Improving Energy-Efficiency Using Successively Reordered Transmissions and Feedback

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***Abstract*—For the binary hypothesis testing problem, we propose a novel feedback-enhanced successively reordered transmissions scheme (FE-SRTS), in which the nodes change the order in which they transmit based on the feedback from the fusion node (FN) in each step. It can be implemented in a distributed manner using the timer scheme without any node knowing the measurement of any other node. We derive novel decision rules for it that enable the FN to decide on a hypothesis after receiving only a subset of measurements. For the Bayesian detection framework, FE-SRTS achieves the same optimal error probability as the unordered transmissions scheme (UTS), in which all the nodes transmit their log-likelihood ratios to the FN. However, it requires far fewer nodes to transmit, on average, than UTS, which leads to a much higher energy-efficiency. As the signal-to-noise ratio increases, the average number of transmissions of FE-SRTS decreases to two. This is much lower than the average number of transmissions of the conventional ordered transmissions scheme, which does not employ feedback and does not update the order in which the nodes transmit.**

I. Introduction

Distributed detection in wireless sensor networks (WSNs) has applications in diverse fields such as surveillance, healthcare, environmental monitoring, inventory management, product quality monitoring, transportation, and logistics. In these networks, geographically distributed sensor nodes make measurements and communicate some or all of them to a fusion node (FN), which makes a decision. While the FN is often connected to the power grid, the sensor nodes are powered by batteries whose energies drain out over time. Therefore, improving the sensor nodes' energy-efficiency, which leads to a longer network lifetime, is a critical issue in these applications. Techniques such as censoring, on-off keying, duty cycling, and clustering improve the energy-efficiency by curtailing the number of nodes that transmit [1]. However, this invariably leads to a degradation in performance.

A notable exception is the ordered transmissions scheme (OTS) in which fewer nodes transmit on average without any degradation in the error probability, which is the probability that an incorrect hypothesis is detected [2]–[8]. In OTS, the nodes transmit in the decreasing order of the absolute value of their log-likelihood ratio (LLR). This is implemented in a decentralized manner using the timer scheme, which ensures that no node, including the FN, has to know the LLR of any other node [9], [10]. Every time the FN receives an LLR from a node, it decides on a hypothesis or waits for the next transmission. Once the FN decides, it broadcasts a control signal to stop the other nodes from transmitting.

In [3], OTS is combined with power control to improve the energy efficiency. Nodes transmit over different coherence intervals and OTS is used to determine which node transmits in a coherence interval. In [4], an ordered transmissions-based sequential detection scheme is used for spectrum sensing. In [5], a decentralized nearest-neighbor regression is implemented using OTS. In [6], a combination of OTS and slotted Aloha is implemented for detecting a shift in the mean for the binary hypothesis testing problem. In [7], OTS is adapted for energy harvesting sensor networks, where some sensors can fail to transmit to the FN due to lack of energy. In [8], OTS is generalized to handle correlated sensor measurements.

A. Contributions

We propose a novel feedback-enhanced successively reordered transmissions scheme (FE-SRTS) for the binary hypothesis testing problem. In this scheme, the node with the highest absolute value of LLR first transmits its LLR to the FN. The FN either decides or broadcasts the LLR it received to all the nodes. The feedback is used to reorder the sequence of nodes that transmit. The node whose LLR is the farthest from the fed back LLR transmits next. The FN either decides or feeds back, as above, its last received LLR. This process continues until the FN decides. As in OTS, in each step, this ordering of transmissions is implemented in a distributed manner using the timer scheme. When the FN decides, it broadcasts a signal to the nodes to stop their timers.

We derive novel decision rules for the FN that require fewer nodes that transmit on average, but with same error probability as the optimal unordered transmissions scheme (UTS) in which all the nodes transmit their LLRs to the FN. We then derive an expression for the average number of transmissions. Our numerical results show that FE-SRTS requires markedly fewer transmissions than both UTS and OTS for the same error probability. We also prove that the average number of transmissions decreases to two as the signal-to-noise ratio (SNR) increases for any number of sensor nodes and prior probability. This is much lower than OTS, in which the number of transmissions decreases only by a factor of two at larger SNRs.

Feedback for the detection problem has received less attention in the literature [11]–[13]. In [11], the FN feeds back its decision to the nodes after receiving local decisions from them. The nodes then update their local decisions and send them back to the FN. In [12], all sensors in a first

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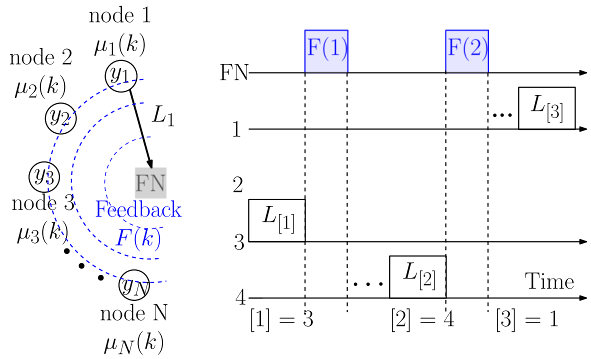

Fig. 1. System model showing measurements, LLRs, metrics that determine which node transmits first, and feedback by the FN.

subset make a decision based on their local measurements and send them to the FN. The nodes in a second subset then make individual decisions using their measurements and an intermediate decision fed back by the FN. In [13], the nodes transmit their decisions to an intermediate detector, which then feeds back its decision to all the nodes. The nodes update their decisions and transmit them to the FN, which makes the final decision. Unlike FE-SRTS, the above schemes require each node to transmit at least once. Furthermore, our application of feedback to successively reorder transmissions is novel.

B. Organization and Notations

Section II presents the system model. Section III specifies FE-SRTS and its decision rules. We present simulation results in Section IV, and our conclusions follow in Section V.

Notations: The probability of an event A is denoted by $\Pr(A)$, and the conditional probability of A given event B by $\Pr(A|B)$. The expectation with respect to a random variable (RV) X is denoted by $\mathbb{E}_X[\cdot]$. We denote vectors in bold font. $\mathbb{1}_{[A]}$ denotes the indicator function; it equals 1 if A is true and is 0 otherwise. The notation $y_i \sim \mathcal{N}(\mu, \sigma^2)$ means that y_i is a Gaussian RV with mean μ and a variance σ^2. For N continuous RVs $X_1, X_2, \ldots, X_N$, $r:N$ denotes the index of the RV with the r^{th} largest realization. Thus, $X_{1:N} > X_{2:N} > \cdots > X_{N:N}$.

II. System Model

We consider a WSN that consists of an FN and N sensor nodes. Time is divided into measurement cycles. In each cycle, based on measurements made by the nodes, the FN needs to make a decision. The node measurement model and detection framework are as follows.

A. Measurement Model and Detection Framework

The sensor measurements follow Gaussian statistics, which is commonly assumed in the literature [2]–[7]. Measurements by the nodes are mutually independent and identically distributed (i.i.d.) when conditioned on the hypotheses H_0 and H_1. Here, H_0 represents the null hypothesis and H_1 the true hypothesis. For example, in target detection, H_0 models the absence of a target, while H_1 models its presence. The measurement y_i by node i is given by

$$y_i \sim \mathcal{N}\left(0, \sigma_0^2\right), \text{ under hypothesis } H_0, \quad (1)$$

$$y_i \sim \mathcal{N}\left(0, \sigma_0^2 + \sigma_s^2\right), \text{ under hypothesis } H_1, \quad (2)$$

where σ_0^2 and σ_s^2 denote the received noise and signal variance, respectively. The LLR L_i of node i given by [2]

$$L_i = \log\left(\frac{f(y_i|H_1)}{f(y_i|H_0)}\right), \quad (3)$$

where $f(y_i|H_h)$ denotes the probability density function (PDF) of y_i conditioned on the hypothesis H_h, for $h \in \{0,1\}$. From (3), L_i is given by [7]

$$L_i = \log\left(\frac{\sigma_0}{\sqrt{\sigma_0^2+\sigma_s^2}}\right) + \frac{\sigma_s^2}{2\sigma_0^2\left(\sigma_0^2+\sigma_s^2\right)} y_i^2. \quad (4)$$

The SNR is defined as σ_s^2/σ_0^2.

According to the Bayesian detection framework, the optimum decision rule that minimizes the error probability is given by [14, Ch. III.A]

$$\sum_{i=1}^{N} L_i \underset{H_0}{\overset{H_1}{\gtrless}} \beta, \quad (5)$$

where $\beta = \log\left(\frac{(c_{10}-c_{00})\zeta_0}{(c_{01}-c_{11})\zeta_1}\right)$ and c_{uv} is the cost incurred if hypothesis H_u is chosen while H_v is true, and $\zeta_h > 0$ is the prior probability of H_h.

B. Recap: OTS and Timer Scheme

In OTS, the nodes transmit in the decreasing order of the absolute value of their LLRs. Let (k) denote the index of the node that transmits in the k^{th} step. Therefore, its LLR is $L_{(k)}$.

To implement this, node i sets a timer and transmits its LLR to the FN when its timer expires. The timer is a monotonically non-increasing function of a locally computed non-negative real number called *metric*. In OTS, the metric μ_i of node i is $|L_i|$. This ensures that node (1) with the highest absolute value of the LLR transmits first to the FN. Then, node (2) with the second highest metric transmits, and so on. As in [2], we shall henceforth assume that collisions due to timers expiring close to each other occur with negligible probability.

When the FN receives an LLR from node (k), it uses the following decision rule:

$$\text{If } \sum_{i=1}^{k} L_{(i)} > \beta + (N-k)\left|L_{(k)}\right| : \text{Decide } H_1, \quad (6)$$

$$\text{If } \sum_{i=1}^{k} L_{(i)} < \beta - (N-k)\left|L_{(k)}\right| : \text{Decide } H_0, \quad (7)$$

Else, wait for next transmission.

The nodes transmit one by one until the FN makes a decision. After the FN decides, it broadcasts a signal to the nodes to stop their timers. The process repeats in the next measurement cycle in which nodes obtain new measurements and the FN makes a new decision. It was shown in [2] that the above decision rules achieve the same error probability as UTS.

III. Feedback-Enhanced SRTS

In FE-SRTS, each node sets a timer as a function of its metric and transmits its LLR to the FN when its timer expires. As before, the timer is a monotone non-increasing function of the metric, which ensures that the node with the highest metric transmits first. A key innovation in FE-SRTS is that the metric changes in each step, as we specify below.

Let $\mu_i(k)$ denote the metric of node i at the beginning of the k^{th} step. Let $[k]$ denote the node that transmits in the k^{th} step of FE-SRTS. Thus, its LLR is $L_{[k]}$. The FN broadcasts $F(k)$ to all the nodes if it cannot decide after receiving $L_{[k]}$.[1] Let $\mathcal{S}_k$ denote the set of all nodes that have not transmitted at the beginning of the k^{th} step. $\mathcal{S}_1$ is initialized to $\{1, 2, \ldots, N\}$.

In the first step, node i sets its metric as $\mu_i(1) = |L_i|$. Thus, $[1] = \arg\max_{i \in \mathcal{S}_1} \{\mu_i(1)\}$. Node $[1]$ is the first to transmit its LLR to the FN. The FN either decides on a hypothesis or broadcasts $F(1) = L_{[1]}$ to the nodes. And, $\mathcal{S}_2 = \mathcal{S}_1 \setminus \{[1]\}$.

If $i \in \mathcal{S}_2$, then node i sets its metric as $\mu_i(2) = |L_i - F(1)|$. Now, the nodes in $\mathcal{S}_2$ run the timer scheme. Hence, $[2] = \arg\max_{i \in \mathcal{S}_2} \{\mu_i(2)\}$. Thus, node $[2]$ transmits in the second step. The FN either decides or broadcasts $F(2) = L_{[2]}$ to all the nodes. Then, $\mathcal{S}_3 = \mathcal{S}_2 \setminus \{[2]\}$.

In general in the k^{th} step ($k \geq 2$), node i, for $i \in \mathcal{S}_k$, sets its metric as $\mu_i(k) = |L_i - F(k-1)| = \left|L_i - L_{[k-1]}\right|$. The nodes in $\mathcal{S}_k$ run the timer scheme. The FN either decides or it broadcasts $F(k) = L_{[k]}$ to all the nodes. And, $\mathcal{S}_{k+1} = \mathcal{S}_k \setminus \{[k]\}$. In any step, once the FN decides, it broadcasts a signal to all the nodes to stop their timers to prevent any further transmission. This is illustrated in Fig. 1. In effect, among the LLRs that remain to be transmitted, FE-SRTS alternates between the LLRs to the extreme right and extreme left of the origin.

A. Decision Rule

To derive the decision rules, we first characterize the order in which the nodes transmit in FE-SRTS.

Lemma *1:* For $2 \leq k \leq N-1$ and $k+1 \leq i \leq N$, the following statements hold:

- If $L_{[1]} \geq 0$ and k is even, or $L_{[1]} < 0$ and k is odd, then $L_{[k]} \leq L_{[i]} \leq L_{[k-1]}$.
- If $L_{[1]} \geq 0$ and k is odd, or $L_{[1]} < 0$ and k is even, then $L_{[k-1]} \leq L_{[i]} \leq L_{[k]}$.

Proof: The proof is given in Appendix A. ■

We now derive the following decision rules for FE-SRTS that ensure the same optimal error probability as UTS.

Result *1:* For $k = 1$, the FN decides as follows:

$$\text{If } L_{[1]} > \beta + (N-1)\left|L_{[1]}\right|: \text{Decide } H_1, \tag{8}$$

$$\text{If } L_{[1]} < \beta - (N-1)\left|L_{[1]}\right|: \text{Decide } H_0, \tag{9}$$

Else, broadcast $F(1) = L_{[1]}$ and wait.

For $2 \leq k \leq N-1$, the FN decides as follows:

- If $L_{[1]} \geq 0$ and k is even, or $L_{[1]} < 0$ and k is odd, then

$$\text{If } \sum_{i=1}^{k} L_{[i]} > \beta - (N-k)L_{[k]}: \text{Decide } H_1, \tag{10}$$

$$\text{If } \sum_{i=1}^{k} L_{[i]} < \beta - (N-k)L_{[k-1]}: \text{Decide } H_0, \tag{11}$$

Else, broadcast $F(k) = L_{[k]}$ and wait.

- If $L_{[1]} \geq 0$ and k is odd, or $L_{[1]} < 0$ and k is even, then

$$\text{If } \sum_{i=1}^{k} L_{[i]} > \beta - (N-k)L_{[k-1]}: \text{Decide } H_1, \tag{12}$$

$$\text{If } \sum_{i=1}^{k} L_{[i]} < \beta - (N-k)L_{[k]}: \text{Decide } H_0, \tag{13}$$

Else, broadcast $F(k) = L_{[k]}$ and wait.

For $k = N$, the FN uses the rule in (5) to decide.

Proof: The proof is given in Appendix B. ■

The choice of the metric and the decision rule for $k = 1$ are the same as OTS. However, the metric and the decision rule are different for $k \geq 2$.

B. Analysis: Average Number of Transmissions

We now derive an expression for the average number of transmissions required by FE-SRTS. Let $\boldsymbol{L} = (L_1, \ldots, L_N)$ and $\boldsymbol{l} = (l_1, \ldots, l_N)$ be a realization of $\boldsymbol{L}$.

Result *2:* The average number of transmissions $\bar{N}_{\text{tx}}$ is given by

$$\bar{N}_{\text{tx}} = \sum_{h \in \{0,1\}} \sum_{t=1}^{N} t \Pr\left(\boldsymbol{L} \in \Gamma_t | H_h\right) \zeta_h, \tag{14}$$

where $\Gamma_t = \Phi_1^c \cap \ldots \cap \Phi_{t-1}^c \cap \Phi_t$ and Φ_t is defined as follows:

1) *For $t = 1$:*

$$\Phi_1 = \left\{\boldsymbol{l} : l_{[1]} - (N-1)\left|l_{[1]}\right| > \beta\right\} \cup \left\{\boldsymbol{l} : l_{[1]} + (N-1)\left|l_{[1]}\right| < \beta\right\}. \tag{15}$$

2) *For $2 \leq t \leq N-1$:*

- *For $l_{[1]} \geq 0$ and t is even, or $l_{[1]} < 0$ and t is odd:*

$$\Phi_t = \left\{\boldsymbol{l} : \sum_{i=1}^{t} l_{[i]} + (N-t)l_{[t]} > \beta\right\} \cup \left\{\boldsymbol{l} : \sum_{i=1}^{t} l_{[i]} + (N-t)l_{[t-1]} < \beta\right\}. \tag{16}$$

- *For $l_{[1]} \geq 0$ and t is odd, or $l_{[1]} < 0$ and t is even:*

$$\Phi_t = \left\{\boldsymbol{l} : \sum_{i=1}^{t} l_{[i]} + (N-t)l_{[t-1]} > \beta\right\} \cup \left\{\boldsymbol{l} : \sum_{i=1}^{t} l_{[i]} + (N-t)l_{[t]} < \beta\right\}. \tag{17}$$

3) *For $t = N$, $\phi_N = \mathbb{R}^N$.*

[1] When a node transmits its LLR or the FN transmits its feedback, we assume that the effect of quantization is negligible [2].

Proof: The proof is given in Appendix C. ■

Result 1 provides an independent way of verifying the numerical results. It also leads to the following insightful corollary, whose proof we skip due to space constraints.

Corollary *1:* As $\sigma_s \to \infty$, $\bar{N}_{\text{tx}} \to 2$ for all ζ_1 and $N \geq 2$.

As $\sigma_s \to \infty$, the probability of deciding H_h after the second transmission can be shown to be equal to ζ_h, for $h \in \{0, 1\}$. Hence, the probability of making a decision after the second transmission becomes 1 at larger SNR. Thus, as the SNR increases, $\bar{N}_{\text{tx}}$ for FE-SRTS always decreases to two regardless of the number of sensor nodes. This is much lower than OTS, in which $\bar{N}_{\text{tx}}$ is close to $\lceil \frac{N}{2} \rceil$, where $\lceil \cdot \rceil$ denotes the ceiling function [2].

Evaluating $Pr(\boldsymbol{L} \in \Gamma_t | H_h)$: The probability in (14) can be numerically computed using Monte Carlo techniques. For a given t and H_h, we generate M i.i.d. realizations of $\boldsymbol{L}$, which we denote by $\boldsymbol{l}_1, \ldots, \boldsymbol{l}_M$. For a realization $\boldsymbol{l}_m$, we first order it using Lemma 1. If the realization satisfies the condition $\boldsymbol{l}_m \in \Gamma_t$, then $\mathbb{1}_{[\boldsymbol{l}_m \in \Gamma_t]} = 1$, and is 0 otherwise. Its empirical mean converges almost surely to $\Pr(\boldsymbol{L} \in \Gamma_t | H_h)$. The error in this computation decreases as $O\left(1/\sqrt{M}\right)$ [15, Ch. 2].

IV. Numerical Results

We compare the average number of transmissions of FE-SRTS, OTS, and UTS since they all achieve the same error probability.[2] A smaller average number of transmissions implies a more energy-efficient scheme. To understand this, we first define energy-efficiency γ. It is the ratio of the average energy consumed by UTS to the average energy consumed by the scheme [7]. When the nodes transmit with a fixed power, the average energies consumed by FE-SRTS and UTS are $\bar{N}_{\text{tx}} E_{\text{tx}}$ and $N E_{\text{tx}}$, respectively. Hence,

$$\gamma = \frac{N E_{\text{tx}}}{\bar{N}_{\text{tx}} E_{\text{tx}}} = \frac{N}{\bar{N}_{\text{tx}}}. \tag{18}$$

The larger the value of γ, the more energy-efficient is the scheme. Note that the energy spent by the FN for feedback is not considered since the FN is often plugged into the power grid, while the nodes are not. We average over 10^4 measurement cycles. We set $c_{uv} = 1$ if $u \neq v$, and $c_{uv} = 0$ if $u = v$.

Fig. 2 plots $\bar{N}_{\text{tx}}$ of OTS and FE-SRTS as a function of the SNR for $N = 10$ and 20 nodes and $\zeta_1 = 0.3$. For UTS, the average number of transmissions is N. We do not show it to avoid clutter. We observe that FE-SRTS requires markedly fewer transmissions than both UTS and OTS. For example, for $N = 20$ and SNR $= 10$ dB, the average number of transmissions of FE-SRTS is 21.8% of that of UTS and 46.9% of that of OTS. As the SNR increases, $\bar{N}_{\text{tx}}$ of FE-SRTS decreases to 2 while that of OTS is close to $\lceil \frac{N}{2} \rceil$. The analysis and simulation results match well.

Fig. 3 plots $\bar{N}_{\text{tx}}$ of OTS and FE-SRTS as a function of ζ_1 for three SNRs. In OTS, as ζ_1 increases, $\bar{N}_{\text{tx}}$ increases, reaches a peak, and then decreases. This can be understood as follows. As ζ_1 increases, $\beta = \log\left((1 - \zeta_1)/\zeta_1\right)$ decreases. Thus, the odds that $\sum_{i=1}^{k} L_{(i)}$ in (6) exceeds $\beta + (N-k)L_{(k)}$ increase and fewer steps are needed when H_1 occurs. On the other hand, when H_0 occurs, we can infer from (7) that as β decreases, more transmissions are needed. At lower values of ζ_1, the effect due to H_0 dominates, while the effect due to H_1 dominates at larger values of ζ_1. We observe the same trend in FE-OTS, but the variation in $\bar{N}_{\text{tx}}$ is smaller. Its average number of transmissions for all the prior probabilities and SNRs is substantially lower than that of OTS.

[2]A comparison with techniques such as censoring, on-off keying, duty cycling, and clustering is not shown as their error probabilities are larger.

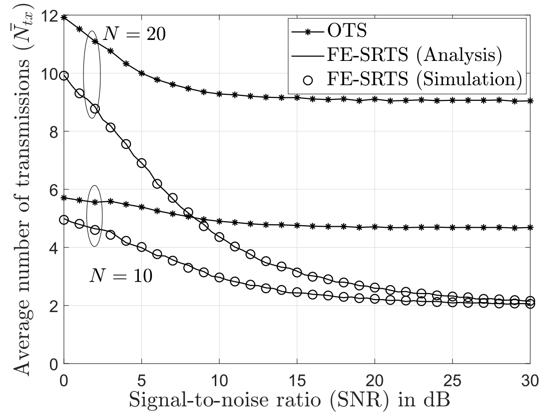

Fig. 2. Average number of transmissions of as a function of SNR ($\zeta_1 = 0.3$).

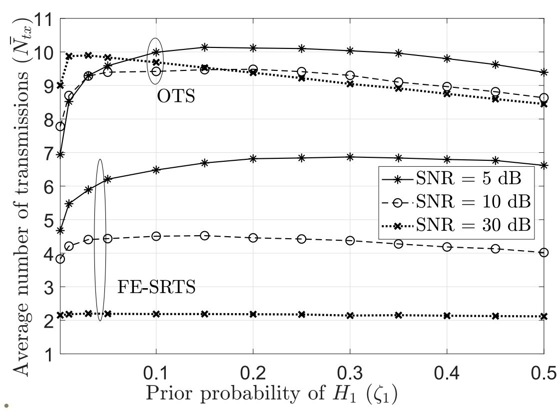

Fig. 3. Average number of transmissions as a function of ζ_1 ($N = 20$).

Fig. 4 plots the energy-efficiency γ of FE-SRTS, OTS, and UTS as a function of N. The energy-efficiency of FE-SRTS and OTS increases as N increases, while that of UTS is always one. While the energy-efficiency of FE-SRTS increases linearly, that of OTS saturates close to 2.

V. Conclusions

We proposed FE-SRTS, in which the metrics of the nodes, which determined the order in which the nodes transmitted, were updated based on the feedback from the FN. We derived novel decision rules for the scheme that guaranteed the same optimal error probability as the conventional UTS. We also derived an expression for average number of transmissions.

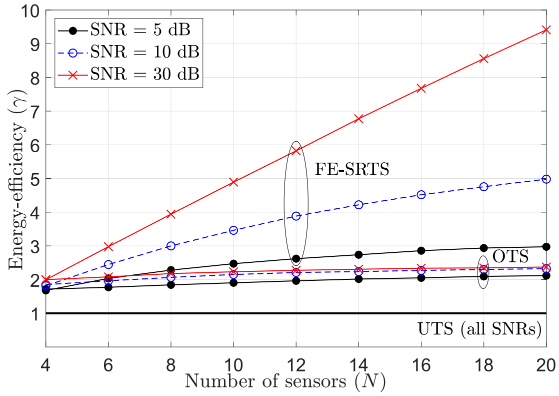

Fig. 4. Energy-efficiency of FE-SRTS, OTS, and UTS as a function of N ($\zeta_1 = 0.5$).

FE-SRTS required markedly fewer transmissions on average than OTS and UTS. While the average number of transmissions of FE-SRTS decreased to two as the SNR increased, that of OTS decreased close to $\lceil \frac{N}{2} \rceil$. This implied that the energy-efficiency of FE-SRTS increased linearly with the number of nodes, while that of OTS saturated close to 2.

APPENDIX

A. Proof of Lemma 1

We use mathematical induction. Consider first the scenario where $L_{[1]} \geq 0$ and k is even. We first prove the lemma for the initial case $k = 2$ and then for the general case.

When $k = 2$: Since $\mu_i(1) = |L_i|$, for $1 \leq i \leq N$, and [1] is the node with the highest metric, we have

$$L_{[i]} \leq L_{[1]}, \text{ for } 2 \leq i \leq N. \tag{19}$$

In the second step, the metric of node i is $\mu_i(2) = |L_i - F(1)| = |L_i - L_{[1]}|$. By the definition of [2], we have $|L_{[2]} - L_{[1]}| \geq |L_{[i]} - L_{[1]}|$, for $3 \leq i \leq N$. Therefore,

$$L_{[1]} - |L_{[2]} - L_{[1]}| \leq L_{[i]} \leq L_{[1]} + |L_{[2]} - L_{[1]}|. \tag{20}$$

From (19), we have $L_{[2]} \leq L_{[1]}$. Hence, $|L_{[2]} - L_{[1]}| = L_{[1]} - L_{[2]}$. Substituting this in the upper bound in (20) yields

$$L_{[2]} \leq L_{[i]}, \text{ for } 3 \leq i \leq N. \tag{21}$$

Combining (19) and (21), we get $L_{[2]} \leq L_{[i]} \leq L_{[1]}$, for $3 \leq i \leq N$.

When $k = l$ and l is Even: Let Lemma 1 be true up to $k = l-1$. We now prove that it is true for $k = l$. Since l is even, we are given that

$$L_{[l-2]} \leq L_{[i]} \leq L_{[l-1]}, \text{ for } l \leq i \leq N. \tag{22}$$

In the l^{th} step, the metric of node i is $\mu_i(l) = |L_i - F(l-1)| = |L_i - L_{[l-1]}|$. From the definition of $[l]$,

$$|L_{[l]} - L_{[l-1]}| \geq |L_{[i]} - L_{[l-1]}|, \text{ for } l+1 \leq i \leq N. \tag{23}$$

Therefore, for $l+1 \leq i \leq N$, we have

$$L_{[l-1]} - |L_{[l]} - L_{[l-1]}| \leq L_{[i]}. \tag{24}$$

From (22), we know that $L_{[l]} \leq L_{[l-1]}$. Hence, $|L_{[l]} - L_{[l-1]}| = L_{[l-1]} - L_{[l]}$. Substituting this in (24), we get

$$L_{[l]} \leq L_{[i]}, \text{ for } l+1 \leq i \leq N. \tag{25}$$

Combining (22) and (25), we get $L_{[l]} \leq L_{[i]} \leq L_{[l-1]}$, for $l+1 \leq i \leq N$.

When k is odd, the initial case is $k = 3$. The proof for it follows a similar logic and is not shown. The proof for $L_{[1]} < 0$ also follows a similar logic, and is skipped.

B. Decision Rules for FE-SRTS

We consider the three cases $k = 1$, $2 \leq k \leq N-1$, and $k = N$ separately below.

1) *When $k = 1$:* Since $L_{[1]}, L_{[2]}, \ldots, L_{[N]}$ is a permutation of $L_1, \ldots, L_N$, the decision rule in (5) can be recast as

$$\sum_{i=1}^{N} L_{[i]} \underset{H_0}{\overset{H_1}{\gtrless}} \beta \tag{26}$$

Since $|L_{[i]}| \leq |L_{[1]}|$, for $2 \leq i \leq N$, we get $-|L_{[1]}| \leq L_{[i]} \leq |L_{[1]}|$. This implies that $-(N-1)|L_{[1]}| \leq \sum_{i=2}^{N} L_{[i]} \leq (N-1)|L_{[1]}|$. Hence, $\sum_{i=1}^{N} L_{[i]}$ can be bounded as

$$L_{[1]} - (N-1)|L_{[1]}| \leq \sum_{i=1}^{N} L_{[i]} \leq L_{[1]} + (N-1)|L_{[1]}|. \tag{27}$$

Thus, if $L_{[1]} + (N-1)|L_{[1]}| < \beta$, it follows that $\sum_{i=1}^{N} L_{[i]} < \beta$. Therefore, the FN can decide H_0. Similarly, if $L_{[1]} - (N-1)|L_{[1]}| > \beta$, then it follows that $\sum_{i=1}^{N} L_{[i]} > \beta$. Hence, the FN can decide H_1. This yields the decision rule in (8) and (9).

2) *When $2 \leq k \leq N$:* We consider the following two cases:

a) *$L_{[1]} \geq 0$ and k is Even, or $L_{[1]} < 0$ and k is Odd:* From Lemma 1, we have

$$L_{[k]} \leq L_{[i]} \leq L_{[k-1]}, \text{ for } k+1 \leq i \leq N. \tag{28}$$

Therefore,

$$(N-k)L_{[k]} \leq \sum_{i=k+1}^{N} L_{[i]} \leq (N-k)L_{[k-1]}. \tag{29}$$

Hence, $\sum_{i=1}^{N} L_{[i]}$ can be bounded as $\sum_{i=1}^{k} L_{[i]} + (N-k)L_{[k]} \leq \sum_{i=1}^{N} L_{[i]} \leq \sum_{i=1}^{k} L_{[i]} + (N-k)L_{[k-1]}$. If $\sum_{i=1}^{k} L_{[i]} + (N-k)L_{[k-1]} < \beta$, then we know that $\sum_{i=1}^{N} L_{[i]} < \beta$. Hence, the FN can decide H_0. Similarly, if $\sum_{i=1}^{k} L_{[i]} + (N-k)L_{[k]} > \beta$, then $\sum_{i=1}^{N} L_{[i]} > \beta$. Hence, the FN can decide H_1. This yields the decision rule in (10) and (11).

b) *$L_{[1]} \geq 0$ and k is Odd, or $L_{[1]} < 0$ and k is Even:* The proof follows a similar logic and is skipped.

3) *When $k = N$:* In this case, the FN knows all N LLRs. Hence, the optimal decision rule in (5) applies.

C. Derivation of Result 2

From the law of total expectation, we have

$$\bar{N}_{\text{tx}} = \sum_{h\in\{0,1\}} \sum_{t=1}^{N} t\Pr(N_{\text{tx}} = t|H_h)\zeta_h. \quad (30)$$

In terms of the joint distribution of the LLRs $L_{[1]}, \ldots, L_{[N]}$, the probability term above is given by

$$\Pr(N_{\text{tx}} = t|H_h) = \int_{\boldsymbol{l}' \in \mathbb{R}^N} f_{L_{[1]},\ldots,L_{[N]}}(\boldsymbol{l}'|H_h)\mathbb{1}_{[\boldsymbol{l}'\in\Gamma_t]}\, d\boldsymbol{l}', \quad (31)$$

where $\boldsymbol{l}' = \left(l_{[1]}, l_{[2]}, \ldots, l_{[N]}\right)$ is a realization of $L_{[1]}, L_{[2]}, \ldots, L_{[N]}$. In (31), Γ_t is the region such that if $\boldsymbol{l}' \in \Gamma_t$, then the FN decides in favor of one of the hypotheses after exactly t transmissions. Let Φ_t be the region of LLRs such that if $\boldsymbol{l}' \in \Phi_t$, the FN decides within t transmissions. Then, it follows that $\Gamma_t = \Phi_1^c \cap \Phi_2^c \cap \ldots \cap \Phi_{t-1}^c \cap \Phi_t$.

From the decision rules for $k = 1$ in (8) and (9), we see that the FN decides H_1 if $l_{[1]} - (N-1)\left|l_{[1]}\right| > \beta$ or H_0 if $l_{[1]} + (N-1)\left|l_{[1]}\right| < \beta$. This yields the definition for Φ_1 in (15). Similarly, from the decision rules in (10) and (11), the definition for Φ_t in (16) follows. From the decision rules in (12) and (13), the definition for Φ_t in (17) follows. Lastly, we have $\Phi_N = \mathbb{R}^N$ since the FN always decides when $k = N$.

For $L_{[1]} \geq 0$, we can show from Lemma 1 that $[1] = 1:N$, $[2] = N:N$, $[3] = 2:N$, and so on. In general, we have

$$[k] = \begin{cases} N - \frac{k}{2} + 1 : N, & \text{if } L_{[1]} \geq 0 \text{ and } k \text{ is even}, \\ \frac{k+1}{2} : N, & \text{if } L_{[1]} \geq 0 \text{ and } k \text{ is odd}, \\ \frac{k}{2} : N, & \text{if } L_{[1]} < 0 \text{ and } k \text{ is even}, \\ N - \frac{k-1}{2} : N, & \text{if } L_{[1]} < 0 \text{ and } k \text{ is odd}. \end{cases} \quad (32)$$

Substituting this in (31) for N even, we get

$$\Pr(N_{\text{tx}} = t|H_h) = \int_{\boldsymbol{l}' \in \mathbb{R}^N} f_{L_{1:N}, L_{N:N}, \ldots, L_{\frac{N}{2}+1:N}}(\boldsymbol{l}'|H_h) \times \mathbb{1}_{[\boldsymbol{l}'\in\Gamma_t]}\, d\boldsymbol{l}'. \quad (33)$$

Similarly, we can show for N odd that

$$\Pr(N_{\text{tx}} = t|H_h) = \int_{\boldsymbol{l}' \in \mathbb{R}^N} f_{L_{1:N}, L_{N:N}, \ldots, L_{\frac{N+1}{2}:N}}(\boldsymbol{l}'|H_h) \times \mathbb{1}_{[\boldsymbol{l}'\in\Gamma_t]}\, d\boldsymbol{l}'. \quad (34)$$

Rearranging terms, we get

$$\Pr(N_{\text{tx}} = t|H_h) = \int_{\boldsymbol{l} \in \mathbb{R}^N} f_{L_{1:N},\ldots,L_{N:N}}(\boldsymbol{l}|H_h)\mathbb{1}_{[\boldsymbol{l}\in\Gamma_t]}\, d\boldsymbol{l}, \quad (35)$$

where $\boldsymbol{l}$ is a permutation of $\boldsymbol{l}'$. Here, we have used the fact that for any permutation, we have $\mathbb{1}_{[\boldsymbol{l}'\in\Gamma_t]} = \mathbb{1}_{[\boldsymbol{l}\in\Gamma_t]}$.

The joint PDF of the ordered RVs $L_{1:N}, \ldots, L_{N:N}$ can be written in closed-form as [16, Ch. II.2.2]

$$f_{L_{1:N},\ldots,L_{N:N}}(\boldsymbol{l}|H_h) = N!\,\mathbb{1}_{[\boldsymbol{l}\in\Delta]}\left[\prod_{n=1}^{N} f\left(l_n|H_h\right)\right], \quad (36)$$

where $\Delta = \{\boldsymbol{l} : l_1 \geq l_2 \geq \cdots \geq l_N\}$. Substituting (36) in (35), we obtain

$$\Pr(N_{\text{tx}} = t|H_h) = N! \int_{\boldsymbol{l} \in \mathbb{R}^N} \mathbb{1}_{[\boldsymbol{l}\in(\Gamma_t\cap\Delta)]}\left[\prod_{n=1}^{N} f\left(l_n|H_h\right)\right] d\boldsymbol{l}, \quad (37)$$

since $\mathbb{1}_{[\boldsymbol{l}\in\Delta]}\mathbb{1}_{[\boldsymbol{l}\in\Gamma_t]} = \mathbb{1}_{[\boldsymbol{l}\in(\Gamma_t\cap\Delta)]}$. This implies that

$$\Pr(N_{\text{tx}} = t|H_h) = N!\Pr\left(\boldsymbol{L} \in (\Gamma_t \cap \Delta)\,|H_h\right). \quad (38)$$

A key observation is that the events $\boldsymbol{L} \in \Gamma_t$ and $\boldsymbol{L} \in \Delta$ are independent when conditioned on the hypothesis, since $\boldsymbol{L} \in \Gamma_t$ does not depend on any ordering of the elements of $\boldsymbol{L}$. Also, the event $\boldsymbol{L} \in \Delta$ is independent of H_h. Lastly, $\Pr\left(\boldsymbol{L} \in \Delta\right) = \frac{1}{N!}$ since all the $N!$ permutations are equally likely. We can then show that $N!\Pr\left(\boldsymbol{L} \in (\Gamma_t \cap \Delta)\,|H_h\right) = \Pr\left(\boldsymbol{L} \in \Gamma_t|H_h\right)$. This implies that

$$\Pr(N_{\text{tx}} = t|H_h) = \Pr\left(\boldsymbol{L} \in \Gamma_t|H_h\right). \quad (39)$$

Substituting this in (30) yields (14).

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