A Novel Interference-Aware, Optimal Gain Adaptation Policy For a Non-regenerative, Underlay Cognitive Radio Relay

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***Abstract*—A relay that operates in an underlay cognitive radio (CR) system is subject to tight constraints on the interference its transmissions cause to the primary receiver (P_{Rx}). While several appealing non-regenerative amplify-and-forward (AF) relaying schemes have been proposed for underlay CR, they are simple, ad hoc adaptations of conventional AF relaying schemes. We present a novel and optimal relay gain adaptation policy (ORGAP) in which the interference-aware relay optimally adapts its gain as a function of its local channel gains. It minimizes the symbol error probability (SEP) at the secondary receiver subject to a constraint on the average interference caused to the P_{Rx}. We also analyze the SEP of MPSK of ORGAP, which serves as a fundamental theoretical benchmark for the gains achievable by AF relaying in CR systems. Extensive numerical results quantify the extent to which ORGAP outperforms several known conventional AF relaying schemes. We also present a near-optimal, simpler relay gain adaptation policy (SRGAP) that is easier to implement.**

I. Introduction

Cognitive radio (CR) is a promising technology to improve the utilization of scarce radio spectrum and addresses the urgent need for more bandwidth and higher data rates [1], [2]. In one common paradigm of CR called underlay CR, secondary users (SUs) can transmit even when the primary user (PU) is transmitting. However, the transmissions by the SUs are subject to strict constraints on the interference they cause to the primary receiver (P_{Rx}) [1]. These constraints can severely limit the data rate and coverage achievable by an SU.

The use of cooperative relaying in CR, in which a secondary relay node helps forward data from a secondary transmitter (S_{Tx}) to a secondary receiver (S_{Rx}), is a promising approach that exploits spatial diversity to mitigate this problem. Several conventional cooperative protocols have also been investigated for underlay CR. These include decode-and-forward (DF) relaying [2], [3], in which the relay (R) decodes the information transmitted by the S_{Tx} and then regenerates it for transmission to the S_{Rx}, and non-regenerative amplify-and-forward (AF) relaying [4]–[6], in which the relay simply amplifies the signal it receives from the S_{Tx} and forwards it to the S_{Rx}. In this paper, we focus on AF relaying given its simplicity and effectiveness.

We summarize below the literature on AF relaying for underlay CR. This leads to some key observations about the suitability of the approaches that have been pursued thus far.

A. Literature Survey and Observations

In [7], [8], the outage probability of an underlay AF relay network, in which the relay transmit power is a function of the R-P_{Rx} link channel gain is investigated. The relay is subject to peak interference and peak transmit power constraints. In [6], the outage and bit error probabilities for the fixed-power relaying model is analyzed, assuming each relay knows the channel gain of the link between itself and the P_{Rx}. The S_{Tx} and relay are subject to a peak interference constraint. On the other hand, fixed-gain relaying for underlay CR, in which a relay's gain is a constant and is, thus, not dependent on the relay's local channel gains, is investigated in [5]. The average bit error rate, outage probability, and average channel capacity of a CR system with multiple fixed-gain AF relays are evaluated. In [9], a different AF protocol in which the S_{Tx} transmits data to the relay and S_{Rx} over orthogonal channels is considered. The S_{Tx} and R are each subject to peak interference constraints.

A key point to note is that the above papers employ simple modifications of the classical fixed-gain relaying or fixed-power relaying models. Specifically, in [7], [8], the relay amplifies the signal it receives by a factor $\sqrt{\beta}$, where

$$\beta = \frac{I_{\text{av}}}{\gamma_{rp}\left(P_s\gamma_{sr}+1\right)}, \tag{1}$$

γ_{rp} is the R-P_{Rx} channel power gain, γ_{sr} is the S_{Tx}-R channel power gain, and P_s is the S_{Tx} power. This ensures that an interference of I_{av} is caused to the P_{Rx} at any time, and is a simple modification of the classical fixed-power policy [10]. We shall refer to this as interference-power relaying. In [6], β is set as

$$\beta = \frac{Q}{P_s\gamma_{sr}+1}, \tag{2}$$

where Q is a constant. Thus, the relay gain only depends on γ_{sr}. This is similar to classical fixed-power AF relaying. In [4], [5], β is fixed, which is similar to the classical fixed-gain policy [11]. Thus, the relay gain is not a function of γ_{sr}, γ_{rd}, and γ_{rp}.[1]

The interference constraint imposed by underlay CR fundamentally alters the way a relay should amplify its received signal. How to optimally adapt the relay gain for underlay

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[1] We note, however, that in the relay selection rules considered in [4]–[6], whether the relay is considered for selection or not does depend on γ_{rp}.

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CR as a function of the local channel state information (CSI) available at the relay has been an open problem, and one that we solve in this paper. We also show that the above ad hoc modifications can be much improved upon.

B. Contributions

We make the following three contributions in this paper.

First, we present a novel non-regenerative AF relaying model for underlay CR in which the interference-aware underlay relay optimally adjusts both its gain and transmit power as a function of the channel power gains of its local $\mathrm{S_{Tx}}$-R and R-$\mathrm{S_{Rx}}$ links, over which the signal is transmitted, and its local R-$\mathrm{P_{Rx}}$ link, through which it causes interference to the $\mathrm{P_{Rx}}$. We derive the optimal relay gain adaptation policy (ORGAP) that minimizes the symbol error probability (SEP) at the $\mathrm{S_{Rx}}$ while ensuring that the average interference caused to the $\mathrm{P_{Rx}}$ is constrained to lie below I_{av}. While we focus on MPSK transmissions in this paper, our approach generalizes to other constellations such as MQAM.

ORGAP, given its optimality, serves as a new and fundamental theoretical benchmark for non-regenerative relaying in CR systems. It turns out to be functionally quite different from the aforementioned modifications of classical AF relaying models. For example, in an insightful asymptotic regime, we show that the optimal gain β is proportional to ($\propto$) $\frac{1}{\gamma_{sr}\sqrt{\gamma_{rd}\gamma_{rp}}}$. This behavior is different from fixed-power relaying in which $\beta \propto \frac{1}{\gamma_{sr}}$, fixed-gain relaying in which $\beta \propto 1$, and interference-power relaying in which $\beta \propto \frac{1}{\gamma_{rp}\gamma_{sr}}$.

The second contribution is an SEP analysis of ORGAP, which is quite involved and different from that for the above ad hoc models. We also present numerical results to benchmark ORGAP with the classical models pursued in the literature; these bring out the significant gains that ORGAP can deliver.

The third contribution is the development of a novel, near-optimal, and practically amenable simpler relay gain adaptation policy (SRGAP), which specifies the relay gain and power in terms of elementary functions of γ_{sr}, γ_{rd}, and γ_{rp}.

Comments: While the AF relay gain and power were also adapted in [12], [13], there are several important differences. In both [12], [13], the underlay CR interference constraint is not considered. Therefore, the relaying models considered in them need not even be feasible in our model. In [13], the optimal relay gain that maximizes the average SNR at the $\mathrm{S_{Rx}}$ is determined numerically using techniques that exploit quasi-concavity. Instead, in our paper, the optimal relay gain and power for ORGAP is shown to be the unique solution of a transcendental equation. This provides valuable analytical insights about its behavior, such as an explicit characterization of when the relay should shut off as a function of the local CSI. It also facilitates its SEP analysis. It also enables us to propose SRGAP, which is a simpler, near-optimal policy that is easier to implement. Note also that given the non-linear relationship between the SNR and the SEP, an optimal solution that maximizes the average SNR is different from one that minimizes the average SEP.

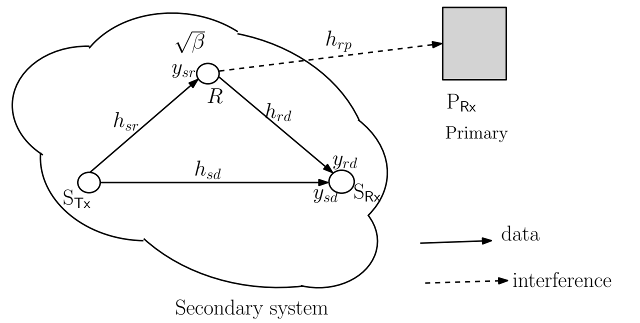

Fig. 1. Underlay CR system that consists of a secondary transmitter ($\mathrm{S_{Tx}}$) that communicates with a secondary receiver ($\mathrm{S_{Rx}}$) with the help of an AF relay in the presence of a primary receiver ($\mathrm{P_{Rx}}$).

The paper is organized as follows. Section II describes the system model. Section III derives the ORGAP policy and its SEP. Our results and conclusions follow in Sections IV and V, respectively.

II. System Model

Figure 1 shows an underlay CR system that consists of a source $\mathrm{S_{Tx}}$ that sends data to a destination $\mathrm{S_{Rx}}$ with the help of a half-duplex AF relay R, which interferes with a primary receiver $\mathrm{P_{Rx}}$. Each node is equipped with a single transmit or receive antenna. The $\mathrm{S_{Tx}}$-$\mathrm{S_{Rx}}$, R-$\mathrm{P_{Rx}}$, R-$\mathrm{S_{Rx}}$, and $\mathrm{S_{Tx}}$-$\mathrm{S_{Rx}}$ channels undergo frequency-flat Rayleigh fading, and are mutually independent. However, they need not be statistically identical. All transmissions occur over the same bandwidth. The relay is assumed to know the channel power gains of its local links $\mathrm{S_{Tx}}$-R, R-$\mathrm{S_{Rx}}$, and R-$\mathrm{P_{Rx}}$, as has also been assumed in [13], [14]. It can estimate these by exploiting reciprocity and overhearing the transmissions from the $\mathrm{S_{Tx}}$, $\mathrm{S_{Rx}}$, and $\mathrm{P_{Rx}}$. Note that it need not know the phases of the complex baseband channel gains of any of these links. Thus, simple receive energy based estimation techniques can be employed by the relay. The relay requires no information about the state of the direct $\mathrm{S_{Tx}}$-$\mathrm{S_{Rx}}$ link.

We use the following notation henceforth. The complex conjugate of z is denoted by z^* and the probability of an event B is denoted by $\Pr(B)$. For a random variable (RV) Y, its probability density function (PDF), expectation, and variance are denoted by $p_Y(y)$, $\mathbf{E}[Y]$, and $\mathbf{var}[Y]$, respectively. $X \sim \mathcal{CN}(\sigma^2)$ means that X is a zero-mean circular symmetric complex Gaussian RV with variance σ^2.

A. Non-regenerative Relaying Protocol

The protocol takes two time slots, as is also the case for fixed-gain and fixed-power relaying. The difference lies in how the relay gain and power are set, as we specify below.

$\mathrm{S_{Tx}}$ Transmission: In the first slot, $\mathrm{S_{Tx}}$ broadcasts a data symbol α that is drawn with equal probability from the MPSK constellation. The complex baseband received signals y_{sd} and y_{sr} at the $\mathrm{S_{Rx}}$ and relay, respectively, are given by

$$y_{sd} = \sqrt{P_s} h_{sd}\alpha + n_{sd}, \tag{3}$$

$$y_{sr} = \sqrt{P_s} h_{sr}\alpha + n_{sr}, \tag{4}$$

where P_s is the $\mathrm{S_{Tx}}$ transmit power, $|\alpha|^2 = 1$, $h_{sd} \sim \mathcal{CN}(\sigma_{sd}^2)$ is the $\mathrm{S_{Tx}}$-$\mathrm{S_{Rx}}$ baseband channel gain, and $h_{sr} \sim \mathcal{CN}(\sigma_{sr}^2)$ is the $\mathrm{S_{Tx}}$-R channel gain. Further, without loss of generality (w.l.o.g.), the additive noise terms n_{sr} and n_{sd}, which also include the interference from primary transmissions at the relay and $\mathrm{S_{Rx}}$, respectively, are modeled as $\mathcal{CN}(1)$ RVs, and are independent of each other and the channel gains. This corresponds to a worst case model for the interference that is often used in the underlay CR literature [15].

Relay Transmission: In the second slot, the relay amplifies the signal it receives, y_{sr}, by a factor $\sqrt{\beta}$. Therefore, the $\mathrm{S_{Rx}}$ receives the signal y_{rd}, which is given by

$$y_{rd} = \sqrt{\beta P_s} h_{sr} h_{rd} \alpha + \sqrt{\beta} h_{rd} n_{sr} + n_{rd}. \tag{5}$$

The relay transmit power is equal to $\beta \left(P_s \left| h_{sr} \right|^2 + 1 \right)$.

Reception: The $\mathrm{S_{Rx}}$ employs coherent detection to determine the transmitted symbol α using its two observables y_{sd} and y_{rd} [6], [9], [10]. The output SNR γ_E at the $\mathrm{S_{Rx}}$ when it employs maximal ratio combining is given by [5], [10]

$$\gamma_E = P_s \gamma_{sd} + \frac{P_s \gamma_{sr} \gamma_{rd}}{\gamma_{rd} + \beta^{-1}}. \tag{6}$$

B. Interference at P_{Rx}

The interference signal i_p seen by $\mathrm{P_{Rx}}$ from the relay is

$$i_p = \sqrt{\beta} y_{sr} h_{rp}, \tag{7}$$

where $h_{rp} \sim \mathcal{CN}(\sigma_{rp}^2)$ is the RP channel gain. Let $\gamma_{sd} \triangleq |h_{sd}|^2$, $\gamma_{sr} \triangleq |h_{sr}|^2$, $\gamma_{rd} \triangleq |h_{rd}|^2$, and $\gamma_{rp} = |h_{rp}|^2$.

In our model, the relay adjusts β as a function of its local $\mathrm{S_{Tx}}$-R, R-$\mathrm{P_{Rx}}$, and R-$\mathrm{S_{Rx}}$ channel power gains subject to the average interference power constraint, which requires that $\mathbf{E}\left[\beta \gamma_{rp} \left(P_s \gamma_{sr} + 1 \right)\right] = I_{\mathrm{av}}$.

Comments and Extensions: In order to focus on the impact of the average interference constraint on AF relaying, we do not impose an average transmit power constraint on the relay, as has also been done for underlay relays in [3], [16] and for underlay transmitters in [17], [18]. Since the $\mathrm{S_{Tx}}$ and the relay are spatially separated, transmit in orthogonal time slots, and have different local CSI, jointly adapting the $\mathrm{S_{Tx}}$ transmit power and relay powers is beyond the scope of this paper.

III. Interference-Aware Gain Adaptation

We now derive the optimum relay gain β as a function of γ_{sr}, γ_{rd}, and γ_{rp}. To avoid clutter, the dependence of β on these channel power gains is not explicitly shown.

The fading-averaged SEP for MPSK at the $\mathrm{S_{Rx}}$ is given by [19, (8.23)]

$$\mathrm{SEP} = \frac{1}{\pi} \int_0^{\left(\frac{M-1}{M}\right)\pi} \mathbf{E}\left[\exp\left(-\frac{m}{\sin^2\theta} \gamma_E \right) \right] d\theta, \tag{8}$$

where $m = \sin^2\left(\frac{\pi}{M}\right)$. Averaging over γ_{sd}, which is an exponential RV that is independent of γ_{sr}, γ_{rp}, γ_{rd}, and β, we get

$$\mathrm{SEP} = \frac{1}{\pi} \int_0^{\left(\frac{M-1}{M}\right)\pi} \frac{\mathbf{E}\left[\exp\left(-\frac{P_s \gamma_{sr} \gamma_{rd}}{\gamma_{rd} + \beta^{-1}} \frac{m}{\sin^2\theta} \right) \right]}{1 + P_s \overline{\gamma}_{sd} \frac{m}{\sin^2\theta}} d\theta, \tag{9}$$

$$\leq \mathrm{SEP}_0 \mathbf{E}\left[\exp\left(-\frac{m P_s \gamma_{sr} \gamma_{rd}}{\gamma_{rd} + \beta^{-1}} \right) \right]. \tag{10}$$

Equation (10) is obtained by replacing $\sin^2\theta$ in the term inside the expectation by its upper bound of unity, and then solving the resulting integral. Here, $\mathrm{SEP}_0 = \frac{M-1}{M} - \frac{1}{\sqrt{1 + \frac{m^{-1}}{P_s \overline{\gamma}_{sd}}}} \left(\frac{1}{2} + \frac{1}{\pi} \arctan \sqrt{\frac{1-m}{m + \frac{1}{P_s \overline{\gamma}_{sd}}}} \right)$ captures the contribution of the direct SD link.

Minimizing the exact SEP expression in (9) is intractable given that it is in the form of an integral. We, therefore, minimize the above Chernoff upper bound on the SEP in (10) given its integral-free form. This approach is valuable given the significant analytical insights it provides, and has been used in other wireless problems [20], [21].

A. Optimization Problem and Solution

Since SEP_0 does not depend on the $\mathrm{S_{Tx}}$-R, R-$\mathrm{S_{Rx}}$, and R-$\mathrm{P_{Rx}}$ channel power gains, the minimization problem can be formally stated as:

$$\min_{\beta} \quad \mathbf{E}\left[\exp\left(-\frac{m P_s \gamma_{sr} \gamma_{rd}}{\gamma_{rd} + \beta^{-1}} \right) \right] \tag{11}$$

$$\text{s.t.} \quad \mathbf{E}\left[\beta \gamma_{rp} \left(P_s \gamma_{sr} + 1 \right) \right] \leq I_{\mathrm{av}}, \text{ and} \tag{12}$$

$$\beta \geq 0, \quad \forall\, \gamma_{sr} \geq 0, \gamma_{rp} \geq 0, \gamma_{rd} \geq 0. \tag{13}$$

The optimal relay gain adaptation policy for underlay CR is as follows.

Result *1:* Let

$$\mathcal{B}(\gamma_{sr}, \gamma_{rp}) \triangleq \frac{\lambda \gamma_{rp}}{m} \left(1 + \frac{1}{P_s \gamma_{sr}} \right). \tag{14}$$

Then,

$$\beta_{\mathrm{opt}} = 0, \quad \gamma_{rd} < \mathcal{B}(\gamma_{sr}, \gamma_{rp}). \tag{15}$$

Else, for $\gamma_{rd} \geq \mathcal{B}(\gamma_{sr}, \gamma_{rp})$, β_{opt} is the unique positive root of the following transcendental equation in x:

$$\exp\left(\frac{m P_s \gamma_{sr} \gamma_{rd}}{\gamma_{rd} + x^{-1}} \right) \left(x \gamma_{rd} + 1 \right)^2 = \frac{m P_s \gamma_{sr} \gamma_{rd}}{\lambda \gamma_{rp} (P_s \gamma_{sr} + 1)}. \tag{16}$$

Here, λ is a positive constant that is chosen to satisfy the average interference power constraint in (12) with equality, and such a choice always exists.

Proof: The proof is relegated to Appendix A. ■

Comments about the optimal policy:

1) From (15), we see that $\mathcal{B}(\gamma_{sr}, \gamma_{rp})$ defines the boundary of a region in which the relay shuts down if γ_{rd} is weak. Furthermore, $\mathcal{B}(\gamma_{sr}, \gamma_{rp})$ is linearly proportional to γ_{rp}. Thus, the interference-aware relay shuts down over a larger range of values of γ_{rd} when the R-$\mathrm{P_{Rx}}$ link is strong so as not to cause more interference at the $\mathrm{P_{Rx}}$.
2) The constant λ needs to be computed numerically, as is typical of several constrained adaptation problems that

arise in wireless communications [21]. However, this needs to be done only once at the beginning.

Note that (16) needs to be solved for each realization of γ_{sr}, γ_{rp}, and γ_{rd}. This is cumbersome to implement in an AF relay in real-time. We now present a simpler policy that circumvents this difficulty, and is amenable to practical implementation.

B. Bound, Asymptotic Insights, and SRGAP

We first derive an upper bound for β_{opt}.

Result *2:* $\beta_{\text{opt}} \leq \beta_u$, where

$$\beta_u = \begin{cases} \frac{b_1 - 1 + \sqrt{b_2 \gamma_{rd}}}{\gamma_{rd}}, & \gamma_{rd} \geq \mathcal{B}(\gamma_{sr}, \gamma_{rp}), \\ 0, & \text{else}, \end{cases} \tag{17}$$

and $b_1 = \frac{m^2 P_s^2 \gamma_{sr}^2}{1+m^2 P_s^2 \gamma_{sr}^2}$ and $b_2 = \frac{1}{\mathcal{B}(\gamma_{sr},\gamma_{rp})(1+m^2 P_s^2 \gamma_{sr}^2)}$.

Proof: The proof is given in Appendix B. ■

This bound provides analytical insights about the dependence of the optimal relay gain on the channel gains. Further, in the asymptotic regime $\frac{\gamma_{rd}}{\gamma_{rp}} \gg 1$, it can be shown that $\beta_u \propto \frac{1}{\gamma_{sr}\sqrt{\gamma_{rd}\gamma_{rp}}}$. A lower bound β_l for β_{opt} can also be derived, and exhibits a similar behavior [22].

β_u can be used as an approximation for β_{opt}. We shall refer to this policy as SRGAP. It is practically amenable because of its closed-form characterization in terms of elementary functions. We shall evaluate its efficacy in Sec. IV.

C. SEP Analysis

We now analyze the SEP of ORGAP. This is challenging because expanding the expectation term in (9), we get a four-fold integral in γ_{sr}, γ_{rd}, γ_{rp}, and θ. We circumvent this by deriving simpler SEP bounds. Let $\overline{\gamma}_{sr} = \mathbf{E}[\gamma_{sr}]$, $\overline{\gamma}_{rd} = \mathbf{E}[\gamma_{rd}]$, $\overline{\gamma}_{sd} = \mathbf{E}[\gamma_{sd}]$, and $\overline{\gamma}_{rp} = \mathbf{E}[\gamma_{rp}]$.

Result *3:* The SEP of ORGAP is upper bounded as

$$\begin{aligned} \text{SEP} \leq \text{SEP}_u &\triangleq I_1 + \frac{\text{SEP}_0}{\overline{\gamma}_{rd}\overline{\gamma}_{rp}\overline{\gamma}_{sr}} \int_0^\infty \Bigg[\frac{b_1^2 \overline{\gamma}_{rd}^2}{\mathcal{B}'(\gamma_{sr})} \\ &\times \left(-\frac{1}{\frac{\overline{\gamma}_{rd}}{\mathcal{B}'(\gamma_{sr})\overline{\gamma}_{rp}} \left(\frac{\overline{\gamma}_{rd}}{\mathcal{B}'(\gamma_{sr})\overline{\gamma}_{rp}} + 1 \right)} + \frac{\ln\left(1 + \left(\frac{\overline{\gamma}_{rd}}{\mathcal{B}'(\gamma_{sr})\overline{\gamma}_{rp}}\right)\right)}{\left(\frac{\overline{\gamma}_{rd}}{\mathcal{B}'(\gamma_{sr})\overline{\gamma}_{rp}}\right)^2} \right) \\ &+ \frac{2m^2 P_s^2 \gamma_{sr}^2 \overline{\gamma}_{rd}^2}{\mathcal{B}'(\gamma_{sr})\left(1 + m^2 P_s^2 \gamma_{sr}^2\right)^{\frac{3}{2}}} \\ &\times \left(\frac{\tan^{-1}\left(\sqrt{\frac{\overline{\gamma}_{rd}}{\mathcal{B}'(\gamma_{sr})\overline{\gamma}_{rp}}}\right)}{\left(\frac{\overline{\gamma}_{rd}}{\mathcal{B}'(\gamma_{sr})\overline{\gamma}_{rp}}\right)^{\frac{3}{2}}} - \frac{1}{\frac{\overline{\gamma}_{rd}}{\mathcal{B}'(\gamma_{sr})\overline{\gamma}_{rp}}\left(\frac{\overline{\gamma}_{rd}}{\mathcal{B}'(\gamma_{sr})\overline{\gamma}_{rp}} + 1\right)} \right) \\ &+ \frac{\overline{\gamma}_{rp}\overline{\gamma}_{rd}^2}{(1+m^2 P_s^2 \gamma_{sr}^2)\left(\mathcal{B}'(\gamma_{sr})\overline{\gamma}_{rp} + \overline{\gamma}_{rd}\right)} \Bigg] \exp\left(-\frac{\gamma_{sr}}{\overline{\gamma}_{sr}}\right) d\gamma_{sr}, \end{aligned} \tag{18}$$

where $I_1 = \text{SEP}_0 \left(m_2 \overline{\gamma}_{sr} + \frac{m_2}{m_1} \exp(m_2) E_1(m_2) \right)$, $m_1 = \frac{\lambda\overline{\gamma}_{rp} + m\overline{\gamma}_{rd}}{m\overline{\gamma}_{rd}}$, $m_2 = \frac{\lambda\overline{\gamma}_{rp}}{P_s\overline{\gamma}_{sr}\left(\lambda\overline{\gamma}_{rp}+m\overline{\gamma}_{rd}\right)}$, $\mathcal{B}'(\gamma_{sr}) = \frac{\lambda}{m}\left(1 + \frac{1}{P_s\gamma_{sr}}\right)$, and $E_1(x)$ is the exponential integral [23, (5.1.1)].

Proof: The derivation is given in Appendix C. ■

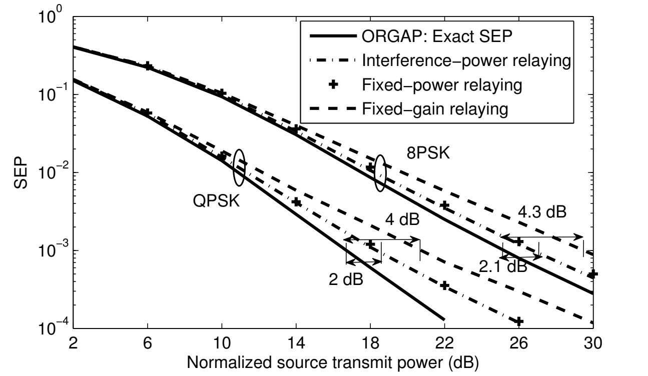

Fig. 2. With S_{Tx}-S_{Rx} link: Comparison of SEPs of ORGAP, fixed gain, and fixed-power relaying for QPSK and 8PSK ($\overline{\gamma}_{sr} = \overline{\gamma}_{rd} = \overline{\gamma}_{rp} = \overline{\gamma}_{sd} = 1$ and $I_{\text{av}} = 15$ dB).

The integral can be easily evaluated using Gauss-quadrature [24]. While the result above is quite involved, it is considerably simpler than the exact SEP expression.

IV. Numerical Results and Performance Benchmarking

We first extensively benchmark ORGAP's performance with interference-power relaying [7], [8] ($\beta = \frac{I_{\text{av}}}{\gamma_{rp}(P_s\gamma_{sr}+1)}$), fixed-power relaying [6] ($\beta = \frac{I_{\text{av}}}{\overline{\gamma}_{rp}(P_s\gamma_{sr}+1)}$), and fixed-gain relaying [4], [5] ($\beta = \frac{I_{\text{av}}}{\overline{\gamma}_{rp}(P_s\overline{\gamma}_{sr}+1)}$). Recall that the noise powers are normalized to unity.

Figure 2 considers the scenario where the S_{Tx}-S_{Rx} link is comparable in strength to the other links. It plots the SEPs of ORGAP and the benchmark schemes as a function of P_s for QPSK and 8PSK constellations. For QPSK, at an SEP of 0.001, ORGAP outperforms interference-power, fixed-power, and fixed-gain relaying by 1.6 dB, 2.0 dB, and 4.0 dB, respectively. For 8PSK, the corresponding gains of ORGAP increase to 1.8 dB, 2.1 dB, and 4.3 dB.

Figure 3 considers the scenario where the S_{Tx}-S_{Rx} link is absent ($\overline{\gamma}_{sd} = 0$). It plots the SEPs of ORGAP and the benchmark schemes as a function of P_s. Also shown is the SEP upper bound SEP_u. Since the direct link is absent, the SEPs are higher than in Figure 2. An error floor, which arises because of the interference constraint, is now visible. Now, ORGAP outperforms the benchmark schemes by an even larger margin than in Figure 2 because the relay influences the signal quality more. Compared to ORGAP, the error floors of fixed-gain, interference-power, and fixed-power relaying are greater by a factor of 15.8, 5.3, and 5.8, respectively.

Effectiveness of SRGAP: Figure 4 compares the SEPs of ORGAP and SRGAP.[2] The cases where the S_{Tx}-S_{Rx} link is comparable to the other links and where it is absent are considered. In both cases, only a small gap exists between the two curves, which shows that SRGAP is nearly optimal.

[2]For SRGAP, λ in (17) is now chosen such that the average interference constraint is satisfied when β_u is used as the relay gain.

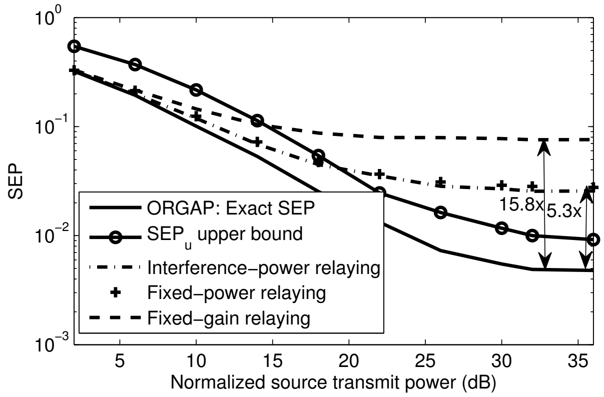

Fig. 3. Without S_{Tx}-S_{Rx} link: Comparison of SEPs of ORGAP, fixed-gain, fixed-power relaying, and interference power relaying. Also shown is the SEP upper bound ($\overline{\gamma}_{sr} = \overline{\gamma}_{rd} = \overline{\gamma}_{rp} = 1$, $I_{\text{av}} = 15$ dB, and QPSK).

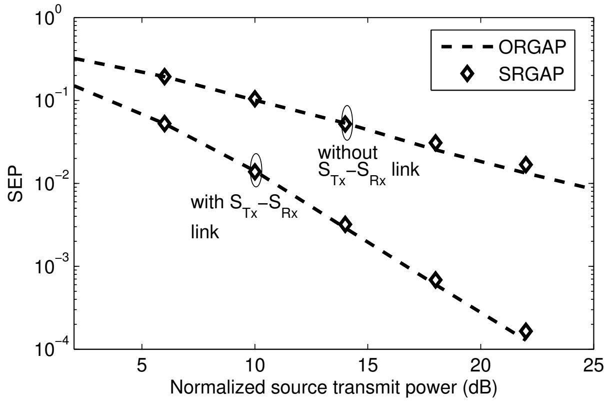

Fig. 4. ORGAP vs. SRGAP: SEP as a function of P_s ($I_{\text{av}} = 15$ dB and QPSK).

Impact of Interference Link: Figure 5 studies the effect of the R-P_{Rx} interference link's mean channel gain on the SEP of ORGAP. For this, results for three different values of $\overline{\gamma}_{rp}$ are shown. As $\overline{\gamma}_{rp}$ increases, i.e., as the R-P_{Rx} link becomes stronger, the SEP increases as expected.

V. Conclusions

We proposed a general and interference-aware optimal relay gain adaptation policy (ORGAP) for cooperative underlay CR systems in which the relay adapts its gain as a function of its local channel gains. It is unlike the ad hoc adaptations of the fixed-power and interference-power (with variable gain) and fixed-gain (with variable power) AF relaying rules that have been extensively used in the literature. We also derived an upper bound for the SEP achieved by ORGAP. We saw that ORGAP outperforms all the benchmark schemes. Furthermore, the weaker the direct S_{Tx}-S_{Rx} link, the more significant are the gains obtained from using ORGAP.

While ORGAP serves a fundamental benchmark, it is difficult to implement in practice. SRGAP circumvents this problem by specifying the relay gain in closed-form, and, yet, incurs a negligible performance penalty. An interesting avenue for future work is developing the optimal policy when the relay is subject to transmit power constraints as well, including relay selection, and modeling imperfect or partial CSI.

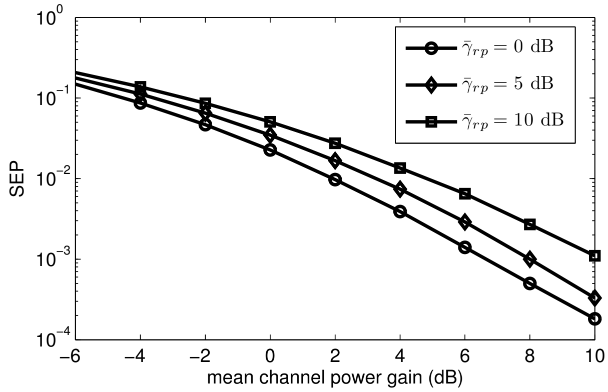

Fig. 5. Effect of R-P_{Rx} link mean channel gain ($\overline{\gamma}_{rp}$): SEP of ORGAP as a function of γ_{av} ($\overline{\gamma}_{sr} = \overline{\gamma}_{rd} = \overline{\gamma}_{sd} \triangleq \gamma_{\text{av}}$, $P_s = 15$ dB, $I_{\text{av}} = 15$ dB, and 8PSK).

Appendix

A. Proof of Result 1

Let $f(x) \triangleq \exp\left(-\frac{mP_s\gamma_{sr}\gamma_{rd}}{\gamma_{rd}+x^{-1}}\right)$. For a policy in which the relay gain is β, define the following auxiliary function $L_\beta(\tilde{\lambda})$:

$$L_\beta(\tilde{\lambda}) = \mathbf{E}\left[f(\beta)\right] + \tilde{\lambda}\mathbf{E}\left[\beta\gamma_{rp}\left(P_s\gamma_{sr}+1\right)\right], \qquad (19)$$

for all $x \geq 0$ and $\tilde{\lambda} \geq 0$.

Consider a policy that sets its relay gain as $\tilde{\beta}$, which is the solution of the following minimization problem:

$$\tilde{\beta} = \arg\min_{x\geq 0}\left\{f(x) + \tilde{\lambda}\left(x\gamma_{rp}\left(P_s\gamma_{sr}+1\right)\right)\right\}. \qquad (20)$$

For this policy, let $\lambda > 0$ denote the value of $\tilde{\lambda}$ such that the constraint in (12) is met with equality.[3] For this specific value of λ, let β_{opt} denote the relay gain policy.

From (20), it follows that $L_{\beta_{\text{opt}}}(\lambda) \leq L_\beta(\tilde{\lambda})$. Substituting this in (19) and rearranging terms, we get

$$\mathbf{E}\left[f(\beta_{\text{opt}})\right] \leq \mathbf{E}\left[f(\beta)\right] + \lambda\left(\mathbf{E}\left[\gamma_{rp}\left(P_s\gamma_{sr}+1\right)\beta\right] - I_{\text{av}}\right). \quad (21)$$

If the policy that uses β as its relay gain is a feasible policy, we know that $\mathbf{E}\left[\gamma_{rp}\left(P_s\gamma_{sr}+1\right)\beta\right] - I_{\text{av}} \leq 0$. Therefore, from (21), $\mathbf{E}\left[f(\beta_{\text{opt}})\right] < \mathbf{E}\left[f(\beta)\right]$. Thus, β_{opt} satisfies the constraints and has the lowest SEP among all feasible policies. Hence, it is optimal.

It is easy to verify that $f(x) + \lambda\left(x\gamma_{rp}\left(P_s\gamma_{sr}+1\right)\right)$ is convex in x. Hence, the optimal solution in (20) is unique. It is the non-negative solution of $f'(x) = -\lambda\gamma_{rp}(P_s\gamma_{sr}+1)$, if it exists, and is 0, otherwise. This yields (15) and (16). The boundary of the region in which $\beta_{\text{opt}} = 0$ is obtained by substituting $x = 0$ in (16), and yields (14).

[3]The existence of such a λ can be proved using the intermediate value theorem. The details are omitted due to space constraints.

B. Proof of Result 2

We know that $\exp\left(\frac{mP_s\gamma_{sr}\gamma_{rd}}{\gamma_{rd}+x^{-1}}\right) \geq 1 + \left(\frac{mP_s\gamma_{sr}\gamma_{rd}}{\gamma_{rd}+x^{-1}}\right)^2$, for $x \geq 0$. Substituting this in (16) and rearranging terms, we can show that

$$\phi(x) \geq \Omega(x),$$

where $\Omega(x) \triangleq \left(\gamma_{rd}^2 + b^2\right)x^2 + 2\gamma_{rd}x + 1 - \frac{b}{\lambda\gamma_{rp}(P_s\gamma_{sr}+1)}$ is a quadratic in x and $b \triangleq mP_s\gamma_{sr}\gamma_{rd}$. From the properties of quadratic equations, it can be verified that when $\gamma_{rd} \geq \mathcal{B}(\gamma_{sr}, \gamma_{rp})$, $\Omega(x)$ has exactly one positive root ζ, which is

$$\zeta = \frac{-1 + \sqrt{\frac{mP_s\gamma_{sr}\gamma_{rd}(1+m^2P_s^2\gamma_{sr}^2)}{\lambda\gamma_{rp}(P_s\gamma_{sr}+1)} - m^2P_s^2\gamma_{sr}^2}}{\gamma_{rd}\left(1 + m^2P_s^2\gamma_{sr}^2\right)}. \quad (22)$$

Since $\phi(x) \geq \Omega(x)$ and since both are convex for $x \geq 0$, it can be proved that $\beta_{\text{opt}} \leq \zeta$. Finally, by dropping the negative $m^2P_s^2\gamma_{sr}^2$ term inside the square root, we get $\zeta \leq \beta_u$, where β_u is given in (17).

Comments: There are two differences between the above derivation and that in [12]. First, the R-P_{Rx} gain γ_{rp} occurs in several terms above, and the form of β_u is simpler than the upper bound in [12, (17)].

C. Brief Derivation of Result 3

To simplify the SEP expression in (8), we make the following observations. First, replacing $\sin^2\theta$ with unity yields an upper bound for the SEP that involves a three-fold integral. Second, for $\gamma_{rd} < \mathcal{B}(\gamma_{sr}, \gamma_{rp})$, $\beta_{\text{opt}} = 0$. Thus, we can replace $\exp\left(\frac{mP_s\gamma_{sr}\gamma_{rd}}{\gamma_{rd}+\beta_{\text{opt}}^{-1}}\right)$ with unity in this region. Third, for $\gamma_{rd} \geq \mathcal{B}(\gamma_{sr}, \gamma_{rp})$, we know from (16) that

$$\exp\left(\frac{mP_s\gamma_{sr}\gamma_{rd}}{\gamma_{rd}+\beta_{\text{opt}}^{-1}}\right) = \frac{mP_s\gamma_{sr}\gamma_{rd}}{\lambda\gamma_{rp}(P_s\gamma_{sr}+1)}\left(\beta_{\text{opt}}\gamma_{rd}+1\right)^{-2}, \quad (23)$$

where, from (17),

$$\beta_{\text{opt}}\gamma_{rd}+1 \leq \frac{m^2P_s^2\gamma_{sr}^2 + \sqrt{\frac{mP_s\gamma_{sr}\gamma_{rd}}{\lambda\gamma_{rp}(P_s\gamma_{sr}+1)}\left(1+m^2P_s^2\gamma_{sr}^2\right)}}{1+m^2P_s^2\gamma_{sr}^2}. \quad (24)$$

Combining all the above observations, we can show that

$$\text{SEP} \leq I_1 + I_2, \quad (25)$$

where

$$I_1 = \frac{\text{SEP}_0}{\overline{\gamma}_{sr}\overline{\gamma}_{rp}\overline{\gamma}_{rd}}\int_0^\infty\int_0^\infty\int_0^{\mathcal{B}(\gamma_{sr},\gamma_{rp})} e^{-\frac{\gamma_{sr}}{\overline{\gamma}_{sr}}}e^{-\frac{\gamma_{rp}}{\overline{\gamma}_{rp}}} \times e^{-\frac{\gamma_{rd}}{\overline{\gamma}_{rd}}}\,d\gamma_{sr}\,d\gamma_{rp}\,d\gamma_{rd}, \quad (26)$$

and

$$I_2 = \frac{\text{SEP}_0}{\overline{\gamma}_{sr}\overline{\gamma}_{rp}\overline{\gamma}_{rd}}\int_0^\infty\int_0^\infty\int_{\mathcal{B}(\gamma_{sr},\gamma_{rp})}^\infty e^{-\frac{\gamma_{sr}}{\overline{\gamma}_{sr}}}e^{-\frac{\gamma_{rp}}{\overline{\gamma}_{rp}}}e^{-\frac{\gamma_{rd}}{\overline{\gamma}_{rd}}} \times \frac{\lambda\gamma_{rp}\left(P_s\gamma_{sr}+1\right)\left(b_1+\sqrt{b_2\gamma_{rd}}\right)^2}{mP_s\gamma_{sr}\gamma_{rd}}\,d\gamma_{sr}\,d\gamma_{rp}\,d\gamma_{rd}. \quad (27)$$

Averaging over γ_{rd} and γ_{rp}, and simplifying I_1 and I_2 further using [24, (8.211.1), (6.227.1), (6.292.1)] yields the desired expression in (18).

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