Optimal Joint Antenna Selection and Power Adaptation in Underlay Cognitive Radios

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***Abstract*—Transmit antenna selection (AS) is a popular, low hardware complexity technique that improves the performance of an underlay cognitive radio system, in which a secondary transmitter can transmit when the primary is on but under tight constraints on the interference it causes to the primary. The underlay interference constraint fundamentally changes the criterion used to select the antenna because the channel gains to the secondary and primary receivers must be both taken into account. We develop a novel and optimal joint AS and transmit power adaptation policy that minimizes a Chernoff upper bound on the symbol error probability (SEP) at the secondary receiver subject to an average transmit power constraint and an average primary interference constraint. Explicit expressions for the optimal antenna and power are provided in terms of the channel gains to the primary and secondary receivers. The SEP of the optimal policy is at least an order of magnitude lower than that achieved by several ad hoc selection rules proposed in the literature and even the optimal antenna selection rule for the case where the transmit power is either zero or a fixed value.**

I. Introduction

Cognitive radio is a promising technology to improve the utilization of scarce radio spectrum. Several modes of CR operation, such as interweave and underlay, have been considered in the literature [1]. In the interweave mode, the secondary users (SUs) only transmit when they sense that the primary is off. In the underlay mode, which is the focus of this paper, a secondary transmitter (STx) can transmit even when the primary is on, but under tight constraints on the interference it causes to the primary receiver (PRx). However, these constraints significantly limit the rates at which the STx can transmit.

Multiple input multiple output (MIMO) antenna techniques can help improve CR performance significantly [2]–[4]. However, in MIMO, each antenna requires a dedicated radio frequency (RF) chain. This increases the hardware complexity and cost of the multiple antenna device. Antenna selection (AS) is a well-studied and effective technique to reduce the hardware complexity of a MIMO system [5]. For example, in single transmit AS, one of the antennas is selected as a function of the channel conditions to transmit data. Given its practical simplicity and effectiveness, transmit AS has been adopted in wireless standards such as IEEE 802.11n and Long Term Evolution (LTE) [6].

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Given its promise, AS has also been considered for CR. AS for SUs that operate in the underlay mode has been studied in [7]–[9]. It has been shown to improve the capacity of an SU in [10]. The interference constraint imposed on the STx that operates in the underlay mode fundamentally alters the basis on which an antenna is selected. Intuitively, even an antenna with a high channel gain to the secondary receiver (SRx) should not be selected if transmitting on it causes significant interference to the PRx.[1] Therefore, the AS rule must take into consideration the channel gains to the SRx as well as the PRx. This is unlike the interweave mode, in which the antenna with the strongest channel gain to the SRx is clearly the one that should be selected, since the STx transmits only when the primary transmitter (PTx) is off.

Literature Survey: Several AS rules have been considered in the literature for underlay CR. These include the minimum interference (MI) rule [7], which selects the antenna with the weakest channel gain to the PRx; the maximum signal power to leak interference power ratio (MSLIR) rule [7], which selects the antenna with the largest ratio of the channel gains to the SRx and PRx; and the difference rule [8], which selects the antenna with the largest weighted difference between the channel gains to the SRx and PRx. However, the STx is assumed to transmit with a fixed power in [7], [8]. For an STx that transmits with a fixed power or with zero power, which we shall refer to as on-off power control, the symbol error probability (SEP)-optimal selection rule is derived in [9].

Joint AS and power control is considered in [11] with the goal of maximizing the SU capacity under a peak interference constraint. However, the STx transmit power is not constrained. Optimal power control to maximize the SU capacity is considered in [12], transmit power or interference power constraints with average or peak power interference constraints are modeled. In [13], power control that minimizes the SEP under an average interference constraint and a peak transmit power constraint is considered assuming that the number of fade states is finite. Optimal power control to maximize ergodic capacity for underlay CR assuming average or peak interference power constraints is also considered in [14], [15]. However, the STx is assumed to know the channel condition between the PTx and PRx. Furthermore, [12]–[15] do not consider AS.

Contributions: In this paper, we derive the optimal joint

[1] By channel gain, we mean the square of the amplitude of the baseband channel response.

antenna selection power adaptation (AS-PA) policy that minimizes a Chernoff upper bound on the SEP for a multiple input single output (MISO) secondary system subject to an average transmit power constraint and an average primary interference power constraint. It exploits the fact that the channel state information required to select an antenna can also help adapt the transmit power of the selected antenna. The optimal choice of the antenna and its transmit power are given explicitly in terms of the STx-to-SRx and STx-to-PRx channel gains. As is typical of optimal solutions to constrained problems in wireless systems, e.g., [16], [17], these are in terms of two constants λ_I and λ_P, which are a function of the channel statistics, constraints, and constellation size, and are determined numerically only once in the beginning.

We also derive an exact expression and a closed-form upper bound for the SEP of the optimal AS-PA policy. Extensive numerical results show that the SEP of the optimal policy is at least one order of magnitude lower than the SEPs of the rules considered thus far in the literature.

Given space constraints, we focus on the case where the STx has two transmit antennas and the SRx has one receive antenna. Doing so helps us highlight the key ideas that lead to the optimal joint AS-PA policy and study how it improves CR performance. Our results generalize to the case where the STx and SRx have an arbitrary number of antennas and to constellations such as MQAM.

The paper is organized as follows. Section II sets up the system model and the optimization problem. The optimal joint AS-PA policy and its SEP are derived in Sec. III. Numerical results and conclusions are presented in Sec. IV and Sec. V, respectively. Mathematical derivations are given in the Appendix.

II. System Model and Problem Statement

As shown in Figure 1, we consider an underlay CR system in which an STx transmits data to an SRx, and in the process interferes with a PRx. The STx has two transmit antennas and one RF chain. The PRx and the SRx have one receive antenna each. For $i \in \{1, 2\}$, h_i denotes the instantaneous channel gain between the i^{th} antenna of the STx and the SRx antenna, and g_i denotes the instantaneous channel gain between the i^{th} antenna of the STx and the PRx antenna. We assume Rayleigh fading. h_1 and h_2 are independent and identically distributed (i.i.d.) exponential random variables (RVs) with mean μ_h. And, g_1 and g_2 are i.i.d. exponential RVs with mean μ_g. Let $\mathbf{h} = [h_1\ h_2]$ and $\mathbf{g} = [g_1\ g_2]$.

A. Selection Options and Data Transmission

Using the selected antenna, the STx transmits with power P a data symbol x, which is drawn with equal probability from an MPSK constellation of size M. The case where $P = 0$ deserves special mention because in this case it does not matter which antenna the STx selects. In this case, we shall say that the STx transmits using a virtual antenna 0, and set $h_0 \triangleq 0$ and $g_0 \triangleq 0$. The SRx does not know when the STx uses the zero-transmit power option. Therefore, the SEP in this case is equal to $m \triangleq 1 - \frac{1}{M}$, since the M symbols are equi-probable [13].

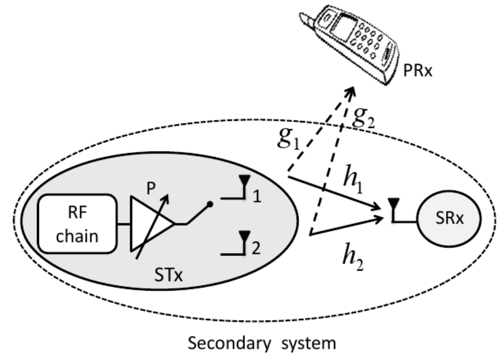

Fig. 1. System model with one PRx and a secondary system consisting of an STx with two transmit antennas and one RF chain that communicates with an SRx with one receive antenna.

Let $s \in \{0, 1, 2\}$ be the antenna selected. The interference signal i_{PRx} seen by the PRx and the signal r_{SRx} received by the SRx are given by

$$r_{\text{SRx}} = \sqrt{P}\sqrt{h_s}e^{j\theta_{h_s}}x + n + w_p, \tag{1}$$

$$i_{\text{PRx}} = \sqrt{P}\sqrt{g_s}e^{j\theta_{g_s}}x, \tag{2}$$

where $|x|^2 = 1$, θ_{g_s} is the phase of the complex baseband channel gain from antenna s of the STx to the PRx, θ_{h_s} is the phase of the complex baseband channel gain from antenna s to the SRx, and n is a circularly symmetric complex additive white Gaussian noise at the SRx. The interference seen by the SRx due to primary transmissions is w_p, and is assumed to be Gaussian. This corresponds to a worst case model for the interference and makes the problem tractable. Therefore, $n + w_p$ is a circular symmetric complex Gaussian RV, whose variance is denoted by σ^2.

We assume that the STx knows $\mathbf{h}$ and $\mathbf{g}$, i.e., its channel gains to the SRx and to the PRx. This has also been assumed in the literature on AS in CR, e.g., [7], [8], [11]. Note that no knowledge of the phases of any complex baseband channel response is required at the STx.[2] The SRx uses a coherent receiver, and only needs to know the complex channel gain from the selected antenna to the receive antenna. It does not know $\mathbf{g}$ or the channel gains of any other antenna. In practice this can be achieved by embedding one or two pilots along with the data symbols in every coherence interval.

B. Problem Statement

An AS-PA policy ϕ is a mapping that, for every realization of $\mathbf{h}$ and $\mathbf{g}$, specifies which of the three antennas 0, 1, or

[2]In the time division duplex (TDD) mode of operation, the STx can exploit reciprocity and estimate $\mathbf{h}$ and $\mathbf{g}$ from the signals it receives from the SRx and PRx when they transmit. Since phase information is not required, simple signal strength based techniques can be used for estimation. We note that these results also serve as bounds on the performance of AS when the STx has partial or imperfect knowledge of $\mathbf{g}$.

2 to transmit from and the corresponding transmit power. For a policy ϕ, the selected antenna and the transmit power, which depend on $\mathbf{h}$ and $\mathbf{g}$, are denoted by $s_\phi(\mathbf{h},\mathbf{g})$ and $P_\phi(\mathbf{h},\mathbf{g})$, respectively. For notational simplicity, we shall drop the arguments $\mathbf{h}$ and $\mathbf{g}$, and denote the selected antenna and transmit power as s_ϕ and P_ϕ, respectively. Using (2), the average interference at the PRx is $\mathbf{E}_{\mathbf{h},\mathbf{g}}\left[P_\phi g_{s_\phi}\right]$ and the average transmit power is $\mathbf{E}_{\mathbf{h},\mathbf{g}}\left[P_\phi\right]$, where $\mathbf{E}_{\mathbf{h},\mathbf{g}}[\cdot]$ denotes the expectation with respect to $\mathbf{h}$ and $\mathbf{g}$.

For a policy ϕ, the SEP for MPSK conditioned on $\mathbf{h}$ and $\mathbf{g}$, which we denote by $\Pr(\text{Err}|\mathbf{h},\mathbf{g})$, is then given by [18]

$$\Pr(\text{Err}|\mathbf{h},\mathbf{g}) = \frac{1}{\pi}\int_0^{m\pi}\exp\left(-\frac{P_\phi h_{s_\phi}}{\eta m \sin^2\theta}\right)d\theta, \tag{3}$$

$$\leq m\exp\left(-\frac{P_\phi h_{s_\phi}}{\eta m}\right), \tag{4}$$

where $\eta = \frac{\sigma^2}{m\sin^2\left(\frac{\pi}{M}\right)}$. The exact SEP expression in (3) is in the form of single integral. To gain analytical insights, we minimize its integral-free Chernoff upper bound given in (4). This form is similar to the integral-free SEP approximations proposed in [17]. The Chernoff upper bound tracks the SEP well, using it to select the antenna is, with high probability, as good as using the exact SEP expression. Thus, for ease of exposition, we shall refer to the policy we derive below as the *SEP-optimal policy* and not the SEP Chernoff upper bound-based-optimal policy.

Our problem can be mathematically stated as the following minimization over all policies ϕ:

$$\min_\phi \quad \mathbf{E}_{\mathbf{h},\mathbf{g}}\left[m\exp\left(-\frac{P_\phi h_{s_\phi}}{\eta m}\right)\right], \tag{5}$$

$$\text{such that} \quad \mathbf{E}_{\mathbf{h},\mathbf{g}}\left[P_\phi g_{s_\phi}\right] \leq I_{\text{ave}}, \tag{6}$$

$$\mathbf{E}_{\mathbf{h},\mathbf{g}}\left[P_\phi\right] \leq P_{\text{ave}}, \tag{7}$$

$$s_\phi \in \{0,1,2\}, \text{ and } P_\phi \geq 0, \ \forall\, \mathbf{h},\mathbf{g}. \tag{8}$$

III. Optimal AS-PA Policy and its SEP analysis

We now present the optimal AS-PA policy, which we denote by ϕ^*. We then analyze its SEP.

It is easy to see that for the optimal policy at least one of the two constraints in (6) and (7) must be active. We first consider the two cases where only one constraint is active. These shall help us specify the optimal policy in all its generality.

Lemma *1:* The optimal AS-PA policy ϕ_I that minimizes the SEP subject only to the average interference constraint in (6) is as follows. The optimal antenna is

$$s_{\phi_I} = \begin{cases} 0, & \frac{h_1}{g_1} \leq \eta\lambda_I, \frac{h_2}{g_2} \leq \eta\lambda_I, \\ \arg\max_{i=1,2}\left\{\frac{h_i}{g_i}\right\}, & \text{otherwise.} \end{cases} \tag{9}$$

The optimal power transmitted from the optimal antenna is

$$P_{\phi_I} = \begin{cases} 0, & s_{\phi_I} = 0, \\ \frac{m\eta}{h_{s_{\phi_I}}}\log_e\left(\frac{h_{s_{\phi_I}}}{\lambda_I \eta g_{s_{\phi_I}}}\right), & \text{otherwise.} \end{cases} \tag{10}$$

Here, $\lambda_I > 0$ is chosen such that the average interference constraint is satisfied with equality.

Lemma *2:* The optimal AS-PA policy ϕ_P that minimizes the SEP when the STx is subject only to the average power constraint in (7) is as follows. The optimal antenna is

$$s_{\phi_P} = \begin{cases} 0, & h_1 \leq \eta\lambda_P, h_2 \leq \eta\lambda_P, \\ \arg\max_{i=1,2}\{h_i\}, & \text{otherwise.} \end{cases} \tag{11}$$

The optimal power transmitted from the optimal antenna is

$$P_{\phi_P} = \begin{cases} 0, & s_{\phi_P} = 0, \\ \frac{m\eta}{h_{s_{\phi_P}}}\log_e\left(\frac{h_{s_{\phi_P}}}{\lambda_P \eta}\right), & \text{otherwise.} \end{cases} \tag{12}$$

Here, $\lambda_P > 0$ is chosen such that the average power constraint is satisfied with equality.

The proofs of Lemmas 1 and 2 are simpler versions of the proof for Theorem 1 below. To conserve space, we do not show them first. With these results, we now present the optimal AS-PA policy ϕ^* for any given P_{ave} and I_{ave}. For this, we define a *feasible policy* to be one that satisfies all the constraints (6), (7), and (8).

Theorem *1:* If ϕ_P is feasible, then $\phi^* = \phi_P$. Else, if ϕ_I is feasible, then $\phi^* = \phi_I$. Else, the optimal antenna is

$$s_{\phi^*} = \begin{cases} 0, & X_1 \leq \eta, \ X_2 \leq \eta, \\ \arg\max_{i=1,2}\{X_i\}, & \text{otherwise,} \end{cases} \tag{13}$$

where $X_j \triangleq \frac{h_j}{\lambda_P + \lambda_I g_j}$, for $j = 1,2$. The optimal power transmitted from the selected antenna is

$$P_{\phi^*} = \begin{cases} 0, & s_{\phi^*} = 0, \\ \frac{m\eta}{h_{s_{\phi^*}}}\log_e\left(\frac{X_{s_{\phi^*}}}{\eta}\right), & \text{otherwise,} \end{cases} \tag{14}$$

where $\lambda_I > 0$ and $\lambda_P > 0$ are chosen so as to meet the average power and interference constraints with equality.

Proof: The proof is given in Appendix A. ■

For the general case in which the STx has N_t transmit antennas, determining the optimal antenna simply requires choosing the maximum from the set $\{X_1,\ldots,X_{N_t}\}$. The optimal power expression remains the same as (14).

A. SEP Analysis

Theorem *2:* The exact SEP of the optimal AS-PA policy is

$$\text{SEP} = \frac{2}{\pi}\int_\eta^\infty\int_0^{m\pi}\left(\frac{\lambda_P e^{-\frac{\lambda_P}{\mu_h}x_1}}{x_1\mu_g\lambda_I + \mu_h} + \frac{\mu_h\mu_g\lambda_I e^{-\frac{\lambda_P}{\mu_h}x_1}}{\left(x_1\mu_g\lambda_I + \mu_h\right)^2}\right) \times\left(\frac{\eta}{x_1}\right)^{\csc^2(\theta)}\left(1 - \frac{\mu_h e^{-\frac{x_1\lambda_P}{\mu_h}}}{\mu_h + x_1\mu_g\lambda_I}\right)d\theta\, dx_1 + m\left(1 - \frac{\mu_h e^{-\frac{\eta\lambda_P}{\mu_h}}}{\mu_h + \eta\mu_g\lambda_I}\right)^2. \tag{15}$$

Proof: The proof is given in Appendix B. ■

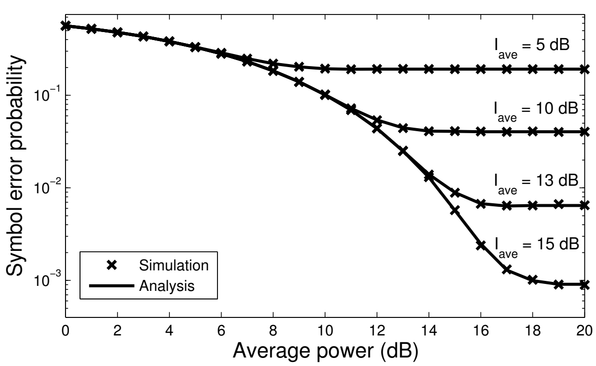

Fig. 2. SEP as a function of P_{ave} for different values of I_{ave} (8-PSK).

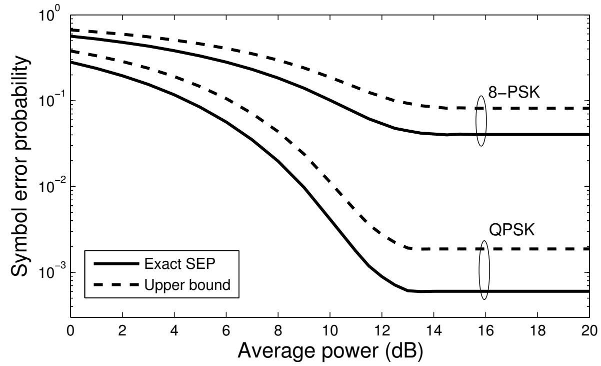

Fig. 3. SEP and its upper bound as a function of P_{ave} for different constellation sizes ($I_{\text{ave}} = 10$ dB).

The above SEP expression is in the form of a double integral. Its Chernoff upper bound SEP_u, obtained by using the inequality $\sin^2(\theta) \leq 1$, has a simple closed-form, as shown below. The derivation is not presented due to space constraints.

Corollary *1:* When the interference constraint is inactive, we have $\text{SEP}_u = \frac{2m\eta\lambda_P}{\mu_h}\left[E_1\left(\frac{\eta\lambda_P}{\mu_h}\right) - E_1\left(\frac{2\eta\lambda_P}{\mu_h}\right)\right] + m\left(1 - e^{-\frac{\eta\lambda_P}{\mu_h}}\right)^2$, where $E_1(z) = \int_z^\infty \frac{e^{-t}}{t}\,dt$ is the exponential integral. When the power constraint is inactive, we have $\text{SEP}_u = \frac{m\eta\mu_g\lambda_I}{\eta\mu_g\lambda_I + \mu_h}$. When both constraints are active, we have

$$\begin{aligned}\text{SEP}_u = \frac{2m\eta}{\mu_h}&\left[(\lambda_P + \mu_g\lambda_I)\left(E_1\left(\frac{\eta\lambda_P}{\mu_h}\right) - E_1\left(\frac{2\eta\lambda_P}{\mu_h}\right)\right)\right.\\ &+ (\mu_g\lambda_I - \lambda_P)\, e^{\frac{2\lambda_P}{\mu_g\lambda_I}} E_1\left(\frac{2\eta\lambda_P}{\mu_h} + \frac{2\lambda_P}{\mu_g\lambda_I}\right)\\ &\left.-\mu_g\lambda_I e^{\frac{\lambda_P}{\mu_g\lambda_I}} E_1\left(\frac{\eta\lambda_P}{\mu_h} + \frac{\lambda_P}{\mu_g\lambda_I}\right)\right] + m\\ &- 2me^{-\frac{\eta\lambda_P}{\mu_h}} + me^{-\frac{2\eta\lambda_P}{\mu_h}}\left(1 + \frac{\eta\mu_g\lambda_I}{\mu_h + \eta\mu_g\lambda_I}\right). \quad (16)\end{aligned}$$

IV. Numerical Results

We now present Monte Carlo simulations using 10^6 samples to verify the analytical results and to benchmark the performance of the optimal policy with several other rules considered in the literature. For illustration, we set $\mu_h = \mu_g = \sigma^2 = 1$.

Figure 2 plots the SEP of the optimal joint AS-PA policy as a function of the average power constraint P_{ave}. This is done for different values of the average interference constraint I_{ave}. We see that the simulation and analysis results are in excellent agreement, which verifies the analysis. The SEP decreases as P_{ave} increases. However, it reaches an error floor that depends on the average interference constraint I_{ave}. The larger the value of I_{ave}, the lower the error floor.

Figure 3 plots the SEP and its upper bound as a function of P_{ave} for $I_{\text{ave}} =$10 dB. This is done for QPSK and 8-PSK constellations. We see that the upper bound is within 2 dB of the exact SEP and tracks the exact SEP well. The figure also shows the dependence of the SEP on the constellation size M. As expected, as M increases, the SEP increases.

A. Benchmark Selection Rules

We now compare the performance of the optimal policy with the following selection rules considered in the literature, all of which use fixed power transmission.

1) Optimal AS with On-off Power Control [9]: This policy minimizes the SEP at the SRx – subject to an average interference constraint – when the STx transmits with a fixed power P or with zero power. The selected antenna s is

$$s = \arg\min_{i=0,1,2}\left\{\varphi g_i + m\exp\left(-\frac{Ph_i}{\eta m}\right)\right\}, \quad (17)$$

where φ is either zero or is chosen such that the average interference constraint is satisfied with equality.

2) Enhanced MI (EMI) Rule: The selected antenna s is

$$s = \begin{cases} 0, & g_1 \geq \tau,\ g_2 \geq \tau, \\ \arg\min_{i=1,2}\{g_i\}, & \text{otherwise}. \end{cases} \quad (18)$$

As above, the STx transmits with a fixed power P or with zero power. The EMI rule generalizes the MI rule of [7], which corresponds to setting $\tau = \infty$ in (18), and ensures that it is feasible for any given P_{ave} and I_{ave}.

3) Enhanced MSLIR (EMSLIR) Rule: The selected antenna s is

$$s = \begin{cases} 0, & \text{if } \frac{h_1}{g_1} \leq \beta,\ \frac{h_2}{g_2} \leq \beta, \\ \arg\max_{i=1,2}\left\{\frac{h_i}{g_i}\right\}, & \text{otherwise}. \end{cases} \quad (19)$$

The STx transmits with a fixed power P or with zero power. This rule generalizes the MSLIR rule of [7], which corresponds to setting $\beta = 0$ in (19).

4) Difference AS (DAS) Rule [8]: The selected antenna s is

$$s = \arg\max_{i=1,2}\{\delta h_i - (1-\delta)g_i\}, \quad (20)$$

where $\delta \in [0, 1]$.

Figure 4 compares the SEP as a function of the average transmit power for the optimal AS-PA policy and the

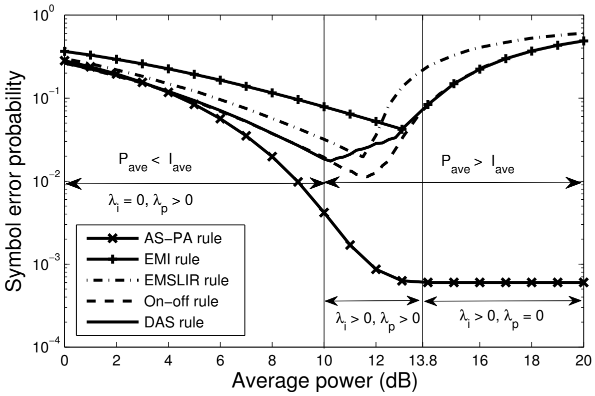

Fig. 4. Comparison of the SEPs of the optimal AS-PA policy and other benchmark rules ($I_{\text{ave}} = 10$ dB, and QPSK).

benchmark rules for $I_{\text{ave}} = 10$ dB. For the EMI rule, P and τ are jointly chosen to meet the average transmit power constraint with equality. Further, the choice is such that the interference constraint is inactive, which happens for smaller P_{ave}, or is otherwise met with equality. The values of P and β for the EMSLIR rule and P and δ for the DAS rule are chosen in a similar manner so as to meet the average power constraint. First, consider the optimal AS-PA policy. When $P_{\text{ave}} \leq I_{\text{ave}} = 10$ dB, only the average power constraint is active. Thus, its SEP decreases as P_{ave} increases. When $10 \text{ dB} < P_{\text{ave}} \leq 13.8$ dB, both the constraints are active. In this regime, the SEP curve begins to flatten. When $P_{\text{ave}} > 13.8$ dB, only the average interference power constraint is active. Thus, the SEP becomes independent of P_{ave}, and an error floor occurs.

The minimum SEP of the optimal policy is lower by a factor of 70.2, 32.5, 28.9, and 20 than the minimum SEPs of the EMI, EMSLIR, DAS, and optimal on-off power control rules, respectively, which demonstrates the effectiveness of the optimal AS-PA policy.

V. Conclusions

We derived the SEP-optimal joint antenna selection and transmit power adaptation policy for a secondary transmitter that operates in the underlay mode and is subject to constraints on its average transmit power and average interference it causes to the primary. The optimal policy turned out to be functionally different from the rules considered thus far in the literature. The optimal and combined use of antenna selection and power adaptation led to a substantial reduction in the SEP by at least one order of magnitude compared to the benchmark schemes. An interesting avenue for future work is determining the optimal policy when the secondary transmitter is subject to a constraint on the probability that the interference exceeds a threshold, instead of an average interference constraint. Extensions to multiple secondary nodes is another interesting problem.

Appendix

A. Brief Proof of Theorem 1

Since ϕ_P is the optimal solution of a problem with fewer constraints, it must be optimal if it is feasible. Similarly, ϕ_I must be optimal if it is feasible. Otherwise, consider the case where both ϕ_P and ϕ_I are not feasible.[3]

For $i = 0, 1, 2$, let

$$\Omega_{\mathbf{h},\mathbf{g}}(i, P) = \text{SEP}\,(h_i, P) + \lambda_P P + \lambda_I P g_i, \tag{21}$$

where $\text{SEP}\,(h_i, P) = m \exp\left(-\frac{P h_i}{m\eta}\right)$ and $\lambda_I \geq 0$ and $\lambda_P \geq 0$ are constants. Given λ_I and λ_P, let ϕ^* be the following policy for each realization of $\mathbf{h}$ and $\mathbf{g}$:

$$(s_{\phi^*}, P_{\phi^*}) = \arg \min_{\{(i,P): i=0,1,2, P\geq 0\}} \Omega_{\mathbf{h},\mathbf{g}}(i, P). \tag{22}$$

Furthermore, for any feasible policy ϕ, let

$$L_\phi = \mathbf{E}_{\mathbf{h},\mathbf{g}}\left[\Omega_{\mathbf{h},\mathbf{g}}(s_\phi, P_\phi)\right]. \tag{23}$$

Clearly, from the definition of ϕ^*, for any feasible policy ϕ, we have $L_{\phi^*} \leq L_\phi$. Rearranging terms, we get

$$\begin{aligned}\mathbf{E}_{\mathbf{h},\mathbf{g}}\left[\text{SEP}(h_{s_{\phi^*}}, P_{\phi^*})\right] &\leq \mathbf{E}_{\mathbf{h},\mathbf{g}}\left[\text{SEP}(h_{s_\phi}, P_\phi)\right] \\ &\quad + \lambda_P(\mathbf{E}_{\mathbf{h},\mathbf{g}}\left[P_\phi\right] - \mathbf{E}_{\mathbf{h},\mathbf{g}}\left[P_{\phi^*}\right]) \\ &\quad + \lambda_I(\mathbf{E}_{\mathbf{h},\mathbf{g}}\left[P_\phi g_{s_\phi}\right] - \mathbf{E}_{\mathbf{h},\mathbf{g}}\left[P_{\phi^*} g_{s_{\phi^*}}\right]). \end{aligned} \tag{24}$$

Choose λ_P and λ_I such that $\mathbf{E}_{\mathbf{h},\mathbf{g}}\left[P_{\phi^*}\right] = P_{\text{ave}}$ and $\mathbf{E}_{\mathbf{h},\mathbf{g}}\left[P_{\phi^*} g_{s_{\phi^*}}\right] = I_{\text{ave}}$.[4] Thus, ϕ^* is feasible. Upon so choosing, (24) becomes

$$\mathbf{E}_{\mathbf{h},\mathbf{g}}\left[\text{SEP}(h_{s_{\phi^*}}, P_{\phi^*})\right] \leq \mathbf{E}_{\mathbf{h},\mathbf{g}}\left[\text{SEP}(h_{s_\phi}, P_\phi)\right]. \tag{25}$$

Hence, ϕ^* is the SEP-optimal policy.

1) Minimizing $\Omega_{\mathbf{h},\mathbf{g}}(i, P)$*:* Given $\mathbf{h}$ and $\mathbf{g}$, we first determine the optimal power P^* when the selected antenna is $i \in \{1, 2\}$. Equating the derivative of $\Omega_{\mathbf{h},\mathbf{g}}(i, P)$ with respect to P to zero, we get $\exp\left(-\frac{P^* h_i}{m\eta}\right) = \eta \frac{(\lambda_P + \lambda_I g_i)}{h_s}$. We then get $P^* = \frac{m\eta}{h_i} \log_e\left(\frac{X_i}{\eta}\right)$, if $X_i \geq \eta$, and is 0, otherwise. Here, $X_i = \frac{h_i}{\lambda_P + g_i \lambda_I}$. Substituting this in (21), we get

$$\Omega_{\mathbf{h},\mathbf{g}}(i, P^*) = \zeta(X_i), \quad \text{for } P^* > 0, \tag{26}$$

where

$$\zeta(x) = \frac{m\eta}{x}\left(1 + \log_e\left(\frac{x}{\eta}\right)\right). \tag{27}$$

Notice that $\zeta(x)$ has a unique maximum, which equals m and occurs at $x = \eta$.

All that remains now is determining the optimal antenna. Notice that setting the transmit power to 0 is equivalent to selecting antenna 0. Hence, if $X_1 \leq \eta$ and $X_2 \leq \eta$, then $s_{\phi^*} =$

[3]If both ϕ_P and ϕ_I are feasible, then it can be shown that they must be same.

[4]That such a choice of λ_I and λ_P exists can be proved using the intermediate value theorem, the fact that the rule that always selects $s = 0$ is feasible, and that both ϕ_P and ϕ_I are infeasible.

0. Furthermore, in this case, $\Omega_{\mathbf{h},\mathbf{g}}(0, P_{\phi^*}) = m$. Otherwise, three cases arise: (i) If $X_1 > \eta$ and $X_2 \leq \eta$: In this case, the properties of $\zeta(\cdot)$ above imply that $s_{\phi^*} = 1$. (ii) If $X_2 > \eta$ and $X_1 \leq \eta$: In this case, $s_{\phi^*} = 2$. (iii) $X_1 > \eta$ and $X_2 > \eta$: In this case, $s_{\phi^*} = \arg\min_{i=1,2} \zeta(X_i)$. Since $\zeta(x)$ can be shown to be monotonically decreasing for $x \geq \eta$, it follows that $s_{\phi^*} = \arg\max_{i=1,2} X_i$. The above results together can be written in the compact form given in (13).

Comment: One can see that determining ϕ_I is a special case of the derivation above with $\lambda_P = 0$. The expressions given in Lemma 1 follow by substituting $\lambda_P = 0$ in (13) and (14). Similarly, the expressions given in Lemma 2 for ϕ_P follow by substituting $\lambda_I = 0$ in (13) and (14).

B. Proof of Theorem 2

When antenna 0 is selected, the SEP is m. From the law of total probability and since the channel gains of the two transmit antennas are i.i.d., the SEP can be written as a sum of two terms T_1 and T_2, where $T_1 = m\mathbf{E}_{\mathbf{h},\mathbf{g}}\left[\Pr\left(s=0|\mathbf{h},\mathbf{g}\right)\right]$ and $T_2 = 2\mathbf{E}_{\mathbf{h},\mathbf{g}}\left[\Pr\left(s=1|\mathbf{h},\mathbf{g}\right)\mathrm{SEP}\left(h_1, P_1\right)\right]$.

1) Evaluating T_1: Using the fundamental theorem of expectation, it can be shown that

$$T_1 = m\mathbf{E}_{\mathbf{h},\mathbf{g}}\left[\Pr\left(s=0|\mathbf{h},\mathbf{g}\right)\right] = m\Pr\left(s=0\right). \tag{28}$$

From (13), we get

$$\Pr\left(s=0\right) = \Pr\left(X_1 \leq \eta, X_2 \leq \eta\right) = \left(\Pr\left(X_1 \leq \eta\right)\right)^2. \tag{29}$$

From the definition of X_1, $\Pr\left(X_1 \leq \eta\right) = \Pr\left(\frac{h_1}{\lambda_P + \lambda_I g_1} \leq \eta\right)$. Since h_1 and g_1 are exponential RVs, it can be shown that $\Pr\left(X_1 \leq \eta\right) = 1 - \frac{\mu_h e^{-\frac{\eta\lambda_P}{\mu_h}}}{\mu_h + \eta\mu_g\lambda_I}$. Substituting this and (29) in (28) yields the formula for T_1.

2) Evaluating T_2: Let $\mathbf{X} = [X_1,\ X_2]$. Substituting the SEP expression in (3) and writing in terms of $\mathbf{X}$, we get

$$T_2 = \frac{2}{\pi}\int_0^{m\pi} \mathbf{E}_{\mathbf{X}}\left[\Pr\left(s=1|\mathbf{X}\right)\left(\frac{\eta}{X_1}\right)^{\csc^2(\theta)}\right] d\theta. \tag{30}$$

From the fundamental theorem of expectation, it follows that

$$\begin{aligned}\mathbf{E}_{\mathbf{X}}&\left[\Pr\left(s=1|\mathbf{X}\right)\left(\frac{\eta}{X_1}\right)^{\csc^2(\theta)}\right] \\ &= \mathbf{E}_{X_1}\left[\Pr\left(s=1|X_1=x_1\right)\left(\frac{\eta}{x_1}\right)^{\csc^2(\theta)}\right]. \end{aligned} \tag{31}$$

Writing the above expectation over X_1 in terms of its probability density function (PDF) $f_{X_1}(\cdot)$, we get

$$\begin{aligned}T_2 = \frac{2}{\pi}\int_0^{m\pi}\int_0^{\infty} &\Pr\left(s=1|X_1=x_1\right) \\ &\times \left(\frac{\eta}{x_1}\right)^{\csc^2(\theta)} f_{X_1}(x_1)\, dx_1\, d\theta. \end{aligned} \tag{32}$$

From the definition of X_1, it can be shown that

$$f_{X_1}(x_1) = \frac{\lambda_P e^{-\frac{\lambda_P}{\mu_h}x_1}}{x_1\mu_g\lambda_I + \mu_h} + \frac{\mu_h\mu_g\lambda_I e^{-\frac{\lambda_P}{\mu_h}x_1}}{\left(x_1\mu_g\lambda_I + \mu_h\right)^2},\ x_1 \geq 0. \tag{33}$$

From the AS rule in (13), we get

$$\begin{aligned}\Pr\left(s=1|X_1=x_1\right) &= \Pr\left(X_1 > \eta, X_2 < X_1|X_1=x_1\right), \\ &= I_{\{x_1>\eta\}}\Pr\left(X_2 < x_1|X_1=x_1\right), \end{aligned} \tag{34}$$

where $I_{\{a\}}$ denotes the indicator function; it equals 1 if a is true, and is 0 otherwise. It can be shown that $\Pr\left(s=1|X_1=x_1\right) = I_{\{x_1>\eta\}}\left(1 - \frac{\mu_h e^{-\frac{x_1\lambda_P}{\mu_h}}}{\mu_h + x_1\mu_g\lambda_I}\right)$. Substituting this and (33) in (32) yields the expression for T_2. Combining the expressions for T_1 and T_2 yields (15).

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