Correlation-Aware Ordered Transmissions Scheme for Energy-Efficient Detection

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***Abstract*—Ordered transmissions is an energy-efficient scheme that improves the lifetime of a wireless sensor network (WSN) and yet achieves the same performance as the conventional unordered transmissions scheme (UTS) in which all nodes transmit. We present a novel scheme for the binary hypothesis testing problem that exploits ordered transmissions to reduce the number of transmissions for the general and practically important scenario in which the measurements of the sensor nodes are correlated. It differs from the literature that assumes that, conditioned on the hypotheses, the measurements of different nodes are statistically independent or that the covariance matrix has a special structure. In our scheme, the nodes transmit their measurements in the decreasing order of the magnitudes of the measurements. We present two novel approaches to design the decision rules. These address the presence of cross-terms between the measurements of different nodes that now arise in the decision statistic due to correlation and make a distributed implementation challenging. Both our approaches markedly reduce the average number of transmissions compared to UTS and ensure that the error probability remains the same.**

I. Introduction

Energy-efficiency is a critical issue in the design of wireless sensor networks (WSNs). It ensures that these networks have the longevity required by their many compelling and diverse applications such as transportation and logistics, environmental monitoring, military surveillance, and healthcare [1]. Several techniques have been proposed to improve the energy-efficiency of the sensor nodes, such as censoring, on-off keying, duty cycling, and clustering [2]. These improve the energy-efficiency by curtailing the number of times a node transmits. However, they also cause an unwelcome degradation in the performance of the WSN.

A notable exception is the ordered transmissions scheme (OTS), which improves the energy-efficiency without any degradation in performance [3]–[5]. OTS, which was first proposed in [3], exploits the fact that the decision statistic at the fusion node (FN) is separable and is the sum of the log-likelihood ratios (LLRs) of the measurements at the individual sensor nodes. In it, each node sets a timer that is a monotone non-increasing function of the absolute value of its LLR [6]. When a node's timer expires, it transmits its LLR to the FN. Thus, the nodes transmit their LLRs one after another in the decreasing order of the absolute value of the LLRs without knowing the LLR of any other node. Intuitively, the larger the absolute value of the LLR, the more informative it is to the FN. Every time the FN receives an LLR, it decides on a hypothesis or allows the nodes to further decrement their timers. OTS is appealing because it provably reduces the average number of transmissions by at least 50% compared to the conventional unordered transmissions scheme (UTS), in which all the nodes transmit their LLRs to the FN, without any increase in the error rate [3]. While the nodes transmit their LLRs one by one in sequential detection [7, Ch. III.D], they transmit in a random order, which is unlike OTS.

In this paper, we address the more general and practically relevant case in which the measurements of the sensor nodes are correlated. For example, this occurs when the node observations are distorted copies of the source signal due to sensing inaccuracies and noise [8]. In this case, the decision statistic at the FN is no longer the sum of the individual node statistics. Instead, it also consists of cross-terms involving each node's measurement with every other node's measurement. Thus, the OTS schemes considered in the literature that order the individual node LLRs no longer work.

For correlated measurements, OTS for detecting a shift in the mean vector and a shift in the covariance matrix has been recently proposed in [9] and [10], respectively. It assumes a decomposable Gaussian graphical model and exploits its elegant structure to partition the nodes into clusters. Each cluster has a cluster head, which receives the measurements from all of its constituent sensors. Ordering the transmissions of the cluster heads is shown to reduce the average number of transmissions required by the FN to decide.

A. Focus and Contributions

In this paper, we propose a novel correlation-aware OTS (CA-OTS) for the binary hypothesis framework that applies to an arbitrary correlation model in which the covariance matrices of the measurements by the nodes are different under the two hypotheses. It does not presume a decomposable Gaussian graphical model that requires the observations at non-adjacent nodes to be independent conditioned on all other observations. We propose two novel approaches to arrive at the decision rules. Both achieve significant reductions in the average number of transmissions, while achieving the same error probability as UTS, in which all the nodes transmit.

The first approach is called the eigenvalue bound-based approach (EVBA). It uses bounds on the eigenvalues of the covariance matrix of the measurements to arrive at the decision rules. The second approach, which refines this approach further, is called the received measurement-based

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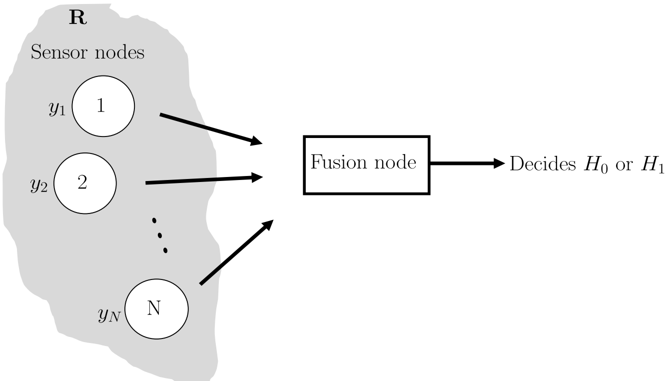

Fig. 1. A WSN consisting of N nodes that transmit their measurements to an FN, which decides in favor of a hypothesis.

EVBA (RM-EVBA). It uses bounds on the eigenvalues of the sub-matrix of the covariance matrix corresponding to the measurements that are yet to be received.

Our benchmarking results show that EVBA and RM-EVBA significantly reduce the average number of transmissions compared to conventional UTS. For example, for the uniform correlation model [11] in which the node measurements are equally correlated with a correlation-coefficient ρ, they reduce the average number of transmissions by 85% and 60% when ρ is 0.2 and 0.8, respectively.

There are two key differences between our approach and those in [9], [10]. First, our approach applies to an arbitrary correlation model. Second, in our approach, the ordering happens across all the sensor nodes. On the other hand, in [9], [10], all the nodes within a cluster transmit and ordering is exploited only across cluster heads.

B. Organization and Notations

Section II presents the system model. Section III specifies CA-OTS. Simulation results are presented in Section IV, and are followed by our conclusions in Section V.

Notations: The probability density function (PDF) and the cumulative distribution function (CDF) of a random variable (RV) X are denoted by $f_X(\cdot)$ and $F_X(\cdot)$, respectively. For a matrix $\mathbf{D}$, $\mathbf{D}^{\mathrm{T}}$ denotes its transpose, $\mathbf{D}_{ij}$ denotes the entry in the i^{th} row and j^{th} column, $\det(\mathbf{D})$ denotes its determinant, and $\lambda_{\min}(\mathbf{D})$ and $\lambda_{\max}(\mathbf{D})$ denotes the smallest and largest eigenvalues, respectively. We shall employ the order statistics notation [12], which is as follows. For N continuous RVs, $X_1, \ldots, X_N$, $X_{[r]}$ denotes the r^{th} largest value and $[r]$ denotes the index of this RV. Therefore, $X_{[1]} > X_{[2]} > \cdots > X_{[N]}$.

II. System Model

Consider a WSN that consists of N sensor nodes and an FN. Time is divided into measurement rounds. In each round, a decision needs to be made by the WSN. The node measurement model, detection framework, and the optimum decision rule are as follows.

Measurement Model and Detection Framework: At the beginning of a round, the sensor node i makes a measurement y_i, where $1 \leq i \leq N$. Within each round, the FN collects measurements from some or all of the sensor nodes and decides in favor of a hypothesis. We consider the binary hypothesis testing framework, which is a fundamental problem in signal detection theory [9], [10], and has several practical applications such as target detection [4], spectrum sensing [5], and fingerprint detection [13]. For Gaussian statistics, the signal models for the two hypotheses are [10]

$$\begin{aligned} H_0 &: \mathbf{y} \sim \mathcal{N}\left(\mathbf{0}, \sigma_0^2 \mathbf{I}_N\right), \\ H_1 &: \mathbf{y} \sim \mathcal{N}(\mathbf{0}, \mathbf{R}), \end{aligned} \tag{1}$$

where $\mathbf{y} = [y_1, y_2, \ldots, y_N]^{\mathrm{T}}$, $\mathbf{I}_N$ denotes the identity matrix of size N, and $\mathbf{R}$ is a symmetric positive definite matrix, which denotes the covariance matrix under hypothesis H_1. Thus, the measurements are i.i.d. with zero mean and variance σ_0^2 under hypothesis H_0 and are correlated with zero mean and a covariance matrix $\mathbf{R}$ under H_1. Our approach also applies to the case in which the covariance matrix for H_0 is another arbitrary positive definite matrix and not just $\mathbf{I}_N$. The results for this are not shown to conserve space. The system model is illustrated in Fig. 1.

Optimum Decision Rule: Let c_{uv} be the cost incurred if hypothesis H_u is chosen when hypothesis H_v is true, and let ζ_0 and ζ_1 be the prior probabilities of H_0 and H_1, respectively. The decision rule that minimizes the error probability is given by [7, Ch. III.A]

$$\log\left(\frac{f_{\mathbf{Y}}(\mathbf{y}|H_1)}{f_{\mathbf{Y}}(\mathbf{y}|H_0)}\right) \underset{H_0}{\overset{H_1}{\gtrless}} \beta, \tag{2}$$

where $f_{\mathbf{Y}}(\mathbf{y}|H_h)$ is the PDF of the measurements conditioned on the hypothesis H_h, where $h \in [0,1]$, and $\beta = \log\left(\frac{(c_{10}-c_{00})\zeta_0}{(c_{01}-c_{11})\zeta_1}\right)$. For the signal model in (1), it can be easily shown that the decision rule in (2) reduces to

$$d(\mathbf{y}) \triangleq \mathbf{y}^{\mathrm{T}} \mathbf{G} \mathbf{y} \underset{H_0}{\overset{H_1}{\gtrless}} \alpha, \tag{3}$$

where

$$\alpha = 2\sigma_0^2 \left[\beta + \frac{1}{2}\log\left(\frac{\det(\mathbf{R})}{\sigma_0^{2N}}\right)\right], \tag{4}$$

$$\mathbf{G} = \mathbf{I}_N - \sigma_0^2 \mathbf{R}^{-1}. \tag{5}$$

We shall refer to $d(\mathbf{y})$ as the decision statistic.

Expanding (3), we get

$$d(\mathbf{y}) = \sum_{i=1}^{N} \mathbf{G}_{ii} y_i^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \mathbf{G}_{ij} y_i y_j. \tag{6}$$

Thus, as mentioned, in the presence of correlation, $d(\mathbf{y})$ also depends on the cross-terms $y_i y_j$, for $i \neq j$.

III. CA-OTS

In CA-OTS, a node i sets a timer that is a monotone non-increasing function of $|y_i|$, which we shall refer to as the metric of the node. Once the timer expires, it transmits its measurement y_i and its identity i to the FN. The timer scheme ensures that the nodes transmit in the decreasing order of their metrics, without any node knowing the metric of any other node a priori.

Every time the FN receives a measurement, it decides between the two hypotheses, or waits for the next measurement. Once the FN makes a decision, it broadcasts a control signal to all the sensor nodes to halt their timers for the rest of the round. Else, the nodes continue to decrement their timers. The process starts afresh at the beginning of every round when a new set of measurements arrives at the nodes. As in OTS, we assume that the measurements are received at the FN with a negligible probability of decoding error [3], [9], [10].

We now present two approaches to derive the decision rules for the above general correlation model.

A. EVBA

The following result, which is derived in Appendix A, compactly specifies the new decision rules for EVBA.

Result *1:* Let the FN have received the measurements $y_{[1]},\ldots,y_{[k]}$ from nodes $[1],\ldots,[k]$. The decision rules depend on the range of values of $\lambda_{\max}(\mathbf{R})$ and $\lambda_{\min}(\mathbf{R})$, and are as follows:

1) If $\lambda_{\min}(\mathbf{R}) \geq \sigma_0^2$:

$$\text{Decide } H_1 \text{ if: } \left(1-\frac{\sigma_0^2}{\lambda_{\min}(\mathbf{R})}\right)\sum_{i=1}^{k} y_{[i]}^2 > \alpha, \tag{7}$$

$$\text{Decide } H_0 \text{ if: } \left(1-\frac{\sigma_0^2}{\lambda_{\max}(\mathbf{R})}\right)\sum_{i=1}^{k} y_{[i]}^2 < \alpha-\left(1-\frac{\sigma_0^2}{\lambda_{\max}(\mathbf{R})}\right)(N-k)y_{[k]}^2, \tag{8}$$

Wait for the next transmission, otherwise.

2) If $\lambda_{\min}(\mathbf{R}) < \sigma_0^2 \leq \lambda_{\max}(\mathbf{R})$:

$$\text{Decide } H_1 \text{ if: } \left(1-\frac{\sigma_0^2}{\lambda_{\min}(\mathbf{R})}\right)\sum_{i=1}^{k} y_{[i]}^2 > \alpha-\left(1-\frac{\sigma_0^2}{\lambda_{\min}(\mathbf{R})}\right)(N-k)y_{[k]}^2, \tag{9}$$

$$\text{Decide } H_0 \text{ if: } \left(1-\frac{\sigma_0^2}{\lambda_{\max}(\mathbf{R})}\right)\sum_{i=1}^{k} y_{[i]}^2 < \alpha-\left(1-\frac{\sigma_0^2}{\lambda_{\max}(\mathbf{R})}\right)(N-k)y_{[k]}^2, \tag{10}$$

Wait for the next transmission, otherwise.

3) If $\lambda_{\max}(\mathbf{R}) \leq \sigma_0^2$:

$$\text{Decide } H_1 \text{ if: } \left(1-\frac{\sigma_0^2}{\lambda_{\min}(\mathbf{R})}\right)\sum_{i=1}^{k} y_{[i]}^2 > \alpha-\left(1-\frac{\sigma_0^2}{\lambda_{\min}(\mathbf{R})}\right)(N-k)y_{[k]}^2, \tag{11}$$

$$\text{Decide } H_0 \text{ if: } \left(1-\frac{\sigma_0^2}{\lambda_{\max}(\mathbf{R})}\right)\sum_{i=1}^{k} y_{[i]}^2 < \alpha, \tag{12}$$

Wait for the next transmission, otherwise.

Remark: As shown in Appendix A, for each case, the decision is the same as UTS, in which the FN knows all the N measurements. The decision rules only require the maximum and minimum eigenvalues of $\mathbf{R}$, which are related to those of $\mathbf{G}$ by $\lambda_{\max}(\mathbf{G}) = 1-\left(\sigma_0^2/\lambda_{\max}(\mathbf{R})\right)$ and $\lambda_{\min}(\mathbf{G}) = 1-\left(\sigma_0^2/\lambda_{\min}(\mathbf{R})\right)$.

B. RM-EVBA

We now present a more refined version of EVBA that considers the sub-matrices of $\mathbf{R}$ corresponding to the received and the yet-to-be-received measurements. In terms of the ordered-statistics notation, (6) can be expressed as

$$d(\mathbf{y}) = \sum_{i=1}^{N}\mathbf{G}_{[i][i]}y_{[i]}^2 + \sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}\mathbf{G}_{[i][j]}y_{[i]}y_{[j]}. \tag{13}$$

When the FN has received the measurements from nodes $[1],\ldots,[k]$, we write (13) in a compact matrix notation as follows. Let $\mathbf{r}_k = \left[y_{[1]},y_{[2]},\ldots,y_{[k]}\right]^{\mathrm{T}}$. Let $\mathbf{P}_k$ be a $k\times k$ matrix that is given by

$$\mathbf{P}_k = \left[\mathbf{G}_{[i][j]}\right], \text{ for } 1\leq i,j\leq k. \tag{14}$$

Let $\mathbf{Q}_{N-k}$ be an $(N-k)\times(N-k)$ matrix that is given by

$$\mathbf{Q}_{N-k} = \left[\mathbf{G}_{[k+i][k+j]}\right], \text{ for } 1\leq i,j\leq N-k. \tag{15}$$

Lastly,

$$\mathbf{v}_{N-k} = \left[\sum_{i=1}^{k}\mathbf{G}_{[i][k+1]}y_{[i]},\sum_{i=1}^{k}\mathbf{G}_{[i][k+2]}y_{[i]},\ldots,\sum_{i=1}^{k}\mathbf{G}_{[i][N]}y_{[i]}\right]^{\mathrm{T}}. \tag{16}$$

Thus, $\mathbf{r}_k$ denotes the received measurements, $\mathbf{P}_k$ consists of the elements of $\mathbf{G}$ that correspond to the received measurements, $\mathbf{Q}_{N-k}$ consists of the elements of $\mathbf{G}$ that correspond to the yet-to-be-received measurements, and $\mathbf{v}_{N-k}$ accounts for the cross-terms between the received and yet-to-be-received measurements. Let $\mathbf{u}_{N-k} = \left[y_{[k+1]},\ldots,y_{[N]}\right]^{\mathrm{T}}$ be the vector of measurements that are yet to be received. Hence, (13) can be recast as

$$d(\mathbf{y}) = \mathbf{r}_k^{\mathrm{T}}\mathbf{P}_k\mathbf{r}_k + \mathbf{u}_{N-k}^{\mathrm{T}}\mathbf{Q}_{N-k}\mathbf{u}_{N-k} + 2\mathbf{v}_{N-k}^{\mathrm{T}}\mathbf{u}_{N-k}. \tag{17}$$

The decision rules for RM-EVBA, which are derived in Appendix B, are as follows.

Result *2:* When the measurements from the nodes $[1],\ldots,[k]$ have been received, the decision rules, which depend on the eigenvalues of $\mathbf{Q}_{N-k}$, are as follows:

1) If $\lambda_{\min}(\mathbf{Q}_{N-k}) \geq 0$:

$$\text{Decide } H_1 \text{ if: } \mathbf{r}_k^{\mathrm{T}}\mathbf{P}_k\mathbf{r}_k > \alpha + 2|y_{[k]}| \times \sum_{i=1}^{k}\sum_{j=k+1}^{N}\left|\mathbf{G}_{[i][j]}y_{[i]}\right|, \tag{18}$$

$$\text{Decide } H_0 \text{ if: } \mathbf{r}_k^{\mathrm{T}}\mathbf{P}_k\mathbf{r}_k < \alpha - \lambda_{\max}(\mathbf{Q}_{N-k})(N-k)y_{[k]}^2 - 2|y_{[k]}|\sum_{i=1}^{k}\sum_{j=k+1}^{N}\left|\mathbf{G}_{[i][j]}y_{[i]}\right|, \tag{19}$$

Wait for the next transmission, otherwise.

2) If $\lambda_{\min}(\mathbf{Q}_{N-k}) < 0 \leq \lambda_{\max}(\mathbf{Q}_{N-k})$:

Decide H_1 if: $$\mathbf{r}_k^{\mathrm{T}}\mathbf{P}_k\mathbf{r}_k > \alpha - \lambda_{\min}(\mathbf{Q}_{N-k})(N-k)y_{[k]}^2 + 2|y_{[k]}|\sum_{i=1}^{k}\sum_{j=k+1}^{N}|\mathbf{G}_{[i][j]}y_{[i]}|, \quad (20)$$

Decide H_0 if: $$\mathbf{r}_k^{\mathrm{T}}\mathbf{P}_k\mathbf{r}_k < \alpha - \lambda_{\max}(\mathbf{Q}_{N-k})(N-k)y_{[k]}^2 - 2|y_{[k]}|\sum_{i=1}^{k}\sum_{j=k+1}^{N}|\mathbf{G}_{[i][j]}y_{[i]}|, \quad (21)$$

Wait for the next transmission, otherwise.

3) If $\lambda_{\max}(\mathbf{Q}_{N-k}) \leq 0$:

Decide H_1 if: $$\mathbf{r}_k^{\mathrm{T}}\mathbf{P}_k\mathbf{r}_k > \alpha - \lambda_{\min}(\mathbf{Q}_{N-k})(N-k)y_{[k]}^2 + 2|y_{[k]}|\sum_{i=1}^{k}\sum_{j=k+1}^{N}|\mathbf{G}_{[i][j]}y_{[i]}|, \quad (22)$$

Decide H_0 if: $$\mathbf{r}_k^{\mathrm{T}}\mathbf{P}_k\mathbf{r}_k < \alpha - 2|y_{[k]}| \times \sum_{i=1}^{k}\sum_{j=k+1}^{N}|\mathbf{G}_{[i][j]}y_{[i]}|, \quad (23)$$

Wait for the next transmission, otherwise.

Remark: RM-EVBA yields the same error probability as UTS.

IV. Numerical Results

We now benchmark the performance of EVBA and RM-EVBA. Since they ensure that the error probability is the same as UTS, we compare their average number of transmissions. The smaller this number compared to UTS, the more energy-efficient the scheme is. We simulate the WSN for a duration of 10^6 rounds. We consider the uniform cost model with $c_{uv} = 1$, if $u \neq v$, and $c_{uv} = 0$, if $u = v$ [3], [10], and $\zeta_1 = 0.5$.

To illustrate the generality of our approach, we first show results for the correlation model in [14], where $\mathbf{R}_{ij} = \sigma_1^2\theta_i\theta_j$. We set $\theta_i = \theta_{\max} - (i-1)((\theta_{\max} - \theta_{\min})/(N-1))$, for $1 \leq i \leq N$. Thus, the correlation-coefficient of the measurements between two sensors i and j is $\theta_i\theta_j$, and $\theta_1, \ldots, \theta_N$ are obtained by uniformly sampling the interval $[\theta_{\min}, \theta_{\max}]$.

Fig. 2 compares the average number of transmissions as a function of the signal-to-noise ratio (SNR) σ_1^2/σ_0^2 of EVBA and RM-EVBA with UTS for a WSN with $N = 20$ sensor nodes. The number of transmissions for UTS is always N. As the SNR increases, both EVBA and RM-EVBA require far fewer transmissions to make a decision as compared to UTS. RM-EVBA, which uses a more refined approach, requires fewer transmissions than EVBA, especially for smaller SNRs. However, as seen from (6), the coefficients of the cross terms are negligible at higher SNRs. Hence, in this regime, the two schemes exhibit the same behavior.

To gain further insights about the effect of correlation, we now consider the widely studied uniform correlation model [11], which is defined by a single parameter $\rho \in [-1, 1]$ as follows: $\mathbf{R}_{ij} = \sigma_1^2\rho$, if $i \neq j$, and is σ_1^2, if $i = j$. Thus, ρ is the correlation-coefficient between the measurements of any two nodes. For this model, the corresponding probabilistic graphical model has a single maximal clique. Consequently, the OTS approach in [10], which requires all the nodes in a maximal clique to transmit to the cluster head, does not achieve any reduction in the average number of transmissions.

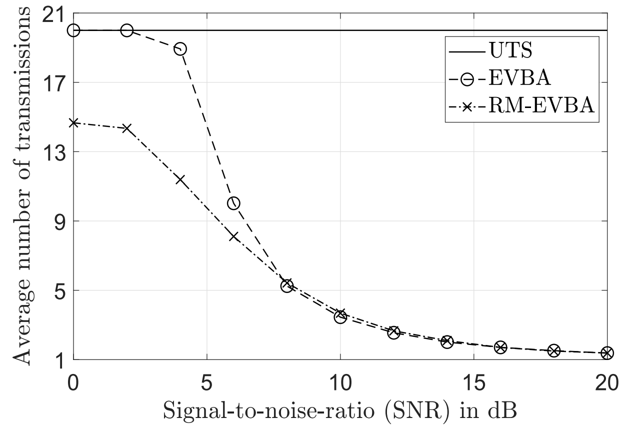

Fig. 2. Average number of transmissions as a function of the SNR σ_1^2/σ_0^2 for arbitrary correlation ($N = 20$, $\theta_{\max} = 0.75$, and $\theta_{\min} = 0.25$).

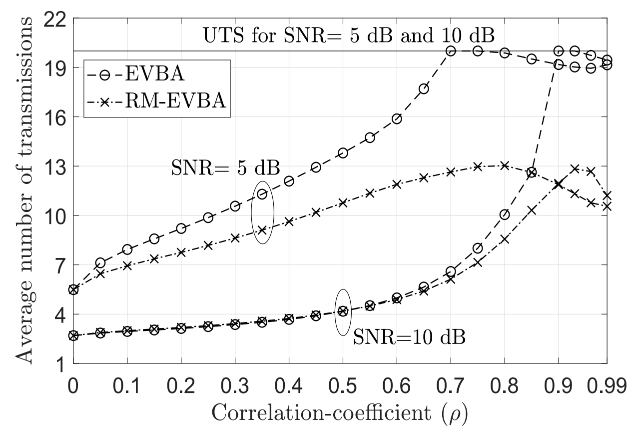

Fig. 3. Average number of transmissions as a function of ρ for different values of SNR for uniform correlation ($N = 20$).

Fig. 3 compares the average number of transmissions as a function of ρ of EVBA, RM-EVBA, and UTS for $N = 20$ and two values of SNR. Both EVBA and RM-EVBA require significantly fewer transmissions compared to UTS, for which the average number of transmissions is always N. Consider, for example, the SNR of 5 dB. As ρ increases, the average number of transmissions of EVBA and RM-EVBA initially increase. This can be understood using the notion of generalized variance, which is a measure of dispersion for a multivariate distribution and is the determinant of the covariance matrix $\mathbf{R}$ [15, Ch. 7.5]. From [11], $\det(\mathbf{R}) = \sigma_1^{2N}(1-\rho)^{N-1}(1+(N-1)\rho)$. As ρ increases, $\det(\mathbf{R})$ decreases and H_1 becomes less dispersed compared to H_0, the corresponding determinant for which is $\det(\sigma_0^2\mathbf{I}_N) = \sigma_0^{2N}$. Hence, more transmissions are needed to make a decision.

When ρ increases further, the average number of transmissions of both schemes decrease. This is because $\det(\mathbf{R})$

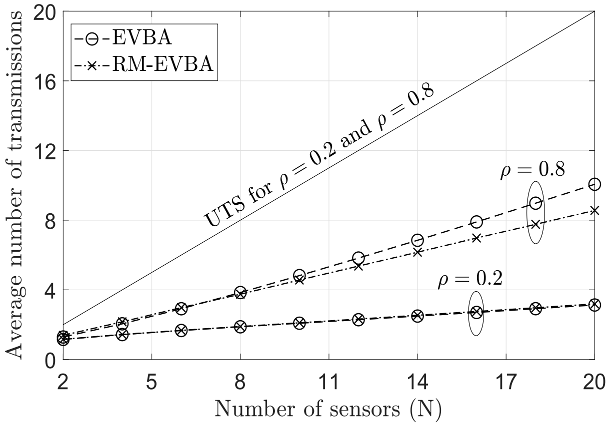

Fig. 4. Average number of transmissions as a function of N for uniform correlation for different values of ρ (SNR $= 10$ dB).

becomes significantly less than σ_0^{2N} and, thus, H_0 is more dispersed than H_1. Since the relative dispersion between the two hypotheses increases, fewer transmissions are needed to make a decision. Similar behavior is seen for the SNR of 10 dB, except that fewer transmissions occur for $\rho \leq 0.85$.

Fig. 4 plots the average number of transmissions as a function of N for $\rho = 0.2$ and 0.8. We see that as N increases, the average number of transmissions for all the schemes increases. However, the rate at which it increases is much smaller for EVBA and RM-EVBA than UTS.

V. Conclusions

We proposed a novel ordered transmissions scheme for detecting a shift in the covariance matrix. It applied to any general positive definite covariance matrix and addressed the presence of cross-terms involving measurements of different nodes that arose in the decision statistic. In it, the nodes transmitted their measurements to the FN in the decreasing order of the magnitudes of their measurements. To develop the decision rules, EVBA considered the smallest and largest eigenvalues of the entire covariance matrix, while RM-EVBA considered the smallest and largest eigenvalues of the submatrix of the covariance matrix corresponding to the measurements that are yet to be received by the FN. Both approaches markedly reduced the average number of transmissions required by the FN to decide without any increase in the error rate compared to UTS, in which all nodes transmitted. Due to its more refined approach, RM-EVBA outperformed EVBA.

Future work includes extending the approach to address the shift in mean hypothesis testing problem, analytically characterizing the average number of transmissions for the proposed decision rules, and incorporating the impact of channel fading.

Appendix

A. Decision Rules for EVBA

As $\mathbf{G}$ is a symmetric matrix, from [16, Ch. A.5.2], we get

$$\lambda_{\min}(\mathbf{G})\,\mathbf{y}^{\mathrm{T}}\mathbf{y} \leq d(\mathbf{y}) \leq \lambda_{\max}(\mathbf{G})\,\mathbf{y}^{\mathrm{T}}\mathbf{y}. \tag{24}$$

From (5), the maximum and minimum eigenvalues of $\mathbf{G}$ and $\mathbf{R}$ are related as

$$\lambda_{\max}(\mathbf{G}) = 1 - \frac{\sigma_0^2}{\lambda_{\max}(\mathbf{R})}, \tag{25}$$

$$\lambda_{\min}(\mathbf{G}) = 1 - \frac{\sigma_0^2}{\lambda_{\min}(\mathbf{R})}. \tag{26}$$

Writing $\mathbf{y}^{\mathrm{T}}\mathbf{y}$ in terms of the measurements received thus far from the nodes $[1], \ldots, [k]$, and the ones not yet received from the nodes $[k+1], \ldots, [N]$, we get

$$\mathbf{y}^{\mathrm{T}}\mathbf{y} = \sum_{i=1}^{N} y_{[i]}^2 = \sum_{i=1}^{k} y_{[i]}^2 + \sum_{j=k+1}^{N} y_{[j]}^2. \tag{27}$$

Since the nodes transmit in the decreasing order of the absolute values of their measurements, we have

$$0 \leq y_{[j]}^2 < y_{[k]}^2, \text{ for } 1 \leq k < j \leq N. \tag{28}$$

To apply this inequality in (24), we need to consider the signs of $\lambda_{\min}(\mathbf{G})$ and $\lambda_{\max}(\mathbf{G})$, which are determined by the values of $\lambda_{\min}(\mathbf{R})$ and $\lambda_{\max}(\mathbf{R})$. This gives rise to the following three cases:

1. If $\lambda_{\min}(\mathbf{R}) \geq \sigma_0^2$: From (26), this implies $\lambda_{\min}(\mathbf{G}) \geq 0$. Hence, (24), (27), and (28) imply the following:

$$\lambda_{\min}(\mathbf{G}) \left(\sum_{i=1}^{k} y_{[i]}^2 \right) < d(\mathbf{y}), \tag{29}$$

$$d(\mathbf{y}) < \lambda_{\max}(\mathbf{G}) \left(\sum_{i=1}^{N} y_{[i]}^2 + (N-k) y_{[k]}^2 \right). \tag{30}$$

Therefore, if

$$\lambda_{\min}(\mathbf{G}) \left(\sum_{i=1}^{k} y_{[i]}^2 \right) > \alpha, \tag{31}$$

then from (29), we get $d(\mathbf{y}) > \alpha$. Hence, the FN should decide H_1, just as UTS would after receiving all N measurements. Substituting (26) in (31) yields (7). Similarly, if

$$\lambda_{\max}(\mathbf{G}) \left(\sum_{i=1}^{k} y_{[i]}^2 \right) < \alpha - \lambda_{\max}(\mathbf{G})\,(N-k) y_{[k]}^2, \tag{32}$$

then from (30), $d(\mathbf{y}) < \alpha$ and the FN should decide H_0.

2. If $\lambda_{\min}(\mathbf{R}) < \sigma_0^2 \leq \lambda_{\max}(\mathbf{R})$: From (25) and (26), we get $\lambda_{\min}(\mathbf{G}) < 0 \leq \lambda_{\max}(\mathbf{G})$. Thus, (24), (27), and (28) imply

$$\lambda_{\min}(\mathbf{G}) \left(\sum_{i=1}^{k} y_{[i]}^2 + (N-k) y_{[k]}^2 \right) < d(\mathbf{y}). \tag{33}$$

Therefore, if $\lambda_{\min}(\mathbf{G}) \left(\sum_{i=1}^{k} y_{[i]}^2 + (N-k) y_{[k]}^2 \right) > \alpha$, then from (33), $d(\mathbf{y}) > \alpha$ and the FN should decide H_1. Substituting (26) and rearranging terms yields (9). The decision rule for H_0 is obtained using a similar logic.

3. If $\lambda_{\max}(\mathbf{R}) \leq \sigma_0^2$: From (25), we have $\lambda_{\max}(\mathbf{G}) \leq 0$. From (24), (27), and (28), we get

$$d(\mathbf{y}) < \lambda_{\max}(\mathbf{G}) \left(\sum_{i=1}^{k} y_{[i]}^2 \right). \tag{34}$$

Thus, the FN will decide H_0 if $\lambda_{\max}(\mathbf{G})\left(\sum_{i=1}^{k} y_{[i]}^2\right) < \alpha$. Substituting (25) yields the decision rule in (12). The decision rule for H_1 can be derived in a similar manner.

We note that while the bounds in (24) are classical in nature, their application to the design of the decision rules for OTS is novel to the best of our knowledge.

B. Decision Rules for RM-EVBA

In (17), the decision statistic is a sum of three terms. The first term $\mathbf{r}_k^{\mathrm{T}}\mathbf{P}_k\mathbf{r}_k$ is in terms of measurements that are known to the FN. The second term $\mathbf{u}_{N-k}^{\mathrm{T}}\mathbf{Q}_{N-k}\mathbf{u}_{N-k}$ can be bounded, along lines similar to (24), as follows:

$$\lambda_{\min}(\mathbf{Q}_{N-k})\left(\sum_{j=k+1}^{N} y_{[j]}^2\right) \leq \mathbf{u}_{N-k}^{\mathrm{T}}\mathbf{Q}_{N-k}\mathbf{u}_{N-k} \leq \lambda_{\max}(\mathbf{Q}_{N-k})\left(\sum_{j=k+1}^{N} y_{[j]}^2\right). \quad (35)$$

To bound the third term $\mathbf{v}_{N-k}^{\mathrm{T}}\mathbf{u}_{N-k}$, we note that $-|y_{[k]}| < y_{[j]} < |y_{[k]}|$ and $0 < \sum_{j=k+1}^{N} y_{[j]}^2 < (N-k)y_{[k]}^2$, for $k+1 \leq j \leq N$. Using these inequalities, we can show that

$$-|y_{[k]}|\sum_{i=1}^{k}\sum_{j=k+1}^{N}|\mathbf{G}_{[i][j]}y_{[i]}| < \mathbf{v}_{N-k}^{\mathrm{T}}\mathbf{u}_{N-k} < |y_{[k]}|\sum_{i=1}^{k}\sum_{j=k+1}^{N}|\mathbf{G}_{[i][j]}y_{[i]}|. \quad (36)$$

As in Appendix A, the following three cases arise:

1. If $\lambda_{\min}(\mathbf{Q}_{N-k}) \geq 0$: Using (35) and (36) in (17) yields

$$d(\mathbf{y}) < \mathbf{r}_k^{\mathrm{T}}\mathbf{P}_k\mathbf{r}_k + \lambda_{\max}(\mathbf{Q}_{N-k})\,(N-k)y_{[k]}^2 + 2|y_{[k]}|\sum_{i=1}^{k}\sum_{j=k+1}^{N}|\mathbf{G}_{[i][j]}y_{[i]}|, \quad (37)$$

$$d(\mathbf{y}) > \mathbf{r}_k^{\mathrm{T}}\mathbf{P}_k\mathbf{r}_k - 2|y_{[k]}|\sum_{i=1}^{k}\sum_{j=k+1}^{N}|\mathbf{G}_{[i][j]}y_{[i]}|. \quad (38)$$

Therefore, if $\mathbf{r}_k^{\mathrm{T}}\mathbf{P}_k\mathbf{r}_k < \alpha - \lambda_{\max}(\mathbf{Q}_{N-k})\,(N-k)y_{[k]}^2 - 2|y_{[k]}|\sum_{i=1}^{k}\sum_{j=k+1}^{N}|\mathbf{G}_{[i][j]}y_{[i]}|$, then it follows from from (37) that $d(\mathbf{y}) < \alpha$. Thus, the FN should decide H_0. Similarly, if $\mathbf{r}_k^{\mathrm{T}}\mathbf{P}_k\mathbf{r}_k > \alpha + 2|y_{[k]}|\sum_{i=1}^{k}\sum_{j=k+1}^{N}|\mathbf{G}_{[i][j]}y_{[i]}|$, then from (38), $d(\mathbf{y}) > \alpha$ and the FN should decide H_1.

2. If $\lambda_{\min}(\mathbf{Q}_{N-k}) < 0 \leq \lambda_{\max}(\mathbf{Q}_{N-k})$: From (35) and (36), we have

$$d(\mathbf{y}) > \mathbf{r}_k^{\mathrm{T}}\mathbf{P}_k\mathbf{r}_k + \lambda_{\min}(\mathbf{Q}_{N-k})\,(N-k)y_{[k]}^2 - 2|y_{[k]}|\sum_{i=1}^{k}\sum_{j=k+1}^{N}|\mathbf{G}_{[i][j]}y_{[i]}|. \quad (39)$$

In this case, the upper bound on $d(\mathbf{y})$ is the same as (37). Hence, it can be shown that the condition for deciding H_0 will be exactly the same as that of the first case. This yields (21). Similarly, if

$$\mathbf{r}_k^{\mathrm{T}}\mathbf{P}_k\mathbf{r}_k > \alpha - \lambda_{\min}(\mathbf{Q}_{N-k})\,(N-k)y_{[k]}^2 + 2|y_{[k]}|\sum_{i=1}^{k}\sum_{j=k+1}^{N}|\mathbf{G}_{[i][j]}y_{[i]}|, \quad (40)$$

then from (39), $d(\mathbf{y}) > \alpha$. Thus, the FN should decide H_1.

3. If $\lambda_{\max}(\mathbf{Q}_{N-k}) \leq 0$: We have

$$d(\mathbf{y}) < \mathbf{r}_k^{\mathrm{T}}\mathbf{P}_k\mathbf{r}_k + 2|y_{[k]}|\sum_{i=1}^{k}\sum_{j=k+1}^{N}|\mathbf{G}_{[i][j]}y_{[i]}|. \quad (41)$$

Therefore, if $\mathbf{r}_k^{\mathrm{T}}\mathbf{P}_k\mathbf{r}_k < \alpha - 2|y_{[k]}|\sum_{i=1}^{k}\sum_{j=k+1}^{N}|\mathbf{G}_{[i][j]}y_{[i]}|$, then from (41), $d(\mathbf{y}) < \alpha$ and the FN should decide H_0.

The lower bound on $d(\mathbf{y})$ is the same as (39). Hence, it can be shown that the condition for deciding H_1 will be exactly the same as that of the second case.

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