Optimal Joint Antenna Selection and Power Adaptation for Underlay Spectrum Sharing

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***Abstract*—Underlay spectrum sharing improves spectral utilization by allowing a secondary user to transmit concurrently with a primary user. However, the secondary user's performance is limited by the interference constraint that is imposed on it to protect the primary user. Transmit antenna selection overcomes this limitation with a hardware complexity comparable to a single-antenna system. We present a novel and optimal joint antenna selection and power adaptation rule for a secondary system that is subject to the practically motivated interference-outage constraint, which is more general than the widely studied peak interference constraint. The rule provably minimizes the average symbol error probability (SEP) of the secondary user. We show that it has a fundamentally different and novel structure compared to the ones studied in the literature. We present key geometric insights about its novel structure. We use these to propose a simpler, linearized, and near-optimal variant. Compared to the rules considered in the literature, the proposed rules reduce the average SEP by an order of magnitude.**

I. INTRODUCTION

Availability of sufficient spectrum is crucial for the success of new wireless technologies such as 5G and IEEE 802.11be [1], which target high data rates and massive connectivity using larger bandwidths. While the sub-6 GHz spectrum has favorable propagation characteristics, it is already crowded. Regulatory authorities are, therefore, opening up pre-allocated spectrum bands for the shared and unlicensed use. For example, the Federal Communications Commission (FCC) in the USA has opened up 3.5 GHz band for the shared use and the 6 GHz band for unlicensed secondary users [2]. These users can share the spectrum so long as they do not cause excessive interference to the existing primary users.

In the underlay mode of spectrum sharing for which practical demonstrations now exist, the secondary user transmits concurrently in the same spectrum as the primary user, but is subject to a constraint on the interference it causes to the primary receiver (PRx) [3], [4]. While the interference constraint protects the PRx from excessive interference, it also can significantly degrade the secondary user's performance. Furthermore, it plays a key role in determining the optimal transmission strategy of the secondary user. Though the underlay mode has been extensively studied in the context of cognitive radio, fundamental questions such as the right interference constraint to impose and the corresponding optimal transmission policy for the secondary users are still open.

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Transmit antenna selection (TAS) is a technology that improves the secondary user's performance [5]–[9]. In it, the secondary transmitter (STx) dynamically selects one antenna among multiple antennas, connects it to the radio frequency (RF) chain, and transmits data to the secondary receiver (SRx). It is appealing as it exploits the spatial diversity of multiple antennas but with a hardware complexity comparable to a single-antenna system.

In TAS in conventional interference-unconstrained systems, the antenna selected at the transmitter depends only on the channel gains from the transmitter to the receiver. For example, [10] selects the antenna with the highest instantaneous channel power gain to the receiver. However, in an underlay secondary system, TAS must also consider the STx to PRx (STx-PRx) channel gains along with the STx to SRx (STx-SRx) channel gains since it simultaneously needs to control the interference it causes to the PRx. For example, for a secondary system that is subject to the peak interference constraint, which limits the instantaneous interference power at the PRx to lie below a threshold, the STx in [7] selects the antenna with the highest ratio of the STx-SRx and STx-PRx channel power gains. Its transmit power is inversely proportional to the STx-PRx channel power gain. Variations of this idea are also considered in [6], [8].

However, the antenna selection and power adaptation (ASPA) rules turn out to be very different for stochastic constraints such as the average interference constraint [5], [9] and the interference-outage constraint [11]. The average interference constraint limits the fading-averaged interference power at the PRx. Instead, the interference-outage constraint limits the probability that the interference power at the PRx exceeds a threshold [11], [12]. The optimal ASPA rule for the average interference constraint [5] has a very different form than the one developed for the peak interference constraint [6]. However, the optimal ASPA rule is not known in the literature for the interference-outage constraint.

Studying the interference-outage constraint is theoretically and practically important in the underlay spectrum sharing systems for the following reasons. Firstly, it is a generalization of the widely studied peak interference constraint [13]. Secondly, to satisfy the peak interference constraint, the STx needs to perfectly know the instantaneous STx-PRx channel power gains, which can be difficult in practical systems that suffer from estimation errors or encounter time-variations in the channel. However, this is not so for the interference-outage constraint given its stochastic nature. Thirdly, it is well suited for primary systems that offer delay or disruption-tolerant

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services and are designed to tolerate co-channel interference-induced outages [11], [12].

A. Focus and Contributions

In this paper, we address the problem of joint antenna selection and power adaptation for an underlay secondary system that is subject to the interference-outage constraint and a peak transmit power constraint. The peak transmit power constraint is motivated by the RF circuit output limitations [6].

We make the following contributions:

- We present an optimal ASPA rule that minimizes the average symbol error probability (SEP) of a interference-outage constrained secondary user. Given the fundamental role that the interference constraint plays in an underlay spectrum sharing system, considering this interference constraint leads to a different optimization problem and an altogether different optimal rule.
- We present a novel and closed-form specification of the optimal antenna and its transmit power. We then present an insightful geometric interpretation of the optimal rule in terms of three regions in which the STx-SRx and STx-PRx channel power gains can lie.
- We exploit the above geometric interpretation to present a new, simpler rule called the *linear rule*. We prove that its average SEP lower bounds that of the optimal rule. We also derive a closed-form upper bound for its interference-outage probability.
- Our performance benchmarking with several other ASPA rules shows that the optimal rule reduces the average SEP by an order of magnitude and that the practically amenable linear rule is near-optimal.

The optimal ASPA rule markedly differs from the rules in [5], in which the transmit power is a logarithmic function of a ratio of the STx-SRx and STx-PRx channel power gains, and in [6]–[8], in which the transmit power is independent of the STx-SRx channel power gain. It also differs from the rule in [11], in which the net cost of an antenna is a continuous function of its STx-SRx channel power gain.

B. Outline and Notation

Section II presents the system model and the problem statement. The optimal and linear rules are developed in Sections III and IV, respectively. Numerical results are shown in Section V, and are followed by our conclusions in Section VI.

Notation: Scalar variables are written in normal font and vector variables in bold font. The probability of an event A and the conditional probability of A given B are denoted by $\Pr(A)$ and $\Pr(A|B)$, respectively. $\mathbb{E}_X[\cdot]$ denotes expectation with respect to a random variable (RV) X. $I_{\{a\}}$ denotes the indicator function; it is 1 if a is true and is 0 otherwise.

II. System Model and Problem Statement

The system model is shown in Fig. 1. It consists of an STx that communicates with an SRx and interferes with the PRx. The STx dynamically selects one among the N_t transmit antennas and connects it to the RF chain. The SRx and PRx are each equipped with one antenna each. For $k \in \{1, 2, \dots, N_t\}$, h_k denotes the instantaneous channel power gain from the k^{th} antenna of the STx to the SRx and g_k denotes the instantaneous channel power gain from the k^{th} antenna of the STx to the PRx. They undergo Rayleigh fading. We assume that the STx-SRx channels are independent and identically distributed (i.i.d.) RVs, and so are the STx-PRx channels. This occurs when the antennas are sufficiently spaced apart, and is commonly assumed in the related literature [6]–[9]. Thus, h_k and g_k are i.i.d. exponential RVs with means μ_h and μ_g, respectively. Let $\mathbf{h} \triangleq [h_1, \dots, h_{N_t}]$ and $\mathbf{g} \triangleq [g_1, \dots, g_{N_t}]$. The STx selects an antenna s and transmits with power P_s. Note that both s and P_s are functions of $\mathbf{h}$ and $\mathbf{g}$.

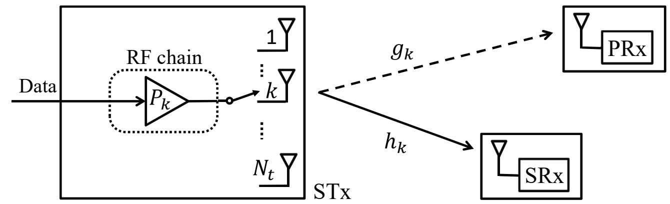


Fig. 1. System model that consists of an STx with N_t antennas and one RF chain. It transmits data to an SRx, which causes interference to a PRx.

Let $\mathsf{S}(P_k, h_k)$ denote the instantaneous SEP when antenna k transmits with power P_k. It is given by [14, (9.7)]

$$\mathsf{S}(P_k, h_k) \approx c_1 \exp\left(-c_2 \frac{P_k h_k}{\sigma^2}\right), \text{ for } 1 \leq k \leq N_t, \quad (1)$$

where c_1 and c_2 are modulation-dependent parameters and σ^2 is sum of the noise power at the SRx and the interference power from the primary transmissions [5], [9], [12]. The above formula applies to many constellations, for example, differential BPSK, QPSK, 8-PSK, and 16-QAM [6], [11].

Our channel state information (CSI) model is similar to that in [5], [6], [9], [11]. Specifically, the STx knows the STx-SRx channel power gains $\mathbf{h}$ and the STx-PRx channel power gains $\mathbf{g}$. It does not need phase information of any of these channels. When the primary and secondary systems operate in the time division duplexing mode, the STx can estimate $\mathbf{h}$ and $\mathbf{g}$ by making use of reciprocity by periodically sensing the signal from the SRx and PRx, respectively. Alternately, in the frequency division duplexing mode, the STx can obtain $\mathbf{h}$ using feedback and $\mathbf{g}$ using the hidden power feedback loop technique [15]. The SRx uses pilot symbols embedded in the data to estimate the complex baseband channel from the antenna s to itself and performs coherent demodulation.

A. Constraints and Problem Statement

The STx is subject to the following two constraints:

1) *Interference-Outage Constraint [11], [12]:* The instantaneous interference power at the PRx is equal to $P_s g_s$. An interference-outage happens when $P_s g_s$ exceeds the interference power threshold τ. Thus, the constraint can be formally stated as $\Pr(P_s g_s > \tau) \leq O_{\max}$, where $O_{\max}$ is the maximum allowed interference-outage probability. For the PRx, this is

a stochastic constraint because the antenna s selected and its transmit power P_s, which depend on $\mathbf{h}$ and $\mathbf{g}$, are RVs for it.

2) *Peak Transmit Power Constraint [6], [7]:* It is given by $P_s \leq P_{\max}$, where $P_{\max}$ is the peak transmit power.

ASPA Rule Definition: Formally, an ASPA rule ϕ maps $(\mathbf{h}, \mathbf{g})$ to an antenna s in the set $\{1, 2, \ldots, N_t\}$ and a transmit power P_s in the interval $[0, P_{\max}]$, i.e., $(s, P_s) = \phi(\mathbf{h}, \mathbf{g})$.

Our goal is to derive an optimal ASPA rule ϕ^* that minimizes the average SEP of a secondary user that is subject to the above constraints. Our problem $\mathcal{P}$ is as follows:

$$\mathcal{P}: \quad \min_{\phi} \quad \mathbb{E}_{\mathbf{h},\mathbf{g}}\left[\mathsf{S}(P_s, h_s)\right], \tag{2}$$

$$\text{s.t.} \quad \Pr\left(P_s g_s > \tau\right) \leq O_{\max}, \tag{3}$$

$$0 \leq P_s \leq P_{\max}, \tag{4}$$

$$(s, P_s) = \phi(\mathbf{h}, \mathbf{g}). \tag{5}$$

III. Optimal Rule and Its Behavior

A. Optimal Rule

First, consider the scenario in which the interference-outage constraint is inactive. Since the instantaneous SEP is a monotonically decreasing function of $P_s h_s$, it is easy to see that the optimal rule selects the antenna with the highest STx-SRx channel power gain and transmits with peak power $P_{\max}$:

$$s = \underset{1 \leq k \leq N_t}{\arg\max}\,\{h_k\} \text{ and } P_s = P_{\max}. \tag{6}$$

We shall refer to this as the *unconstrained* (UC) rule. Its interference-outage probability O_{u} is given by

$$O_{\mathrm{u}} = \Pr\left(P_{\max} g_s > \tau\right) = \Pr\left(P_{\max} g_1 > \tau\right) = e^{-\frac{\tau}{\mu_g P_{\max}}}. \tag{7}$$

Here, the second equality follows because the antenna selected by the UC rule is independent of $\mathbf{g}$ and $g_1, \ldots, g_{N_t}$ are i.i.d. When $O_{\mathrm{u}} \leq O_{\max}$, the UC rule satisfies the constraint in (3) and is optimal. We shall refer to this as the *unconstrained regime*. However, when $O_{\mathrm{u}} > O_{\max}$, the UC rule does not satisfy the constraint in (3). We shall refer to this as the *constrained regime* and present the optimal rule for it below.

Result *1:* In the constrained regime, the optimal ASPA rule selects the antenna

$$s^* = \underset{k \in \{1,2,\ldots,N_t\}}{\arg\min}\,\{\mathsf{NC}_k\}, \tag{8}$$

where

$$\mathsf{NC}_k \triangleq \mathsf{S}(P_k, h_k) + \lambda I_{\{P_k g_k > \tau\}}, \tag{9}$$

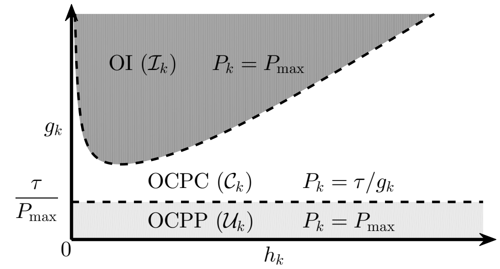
and

$$P_k = \begin{cases} P_{\max}, & \text{if } P_{\max} g_k \leq \tau, \\ P_{\max}, & \text{if } \mathsf{S}\left(\tau/g_k, h_k\right) > \mathsf{S}\left(P_{\max}, h_k\right) + \lambda, \\ \tau/g_k, & \text{else.} \end{cases} \tag{10}$$

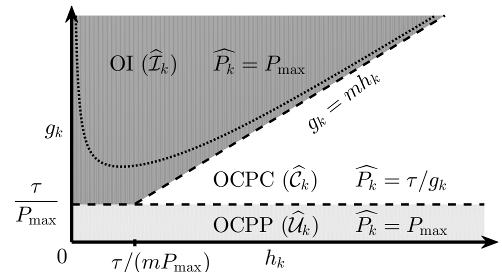
It transmits with power P_{s^*}. Here, λ is a penalty factor that is set to be equal to $\lambda^* \in (0, c_1)$, such that $\Pr\left(P_{s^*} g_{s^*} > \tau\right) = O_{\max}$. And, a unique choice of λ^* always exists.

Proof: The proof is given in Appendix A. ■

Here, for an antenna k, NC_k can be interpreted as its *net cost* and P_k as its transmit power if it were selected.



(a) Optimal rule



(b) Linear rule

Fig. 2. Structure of optimal rule: OCPP, OCPC, and OI regions and the transmit power of antenna k as a function of h_k and g_k.

Therefore, the optimal rule selects the antenna s^* with the smallest net cost and transmits with power P_{s^*}. We note that the above net cost NC_k is a discontinuous function of both h_k and g_k unlike the net costs of the optimal rules for the peak interference constraint [6, (4)] and the average interference constraint [5, (14)]. We also note that P_k is different from that in [6, (1)], which is independent of h_k, and that in [5, (16)], which is a logarithmic function of h_k and g_k. The optimal penalty factor λ^* is computed numerically, as is typical in several constrained optimization problems [5], [9], [14]. In Section IV, we present a closed-form bound based approach to directly compute it analytically.

B. Behavior of the Optimal Rule

A key insight from (10) is that the optimal transmit power P_k of antenna k is different in the following three regions:

1) If $P_{\max} g_k \leq \tau$, it transmits with peak power $P_{\max}$ and does not cause an interference-outage. Hence, we call it an *outage-compliant peak power* (OCPP) antenna.
2) If $\mathsf{S}\left(\tau/g_k, h_k\right) > \mathsf{S}\left(P_{\max}, h_k\right) + \lambda$, it transmits with $P_{\max}$ but causes an interference-outage because $P_{\max} g_k > \tau$. Hence, we call it an *outage-inducing* (OI) antenna.
3) Else, it transmits with power $P_k = \tau/g_k < P_{\max}$ and does not cause an interference-outage. Hence, we call it an *outage-compliant power constrained* (OCPC) antenna.

We denote the regions in which antenna k is an OCPP, OCPC, and OI antenna as $\mathcal{U}_k$, $\mathcal{C}_k$, and $\mathcal{I}_k$, respectively. These are shown in Fig. 2a. Using (8) and (10), it can be shown that for $\lambda = 0$, the optimal rule becomes equivalent to the UC rule in (6). For $\lambda = c_1$, it computes $P_k = \min\{P_{\max}, \tau/g_k\}$ and selects $s^* = \arg\max_{1 \leq i \leq N_t}\{P_i h_i\}$, which is the same as the ASPA rule in [6].

IV. Linear Rule and Its Analysis

In Fig. 2a, we see that the boundary between $\mathcal{C}_k$ and $\mathcal{I}_k$ is a non-linear function of h_k and g_k. This makes it difficult to analyze the performance of the optimal rule and gain rigorous insights about its performance. To address this, we now propose the linear rule. We shall show that it lower bounds the average SEP of the optimal rule. We also derive a closed-form upper bound for its interference-outage probability, which makes it easy to practically implement the linear rule.

In the linear rule, we classify an antenna k as OCPP antenna if $P_{\max} g_k \leq \tau$, as an OI antenna if $P_{\max} g_k > \tau$ and $\mathsf{S}(\tau/g_k, h_k) > \lambda$ (which is obtained by dropping the $\mathsf{S}(P_{\max}, h_k)$ term from the inequality in the definition of an OI antenna for the optimal rule), and as an OCPC antenna otherwise. Using (1) and algebraic simplifications, the three regions for the linear rule can be written as

$$\text{OCPP}: \widehat{\mathcal{U}}_k = \{(h_k, g_k) : P_{\max} g_k \leq \tau\}, \quad (11a)$$

$$\text{OCPC}: \widehat{\mathcal{C}}_k = \{(h_k, g_k) : P_{\max} g_k > \tau, g_k \leq m h_k\}, \quad (11b)$$

$$\text{OI}: \widehat{\mathcal{I}}_k = \{(h_k, g_k) : P_{\max} g_k > \tau, g_k > m h_k\}. \quad (11c)$$

Here, $m = -c_2 \tau / \left(\sigma^2 \ln(\lambda/c_1)\right)$ is the slope of the line that divides $\widehat{\mathcal{C}}_k$ and $\widehat{\mathcal{I}}_k$, as shown in Fig. 2b. The linear rule in terms of the these three regions can be interpreted as follows. It first computes the power $\widehat{P}_k$ of antenna k as follows:

$$\widehat{P}_k = \begin{cases} \tau/g_k, & \text{if } (h_k, g_k) \in \widehat{\mathcal{C}}_k, \\ P_{\max}, & \text{else.} \end{cases} \quad (12)$$

It then selects antenna $s = \arg\min \left\{ \widehat{\mathsf{NC}}_1, \ldots, \widehat{\mathsf{NC}}_{N_t} \right\}$, where

$$\widehat{\mathsf{NC}}_k \triangleq \mathsf{S}(\widehat{P}_k, h_k) + \lambda I_{\left\{\widehat{P}_k g_k > \tau\right\}}, \text{ for } 1 \leq k \leq N_t, \quad (13)$$

and transmits with power $\widehat{P}_s$. Note that for $\lambda = 0$ and $\lambda = c_1$, the linear rule is equivalent to the optimal rule.

Result *2:* For a given $\lambda \in (0, c_1)$, the average SEP of the linear rule lower bounds the average SEP of the optimal rule.

Proof: The proof is given in Appendix B. ■

Similarly, it can be shown that the interference-outage probability O_λ of the linear rule upper bounds that of the optimal rule. We now derive an upper bound for O_λ.

Result *3:* O_λ can be upper bounded as follows:

$$O_\lambda \leq B_\lambda = \left[1 - \left(\frac{\lambda}{c_1}\right)^{\frac{1}{c_2\Omega}} + \left(\frac{\lambda}{c_1}\right)^{\frac{1}{c_2\Omega}} \frac{O_{\mathrm{u}} \mu_g}{\mu_g + m\mu_h}\right]^{N_t} - \left[(1 - O_{\mathrm{u}}) \left(1 - \left(\frac{\lambda}{c_1}\right)^{\frac{1}{c_2\Omega}}\right)\right]^{N_t}, \quad (14)$$

where $\Omega = P_{\max} \mu_h / \sigma^2$.

Proof: The proof is given in Appendix C. ■

The interference-outage probability decreases as λ increases and B_λ upper bounds O_λ. This implies that the linear rule with its penalty factor set to the upper bound of λ that is obtained by solving $B_\lambda = O_{\max}$ satisfies the interference-outage constraint. Furthermore, B_λ becomes tighter as $P_{\max}$ increases. From (7), we get $O_{\mathrm{u}} \to 1$ as $P_{\max}$ increases. Substituting this limit in (14), we get $B_\lambda = (\mu_g/(\mu_g + m\mu_h))^{N_t}$ for large $P_{\max}$. Equating this with $O_{\max}$ yields the following closed-form expression for λ:

$$\lambda = c_1 \exp\left(-\frac{c_2 \tau \mu_h}{\sigma^2 \mu_g} \frac{(O_{\max})^{1/N_t}}{\left(1 - (O_{\max})^{1/N_t}\right)} \right). \quad (15)$$

We see that λ decreases as N_t or $O_{\max}$ increases.

V. Numerical Results and Benchmarking

We now present Monte Carlo simulations to study and compare the performance of the proposed ASPA rules with several other rules in the literature. We set $\mu_h = -114$ dB, $\mu_g = -121$ dB, and $\sigma^2 = -109$ dBm, which leads to a peak fading-averaged signal-to-noise ratio (SNR) $\Omega \left(P_{\max}\mu_h/\sigma^2\right) = 10$ dB for a peak transmit power ($P_{\max}$) of 15 dBm.[1]

Performance Benchmarking: We compare the proposed optimal rule with the following ASPA rules: minimum interference (MI) rule [8], maximum ratio (MR) rule [7], and maximum signal power (MSP) rule [6]. As originally proposed, these rules set the transmit power as $P_k = \min\{P_{\max}, \tau/g_k\}$, which leads to zero interference-outage probability. Therefore, to enable them to better exploit the leeway allowed by the interference constraint, we use the following more general transmit power policy: $P_k = P_{\max}$ if $P_{\max} g_k \leq \tau$; else,

$$P_k = \begin{cases} P_{\max}, & \text{with probability } q, \\ \tau/g_k, & \text{with probability } 1 - q. \end{cases} \quad (16)$$

We set q numerically such that the interference-outage probability is equal to $O_{\max}$.

The MI rule selects the antenna $s = \arg\min_{1 \leq k \leq N_t} \{g_k\}$ and transmits with power P_s as per (16). The MR rule selects the antenna $s = \arg\max_{1 \leq k \leq N_t} \{h_k/g_k\}$ and transmits with power P_s given by (16). The MSP rule first computes P_k as per (16) for each antenna k. It then selects the antenna $s = \arg\max_{1 \leq k \leq N_t} \{P_k h_k\}$ and transmits with power P_s.

In addition, to evaluate the gains from transmit power adaptation, we also compare with the on-off power adaptation rule [11, (9)] in which P_s is either 0 or $P_{\max}$. It selects

$$s = \underset{k \in \{0,1,\ldots,N_t\}}{\arg\min} \{\mathsf{S}(P_{\max}, h_k) + \alpha I_{\{P_{\max} g_k > \tau\}}\}. \quad (17)$$

Here, $s = 0$ represents the case when the STx transmits with zero power in order not to cause interference-outage at the PRx and $h_0 = g_0 \triangleq 0$. Hence, $P_s = 0$ if $s = 0$; else, $P_s = P_{\max}$. The parameter α is chosen such that $\Pr(P_s g_s > \tau) = O_{\max}$ in the constrained regime. Else, $\alpha = 0$.

Fig. 3 compares the average SEP as a function of the peak fading-averaged SNR Ω for all the above rules. The behavior is different in the following two regimes: (i) *Unconstrained regime ($\Omega \leq 2.9$ dB)*: Here, the average SEPs of the optimal

[1]These values correspond to a system bandwidth of 1 MHz, a carrier frequency of 2.4 GHz, a path-loss exponent of 3.7, a noise figure of 5 dB, at 300 K temperature. We considered a reference distance of 1 m, a distance of 100 m between the STx and SRx, and a distance of 150 m between the STx and PRx as per the simplified path-loss model [14, Chap. 2.6].

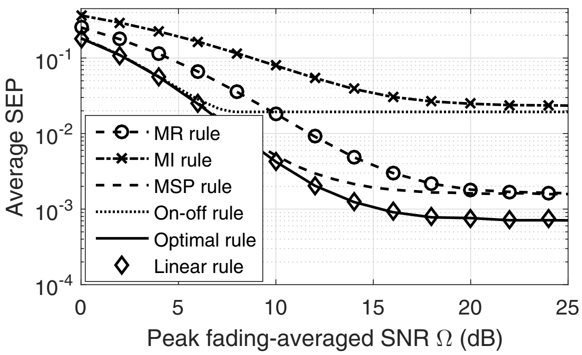


Fig. 3. Performance benchmarking: Average SEP as a function of Ω for different ASPA rules ($N_t = 4$, $O_{\max} = 0.01$, $\tau/\sigma^2 = 3$ dB, and QPSK with $c_1 = 0.5$ and $c_2 = 0.6$).

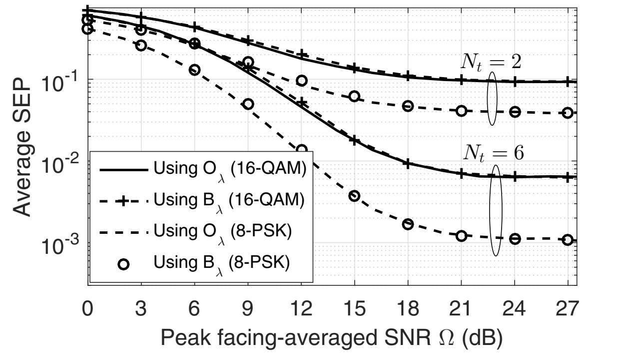


Fig. 4. Linear rule: Average SEP as a function of Ω for 8-PSK with $c_1 = 0.6$ and $c_2 = 0.18$ and 16-QAM with $c_1 = 0.8$ and $c_2 = 0.12$ ($O_{\max} = 0.1$ and $\tau/\sigma^2 = 3$ dB).

rule ($\lambda^* = 0$), the linear rule ($\lambda = 0$), the MSP rule ($q = 0$), and the on-off rule ($\alpha = 0$) are the same and decrease as Ω increases. (ii) *Constrained regime ($\Omega > 2.9$ dB):* From (7), it can be shown that this regime corresponds to $\Omega > -\tau\mu_h/(\mu_g\sigma^2 \ln(O_{\max}))$ in general. Here, the penalty factors of the optimal rule ($\lambda^* > 0$), the linear rule ($\lambda > 0$), and the other ASPA rules are chosen to meet the interference-outage constraint with equality. The average SEPs of all the rules decrease as Ω increases and reach error floors. This is because, for large Ω, the average SEP is dominated by the event in which the STx transmits with power τ/g_k. The error floor of the optimal rule is lower than that of the MI, MSP, and MR rules by a factor of 32.3, 2.3, and 2.3, respectively. It is lower than that of the on-off rule in (17) by a factor of 27, which shows that our power adaptation exploits the CSI much more effectively. Also, the linear rule is near-optimal.

Fig. 4 plots the average SEP of the linear rule as a function of Ω for two constellations and different values of N_t. We compare its performance when the penalty factor λ is obtained by equating the exact interference-outage probability O_λ with $O_{\max}$ and when it is obtained by equating the interference-outage upper bound B_λ in (14) with $O_{\max}$. As mentioned in Section IV, using B_λ ensures that the interference-outage probability is lower than $O_{\max}$. We see that the average SEP so obtained is only marginally more than that obtained using O_λ; the difference between the two vanishes as Ω increases. This is because B_λ is tight for small Ω and becomes exact for large Ω. Thus, B_λ helps in implementing the linear rule in a near-optimal manner with much lower complexity. We also see that the error floor decreases significantly as N_t increases.

VI. Conclusions

We proposed a novel and optimal ASPA rule that minimized the average SEP of an underlay spectrum sharing system that was subject to the interference-outage and peak transmit power constraints. We observed that the optimal transmit power and the net cost of each antenna were discontinuous functions of both STx-SRx and STx-PRx channel power gains and were very different from other rules in the literature. This brings out the fundamental role that the interference constraint plays in an underlay spectrum sharing system. We saw that the interference-outage probability of the linear rule could be bounded in closed-form, which made implementing it easy in practice. It also led to a closed-form expression for λ when $P_{\max}$ was large. We showed that both optimal and linear rules reduced the average SEP by a significant margin compared to other ASPA rules in the literature.

Appendix

A. Proof of Result 1

We shall say that an ASPA rule ϕ is feasible if it satisfies both constraints in (3) and (4). For any feasible rule ϕ, define

$$L_\phi(\lambda) \triangleq \mathbb{E}\left[\mathsf{S}(P_s, h_s) + \lambda I_{\{P_s g_s > \tau\}}\right], \tag{18}$$

where the expectation is over $\mathbf{h}$ and $\mathbf{g}$. Consider ϕ^* defined as

$$(s^*, P_{s^*}) \triangleq \underset{\{(k,P_k): k=\{1,2,\ldots,N_t\},\ P_k \in [0,P_{\max}]\}}{\arg\min} \{\mathsf{NC}_k\}, \tag{19}$$

where $\mathsf{NC}_k = \mathsf{S}(P_k, h_k) + \lambda I_{\{P_k g_k > \tau\}}$ and λ is set to be equal to $\lambda^* > 0$ such that $\Pr(P_{s^*} g_{s^*} > \tau) = O_{\max}$.[2] Clearly, by construction, ϕ^* is a feasible rule.

From (18), it is clear that $L_{\phi^*}(\lambda^*) \leq L_\phi(\lambda^*)$. Thus,

$$\mathbb{E}\big[\mathsf{S}(P_{s^*}, h_{s^*}) + \lambda^* I_{\{P_{s^*} g_{s^*} > \tau\}}\big] \leq \mathbb{E}\big[\mathsf{S}(P_s, h_s) + \lambda^* I_{\{P_s g_s > \tau\}}\big].$$

Using $\mathbb{E}\left[I_{\{a\}}\right] = \Pr(a)$, $\Pr(P_{s^*} g_{s^*} > \tau) = O_{\max}$, and rearranging the terms in the above inequality, we get

$$\mathbb{E}\left[\mathsf{S}(P_{s^*}, h_{s^*})\right] \leq \mathbb{E}\left[\mathsf{S}(P_s, h_s)\right] + \lambda^*(\Pr(P_s g_s > \tau) - O_{\max}).$$

As $\lambda^* > 0$ and $\Pr(P_s g_s > \tau) \leq O_{\max}$, it is clear that $\mathbb{E}\left[\mathsf{S}(P_{s^*}, h_{s^*})\right] \leq \mathbb{E}\left[\mathsf{S}(P_s, h_s)\right]$. Thus, ϕ^* is SEP-optimal.

Value of P_k that Minimizes NC_k: We consider two cases:

1) $P_{\max} g_k \leq \tau$: Here, for all $P_k \in [0, P_{\max}]$, we have $I_{\{P_k g_k > \tau\}} = 0$. As $\mathsf{S}(P_k, h_k)$ is a monotonically decreasing function of P_k, NC_k attains its minimum at $P_{\max}$.

[2]It can be shown that $\Pr(P_{s^*} g_{s^*} > \tau)$ is a monotonically decreasing and continuous function of λ. Then, the existence of a unique λ^* follows from the intermediate value theorem.

2) $P_{\max} g_k > \tau$: Here, for $P_k \in [0, \tau/g_k]$, we have $I_{\{P_k g_k > \tau\}} = 0$. Thus, $\mathsf{NC}_k = \mathsf{S}(P_k, h_k)$, where it takes the smallest value at $P_k = \tau/g_k$. Similarly, for $P_k \in (\tau/g_k, P_{\max}]$, we have $I_{\{P_k g_k > \tau\}} = 1$. Thus, $\mathsf{NC}_k = \mathsf{S}\left(P_{\max}, h_k\right) + \lambda$, where it takes the smallest value at $P_k = P_{\max}$. Thus, the value of P_k that minimizes NC_k can be written as

$$P_k = \begin{cases} P_{\max}, & \text{if } \mathsf{S}\left(\tau/g_k, h_k\right) > \mathsf{S}\left(P_{\max}, h_k\right) + \lambda, \\ \tau/g_k, & \text{else.} \end{cases} \quad (20)$$

Combining the above two cases yields (10).

B. Proof of Result 2

As the region $\widehat{\mathcal{I}}_k$ is obtained by dropping a positive term $\mathsf{S}\left(P_{\max}, h_k\right) > 0$, it can be shown that $\mathcal{I}_k \subset \widehat{\mathcal{I}}_k$, as can be seen from Fig. 2b. Thus, when $P_{\max} g_k > \tau$, the linear rule transmits with the peak power $P_{\max}$ with a higher probability than the optimal rule. Hence, its average SEP lower bounds that of the optimal rule.

C. Brief Proof of Result 3

Let $\widehat{U}_k \triangleq \{(h_k, g_k) \in \widehat{\mathcal{U}}_k\}$ and $\widehat{I}_k \triangleq \{(h_k, g_k) \in \widehat{\mathcal{I}}_k\}$ denote the events in which (h_k, g_k) belongs to the OCPP and OI regions, respectively. An interference-outage happens only when the linear rule selects an antenna s in the OI region. Thus, we can write

$$O_\lambda = \Pr\left(\widehat{I}_s\right) = N_t \Pr\left(s = 1, \widehat{I}_1\right), \quad (21)$$

where the second equality follows by symmetry. Using (13) and the definitions of the OCPC and OI regions in (11b) and (11c), respectively, it can be shown that antenna 1 in the OI region is selected only if no other antenna is in the OCPC region. Therefore, among the antennas $\{2, \ldots, N_t\}$, let k of them be in the OI region and the remaining be in the OCPP region. There are $\binom{N_t-1}{k}$ possible combinations, which by symmetry are equally likely.

Let $E_k = \{\widehat{I}_2, \ldots, \widehat{I}_{k+1}, \widehat{U}_{k+2}, \ldots, \widehat{U}_{N_t}\}$ be one such event where the antennas $2, \ldots, k+1$ are in the OI region and the antennas $k+2, \ldots, N_t$ are in the OCPP region. Then,

$$\Pr\left(s = 1, \widehat{I}_1\right) = \sum_{k=0}^{N_t-1} \binom{N_t - 1}{k} \Pr\left(s = 1, \widehat{I}_1, E_k\right), \quad (22)$$

For $1 \leq i \leq k+1$, from (13), we know that $\widehat{\mathsf{NC}}_i = \mathsf{S}\left(P_{\max}, h_i\right) + \lambda$. Similarly, for $k+2 \leq i \leq N_t$, $\widehat{\mathsf{NC}}_i = \mathsf{S}\left(P_{\max}, h_i\right)$. Therefore, the summand in (22) becomes

$$\begin{aligned} \Pr\left(s = 1, \widehat{I}_1, E_k\right) = \Pr\Big(&\widehat{I}_1, h_2 < h_1, \widehat{I}_2, \ldots, h_{k+1} < h_1, \\ &\widehat{I}_{k+1}, \mathsf{S}\left(P_{\max}, h_1\right) + \lambda < \mathsf{S}\left(P_{\max}, h_{k+2}\right), \widehat{U}_{k+2}, \ldots, \\ &\mathsf{S}\left(P_{\max}, h_1\right) + \lambda < \mathsf{S}\left(P_{\max}, h_{N_t}\right), \widehat{U}_{N_t}\Big). \end{aligned} \quad (23)$$

Conditioning on h_1, and using the i.i.d. assumption, we get

$$\Pr\left(s = 1, \widehat{I}_1, E_k | h_1 = x\right) = T_1(x) [T_2(x)]^k [T_3(x)]^{N_t - 1 - k}, \quad (24)$$

where $T_1(x) = \Pr\left(\widehat{I}_1 | h_1 = x\right)$, $T_2(x) = \Pr\left(h_2 < x, \widehat{I}_2\right)$, and $T_3(x) = \Pr\left(\mathsf{S}(P_{\max}, x) + \lambda < \mathsf{S}\left(P_{\max}, h_{k+2}\right), \widehat{U}_{k+2}\right)$.

We now evaluate these three terms separately.

1) *$T_1(x)$:* If $mx \leq \tau/P_{\max}$, then $T_1(x) = \Pr\left(P_{\max} g_1 > \tau\right) = O_{\mathrm{u}}$. Else, $T_1(x) = \Pr\left(g_1 > mx\right) = \exp\left(-mx/\mu_g\right)$.

2) *$T_2(x)$:* If $mx \leq \tau/P_{\max}$, $T_2(x) = O_{\mathrm{u}}\left(1 - \exp(-x/\mu_h)\right)$. Else, it can be simplified as

$$T_2(x) = O_{\mathrm{u}} - \frac{m \mu_h O_{\mathrm{u}}}{\mu_g + m \mu_h} \left(\frac{\lambda}{c_1}\right)^{\frac{1}{c_2 \Omega}} - \frac{\mu_g e^{-\left(\frac{m}{\mu_g} + \frac{1}{\mu_h}\right) x}}{\mu_g + m \mu_h}. \quad (25)$$

3) *$T_3(x)$:* We upper bound $T_3(x)$ by dropping $\mathsf{S}(P_{\max}, x)$ inside the probability term. Thus, we get $T_3(x) \leq \Pr\left(\lambda < \mathsf{S}\left(P_{\max}, h_{k+2}\right), P_{\max} g_{k+2} \leq \tau\right)$. Using the SEP expression in (1), we get $T_3(x) \leq \left(1 - O_{\mathrm{u}}\right)\left(1 - (\lambda/c_1)^{\frac{1}{c_2 \Omega}}\right)$.

Substituting $T_1(x)$, $T_2(x)$, and the bound for $T_3(x)$ in (24) and averaging over h_1 yields the bound for $\Pr\left(s = 1, \widehat{I}_1, E_k\right)$. Substituting it in (22) and (21), and simplifying yields (14).

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