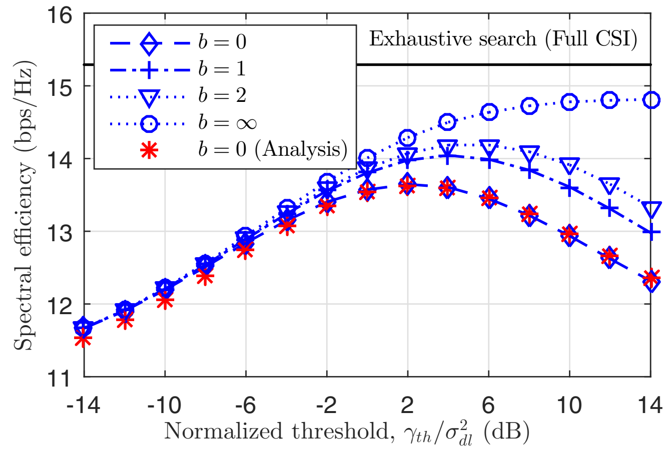


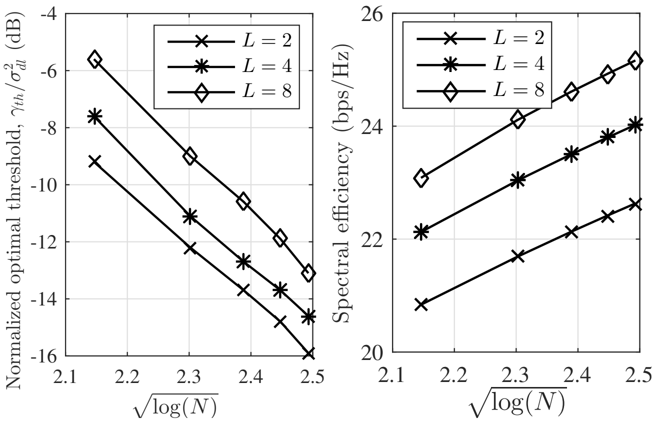
Fig. 2: Single-cell scenario: Sum rate as a function of the normalized threshold $\gamma_{\text{th}}/\sigma_{\text{dl}}^2$ ($N = 8$, $L = 4$, and $\Delta = \infty$).

A. Single-Cell Scenario

Fig. 2 plots the sum rate of UPSMA as a function of the normalized threshold $\gamma_{\text{th}}/\sigma_{\text{dl}}^2$. When $P_{\text{BS}} = P_{\text{U}}$, it follows from (8) that the uplink first case occurs. For index-only feedback ($b = 0$), $b = 1$, and $b = 2$, the sum rate increases as γ_{th} increases, reaches a peak, and then decreases. This is because when γ_{th} is small, the feedback set $\mathcal{B}_f$ is often not filled as there are few users with inter-user interference below γ_{th}. As a result, the BS has less choice in scheduling the second user. On the other hand, when γ_{th} is large, $|\mathcal{B}_f|$ is L with high probability, but the inter-user interference becomes significant and reduces the downlink SINR. For $b = \infty$, γ_{th} affects only the feedback set size. Hence, the sum rate saturates. At the optimal γ_{th}, the sum rate is 10.9%, 8.2%, and 7.2% of that of exhaustive search for $b = 0$, 1, and 2. Also, the analytical curve is close to the simulation curve, which verifies Result 1.

Fig. 3 studies the asymptotic scaling behavior of UPSMA. Fig. 3a plots the optimal $\gamma_{\text{th}}/\sigma_{\text{dl}}^2$, which is found numerically,

[2]We use the moment generation function matching method [12] with parameters $s_1 = 0.35$ and $s_2 = 0.55$ for this approximation.



(a) Threshold γ_{th} (b) Sum rate

Fig. 3: Single-cell scenario: Asymptotic scaling of sum rate and threshold γ_{th} as a function of N ($\mu' = -94$ dB, $\mu'' = -105.1$ dB, and $\Delta = \infty$).

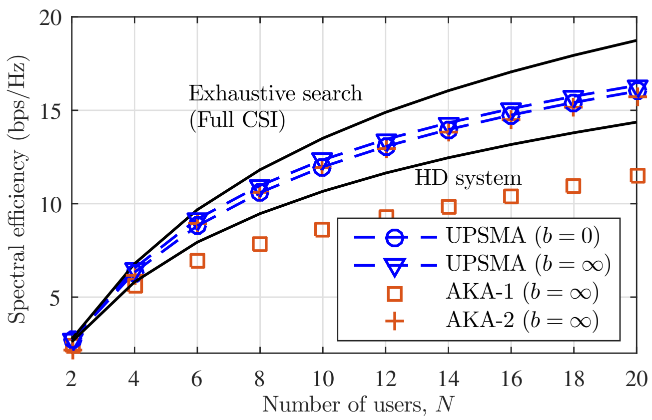


Fig. 4: Multi-cell scenario: Benchmarking of sum rate as a function of N ($L = 4$ and $\Delta = 110$ dB).

as a function of $\sqrt{\log(N)}$. The optimal γ_{th} increases as L increases. This is to increase the odds that there are sufficiently many users whose inter-user interferences are below γ_{th}. Fig. 3b plots the sum rate as a function of $\sqrt{\log(N)}$ for different L. This is done using the optimal γ_{th} from Fig. 3a. The sum rate increases as L increases because more CSI is fed back. The near-linear behavior of the curves in the figure verifies the scaling laws derived in Lemma 1 and Result 2.

B. Multi-cell Scenario and Performance Benchmarking

In the multi-cell scenario, 19 cells are laid out in a two-tier hexagonal cellular layout. Now, CCI occurs at the BS and the users. We also model imperfect SI cancellation ($\Delta < \infty$). As a result, the uplink and downlink first cases can both occur for any value of P_{U} and P_{BS}.

Fig. 4 plots the sum rate as a function of N. UPSMA outperforms the HD system, AKA-1, and AKA-2 even though AKA-2 has CSI with infinite resolution. This is because AKA-2 always chooses an uplink user first and it does not account for the inter-cell interference while selecting the downlink user. AKA-1 performs the worst because it ignores inter-user interference while selecting the users. The sum rate of AKA-3 is similar to AKA-2, and is not shown. As before, the sum rate of UPSMA increases as b and N increase. It is within 15% of exhaustive search.

VI. Conclusions

We proposed a user scheduling and mode selection algorithm and a feedback scheme that fed back at most L inter-user interferences that were below the threshold γ_{th}. This enabled the BS to lower bound the downlink SINR even when it was yet to determine the uplink user. The asymptotic analysis revealed how to choose the optimal threshold and the scaling of the uplink and downlink rates. This highlighted the following tradeoff. A larger γ_{th} increased the number of users in the feedback set, but also led to a larger interference. A smaller γ_{th} increased the downlink SINR, but it reduced the number of users that the scheduler could choose from. Simulations showed that UPSMA achieved a sum rate close to that of exhaustive search and other algorithms that assumed the availability of more CSI at the BS.

Appendix

A. Brief Proof of Result 1

i) C_{UL}: Let γ_{UL} represent SINR of the uplink. In the uplink first case, the first scheduled user f has the largest FD mode uplink SINR among all the N users. Hence, the CDF $F_{\gamma_{\text{UL}}}(\gamma)$ of γ_{UL} is given by

$$F_{\gamma_{\text{UL}}}(\gamma) = \mathbb{P}\left(\max_{i\in\mathbb{N}} \left\{\gamma_{\text{UL}}^{\text{FD}}(i)\right\} \leq \gamma\right) = \prod_{i\in\mathbb{N}} F_{\gamma_{\text{UL}}^{\text{FD}}(i)}(\gamma). \tag{17}$$

Differentiating $F_{\gamma_{\text{UL}}}(\gamma)$ with respect to γ and substituting it in the formula $C_{\text{UL}} = \int_0^\infty \log_2(1+\gamma) f_{\gamma_{\text{UL}}}(\gamma)\, d\gamma$ yields (12).

ii) C_{DL}: Let γ_{DL} denote SINR of the downlink, $\mathbb{P}(\gamma_{\text{DL}} \leq \gamma, f, \mathcal{B}_f, s)$ denote the probability of the event $\gamma_{\text{DL}} \leq \gamma$, first scheduled user is f, second scheduled user is s, and feedback set is $\mathcal{B}_f$, and $\mathbb{P}(f, \mathcal{B}_f = \emptyset)$ denote the probability that the first scheduled user is f and $\mathcal{B}_f = \emptyset$. If $\mathcal{B}_f = \emptyset$, then no user is scheduled on the downlink, which is equivalent to $\gamma_{\text{DL}} = 0$. Then, from the law of total probability, $F_{\gamma_{\text{DL}}}(\gamma)$ is given by

$$F_{\gamma_{\text{DL}}}(\gamma) = \sum_{f\in\mathbb{N}} \sum_{\mathcal{B}_f\neq\emptyset} \sum_{s\in\mathcal{B}_f} \mathbb{P}(\gamma_{\text{DL}} \leq \gamma, f, \mathcal{B}_f, s) + \sum_{f\in\mathbb{N}} \mathbb{P}(f, \mathcal{B}_f = \emptyset). \tag{18}$$

From Bayes' rule, we have

$$\mathbb{P}(\gamma_{\text{DL}} \leq \gamma, f, \mathcal{B}_f, s) = \mathbb{P}(\gamma_{\text{DL}} \leq \gamma, f, s | \mathcal{B}_f)\, \mathbb{P}(\mathcal{B}_f). \tag{19}$$

The downlink user s, which is scheduled second, has the largest downlink SINR among the users in $\mathcal{B}_f$. Therefore,

$$\mathbb{P}(\gamma_{\text{DL}} \leq \gamma, f, s|\mathcal{B}_f) = \mathbb{P}\left(\hat{\gamma}_{\text{DL}}^{\text{FD}}(s) < \gamma, \hat{\gamma}_{\text{DL}}^{\text{FD}}(s) \geq \hat{\gamma}_{\text{DL}}^{\text{FD}}(i), \forall i \in \mathcal{B}_f \setminus \{s\}, \gamma_{\text{UL}}^{\text{FD}}(f) \geq \gamma_{\text{UL}}^{\text{FD}}(i), \forall i \in \mathbb{N} \setminus \{f\}\right). \tag{20}$$

From (8), we see that $\gamma_{\text{UL}}^{\text{FD}}(f) \geq \gamma_{\text{UL}}^{\text{FD}}(i)$ is equivalent to $\hat{\gamma}_{\text{DL}}^{\text{FD}}(f) \geq \hat{\gamma}_{\text{DL}}^{\text{FD}}(i)$. Setting $i = s$, we get $\hat{\gamma}_{\text{DL}}^{\text{FD}}(f) \geq \hat{\gamma}_{\text{DL}}^{\text{FD}}(s)$.

For $i \in \mathcal{B}_f$, the condition $\hat{\gamma}_{\text{DL}}^{\text{FD}}(i) \leq \hat{\gamma}_{\text{DL}}^{\text{FD}}(s)$ implies that $\hat{\gamma}_{\text{DL}}^{\text{FD}}(i) \leq \hat{\gamma}_{\text{DL}}^{\text{FD}}(f)$. Conditioning on $\hat{\gamma}_{\text{DL}}^{\text{FD}}(f) = x$ and $\hat{\gamma}_{\text{DL}}^{\text{FD}}(s) = y$, we get

$$\begin{aligned}\mathbb{P}(\gamma_{\text{DL}} \leq \gamma, f, s|\mathcal{B}_f) = \mathbb{E}\big[&\mathbb{P}\big(\hat{\gamma}_{\text{DL}}^{\text{FD}}(i) \leq y, \forall i \in \mathcal{B}_f \setminus \{s\};\\ &\hat{\gamma}_{\text{DL}}^{\text{FD}}(i) \leq x, \forall i \in \mathbb{N} \setminus (\mathcal{B}_f \cup \{f\}) \,|\hat{\gamma}_{\text{DL}}^{\text{FD}}(f) = x, \hat{\gamma}_{\text{DL}}^{\text{FD}}(s) = y\big)\\ &\times 1_{\{y<\gamma\}} 1_{\{x>y\}}\big]. \end{aligned} \quad (21)$$

Since $\hat{\gamma}_{\text{DL}}^{\text{FD}}(1), \hat{\gamma}_{\text{DL}}^{\text{FD}}(2), \ldots, \hat{\gamma}_{\text{DL}}^{\text{FD}}(N)$ are mutually independent, the above expression can be shown to simplify to

$$\begin{aligned}\mathbb{P}(\gamma_{\text{DL}} \leq \gamma, f, s|\mathcal{B}_f) = &\int_0^{\gamma} f_{\hat{\gamma}_{\text{DL}}^{\text{FD}}(s)}(y) \left[\prod_{i \in \mathcal{B}_f \setminus \{s\}} F_{\hat{\gamma}_{\text{DL}}^{\text{FD}}(i)}(y)\right]\\ &\times \int_y^{\infty} f_{\hat{\gamma}_{\text{DL}}^{\text{FD}}(f)}(x) \left[\prod_{i \in \mathbb{N} \setminus (\mathcal{B}_f \cup \{f\})} F_{\hat{\gamma}_{\text{DL}}^{\text{FD}}(i)}(x)\right] dx\, dy. \end{aligned} \quad (22)$$

Combining (18), (19), and (22), we get $F_{\gamma_{\text{DL}}}(\gamma)$. Differentiating it with respect to γ to get $f_{\gamma_{\text{DL}}}(\gamma)$ and substituting it in the expression $C_{\text{DL}} = \int_0^{\infty} \log_2(1+\gamma) f_{\gamma_{\text{DL}}}(\gamma)\, d\gamma$ yields (13).

B. Brief Proof of Result 2

i) Scaling of C_{UL}: Let $a(N) = \log(\log(N))$ and

$$\Gamma_L \triangleq \frac{P_{\text{U}}}{\sigma_{\text{ul}}^2} \exp\left(\frac{\tilde{\mu}'}{\xi} + \frac{\tilde{\sigma}_{\text{dB}}}{\xi}\sqrt{2[\log(N) - a(N)]}\right), \quad (23)$$

$$\Gamma_U \triangleq \frac{P_{\text{U}}}{\sigma_{\text{ul}}^2} \exp\left(\frac{\tilde{\mu}'}{\xi} + \frac{\tilde{\sigma}_{\text{dB}}}{\xi}\sqrt{2[\log(N) + a(N)]}\right). \quad (24)$$

We can show that $\mathbb{P}(\Gamma_L < \gamma_{\text{UL}} \leq \Gamma_U) \to 1$. Therefore, it follows from (23) and (24) that

$$C_{\text{UL}} = \frac{\tilde{\sigma}_{\text{dB}} \log_2(e)}{\xi}\sqrt{2\log(N)} + \log_2\left(\frac{P_{\text{U}}}{\sigma_{\text{ul}}^2} e^{\frac{\tilde{\mu}'}{\xi}}\right) + O(\log(\log(N))). \quad (25)$$

ii) Scaling of C_{DL}: From the law of total probability,

$$C_{\text{DL}} = \sum_{f \in \mathbb{N}} \sum_{\mathcal{B}_f} \mathbb{P}(f, \mathcal{B}_f)\, T_{\text{DL}}(f, \mathcal{B}_f), \quad (26)$$

where $T_{\text{DL}}(f, \mathcal{B}_f)$ is the downlink rate conditioned on the first scheduled user f and the feedback set $\mathcal{B}_f$. It equals

$$\begin{aligned}T_{\text{DL}}(f, \mathcal{B}_f) = \mathbb{E}\Big[&\max_{i \in \mathcal{B}_f}\left\{\log_2\left(1 + \hat{\gamma}_{\text{DL}}^{\text{FD}}(i)\right)\right\}\\ &|\gamma_{\text{UL}}^{\text{FD}}(j) \leq \gamma_{\text{UL}}^{\text{FD}}(f), \forall j \in \mathbb{N} \setminus \{f\}, \mathcal{B}_f\Big]. \end{aligned} \quad (27)$$

Since the downlink SINRs are independent, this simplifies to

$$\begin{aligned}T_{\text{DL}}(f, \mathcal{B}_f) = \mathbb{E}\Big[&\max_{i \in \mathcal{B}_f}\left\{\log_2\left(1 + \hat{\gamma}_{\text{DL}}^{\text{FD}}(i)\right)\right\}\\ &|\gamma_{\text{UL}}^{\text{FD}}(j) \leq \gamma_{\text{UL}}^{\text{FD}}(f), \forall j \in \mathcal{B}_f\Big]. \end{aligned} \quad (28)$$

From above, $\gamma_{\text{UL}}^{\text{FD}}(f) = \Theta\left(\exp\left(\sqrt{\log(N)}\right)\right) \to \infty$. Since $|\mathcal{B}_f| \leq L$, it follows that $\mathbb{P}\left(\gamma_{\text{UL}}^{\text{FD}}(j) \leq \gamma_{\text{UL}}^{\text{FD}}(f), \forall j \in \mathcal{B}_f\right) \to 1$. Hence, the conditioning in (28) can be dropped. This yields

$$T_{\text{DL}}(f, \mathcal{B}_f) = \mathbb{E}\left[\max_{i \in \mathcal{B}_f}\left\{\log_2\left(1 + \hat{\gamma}_{\text{DL}}^{\text{FD}}(i)\right)\right\}\right]. \quad (29)$$

Lower Bound: Since $\log_2(1 + e^x)$ is convex in x, we can show using (8) that

$$T_{\text{DL}}(f, \mathcal{B}_f) \geq \max_{i \in \mathcal{B}_f}\left\{\log_2\left(1 + \frac{P_{\text{BS}}}{\gamma_{\text{th}} + \sigma_{\text{dl}}^2} \exp\left(\mathbb{E}[\log(h_i)]\right)\right)\right\}.$$

Since $h_1, h_2, \ldots, h_N$ are independent and identically distributed (i.i.d.), $\gamma_{\text{th}} \to 0$ (from Lemma 1), and $\mathbb{E}[\log(h_1)] = \tilde{\mu}'/\xi$, we get

$$T_{\text{DL}}(f, \mathcal{B}_f) \geq \log_2\left(1 + \frac{P_{\text{BS}}}{\sigma_{\text{dl}}^2} \exp\left(\frac{\tilde{\mu}'}{\xi}\right)\right). \quad (30)$$

Substituting this in (26) yields the lower bound in (16).

Upper Bound: Since $\max_{i \in \mathcal{B}_f}\left\{\log_2\left(1 + \hat{\gamma}_{\text{DL}}^{\text{FD}}(i)\right)\right\} \leq \sum_{i \in \mathcal{B}_f} \log_2\left(1 + \hat{\gamma}_{\text{DL}}^{\text{FD}}(i)\right)$, we get

$$T_{\text{DL}}(f, \mathcal{B}_f) \leq \mathbb{E}\left[\sum_{i \in \mathcal{B}_f} \log_2\left(1 + \hat{\gamma}_{\text{DL}}^{\text{FD}}(i)\right)\right]. \quad (31)$$

Since $\hat{\gamma}_{\text{DL}}^{\text{FD}}(1), \hat{\gamma}_{\text{DL}}^{\text{FD}}(2), \ldots, \hat{\gamma}_{\text{DL}}^{\text{FD}}(N)$ are i.i.d., $\mathbb{P}(|\mathcal{B}_f| = L) \to 1$, and $\gamma_{\text{th}} \to 0$, we can show that

$$T_{\text{DL}}(f, \mathcal{B}_f) \leq L\mathbb{E}\left[\log_2\left(1 + \frac{P_{\text{BS}}}{\sigma_{\text{dl}}^2} h_1\right)\right], \quad (32)$$

$$\leq L \log_2\left(1 + \frac{P_{\text{BS}}}{\sigma_{\text{dl}}^2} \mathbb{E}[h_1]\right). \quad (33)$$

Substituting $\mathbb{E}[h_1] = \exp\left((\tilde{\mu}'/\xi) + (\tilde{\sigma}_{\text{dB}}^2/2\xi^2)\right)$, we get

$$T_{\text{DL}}(f, \mathcal{B}_f) \leq L \log_2\left(1 + \frac{P_{\text{BS}}}{\sigma_{\text{dl}}^2} \exp\left(\frac{\tilde{\mu}'}{\xi} + \frac{\tilde{\sigma}_{\text{dB}}^2}{2\xi^2}\right)\right). \quad (34)$$

Substituting (34) in (26) yields the upper bound in (16).

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