Rate-Optimal Relay Selection for Average Interference-Constrained Underlay CR

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***Abstract*— Cooperative relaying combined with selection exploits spatial diversity to improve the performance of interference-constrained secondary users in an underlay cognitive radio (CR) network. While a relay improves the signal-to-interference-plus-noise ratio (SINR) of the secondary network, it requires two hops and also generates interference to the primary network. We present a novel, optimal relay selection rule that maximizes the fading-averaged transmission rate of an average interference-constrained underlay secondary network. It differs from the several ad hoc incremental relaying schemes proposed in the literature, while requiring a feedback overhead that is comparable to them. We then analyze the average rate of the optimal rule. We also present insightful high and low SINR asymptotic analyses, which bring out the extent to which the use of the relays improves the average rate as a function of the system parameters. Our numerical results show that the proposed rule outperforms several known relay selection schemes for CR, and also characterize the regimes in which some of these schemes are near-optimal.**

***Index Terms*— Cooperative communications, underlay, cognitive radio, relays, selection, interference constraints, rate, incremental relaying.**

I. Introduction

COGNITIVE radio (CR) is a spectrum sharing technology that promises to significantly improve the utilization of scarce wireless spectrum. Three spectrum access paradigms have been studied for CR, namely, interweave, overlay, and underlay [2]. In the interweave mode, a secondary user (SU) can transmit only when it senses that a higher priority primary user (PU) is off. In the overlay mode, the SU can use the spectrum of the PU simultaneously, while facilitating the latter's transmissions. In the underlay mode, which is the focus of our paper, the SU can simultaneously transmit on the same spectrum as the PU so long as the interference it causes to the PU is tightly constrained [3], [4]. Given its promise, CR is being actively considered for smart grid, public safety, and cellular networks [5], [6].

However, the interference constraint in underlay CR can severely limit the data rates achievable by the SUs. Cooperative relaying with selection is an attractive solution to circumvent this challenge [3], [4], [7]–[9]. In it, a single "best" relay that satisfies the interference constraint is selected to forward a message from a secondary source (S) to a destination (D) based on the instantaneous conditions of the channels, which include the interference channels from the secondary source and relays to the primary receiver. Sometimes, not using any relay may even be preferred. Selection is practically appealing because it avoids the timing synchronization problems that arise when multiple, spatially separated relays have to transmit simultaneously. It is also more spectrally efficient than allocating orthogonal channels to the relays.

While cooperative relaying improves the signal-to-noise ratio (SNR) at D, it requires two time slots instead of one time slot required by a direct transmission. Solutions such as incremental relaying (IR) employ a relay only when the direct transmission fails [10]–[12]. In CR, in addition to this aspect, the use of a relay is also governed by the interference its transmissions cause to the primary receiver. Different constraints on the interference have been considered in the literature. These include: (i) average interference constraint, in which the fading-averaged interference power at the primary receiver must not cross a threshold [13]–[16], (ii) peak interference constraint, in which the instantaneous interference power at the primary receiver must not cross a threshold [3], [4], [7], (iii) interference outage constraint, in which the instantaneous interference power is allowed to exceed a threshold for at most a pre-specified fraction of time [17], and (iv) primary signal-to-interference-plus-noise ratio (SINR)-based outage constraint, in which the instantaneous SINR of the primary signal at the primary receiver must not fall below a threshold [8], [9], [18], [19]. Therefore, new rules are needed for selecting a relay – and whether any relay should even be selected – with the interference constraint, relaying scheme, and optimization objective all playing a crucial role in their design.

A. Literature on Relay Selection (RS) for CR

We categorize the several RS rules proposed in the literature on the basis of the interference constraint used and whether the state of the direct source-to-destination (SD) link influences the choice of the selected relay.

- *Peak Interference Constraint Considering Direct Link State:* In [3], [4], [7], [8], incremental relaying-based RS rules are proposed, in which a relay transmits only if the SINR of the direct SD link falls below a threshold γ_{th}. In [3], incremental opportunistic relaying is studied, in which the decode-and-forward (DF) relay that maximizes

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the minimum of the SINRs of the source-to-relay (SR) and relay-to-destination (RD) links is selected. Instead, in [4], incremental reactive DF relaying is studied, in which among the DF relays that successfully decoded the signal transmitted by *S*, the one that maximizes the RD link SINR is selected. Extension to multiple primary receivers is studied in [7]. Unlike [4], if no DF relay successfully decodes the received signal from *S* in the first time slot, then *S* retransmits the same data in the second time slot in [8].

- *Peak Interference Constraint Without Considering Direct Link State:* In [20], among the fixed-gain amplify-and-forward (AF) relays that satisfy the peak interference constraint, the relay that maximizes the RD link SNR is selected. Instead, the fixed-gain AF relay that maximizes the SR link SNR is selected in [21]. In [22], the AF relay with the largest end-to-end SNR at *D* is selected. Conventional opportunistic relaying is studied in [23], in which the peak interference-constrained DF relay that maximizes the minimum of the SINRs of the SR and RD links is always selected. Conventional reactive DF relaying is studied in [24], in which among the relays that successfully decoded the received signal from *S*, the one that maximizes the RD link SINR is selected. Instead, in [19] and [25], the DF relay that maximizes the instantaneous capacity of the secondary network is selected.
- *Average Interference Constraint Considering Direct Link State:* In [26], direct SD link-aware RS rule that minimizes the average symbol error probability (SEP) of the secondary network is studied.
- *Average Interference Constraint Without Considering Direct Link State:* In [16], an SEP-optimal RS rule is studied, but the state of the direct SD link is ignored in the RS rule. The relay that maximizes the effective capacity or end-to-end SNR is selected in [13], [14], but the direct SD link is assumed to be absent.

To the best of our knowledge, the RS rule that maximizes the average rate of an average interference-constrained underlay CR network and accounts for the direct SD link is not known in the CR literature.

B. Contributions and Novelty Relative to Existing Works

1) Contributions: We make the following contributions:

- We present a novel RS rule and prove that it is optimal in terms of maximizing the average rate between the source and destination of the secondary network that is subject to an average interference constraint. It serves as a fundamental and new performance benchmark to assess the efficacy of the many rules proposed in the literature. We focus on the average interference constraint as it is well motivated from many perspectives. For the secondary network, it is less conservative than the peak interference constraint. In terms of the impact on the primary network, it is justified when the packet transmission duration of the primary network spans multiple coherence intervals of the secondary transmitter-to-primary receiver channel. Depending on the quality-of-service requirements of the primary network, e.g., when its data traffic is delay tolerant, it can be justifiable even for larger coherence times [27]. In [15], it has even been shown that the average interference constraint can increase the capacity of the primary network compared to the peak interference constraint.
- We then analyze the average rate of the optimal RS rule. We also derive simpler and insightful closed-form expressions for the average rate in both high and low SINR asymptotic regimes, for which the trends turn out to be different. The insights gleaned from this are useful for practical system design.
- We also benchmark the performance of the optimal RS rule against various RS rules proposed in the literature. This helps understand, for the first time, the regimes in which these rules are sub-optimal or near-optimal. These results show that the proposed RS rule can lead to a significant improvement in the average rate, or, equivalently, a significant reduction in the required transmit power compared to the existing approaches.

2) Novelty Relative to Existing Works: There are several differences between our work and the existing literature in terms of the interference constraint, performance metric, and RS rule, as we discuss below.

a) Interference constraint: The existing works mostly focus on the peak interference constraint [3], [4], [7], [8], and [20]–[24]. The ones that do focus on the average interference constraint either assume that the direct SD link is absent [13], [14], or ignore its role in the RS rule [16]. Instead, we focus on the average interference constraint and show that the state of the direct SD link plays an important role in the optimal RS rule. Unlike the peak interference constraint, a new trade-off arises under the average interference constraint – the underlay CR network can either immediately use a relay or defer its usage for later when the channel conditions are different.

b) Performance objective: While [3], [4], [7], [8], and [23] analyze the outage probability, we optimize and analyze the average rate, which is a measure of spectral efficiency and is a fundamental performance measure of a communication system. While the average rate is analyzed in [13] and [14], the direct SD link is assumed to be absent. The performance analysis of the optimal RS rule is also more involved than that for the sub-optimal RS rules in [3], [4], [7], and [8] because of the optimal RS rule's non-linear form and the influence of the average interference constraint.

c) RS rule: Unlike the sub-optimal IR variants in [3], [4], [7], [8], and [10], the proposed optimal RS rule does not compare the direct SD link SINR to a fixed threshold. Another issue is that the above threshold needs to be numerically optimized as a function of the source transmission rate and the average SINRs of the various links. The repetition of the same transmission by *S* over two time slots makes the RS rule in [8] sub-optimal. The proposed optimal RS rule also differs from the SEP-optimal RS rules in [16] and [26] because minimizing the average SEP does not maximize the average rate due to the non-linear relationship between SEP and rate.

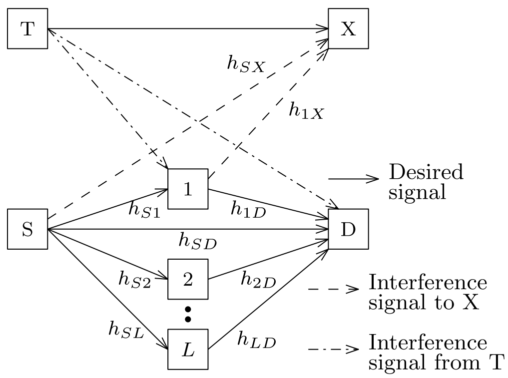

Fig. 1. An underlay CR network with a primary transmitter T, a primary receiver X, a secondary source S, a secondary destination D, and L secondary relays $1, 2, \ldots, L$.

C. Outline and Notation

The paper is organized as follows. Section II develops the system model. The optimal RS rule for underlay CR is presented in Section III. Its average rate is analyzed in Section IV, which also develops the high and low SINR asymptotic analyses. Numerical results are presented in Section V, and our conclusions follow in Section VI.

We shall use the following notation henceforth. The absolute value of a complex number x is denoted by $|x|$. The probability of an event A and the conditional probability of A given event B are denoted by $\Pr(A)$ and $\Pr(A|B)$, respectively. $\mathbb{E}_X[.]$ denotes the expectation with respect to a random variable (RV) X; the subscript is dropped if it is obvious from the context. $X \sim CN(0, \sigma^2)$ means that X is a circularly symmetric zero-mean complex Gaussian RV with variance σ^2, and $1_{\{a\}}$ denotes the indicator function; it is 1 if a is true and is 0 otherwise. Similarly, $X \sim \mathcal{E}\{\mu\}$ means that X is an exponential RV with mean μ.

II. System Model

Our system comprises of a primary network, in which a primary transmitter T sends data to a primary receiver X, and an underlay secondary network, in which S transmits data to D using L DF relays $1, 2, \ldots, L$, as shown in Fig. 1. Each node is equipped with a single antenna. The complex baseband channel gain from S to X is h_{SX}, from S to D is h_{SD}, from S to relay i is h_{Si}, from relay i to D is h_{iD}, and from relay i to X is h_{iX}. Let $\mathbf{h}_S \triangleq [h_{S1}, h_{S2}, \ldots, h_{SL}]$, $\mathbf{h}_D \triangleq [h_{1D}, h_{2D}, \ldots, h_{LD}]$, $\mathbf{h}_X \triangleq [h_{1X}, h_{2X}, \ldots, h_{LX}]$, and $\mathbf{h} \triangleq [\mathbf{h}_S, \mathbf{h}_D, \mathbf{h}_X]$. All channels are assumed to be frequency-flat, block fading channels that remain constant over the duration of relay selection and two data transmissions [10], [12]. h_{SD} is independent of the channel gains of all other links.

A. Data Transmissions

The source can transmit data to D directly, or transmit via a selected DF relay $\beta \in \{1, 2, \ldots, L\}$. We shall denote the case in which no relay transmits, i.e., S transmits data directly to D, by a virtual relay 0 with $h_{S0} = h_{0D} = h_{0X} \triangleq 0$. The choice of β depends on the relay selection rule, which we formally define below, the interference constraint, and the instantaneous channel power gains of the SD, SR, RD, and relay-to-primary receiver (RX) links. We consider the proactive model of relaying, in which RS precedes data transmission by S. This enables S to adapt its transmission rate as a function of the instantaneous SINRs of the SR, RD, and SD links.

If a relay $\beta \neq 0$ is selected, then in the first time slot, S transmits a data symbol x_s with power P_s and the selected relay and D listen. After accounting for the interference at the relays and D due to the transmissions by T, the received signals $y_{S\beta}$ at the selected relay β and y_{SD} at D are given by

$$y_{S\beta} = \sqrt{P_s} h_{S\beta} x_s + n_\beta + w_\beta, \tag{1}$$

$$y_{SD} = \sqrt{P_s} h_{SD} x_s + n_D + w_D. \tag{2}$$

The noises at relay β and D are $n_\beta \sim CN(0, \sigma_0^2)$ and $n_D \sim CN(0, \sigma_0^2)$, respectively, and are mutually independent.

The interferences at relay β and D due to transmissions by T are w_β and w_D, respectively. These are assumed to be Gaussian, which is a worst case statistical model for the interference [29]. Therefore, $w_\beta \sim CN(0, \sigma_1^2)$ and $w_D \sim CN(0, \sigma_2^2)$. The Gaussian interference assumption is widely employed in the CR literature, though it is not often explicitly stated. It is justified even with one primary transmitter when the links from T to the relays and destination undergo Rayleigh fading, and T transmits with a fixed power [17]. With many primary transmitters, it is justified on the basis of the central limit theorem. It also encompasses the simpler model of [3], [8], [19], [23], in which the primary transmitter is assumed to be far away from the relays and destination. We refer the reader to [16] for a comparison of this model with other models assumed in the literature. An alternate interference model, in which w_β and w_D are assumed to be Gaussian after conditioning on the channel gains of the links from T to relay β and destination is considered in [4]. However, we do not employ it in order to ensure analytical tractability.

In the second time slot, the selected relay β retransmits the signal x_s decoded by it to D with power P_r. The received signal $y_{\beta D}$ at D in this slot is given by

$$y_{\beta D} = \sqrt{P_r} h_{\beta D} x_s + n_D' + w_D', \tag{3}$$

where $n_D' \sim CN(0, \sigma_0^2)$ is the noise at D in the second time slot and $w_D' \sim CN(0, \sigma_2^2)$ is the interference from T at D in the second time slot. The destination coherently combines the signals y_{SD} and $y_{\beta D}$ using maximal ratio combining. The instantaneous SINR at D is $\gamma_{SD} + \gamma_{\beta D}$, where $\gamma_{SD} = P_s |h_{SD}|^2 / \left(\sigma_0^2 + \sigma_2^2\right)$ is the SINR of the direct SD link and $\gamma_{\beta D} = P_r |h_{\beta D}|^2 / \left(\sigma_0^2 + \sigma_2^2\right)$ is the SINR of the link between the relay β and D.

The instantaneous rate $C_\beta\left(|h_{SD}|^2, |h_{S\beta}|^2, |h_{\beta D}|^2\right)$ between S and D in bits/sec/Hz when DF relay $\beta \in \{1, 2, \ldots, L\}$ is selected is given by [30]

$$\begin{aligned} C_\beta & \left(|h_{SD}|^2, |h_{S\beta}|^2, |h_{\beta D}|^2\right) \\ &= \min\left\{\frac{1}{2}\log_2\left(1+\gamma_{S\beta}\right), \frac{1}{2}\log_2\left(1+\gamma_{\beta D}+\gamma_{SD}\right)\right\}, \\ &= \frac{1}{2}\log_2\left(1+\min\left\{\gamma_{S\beta}, \gamma_{\beta D}+\gamma_{SD}\right\}\right), \end{aligned} \tag{4}$$

where $\gamma_{S\beta} = P_s|h_{S\beta}|^2/\left(\sigma_0^2+\sigma_1^2\right)$ is the SINR of the link between S and the relay β. The min operator arises because both the selected relay and D need to decode their received signals, and the factor $1/2$ is due to the two time slots needed for a relay-aided transmission.

If no relay is selected ($\beta = 0$), the source sends a new message in the second time slot, in which case the instantaneous rate $C_0\left(|h_{SD}|^2, |h_{S0}|^2, |h_{0D}|^2\right)$ equals

$$C_0\left(|h_{SD}|^2, |h_{S0}|^2, |h_{0D}|^2\right) = \log_2\left(1+\gamma_{SD}\right). \quad (5)$$

Relay Selection Rule and Constraints: An RS rule ϕ is a mapping from the vector consisting of the instantaneous channel power gains $|h_{SD}|^2$, $|h_{Si}|^2$, $|h_{iD}|^2$, and $|h_{iX}|^2$, for $1 \le i \le L$, to an index $\beta \in \{0,1,\ldots,L\}$, which specifies the relay selected. It must ensure that the average interference caused to X due to relay transmissions is below a threshold I_{avg}. The source also adheres to a constraint on the average interference its transmissions cause to X. In addition, S and relays are subject to a maximum transmit power constraint. Thus, the transmit power of S is given by $P_s = \min\left\{P_{\max}, I_{\text{avg}}/\mu_{SX}\right\}$, where $P_{\max}$ is the maximum allowable transmit power and μ_{SX} is the mean channel power gain of the source-to-primary receiver (SX) link. The transmit power of the relay is $P_r = P_{\max}$. Both S and the selected relay transmit with a fixed power.

III. Problem Statement and Optimal RS Rule

Our goal is to find an optimal RS rule ϕ^* that maximizes the average rate of the secondary network while ensuring that the average interference caused to X due to relay transmissions is below a threshold I_{avg}, i.e., $\mathbb{E}_{h_{SD},\mathbf{h}}\left[P_\beta|h_{\beta X}|^2\right] \le I_{\text{avg}}$, where $P_\beta = 0$, for $\beta = 0$, and $P_\beta = P_r$, for $1 \le \beta \le L$. Here, $\beta = \phi(h_{SD},\mathbf{h})$ is itself an RV because it depends on the channel gains h_{SD} and $\mathbf{h}$. Consequently, $C_\beta\left(|h_{SD}|^2, |h_{S\beta}|^2, |h_{\beta D}|^2\right)$, P_β, and $|h_{\beta X}|^2$ are also RVs. Therefore, our problem for any given P_s and P_r can be stated as the following stochastic, constrained optimization:

$$\max_{\phi}\ \mathbb{E}_{h_{SD},\mathbf{h}}\left[C_\beta\left(|h_{SD}|^2, |h_{S\beta}|^2, |h_{\beta D}|^2\right)\right], \quad (6)$$

$$\text{s.t. } \mathbb{E}_{h_{SD},\mathbf{h}}\left[P_\beta|h_{\beta X}|^2\right] \le I_{\text{avg}}, \quad (7)$$

$$\beta = \phi(h_{SD},\mathbf{h}) \in \{0,1,\ldots,L\}. \quad (8)$$

Henceforth, we shall refer to any RS rule that satisfies (7) and (8) as a *feasible RS rule*. Note that in (6) and (7), the average rate and average interference are obtained by averaging the corresponding instantaneous values over the channel gains h_{SD} and $\mathbf{h}$, and inherently over the selected relay β.

A. Optimal RS Rule

Let us first consider the conventional RS rule that maximizes the average rate when the interference constraint in (7) is not active. The optimal RS rule then selects the relay that maximizes the instantaneous rate [30]. Thus, we have

$$\beta = \underset{i\in\{0,1,\ldots,L\}}{\operatorname{argmax}}\left\{C_i\left(|h_{SD}|^2, |h_{Si}|^2, |h_{iD}|^2\right)\right\}. \quad (9)$$

We shall refer to this as the *unconstrained rule*. Let I_{un} denote the average interference caused to X due to transmissions by the relays when this rule is used for RS. Clearly, $I_{\text{un}} = \mathbb{E}_{h_{SD},\mathbf{h}}\left[P_\beta|h_{\beta X}|^2\right]$. When $I_{\text{un}} > I_{\text{avg}}$, the unconstrained rule cannot be optimal as it violates the interference constraint. In general, the optimal RS rule ϕ^* is given as follows. It always satisfies the average interference constraint.

Result 1: The selected relay $\beta^* = \phi^*(h_{SD},\mathbf{h})$ by the optimal RS rule is given by

$$\beta^* = \begin{cases} \underset{i\in\{0,1,\ldots,L\}}{\operatorname{argmax}}\left\{C_i\left(|h_{SD}|^2, |h_{Si}|^2, |h_{iD}|^2\right)\right\}, & I_{\text{un}} \le I_{\text{avg}}, \\ \underset{i\in\{0,1,\ldots,L\}}{\operatorname{argmax}}\left\{C_i\left(|h_{SD}|^2, |h_{Si}|^2, |h_{iD}|^2\right) - \lambda P_i|h_{iX}|^2\right\}, & I_{\text{un}} > I_{\text{avg}}, \end{cases} \quad (10)$$

where $P_i = 0$, for $i = 0$, and $P_i = P_r$, for $1 \le i \le L$. Here, λ is a strictly positive constant that arises only if $I_{\text{un}} > I_{\text{avg}}$. In this case, it is chosen such that the average interference constraint is satisfied with *equality*; such a choice always exists.

Proof: The proof is relegated to Appendix A. ■

Note that this rule applies even when the elements of $\mathbf{h}_S$, $\mathbf{h}_D$, and $\mathbf{h}_X$ are correlated, follow an arbitrary fading distribution, or are non-identically distributed. The term $C_i\left(|h_{SD}|^2, |h_{Si}|^2, |h_{iD}|^2\right) - \lambda P_i|h_{iX}|^2$ can be interpreted as a net payoff, which is the difference between the reward, which is the achieved rate $C_i\left(|h_{SD}|^2, |h_{Si}|^2, |h_{iD}|^2\right)$, and the penalty, which is the instantaneous interference power $P_i|h_{iX}|^2$ at the primary receiver. The result explicitly brings out the manner in which the average interference constraint affects the optimal RS rule. The constant λ is computed numerically, but only once, as is typical in several constrained optimization problems in wireless communications, e.g., optimal rate and power adaption and water-filling [31]. In general, as I_{avg} decreases, λ increases. We treat λ as a system parameter henceforth, and shall compute it as a function of the other system parameters in Section IV-A.

B. Comments About Model, Channel State Information (CSI) Assumptions, and RS Complexity

In the following, we discuss our various modeling and CSI assumptions. We then describe how the optimal RS rule can be implemented in practice, and analyze its feedback overhead.

1) Model: (i) We focus on proactive relaying because then S knows the selected relay and the reliable rate of transmission before it starts transmitting. Another practical advantage is that all relays other than the selected one can enter into an energy-conserving idle mode when S is transmitting. This is unlike reactive relaying, in which RS succeeds data transmission by S, and all the relays have to receive and attempt to decode the signal transmitted by S [32]. (ii) We assume that each node has one antenna because reducing the hardware complexity is one of the motivations for cooperative relaying [3], [4], [7], [8], and the optimal RS rule is novel even for this model. (iii) We focus on two-hop relaying because it constitutes the fundamental building block in a cooperative communication system and the optimal RS rule for the average interference constraint is not known for it in the literature.

2) CSI Assumptions and Acquisition: A relay i is assumed to know the instantaneous channel power gains of its local links, i.e., $|h_{Si}|^2$, $|h_{iD}|^2$, and $|h_{iX}|^2$. It can estimate $|h_{Si}|^2$ and $|h_{iD}|^2$ by using a training protocol and exploiting channel reciprocity; see [33] and the references therein. The relay can use a band manager [7] or it can exploit channel reciprocity to estimate $|h_{iX}|^2$ by periodically sensing the transmitted signal from X whenever X is engaged in a two-way communication with T. The destination is assumed to know the baseband channel gains h_{SD}, $h_{S\beta}$, and $h_{\beta D}$, and the average interference power σ_2^2 from T to itself for coherent demodulation [16]. It can estimate h_{SD} and $h_{\beta D}$ by using a training protocol [33], and it can learn about $h_{S\beta}$ from the selected relay β. We note that the RS rules in [4], [7], and [8] also make similar CSI assumptions.

3) Practical Implementation of RS: The optimal RS rule can be implemented in a distributed, scalable manner using a timer or splitting scheme [34]. For example, in the timer scheme, each relay i sets a timer, which is a monotone non-increasing function of its metric $C_i\left(|h_{SD}|^2, |h_{Si}|^2, |h_{iD}|^2\right) - \lambda g_i$, where $g_i = P_r|h_{iX}|^2$.[1] Upon expiry of its timer, the relay transmits a packet to S containing its identity and metric. The timer scheme guarantees that the first relay to transmit is the best relay. The source then compares the metrics of the best relay and of direct transmission, and selects the best option that maximizes the metric.

Another option is the polling scheme [35], in which each relay i feeds back quantized versions of γ_{iD} and g_i to S. Using a training protocol and by exploiting channel reciprocity, S estimates γ_{SD} and γ_{Si}, for $1 \le i \le L$. It then selects the relay that maximizes $C_i\left(|h_{SD}|^2, |h_{Si}|^2, |h_{iD}|^2\right) - \lambda g_i$, for $0 \le i \le L$.

4) Feedback Overhead Analysis and Comparison: Let O denote the number of feedback bits used to feed back a channel power gain. If the timer scheme is used, then the number of feedback bits needed by the optimal RS rule to feed back γ_{SD} to the relays is O. For incremental opportunistic relaying [3] and incremental reactive DF relaying [4], one bit needs to be fed back to indicate whether γ_{SD} exceeds a threshold or not.

Instead, if the polling scheme is used, then the number of bits needed by the optimal RS rule to feed back γ_{iD} and g_i to S, for $1 \le i \le L$, is $2LO$. Note that γ_{SD} and γ_{Si} do not need to be fed back since reciprocity can be exploited by S. Incremental opportunistic relaying and incremental reactive DF relaying incur the same overhead.

IV. Analysis of Average Rate of Optimal RS Rule

We analyze the average rate of the optimal RS rule when the various links undergo Rayleigh fading and are mutually independent and identically distributed (i.i.d.) [3], [4], [8]. Specifically, $h_{SD} \sim CN(0, \mu_{SD})$, $h_{SX} \sim CN(0, \mu_{SX})$, $h_{Si} \sim CN(0, \mu_{SR})$, $h_{iD} \sim CN(0, \mu_{RD})$, and $h_{iX} \sim CN(0, \mu_{RX})$, for $i \in \{1, 2, \ldots, L\}$. This ensures that the analysis leads to expressions for the average rate that provide valuable insights about the system. A more general model in which the various links are non-identically distributed can also be analyzed, but it leads to extremely involved and uninsightful final expressions. Therefore, the SINRs of the SD, SR, and RD links are $\gamma_{SD} \sim \mathcal{E}\left(\overline{\gamma}_{SD}\right)$, $\gamma_{Si} \sim \mathcal{E}\left(\overline{\gamma}_{SR}\right)$, and $\gamma_{iD} \sim \mathcal{E}\left(\overline{\gamma}_{RD}\right)$, respectively, where $\overline{\gamma}_{SD} = P_s\mu_{SD}/\left(\sigma_0^2 + \sigma_2^2\right)$, $\overline{\gamma}_{SR} = P_s\mu_{SR}/\left(\sigma_0^2 + \sigma_1^2\right)$, and $\overline{\gamma}_{RD} = P_r\mu_{RD}/\left(\sigma_0^2 + \sigma_2^2\right)$. The interference power at X due to a transmission by relay i is $g_i \sim \mathcal{E}(\overline{g})$, where $\overline{g} = P_r\mu_{RX}$.

In the following, we first derive a simple, single-integral form expression for the average rate $\overline{C}$ of the optimal RS rule when $\lambda > 0$, i.e., $I_{\text{un}} > I_{\text{avg}}$. The average rate for the unconstrained rule is a special case, in which $\lambda = 0$.

Result 2: The average rate $\overline{C}$ in the interference-constrained regime ($I_{\text{un}} > I_{\text{avg}}$) is given by

$$\begin{aligned}
\overline{C} &= \frac{1}{\overline{\gamma}_{SD}} \int_0^\infty \log_2(1+\gamma_{SD})\left[\eta(\gamma_{SD})\right]^L e^{-\frac{\gamma_{SD}}{\overline{\gamma}_{SD}}}\, d\gamma_{SD} \\
&\quad + \frac{L e^{\frac{1}{\overline{\gamma}}}}{(2\ln(2))^2\lambda\overline{\gamma}\,\overline{\gamma}_{SD}\overline{g}} \sum_{k=0}^{L-1}\binom{L-1}{k}(-1)^k \\
&\quad \times \int_1^\infty \left[\ln(y)\,\mathrm{E}_{\frac{1}{a\overline{g}}}\left(\frac{y}{\overline{\gamma}}\right) + G^{3,0}_{2,3}\left(\frac{y}{\overline{\gamma}}\middle|\begin{matrix}\frac{1}{a\overline{g}}, \frac{1}{a\overline{g}}\\ 0, \frac{1}{a\overline{g}}-1, \frac{1}{a\overline{g}}-1\end{matrix}\right)\right] \\
&\quad \times e^{-\frac{k(y-1)}{\overline{\gamma}}}\left(1 - \frac{y}{\overline{\gamma}}e^{\frac{y}{\overline{\gamma}}}\mathrm{E}_{\frac{1}{a\overline{g}}}\left(\frac{y}{\overline{\gamma}}\right)\right)^k I_k(y)dy, \qquad (11)
\end{aligned}$$

where

$$I_k(y) = \begin{cases} \dfrac{1 - e^{-(\sqrt{y}-1)\left(\frac{1}{\overline{\gamma}_{SD}} - \frac{k+1}{\overline{\gamma}_{RD}}\right)}}{\frac{1}{\overline{\gamma}_{SD}} - \frac{k+1}{\overline{\gamma}_{RD}}}, & \text{if } (k+1)\overline{\gamma}_{SD} \neq \overline{\gamma}_{RD}, \\ \sqrt{y} - 1, & \text{otherwise}, \end{cases} \qquad (12)$$

$$\begin{aligned}
\eta(\gamma_{SD}) &= 1 - e^{\frac{\gamma_{SD}}{\overline{\gamma}_{RD}} - \frac{\left(2\gamma_{SD}+\gamma_{SD}^2\right)}{\overline{\gamma}}}\Bigg(1 - \frac{(1+\gamma_{SD})^2}{\overline{\gamma}} \\
&\qquad \times e^{\frac{(1+\gamma_{SD})^2}{\overline{\gamma}}}\mathrm{E}_{\frac{1}{a\overline{g}}}\left(\frac{(1+\gamma_{SD})^2}{\overline{\gamma}}\right)\Bigg), \qquad (13)
\end{aligned}$$

$\overline{\gamma} = \overline{\gamma}_{SR}\overline{\gamma}_{RD}/\left(\overline{\gamma}_{SR} + \overline{\gamma}_{RD}\right)$, $a = 2\lambda\ln(2)$, $G^{m,n}_{p,q}\left(x\middle|\begin{matrix}a_1,\ldots,a_p\\ b_1,\ldots,b_q\end{matrix}\right)$ denotes the Meijer's G-function [36, (9.301)], and $\mathrm{E}_k(x)$ denotes the generalized exponential integral function [37, (5.1.4)].

Proof: The proof is relegated to Appendix B. ∎

Using Gauss-Laguerre quadrature [37, (25.4.45)], the above expression in (11) can be further reduced to the following integral-free form:

$$\begin{aligned}
\overline{C} &\approx \sum_{i=1}^N w_i \log_2\left(1 + z_i\overline{\gamma}_{SD}\right)\left[\eta\left(z_i\overline{\gamma}_{SD}\right)\right]^L \\
&\quad + \frac{L e^{\frac{1}{\overline{\gamma}}}}{(2\ln(2))^2\lambda\overline{\gamma}_{SD}\overline{g}} \sum_{k=0}^{L-1}\binom{L-1}{k}\frac{(-1)^k}{k} \\
&\quad \times \sum_{i=1}^N w_i \left[\ln\left(1 + \frac{\overline{\gamma}z_i}{k}\right)\mathrm{E}_{\frac{1}{a\overline{g}}}\left(\frac{k+\overline{\gamma}z_i}{k\overline{\gamma}}\right)\right. \\
&\quad \left. + G^{3,0}_{2,3}\left(\frac{k+\overline{\gamma}z_i}{k\overline{\gamma}}\middle|\begin{matrix}\frac{1}{a\overline{g}}, \frac{1}{a\overline{g}}\\ 0, \frac{1}{a\overline{g}}-1, \frac{1}{a\overline{g}}-1\end{matrix}\right)\right] I_k\left(1 + \frac{\overline{\gamma}z_i}{k}\right) \\
&\quad \times \left(1 - \frac{k+\overline{\gamma}z_i}{k\overline{\gamma}}e^{\frac{k+\overline{\gamma}z_i}{k\overline{\gamma}}}\mathrm{E}_{\frac{1}{a\overline{g}}}\left(\frac{k+\overline{\gamma}z_i}{k\overline{\gamma}}\right)\right)^k, \qquad (14)
\end{aligned}$$

[1] In order to compute its rate, the relay needs to know γ_{SD}, which can be broadcast by D to all the relays. Even one bit of feedback about γ_{SD} makes a significant difference, as was shown in [26]. We do not delve into this aspect further due to space constraints.

where z_i and w_i, for $1 \leq i \leq N$, are the N Gauss-Laguerre abscissas and weights, respectively. As N increases, the approximation error decreases towards zero. We have found that just $N = 5$ terms are sufficient for the range of interest of system parameters to accurately compute (14). Both $\mathrm{E}_k(\cdot)$ and $G^{m,n}_{p,q}(\cdot|\cdot)$ can be evaluated using computationally-efficient routines available in softwares such as Matlab and Mathematica [28].

Insights: In (11), the first term involving the single integral captures the contribution from the direct SD link, and the second term captures the contributions from the L relay links. As L increases, the contribution from the direct link decreases. This is because $\left[\eta(\gamma_{SD})\right]^L$ decreases since $\eta(\gamma_{SD}) < 1$. On the other hand, as λ or $\overline{g}$ increase (i.e., the interference link becomes stronger), the contribution from the direct link increases because the term $\mathrm{E}_{\frac{1}{a\overline{g}}}\left((1+\gamma_{SD})^2/\overline{\gamma}\right)$ in (13) increases.

A. Computing λ and I_{un}

For $\lambda > 0$, the average interference $\overline{I}$ caused to X due to relay transmissions is given by $\overline{I} = \mathbb{E}_{h_{SD},\mathbf{h}}\left[P_\beta |h_{\beta X}|^2\right]$. Since $P_\beta = 0$, for $\beta = 0$, and $P_\beta = P_r$, for $\beta \neq 0$, it can be shown that

$$\begin{aligned}\overline{I} &= \sum_{i=1}^{L} \mathbb{E}_{\gamma_{SD},X_i,g_i}\left[P_r|h_{iX}|^2 \Pr(\beta = i|\gamma_{SD}, X_i, g_i)\right],\\ &= L\mathbb{E}_{\gamma_{SD},X_1,g_1}\left[g_1 \Pr(\beta = 1|\gamma_{SD}, X_1, g_1)\right],\end{aligned} \tag{15}$$

where $X_i = \min\{\gamma_{Si}, \gamma_{iD} + \gamma_{SD}\}$ and $g_i = P_r|h_{iX}|^2$, as defined before. The second equality follows because the various SR links are i.i.d., and so are the various RD and RX links. Substituting the expression for $\Pr(\beta = 1|\gamma_{SD}, X_1, g_1)$ from (35) of Appendix B into (15), averaging over γ_{SD}, X_1, and g_1, simplifying further, and using Gauss-Laguerre quadrature, we get

$$\begin{aligned}\overline{I} \approx\ & \frac{Le^{\frac{1}{\overline{\gamma}}}}{(2\lambda\ln(2))^2\,\overline{\gamma}_{SD}\overline{g}} \sum_{k=0}^{L-1}\binom{L-1}{k}\frac{(-1)^k}{k}\\ &\times \sum_{i=1}^{N} w_i G^{3,0}_{2,3}\left(\frac{k+\overline{\gamma}z_i}{k\overline{\gamma}}\middle|\begin{matrix}\frac{1}{a\overline{g}},\frac{1}{a\overline{g}}\\ 0,\frac{1}{a\overline{g}}-1,\frac{1}{a\overline{g}}-1\end{matrix}\right) I_k\left(1+\frac{\overline{\gamma}z_i}{k}\right)\\ &\times\left(1-\frac{k+\overline{\gamma}z_i}{k\overline{\gamma}}e^{\frac{k+\overline{\gamma}z_i}{k\overline{\gamma}}}\mathrm{E}_{\frac{1}{a\overline{g}}}\left(\frac{k+\overline{\gamma}z_i}{k\overline{\gamma}}\right)\right)^k,\end{aligned} \tag{16}$$

where $I_k(y)$ is defined in (12). Since λ is the solution of the equation $\overline{I} = I_{\text{avg}}$, it can be easily computed using routines such as `fsolve` in Matlab.

Using (15), (9), and the identity in [36, (3.322.2)], the average interference I_{un} caused to X, which corresponds to the special case of $\lambda = 0$, can be shown to be

$$I_{\text{un}} = \frac{\sqrt{\pi\overline{\gamma}}L\overline{g}}{2\overline{\gamma}_{SD}} \sum_{k=0}^{L-1}\binom{L-1}{k}\frac{(-1)^k}{(k+1)^{\frac{3}{2}}}\operatorname{erfc}\left(\sqrt{\delta_1(k)}\right)e^{\delta_1(k)}, \tag{17}$$

where $\delta_1(k) = \overline{\gamma}\left(1/\overline{\gamma}_{SD} + 2(k+1)/\overline{\gamma}_{SR} + (k+1)/\overline{\gamma}_{RD}\right)^2/(4(k+1))$ and $\operatorname{erfc}(x)$ denotes the complementary error function [37, (7.1.2)].

B. Asymptotic Analysis of Average Rate

We now examine the asymptotic behavior of the average rate at high and low average SINRs; these lead to different and new insights about the system.

1) High SINR Analysis: We first consider the regime in which $P_{\max} \to \infty$ with μ_{SD}, μ_{SR}, μ_{RD}, μ_{RX}, μ_{SX}, σ_0^2, σ_1^2, σ_2^2, and λ fixed. Thus, $P_r \to \infty$, $\overline{\gamma}_{RD} \to \infty$, $P_s \to I_{\text{avg}}/\mu_{SX}$, $\overline{\gamma}_{SD} \to I_{\text{avg}}\mu_{SD}/\left(\mu_{SX}\left(\sigma_0^2+\sigma_2^2\right)\right)$, $\overline{\gamma}_{SR} \to I_{\text{avg}}\mu_{SR}/\left(\mu_{SX}\left(\sigma_0^2+\sigma_1^2\right)\right)$, and $\overline{g} \to \infty$. Therefore, the average SINRs of the RD links are high.

Corollary 1: In the high SINR regime, $\overline{C}$ in (11) simplifies to

$$\begin{aligned}\overline{C} \to\ & \frac{e^{\frac{1}{\overline{\gamma}_{SD}}}\mathrm{E}_1\left(\frac{1}{\overline{\gamma}_{SD}}\right)}{\ln(2)} + \frac{L}{2\ln(2)\,\overline{\gamma}_{SR}}\int_1^\infty \ln(t)\,e^{-\frac{(t-1)}{\overline{\gamma}_{SR}}}\\ &\times\left(1-t^{-\frac{1}{a\overline{g}}}\right)\left(1-e^{-\frac{(\sqrt{t}-1)}{\overline{\gamma}_{SD}}}\right)dt.\end{aligned} \tag{18}$$

Proof: The proof is relegated to Appendix C. ■

Using Gauss-Laguerre quadrature, (18) can be further reduced to the following integral-free form:

$$\begin{aligned}\overline{C} \approx\ & \frac{e^{\frac{1}{\overline{\gamma}_{SD}}}\mathrm{E}_1\left(\frac{1}{\overline{\gamma}_{SD}}\right)}{\ln(2)} + \frac{L}{2\ln(2)\,\overline{\gamma}_{SR}}\sum_{i=1}^{N} w_i\ln\left(1+\overline{\gamma}_{SR}z_i\right)\\ &\times\left(1-\left(1+\overline{\gamma}_{SR}z_i\right)^{-\frac{1}{a\overline{g}}}\right)\left(1-e^{-\frac{\left(\sqrt{1+\overline{\gamma}_{SR}z_i}-1\right)}{\overline{\gamma}_{SD}}}\right).\end{aligned} \tag{19}$$

The first term in (18) is the average rate if there were no relay in the system. Since $e^x\mathrm{E}_1(x) \approx \ln(1+1/x)$ [37, (5.1.20)], it can be approximated as $\log_2\left(1+\overline{\gamma}_{SD}\right)$. Furthermore, the second term, which captures the contribution from the relay links, is independent of $\overline{\gamma}_{RD}$ and increases as L increases, or as λ or $\overline{g}$ decrease. Since for any finite $t \geq 1$, $\left(1-t^{-1/(a\overline{g})}\right)$ is very small as $\overline{g} \to \infty$, the first term dominates the second term, and hence, the average rate approaches that of direct transmission.

2) Low SINR Analysis: Next, we analyze the asymptotic behavior at low average SINRs in which $P_{\max} \to 0$ with μ_{SD}, μ_{SR}, μ_{RD}, μ_{RX}, μ_{SX}, σ_0^2, σ_1^2, σ_2^2, and λ fixed. Thus, $P_s \to 0$ and $P_r \to 0$. Therefore, the average SINRs of all the links and the average interference power at the primary receiver due to source and relay transmissions are all low.

Corollary 2: In the low SINR regime, $\overline{C}$ in (11) simplifies to

$$\begin{aligned}\overline{C} \approx\ & \sum_{k=0}^{L}\frac{(-1)^k\binom{L}{k}e^{\delta_2(k)}\mathrm{E}_1(\delta_2(k))}{\ln(2)\left(1+\frac{a\overline{g}}{\overline{\gamma}}\right)^k\overline{\gamma}_{SD}\delta_2(k)}\\ &+L\sum_{k=0}^{L-1}\frac{(-1)^k\binom{L-1}{k}e^{\frac{\delta_2(k+1)}{2}}\mathrm{E}_1\left(\frac{\delta_2(k+1)}{2}\right)}{4\ln(2)\,(k+1)^2\left(1+\frac{a\overline{g}}{\overline{\gamma}}\right)^{k+2}\overline{\gamma}_{SD}\delta_2(k+1)}\\ &\times\Bigg[\left(\frac{\overline{\gamma}}{\overline{\gamma}_{SD}}+4(k+1)-\frac{(k+1)\overline{\gamma}}{\overline{\gamma}_{RD}}\right)\left(1+\frac{a\overline{g}}{\overline{\gamma}}\right)\\ &\quad+(k+1)a\overline{g}\delta_2(k+1)\Bigg],\end{aligned} \tag{20}$$

where $\delta_2(k) = 1/\overline{\gamma}_{SD} + 2k/\overline{\gamma}_{SR} + k/\overline{\gamma}_{RD}$.

Proof: The proof is relegated to Appendix D. ∎

Further Insights for Low SINRs: To get further insights, consider the case $L = 2$ and the average SINRs of the SR and RD links are the same. Then, substituting $\mathrm{E}_1(x) \approx e^{-x}/x$, [37, (5.1.19)] in (20), the ratio of $\overline{C}$ to the average rate of the direct SD link, $\overline{C}_{\mathrm{SD}}$, simplifies to

$$\frac{\overline{C}}{\overline{C}_{\mathrm{SD}}} \approx 1 + \frac{\frac{\overline{\gamma}_{SR}}{2\overline{\gamma}_{SD}}\left(1+\frac{4a\overline{g}}{\overline{\gamma}_{SR}}\right)}{\left(1+\frac{3\overline{\gamma}_{SD}}{\overline{\gamma}_{SR}}\right)\left(1+\frac{2a\overline{g}}{\overline{\gamma}_{SR}}\right)^2} - \frac{\frac{\overline{\gamma}_{SR}}{8\overline{\gamma}_{SD}}\left(1+\frac{6a\overline{g}}{\overline{\gamma}_{SR}}\right)}{\left(1+\frac{6\overline{\gamma}_{SD}}{\overline{\gamma}_{SR}}\right)\left(1+\frac{2a\overline{g}}{\overline{\gamma}_{SR}}\right)^3}, \quad (21)$$

where $\overline{C}_{\mathrm{SD}} = \int_0^\infty \log_2(1+\gamma_{SD}) e^{-\frac{\gamma_{SD}}{\overline{\gamma}_{SD}}}/\overline{\gamma}_{SD}\, d\gamma_{SD} \to \overline{\gamma}_{SD}/\ln 2$. The third term in (21) can be shown to be very small compared to the second term and, hence, can be neglected. Therefore, we see that as the average SINR of the SR link increases compared to that of the SD link, the relative rate gain over the SD link increases by a factor of $\left(\frac{\overline{\gamma}_{SR}}{2\overline{\gamma}_{SD}}\right) \Big/ \left(1+\frac{3\overline{\gamma}_{SD}}{\overline{\gamma}_{SR}}\right)$. Furthermore, as the average SINR of the SR link decreases compared to the average interference power of the RX interference link, i.e., as $\overline{g}/\overline{\gamma}_{SR}$ (or $\overline{g}/\overline{\gamma}_{RD}$) increases, $\left(1+\frac{4a\overline{g}}{\overline{\gamma}_{SR}}\right) \Big/ \left(1+\frac{2a\overline{g}}{\overline{\gamma}_{SR}}\right)^2$ decreases, which results in a reduction in the relative rate gain by this factor. Similarly, as λ increases, the relative rate gain decreases by the same factor.

V. Rate Benchmarking and Numerical Results

In order to verify our analysis and gain quantitative insights, we now present Monte Carlo simulation results of the average rate that are averaged over 10^5 samples. The various links undergo Rayleigh fading. We set $\sigma_1^2 = \sigma_2^2 = 2.16\sigma_0^2$. Let $\overline{\gamma}_{SD}^{\max} = P_{\max}\mu_{SD}/\left(\sigma_0^2+\sigma_2^2\right)$.[2] We shall refer to it as the maximum average SD link SINR, and shall plot it on the x-axis in the figures that follow below.

A. Benchmarking of Average Rate

Fig. 2 compares the average rate of the optimal RS rule with those of: (i) incremental opportunistic relaying [3] and (ii) incremental reactive DF relaying [4] with $\gamma_{\mathrm{th}} = 2$. The trends for the other values of γ_{th} are qualitatively similar. We note that no single value of γ_{th} is optimal for all values of $\overline{\gamma}_{SD}^{\max}$; it needs to be numerically optimized for each value of $\overline{\gamma}_{SD}^{\max}$, which is cumbersome. Also shown for reference are the average rates of: (i) conventional opportunistic relaying [23], (ii) conventional reactive DF relaying [24], and (iii) direct transmission. In order to ensure a fair comparison and focus on the role of the relay selection rule, the source and relays in the benchmarking schemes are also subject to the average interference constraint and the maximum power constraint.

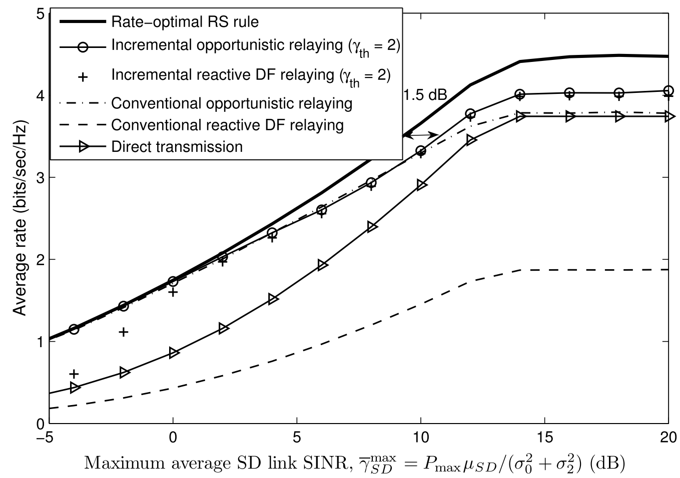

Fig. 2. Comparison of the average rate of the optimal RS rule with several other rules proposed in the literature ($L = 4$, $I_{\mathrm{avg}}/\sigma_0^2 = 15$ dB, $\mu_{SD} = 1$, $\mu_{SR} = \mu_{RD} = 10\mu_{SD}$, and $\mu_{SX} = \mu_{RX} = 0.5\mu_{SD}$).

We observe that the proposed RS rule delivers the highest average rate compared to the other RS rules over the entire range of $\overline{\gamma}_{SD}^{\max}$ considered. For example, at an average rate of 3.5 bits/sec/Hz, the optimal RS rule requires $\overline{\gamma}_{SD}^{\max}$ that is lower by 1.5 dB, 1.7 dB, 2.0 dB, and 3.1 dB compared to incremental opportunistic relaying, incremental reactive DF relaying, conventional opportunistic relaying, and direct transmission, respectively. We observe that incremental opportunistic relaying is near-optimal for $\overline{\gamma}_{SD}^{\max} \leq 2$ dB. Since it considers both the SR and RD links, it outperforms incremental reactive DF relaying, which only considers the RD link for selecting the best relay. The average rate of conventional opportunistic relaying matches that of incremental opportunistic relaying for low-to-mid values of $\overline{\gamma}_{SD}^{\max}$, whereas the former performs worse for higher values of $\overline{\gamma}_{SD}^{\max}$. Similar trends are observed for conventional reactive DF relaying and incremental reactive DF relaying.

B. Effect of Various System Parameters

Fig. 3 plots the average rate of the optimal RS rule as a function of $\overline{\gamma}_{SD}^{\max}$ for different values of $I_{\mathrm{avg}}/\sigma_0^2$. Results from simulations and the analytical expression in (14) are plotted, and they match well. As a reference, the average rates for the interference-unconstrained regime ($I_{\mathrm{avg}}/\sigma_0^2 = \infty$) and direct transmission are also shown. When $P_{\max} \leq I_{\mathrm{avg}}/\mu_{SX}$ or, equivalently, $\overline{\gamma}_{SD}^{\max} \leq I_{\mathrm{avg}}\mu_{SD}/\left(\mu_{SX}\left(\sigma_0^2+\sigma_2^2\right)\right)$, we see that the average rate increases with $\overline{\gamma}_{SD}^{\max}$. However, when $P_{\max} > I_{\mathrm{avg}}/\mu_{SX}$, the average rate saturates and approaches that of direct transmission. In this regime, the average SD link SINR $\overline{\gamma}_{SD} = I_{\mathrm{avg}}\mu_{SD}/\left(\mu_{SX}\left(\sigma_0^2+\sigma_2^2\right)\right)$ becomes a constant and so does the average rate of direct transmission. As $I_{\mathrm{avg}}/\sigma_0^2$ increases, the average rate of the optimal RS rule and the value of $\overline{\gamma}_{SD}^{\max}$ at which it saturates increase due to the more relaxed interference constraint.

[2] A one-to-one correspondence between the average SINR of a link and the distance between its nodes can be made as follows. For the simplified path-loss model [31, Chap. 2.6] with a path-loss exponent of 4, cut-off distance of 10 m, carrier frequency of 900 MHz, distance between the nodes of 205 m, signal bandwidth of 1 MHz, thermal noise temperature of 300 K, noise figure of 10 dB, and an average interference power that is 2.16 times the average noise power, the average SINR is 10 dB when the transmit power is 15 dBmW. At a distance of 115 m, the average SINR increases to 20 dB.

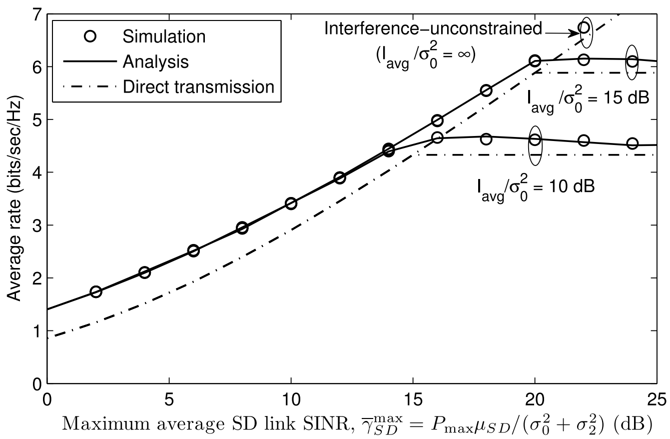

Fig. 3. Average rate of the optimal RS rule as a function of maximum average SD link SINR for different values of $I_{\text{avg}}/\sigma_0^2$ ($L = 4$, $\mu_{SD} = 1$, $\mu_{SR} = \mu_{RD} = 5\mu_{SD}$, $\mu_{RX} = 0.9\mu_{SD}$, and $\mu_{SX} = 0.1\mu_{SD}$).

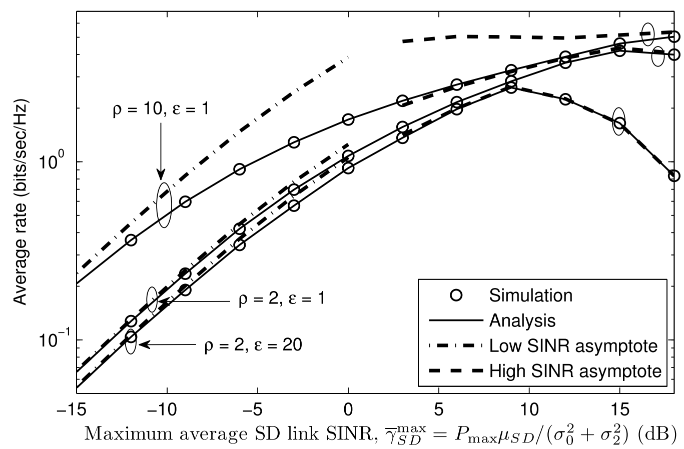

Fig. 5. Role of the strengths of SR, RD, and RX links: Average rate of the optimal RS rule as a function of maximum average SD link SINR and asymptotic curves for different values of ρ and ε ($L = 4$, $\lambda = 0.05$, and $\mu_{SD} = 1$, $\mu_{SX} = 0.01\mu_{SD}$, $\rho = \mu_{SR}/\mu_{SD} = \mu_{RD}/\mu_{SD}$, and $\varepsilon = \mu_{RX}/\mu_{SD}$).

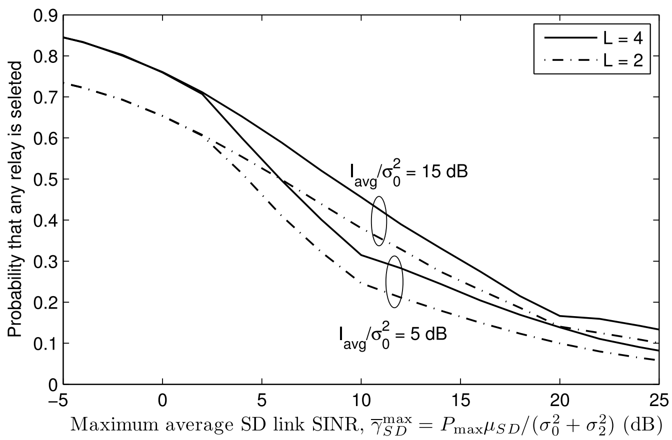

Fig. 4. Probability that any relay is selected by the optimal RS rule as a function of maximum average SD link SINR for different values of L and $I_{\text{avg}}/\sigma_0^2$ ($\mu_{SD} = 1$, $\mu_{SR} = \mu_{RD} = 5\mu_{SD}$, $\mu_{RX} = 0.9\mu_{SD}$, and $\mu_{SX} = 0.1\mu_{SD}$).

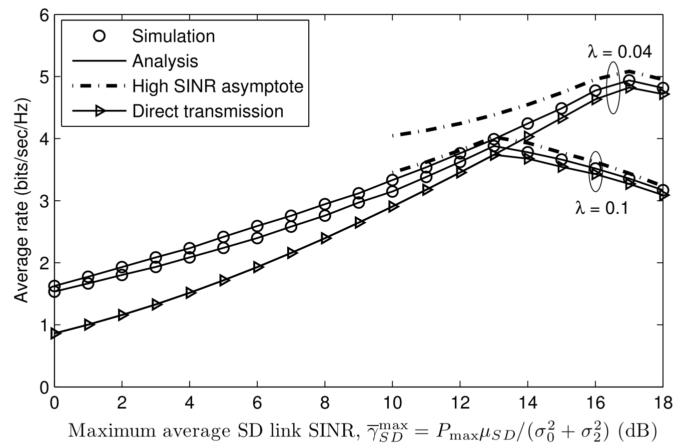

Fig. 6. Role of λ: Average rate of the optimal RS rule as a function of maximum average SD link SINR for different values of λ ($L = 3$, $\mu_{SD} = 1$, $\mu_{SX} = 0.01\mu_{SD}$, $\mu_{SR} = \mu_{RD} = 10\mu_{SD}$, and $\mu_{RX} = 2\mu_{SD}$).

To understand the trends better, Fig. 4 plots the probability that any relay is selected (i.e., $\beta \neq 0$) by the optimal RS rule as a function of $\overline{\gamma}_{SD}^{\max}$ for different values of L and $I_{\text{avg}}/\sigma_0^2$. It decreases as $\overline{\gamma}_{SD}^{\max}$ increases in order to avoid the interference caused to X due to relay transmissions and to save the extra slot required by its use. For a fixed $I_{\text{avg}}/\sigma_0^2$, as L increases, the probability that any relay is selected increases due to increased spatial diversity. And, for a fixed L, as $I_{\text{avg}}/\sigma_0^2$ increases, the probability that any relay is selected increases due to the more relaxed interference constraint.

Fig. 5 studies the effect of the relative strengths of the SR, RD, and RX links compared to the direct SD link on the average rate of the optimal RS rule. For this, we set $\mu_{SR} = \mu_{RD} = \rho\mu_{SD}$ and $\mu_{RX} = \varepsilon\mu_{SD}$, and vary both ρ and ε. This figure also plots the high SINR and low SINR asymptotic results that were derived in (19) and (20), respectively. In order to better distinguish the curves, the y-axis is plotted in log scale. As ρ decreases, the gap between the asymptotic and actual curves decreases because the asymptotics kick in earlier. Given ρ, the average rate decreases as ε increases because the interference link becomes stronger than the direct SD link, and also the SR and RD links. Furthermore, given ε, the average rate increases as ρ increases because the SR and RD links become relatively stronger than the direct SD link. Note that the average rate decreases marginally for higher values of $\overline{\gamma}_{SD}^{\max}$. This is because we have fixed λ in the plot; therefore, as per (16), I_{avg} is a function of $P_{\max}$. As $P_{\max}$ increases, and, thus, $\overline{\gamma}_{SD}^{\max}$ increases, it turns out that the probability that any relay is selected decreases faster than the rate at which $P_{\max}$ increases. Hence, I_{avg}, which is proportional to a product of the two (cf. (15)) decreases. Therefore, P_s tends to I_{avg}/μ_{SX} and decreases; so does the average rate.

Lastly, Fig. 6 studies the effect of λ; the larger its value, the tighter is the interference constraint. We plot the average rate of the optimal RS rule as a function of $\overline{\gamma}_{SD}^{\max}$ for different values of λ. The high SINR asymptote in (19) and the average rate of direct transmission are also shown for reference. For $\lambda = 0.1$ and 0.04, the average rate decreases when $\overline{\gamma}_{SD}^{\max}$ exceeds 13 dB and 17 dB, respectively. This counter-intuitive behavior occurs because λ is fixed. As explained in the previous paragraph, this leads to a reduction in I_{avg}, P_s, and the average rate for higher values of $\overline{\gamma}_{SD}^{\max}$.

VI. Conclusions

We proposed a novel RS rule that provably maximized the average rate between the source and destination in an average interference-constrained underlay cooperative CR network. It optimally traded off between the SINR improvement afforded by the use of a relay and the additional hop required and the interference caused by its use. It also accounted for the state of the direct link. We saw that the proposed rule outperformed several rules proposed in the literature, and the overhead of feeding back the requisite CSI was comparable. We also analyzed the average rate of the optimal RS rule. Its expression simplified considerably in the asymptotic high and low SINR regimes. We saw that for high average SINRs, the direct SD link was selected more often and the contribution to the average rate from the use of the relays increased as the number of relays increased, or as λ or the average channel power gain of the RX links decreased. At low average SINRs, a relay was selected more often and improved the average rate of the underlay CR network compared to direct transmission. Furthermore, we came to know that incremental opportunistic relaying was near-optimal for low average SINRs, whereas incremental reactive DF relaying was sub-optimal for a wide range of average SINRs. Extending our results to a multi-hop network is an interesting avenue for future work.

Appendix

A. Proof of Result 1

When $I_{\text{un}} > I_{\text{avg}}$, the set of all feasible RS rules that satisfy (7) and (8) is a non-empty set. This is because the rule in which no relay transmits causes zero interference to X, and is, therefore, a feasible RS rule. Let ϕ be a feasible RS rule and β be the relay selected by it given h_{SD} and $\mathbf{h}$. For $\lambda > 0$, define an auxiliary function $L_\phi(\lambda)$ associated with ϕ as follows:

$$L_\phi(\lambda) \triangleq \mathbb{E}_{h_{SD},\mathbf{h}}\Big[C_\beta\left(|h_{SD}|^2, |h_{S\beta}|^2, |h_{\beta D}|^2\right) - \lambda P_\beta |h_{\beta X}|^2\Big]. \tag{22}$$

Note that $L_\phi(\lambda)$ is a function of both ϕ and λ.

Further, define a new RS rule ϕ^* in terms of the relay β^* it selects as follows:

$$\beta^* = \underset{i\in\{0,1,\ldots,L\}}{\operatorname{argmax}} \left\{C_i\left(|h_{SD}|^2, |h_{Si}|^2, |h_{iD}|^2\right) - \lambda P_i |h_{iX}|^2\right\}, \tag{23}$$

where λ is chosen such that $\mathbb{E}_{h_{SD},\mathbf{h}}\big[P_{\beta^*}|h_{\beta^* X}|^2\big] = I_{\text{avg}}$. It can proved using the intermediate value theorem that such a unique choice of λ exists. The theorem can be applied on the basis of the following two facts: (i) $0 \le I_{\text{avg}} < I_{\text{un}}$, and (ii) It can be shown that the average interference is a continuous and monotonically decreasing function of λ for $\lambda \ge 0$.

Thus, ϕ^* is a feasible RS rule. We now prove that ϕ^* is the optimal RS rule. From (23), it is clear that $L_{\phi^*}(\lambda) \ge L_\phi(\lambda)$. Therefore, from (22), we get

$$\begin{aligned}\mathbb{E}_{h_{SD},\mathbf{h}}&\Big[C_{\beta^*}\left(|h_{SD}|^2, |h_{S\beta^*}|^2, |h_{\beta^* D}|^2\right)\Big] \\ &\ge \mathbb{E}_{h_{SD},\mathbf{h}}\Big[C_\beta\left(|h_{SD}|^2, |h_{S\beta}|^2, |h_{\beta D}|^2\right)\Big] \\ &\quad - \lambda\left(\mathbb{E}_{h_{SD},\mathbf{h}}\Big[P_\beta|h_{\beta X}|^2\Big] - I_{\text{avg}}\right).\end{aligned} \tag{24}$$

Since ϕ is a feasible RS rule, $\mathbb{E}_{h_{SD},\mathbf{h}}\big[P_\beta|h_{\beta X}|^2\big] \le I_{\text{avg}}$. Therefore, from (24), we get

$$\begin{aligned}\mathbb{E}_{h_{SD},\mathbf{h}}&\Big[C_{\beta^*}\left(|h_{SD}|^2, |h_{S\beta^*}|^2, |h_{\beta^* D}|^2\right)\Big] \\ &\ge \mathbb{E}_{h_{SD},\mathbf{h}}\Big[C_\beta\left(|h_{SD}|^2, |h_{S\beta}|^2, |h_{\beta D}|^2\right)\Big].\end{aligned} \tag{25}$$

Hence, ϕ^* yields the highest average rate among all the feasible RS rules.

B. Proof of Result 2

Using (4) and (5), the average rate for DF relays $\overline{C} = \mathbb{E}_{h_{SD},\mathbf{h}}\big[C_\beta\left(|h_{SD}|^2, |h_{S\beta}|^2, |h_{\beta D}|^2\right)\big]$ simplifies to

$$\overline{C} = A_1 + LA_2, \tag{26}$$

where the factor L arises because the L relays are statistically identical,

$$A_1 = \mathbb{E}_{\gamma_{SD}}\left[\log_2(1+\gamma_{SD})\Pr(\beta = 0|\gamma_{SD})\right], \tag{27}$$

$$A_2 = \frac{1}{2}\mathbb{E}_{\gamma_{SD},X_1,g_1}\big[\log_2(1+X_1)\Pr(\beta = 1|\gamma_{SD}, X_1, g_1)\big], \tag{28}$$

$X_i = \min\{\gamma_{Si}, \gamma_{iD} + \gamma_{SD}\}$, and $g_i = P_r|h_{iX}|^2$, for $1 \le i \le L$. Now, conditioned on γ_{SD}, $X_1, \ldots, X_L$ are i.i.d. RVs, whose conditional probability density function (PDF) $f_{X_i|\gamma_{SD}}(x|\gamma_{SD})$ can be shown to be

$$f_{X_i|\gamma_{SD}}(x|\gamma_{SD}) = \begin{cases} e^{-\frac{x}{\overline{\gamma}_{SR}}}/\overline{\gamma}_{SR}, & 0 \le x \le \gamma_{SD}, \\ e^{-\left(\frac{x}{\overline{\gamma}} - \frac{\gamma_{SD}}{\overline{\gamma}_{RD}}\right)}/\overline{\gamma}, & x > \gamma_{SD}, \end{cases} \tag{29}$$

where $\overline{\gamma} = \overline{\gamma}_{SR}\overline{\gamma}_{RD}/\left(\overline{\gamma}_{SR} + \overline{\gamma}_{RD}\right)$. We now evaluate A_1 and A_2 separately below.

1) Evaluating A_1: From (10), the conditional probability that no relay is selected equals

$$\begin{aligned}\Pr(\beta = 0|\gamma_{SD}) = \Pr\Big(&\log_2(1+\gamma_{SD}) \\ &> \max_{i\in\{1,2,\ldots,L\}}\left\{\frac{1}{2}\log_2(1+X_i) - \lambda g_i\right\} \Big| \gamma_{SD}\Big).\end{aligned} \tag{30}$$

Since $g_1, \ldots, g_L$ are i.i.d. RVs and so are $X_1, \ldots, X_L$ when conditioned on γ_{SD}, (30) becomes

$$\begin{aligned}\Pr(\beta = 0|\gamma_{SD}) &= \left[\Pr\left(g_1 > \frac{1}{a}\ln\left(\frac{1+X_1}{(1+\gamma_{SD})^2}\right) \Big| \gamma_{SD}\right)\right]^L, \\ &= \Bigg(\mathbb{E}_{X_1|\gamma_{SD}}\Bigg[1_{\{X_1 \le 2\gamma_{SD}+\gamma_{SD}^2\}} \\ &\quad + 1_{\{X_1 > 2\gamma_{SD}+\gamma_{SD}^2\}}\left(\frac{1+X_1}{(1+\gamma_{SD})^2}\right)^{-\frac{1}{a\overline{g}}}\Bigg]\Bigg)^L,\end{aligned} \tag{31}$$

where $a = 2\lambda\ln(2)$. Substituting the conditional PDF of X_1 from (29) in (31), we get

$$\begin{aligned}\Pr(\beta = 0|\gamma_{SD}) &= \Bigg[\frac{1}{\overline{\gamma}_{SR}}\int_0^{\gamma_{SD}} e^{-\frac{x_1}{\overline{\gamma}_{SR}}}dx_1 + \frac{1}{\overline{\gamma}}\int_{\gamma_{SD}}^{2\gamma_{SD}+\gamma_{SD}^2} e^{-\frac{x_1}{\overline{\gamma}}+\frac{\gamma_{SD}}{\overline{\gamma}_{RD}}}dx_1 \\ &\quad + \frac{1}{\overline{\gamma}}\int_{2\gamma_{SD}+\gamma_{SD}^2}^{\infty}\left(\frac{1+x_1}{(1+\gamma_{SD})^2}\right)^{-\frac{1}{a\overline{g}}} e^{-\frac{x_1}{\overline{\gamma}}+\frac{\gamma_{SD}}{\overline{\gamma}_{RD}}}dx_1\Bigg]^L.\end{aligned} \tag{32}$$

Employing the variable substitution $(1+x_1)/(1+\gamma_{SD})^2 = t$, substituting (32) into the expression for A_1 in (27), and averaging over the RV $\gamma_{SD} \sim \mathcal{E}\{\overline{\gamma}_{SD}\}$, we get the first term in (11).

2) Evaluating A_2: As in (30), the conditional probability that relay 1 is selected equals

$$\begin{aligned}\Pr(\beta = 1|\gamma_{SD}, X_1, g_1) &= \Pr\Bigg(\frac{1}{2}\log_2(1+X_1) - \lambda g_1 > \log_2(1+\gamma_{SD}),\\ &\quad \frac{1}{2}\log_2(1+X_1) - \lambda g_1 > \frac{1}{2}\log_2(1+X_2) - \lambda g_2,\\ &\quad \ldots, \frac{1}{2}\log_2(1+X_1) - \lambda g_1 > \frac{1}{2}\log_2(1+X_L) - \lambda g_L\\ &\quad \Bigg| \gamma_{SD}, X_1, g_1\Bigg). \end{aligned} \tag{33}$$

Since $g_1, \ldots, g_L$ are i.i.d. RVs and so are $X_1, \ldots, X_L$ when conditioned on γ_{SD}, (33) becomes

$$\begin{aligned}\Pr(\beta = 1|\gamma_{SD}, X_1, g_1) = 1_{\left\{g_1 < \frac{1}{a}\ln\left(\frac{1+X_1}{(1+\gamma_{SD})^2}\right)\right\}} &\times \Bigg[\Pr\Bigg(g_2 > \frac{1}{a}\ln\left(\frac{1+X_2}{(1+X_1)\,e^{-ag_1}}\right)\\ &\quad \Bigg|\gamma_{SD}, X_1, g_1\Bigg)\Bigg]^{L-1}. \end{aligned} \tag{34}$$

Similar to (31) and (32), substituting the PDF of the RV $g_2 \sim \mathcal{E}\{\overline{g}\}$ and the conditional PDF of X_2 from (29) in (34), and simplifying further, we get

$$\begin{aligned}\Pr(\beta = 1|\gamma_{SD}, X_1, g_1) = 1_{\left\{g_1 < \frac{1}{a}\ln\left(\frac{1+X_1}{(1+\gamma_{SD})^2}\right)\right\}} &\times \Bigg[1 - e^{\frac{\gamma_{SD}}{\overline{\gamma}_{RD}} - \frac{(1+X_1)e^{-ag_1}-1}{\overline{\gamma}}}\Bigg(1\\ &- \frac{(1+X_1)\,e^{-ag_1}}{\overline{\gamma}}e^{\frac{(1+X_1)e^{-ag_1}}{\overline{\gamma}}}\\ &\times \mathrm{E}_{\frac{1}{a\overline{g}}}\left(\frac{(1+X_1)\,e^{-ag_1}}{\overline{\gamma}}\right)\Bigg)\Bigg]^{L-1}. \end{aligned} \tag{35}$$

Substituting (35) into the expression for A_2 in (28) and averaging over γ_{SD}, X_1, and g_1, we get

$$\begin{aligned} A_2 = \frac{1}{2\ln(2)\,\overline{\gamma}\,\overline{\gamma}_{SD}\overline{g}} \int_0^\infty \int_{2\gamma_{SD}+\gamma_{SD}^2}^\infty \int_0^{\frac{1}{a}\ln\left(\frac{1+x_1}{(1+\gamma_{SD})^2}\right)} e^{-\frac{\gamma_{SD}}{\overline{\gamma}_{SD}}}\\ \times \ln(1+x_1)\Bigg[1 - e^{\frac{\gamma_{SD}}{\overline{\gamma}_{RD}} - \frac{(1+x_1)e^{-ag_1}-1}{\overline{\gamma}}}\Bigg(1\\ - \frac{(1+x_1)\,e^{-ag_1}}{\overline{\gamma}}e^{\frac{(1+x_1)e^{-ag_1}}{\overline{\gamma}}}\mathrm{E}_{\frac{1}{a\overline{g}}}\left(\frac{(1+x_1)\,e^{-ag_1}}{\overline{\gamma}}\right)\Bigg)\Bigg]^{L-1}\\ \times e^{-\left(\frac{g_1}{\overline{g}} + \frac{x_1}{\overline{\gamma}} - \frac{\gamma_{SD}}{\overline{\gamma}_{RD}}\right)}\,dg_1\,dx_1\,d\gamma_{SD}. \end{aligned} \tag{36}$$

Substituting $(1+x_1)\,e^{-ag_1} = y$ and $e^{ag_1} = t$ in (36), using the identity $\int_1^\infty \ln(t)\,e^{-at}t^{-n}\,dt = G^{3,0}_{2,3}\left(a\middle|^{n,n}_{0,n-1,n-1}\right)$, for $a > 0$, and simplifying further yields

$$\begin{aligned} A_2 = \frac{e^{\frac{1}{\overline{\gamma}}}}{(2\ln(2))^2\,\lambda\overline{\gamma}\,\overline{\gamma}_{SD}\overline{g}} \int_0^\infty \int_{(1+\gamma_{SD})^2}^\infty e^{\left(\frac{\gamma_{SD}}{\overline{\gamma}_{RD}} - \frac{\gamma_{SD}}{\overline{\gamma}_{SD}}\right)}\\ \times \left[\ln(y)\,\mathrm{E}_{\frac{1}{a\overline{g}}}\left(\frac{y}{\overline{\gamma}}\right) + G^{3,0}_{2,3}\left(\frac{y}{\overline{\gamma}}\middle|^{\frac{1}{a\overline{g}},\frac{1}{a\overline{g}}}_{0,\frac{1}{a\overline{g}}-1,\frac{1}{a\overline{g}}-1}\right)\right]\\ \times \zeta^{L-1}(\gamma_{SD}, y)\,dy\,d\gamma_{SD}, \end{aligned} \tag{37}$$

where $\zeta(\gamma_{SD}, y) = 1 - e^{\frac{\gamma_{SD}}{\overline{\gamma}_{RD}} - \frac{y-1}{\overline{\gamma}}}\left(1 - \frac{y}{\overline{\gamma}}e^{\frac{y}{\overline{\gamma}}}\mathrm{E}_{\frac{1}{a\overline{g}}}\left(\frac{y}{\overline{\gamma}}\right)\right)$. Taking the binomial series expansion of $\zeta^{L-1}(\gamma_{SD}, y)$ and interchanging the order of integration yields

$$\begin{aligned} A_2 = \frac{e^{\frac{1}{\overline{\gamma}}}}{(2\ln(2))^2\,\lambda\overline{\gamma}\,\overline{\gamma}_{SD}\overline{g}} \sum_{k=0}^{L-1}\binom{L-1}{k}(-1)^k \int_1^\infty e^{-\frac{k(y-1)}{\overline{\gamma}}}\\ \times \left[\ln(y)\,\mathrm{E}_{\frac{1}{a\overline{g}}}\left(\frac{y}{\overline{\gamma}}\right) + G^{3,0}_{2,3}\left(\frac{y}{\overline{\gamma}}\middle|^{\frac{1}{a\overline{g}},\frac{1}{a\overline{g}}}_{0,\frac{1}{a\overline{g}}-1,\frac{1}{a\overline{g}}-1}\right)\right]\\ \times \left(1 - \frac{y}{\overline{\gamma}}e^{\frac{y}{\overline{\gamma}}}\mathrm{E}_{\frac{1}{a\overline{g}}}\left(\frac{y}{\overline{\gamma}}\right)\right)^k\\ \times \int_0^{\sqrt{y}-1} e^{-\gamma_{SD}\left(\frac{1}{\overline{\gamma}_{SD}} - \frac{k+1}{\overline{\gamma}_{RD}}\right)}\,d\gamma_{SD}\,dy. \end{aligned} \tag{38}$$

The inner integral with respect to γ_{SD} simplifies to $I_k(y)$ as defined in (12). Then, the term LA_2 yields the second term in (11).

C. Proof of Corollary 1

We are given that $P_{\max} \to \infty$ with μ_{SD}, μ_{SR}, μ_{RD}, μ_{RX}, μ_{SX}, σ_0^2, σ_1^2, σ_2^2, and λ fixed. Therefore, $P_s \to I_{\text{avg}}/\mu_{SX}$ and $P_r = P_{\max} \to \infty$. Since $a\overline{g} = 2\lambda\ln(2)\,P_r\mu_{RX} \to \infty$ as $P_r \to \infty$, we have $t^{-1/(a\overline{g})} \to 1$, for any finite $t \geq 1$. Then, the term $\mathrm{E}_{\frac{1}{a\overline{g}}}\left((1+\gamma_{SD})^2/\overline{\gamma}\right)$ in the expression of $\eta(\gamma_{SD})$ after (12) simplifies to

$$\begin{aligned}\mathrm{E}_{\frac{1}{a\overline{g}}}\left(\frac{(1+\gamma_{SD})^2}{\overline{\gamma}}\right) &= \int_1^\infty e^{-\frac{(1+\gamma_{SD})^2 t}{\overline{\gamma}}}t^{-\frac{1}{a\overline{g}}}\,dt,\\ &\to \int_1^\infty e^{-\frac{(1+\gamma_{SD})^2 t}{\overline{\gamma}}}\,dt = \frac{\overline{\gamma}e^{-\frac{(1+\gamma_{SD})^2}{\overline{\gamma}}}}{(1+\gamma_{SD})^2}. \end{aligned} \tag{39}$$

Substituting (39) into the expression of $\eta(\gamma_{SD})$ in (13), we get $\eta(\gamma_{SD}) \to 1$. Therefore, the first term in (11) tends to the first term in (18).

Similarly, the term $\mathrm{E}_{\frac{1}{a\overline{g}}}\left((1+X_1)\,e^{-ag_1}/\overline{\gamma}\right)$ in (35) tends to

$$\mathrm{E}_{\frac{1}{a\overline{g}}}\left(\frac{(1+X_1)\,e^{-ag_1}}{\overline{\gamma}}\right) \to \frac{\overline{\gamma}e^{-\frac{(1+X_1)e^{-ag_1}}{\overline{\gamma}}}}{(1+X_1)\,e^{-ag_1}}.$$

Therefore, (35) simplifies to

$$\Pr(\beta = 1|\gamma_{SD}, X_1, g_1) \to 1_{\left\{g_1 < \frac{1}{a}\ln\left(\frac{1+X_1}{(1+\gamma_{SD})^2}\right)\right\}}. \tag{40}$$

Substituting (40) into the expression for A_2 in (28) and unconditioning over γ_{SD}, X_1, and g_1, we get

$$A_2 \to \frac{1}{2\ln(2)\,\overline{\gamma}\,\overline{\gamma}_{SD}\overline{g}} \int_0^{\infty} \int_{2\gamma_{SD}+\gamma_{SD}^2}^{\infty} \int_0^{\frac{1}{a}\ln\left(\frac{1+x_1}{(1+\gamma_{SD})^2}\right)} \ln(1+x_1) \times e^{-\left(\frac{g_1}{\overline{g}}+\frac{x_1}{\overline{\gamma}}+b\gamma_{SD}\right)}\, dg_1\, dx_1\, d\gamma_{SD}, \quad (41)$$

where $b = 1/\overline{\gamma}_{SD} - 1/\overline{\gamma}_{RD}$.

Integrating with respect to g_1, using the variable substitution $1+x_1 = t$ in (41), and interchanging the order of integrations with respect to t and γ_{SD}, it can be shown that

$$A_2 \to \frac{1}{2\ln(2)\,\overline{\gamma}\,\overline{\gamma}_{SD}} \int_1^{\infty} \ln(t)\, e^{-\frac{(t-1)}{\overline{\gamma}}} \Bigg[\frac{1-e^{-b(\sqrt{t}-1)}}{b} - \left(\Gamma\left(1+\frac{2}{a\overline{g}}, b\right) - \Gamma\left(1+\frac{2}{a\overline{g}}, b\sqrt{t}\right)\right) \times e^{b} b^{-\left(1+\frac{2}{a\overline{g}}\right)} t^{-\frac{1}{a\overline{g}}}\Bigg] dt, \quad (42)$$

where $\Gamma(x, y)$ is the upper incomplete Gamma function [37, (6.5.3)]. Since $1 + 2/(a\overline{g}) \to 1$ as $a\overline{g} \to \infty$, we have $b^{-\left(1+\frac{2}{a\overline{g}}\right)} \to b^{-1}$ and

$$\Gamma\left(1+\frac{2}{a\overline{g}}, b\right) - \Gamma\left(1+\frac{2}{a\overline{g}}, b\sqrt{t}\right) \to e^{-b}\left(1 - e^{-b(\sqrt{t}-1)}\right). \quad (43)$$

Using (43), the above equation in (42) simplifies to

$$A_2 \to \frac{1}{2\ln(2)\,\overline{\gamma}\,\overline{\gamma}_{SD} b} \int_1^{\infty} \ln(t)\, e^{-\frac{(t-1)}{\overline{\gamma}}} \left(1 - t^{-\frac{1}{a\overline{g}}}\right) \times \left(1 - e^{-b(\sqrt{t}-1)}\right) dt. \quad (44)$$

Since $b \to 1/\overline{\gamma}_{SD}$ and $\overline{\gamma} \to \overline{\gamma}_{SR}$ as $\overline{\gamma}_{RD} \to \infty$, it can be shown that the term LA_2 simplifies to the second term in (18).

D. Proof of Corollary 2

We are given that $P_{\max} \to 0$ with μ_{SD}, μ_{SR}, μ_{RD}, μ_{RX}, σ_0^2, σ_1^2, σ_2^2, and λ fixed. Thus, $P_s \to 0$ and $P_r \to 0$. In this regime, using $\ln(1+x) \to x$, for $x \ll 1$, the conditional probability that no relay is selected in (30) tends to

$$\Pr(\beta = 0|\gamma_{SD}) \to \Pr\left(\max_{i\in\{1,2,\ldots,L\}} \{X_i - a g_i\} < 2\gamma_{SD}|\gamma_{SD}\right), \quad (45)$$

where $a = 2\lambda\ln(2)$, as defined before in Appendix B. Since $g_1, \ldots, g_L$ are i.i.d. RVs and so are $X_1, \ldots, X_L$ when conditioned on γ_{SD}, (45) simplifies to

$$\Pr(\beta = 0|\gamma_{SD}) \to \left[\Pr(X_1 - a g_1 < 2\gamma_{SD}|\gamma_{SD})\right]^L, = \left(1 - \frac{e^{-\gamma_{SD}\left(\frac{2}{\overline{\gamma}} - \frac{1}{\overline{\gamma}_{RD}}\right)}}{1+\frac{a\overline{g}}{\overline{\gamma}}}\right)^L. \quad (46)$$

Using this, it follows that A_1 in (27) tends to

$$A_1 \to \frac{1}{\ln(2)\,\overline{\gamma}_{SD}} \int_0^{\infty} \ln(1+\gamma_{SD}) \left(1 - \frac{e^{-\gamma_{SD}\left(\frac{2}{\overline{\gamma}} - \frac{1}{\overline{\gamma}_{RD}}\right)}}{1+\frac{a\overline{g}}{\overline{\gamma}}}\right)^L \times e^{-\frac{\gamma_{SD}}{\overline{\gamma}_{SD}}}\, d\gamma_{SD}. \quad (47)$$

Taking the binomial series expansion of $\left(1 - e^{-\gamma_{SD}\left(\frac{2}{\overline{\gamma}} - \frac{1}{\overline{\gamma}_{RD}}\right)} \Big/ \left(1+\frac{a\overline{g}}{\overline{\gamma}}\right)\right)^L$ and simplifying further yields the first summation in (20).

Similarly, dropping the second order γ_{SD} term in (33) and using $\ln(1+x) \to x$, for $x \ll 1$, the conditional probability that relay 1 is selected tends to

$$\begin{aligned}&\Pr(\beta = 1|\gamma_{SD}, X_1, g_1)\\ &\quad\to \Pr\left(2\gamma_{SD} < X_1 - a g_1, X_2 - a g_2 < X_1 - a g_1, \ldots, X_L - a g_L < X_1 - a g_1|\gamma_{SD}, X_1, g_1\right),\\ &\quad= \left[\Pr(X_2 - a g_2 < X_1 - a g_1|\gamma_{SD}, X_1, g_1)\right]^{L-1} \times 1_{\{X_1 - a g_1 > 2\gamma_{SD}\}}.\end{aligned} \quad (48)$$

Substituting the conditional PDF of X_2 from (29) and the PDF of $g_2 \sim \mathcal{E}\{\overline{g}\}$ in (48), and simplifying further yields

$$\Pr(\beta = 1|\gamma_{SD}, X_1, g_1) \to \left(1 - \frac{e^{\frac{\gamma_{SD}}{\overline{\gamma}_{RD}} - \frac{X_1 - a g_1}{\overline{\gamma}}}}{1+\frac{a\overline{g}}{\overline{\gamma}}}\right)^{L-1} \times 1_{\{X_1 - a g_1 > 2\gamma_{SD}\}}.$$

Using this, it follows that the expression for A_2 in (36) tends to

$$A_2 \to \frac{1}{2\ln(2)\,\overline{\gamma}\,\overline{\gamma}_{SD}\overline{g}} \int_0^{\infty} \int_{2\gamma_{SD}}^{\infty} \int_0^{\frac{x_1 - 2\gamma_{SD}}{a}} \left(1 - \frac{e^{\frac{\gamma_{SD}}{\overline{\gamma}_{RD}} - \frac{x_1 - a g_1}{\overline{\gamma}}}}{1+\frac{a\overline{g}}{\overline{\gamma}}}\right)^{L-1} \times \ln(1+x_1)\, e^{-\left(\frac{g_1}{\overline{g}} + \frac{x_1}{\overline{\gamma}} + \frac{\gamma_{SD}}{\overline{\gamma}_{SD}} - \frac{\gamma_{SD}}{\overline{\gamma}_{RD}}\right)}\, dg_1\, dx_1\, d\gamma_{SD}. \quad (49)$$

Taking the binomial series expansion of $\left(1 - e^{\frac{\gamma_{SD}}{\overline{\gamma}_{RD}} - \frac{x_1 - a g_1}{\overline{\gamma}}} \Big/ \left(1+\frac{a\overline{g}}{\overline{\gamma}}\right)\right)^{L-1}$ and solving the integral for g_1 yields

$$A_2 \to \frac{1}{2\ln(2)\,\overline{\gamma}\,\overline{\gamma}_{SD}\overline{g}} \sum_{k=0}^{L-1} \frac{(-1)^k \binom{L-1}{k}}{\left(1+\frac{a\overline{g}}{\overline{\gamma}}\right)^k \left(\frac{1}{\overline{g}} - \frac{ka}{\overline{\gamma}}\right)} \times \int_0^{\infty} \int_{2\gamma_{SD}}^{\infty} \ln(1+x_1) \left(1 - e^{-\frac{(x_1 - 2\gamma_{SD})\left(\frac{1}{\overline{g}} - \frac{ka}{\overline{\gamma}}\right)}{a}}\right) \times e^{-\left(\left(\frac{1}{\overline{\gamma}_{SD}} - \frac{k+1}{\overline{\gamma}_{RD}}\right)\gamma_{SD} + \frac{(k+1)x_1}{\overline{\gamma}}\right)}\, dx_1\, d\gamma_{SD}. \quad (50)$$

For any $\alpha > 0$, $\int_{2\gamma_{SD}}^{\infty} \ln(1+x) e^{-\alpha x}\, dx = \left[\ln(1+2\gamma_{SD}) e^{-2\alpha\gamma_{SD}} + e^{\alpha} \mathrm{E}_1(\alpha(1+2\gamma_{SD}))\right]/\alpha$. Using $\mathrm{E}_1(x) \approx e^{-x}/x$ [37, (5.1.19)], we get

$$\int_{2\gamma_{SD}}^{\infty} \ln(1+x)\, e^{-\alpha x}\, dx \approx \frac{e^{-2\alpha\gamma_{SD}}}{\alpha}\left[\ln(1+2\gamma_{SD}) + \frac{1}{\alpha(1+2\gamma_{SD})}\right].$$

Substituting this into (50) and simplifying further yields the second sum term in (20).

REFERENCES

[1] P. Das and N. B. Mehta, "Revisiting incremental relaying and relay selection for underlay cognitive radio," in *Proc. IEEE GLOBECOM*, Dec. 2015, pp. 1–6.

[2] A. Goldsmith, S. A. Jafar, I. Maric, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: An information theoretic perspective," *Proc. IEEE*, vol. 97, no. 5, pp. 894–914, Apr. 2009.

[3] K. Tourki, K. A. Qaraqe, and M.-S. Alouini, "Outage analysis for underlay cognitive networks using incremental regenerative relaying," *IEEE Trans. Veh. Technol.*, vol. 62, no. 2, pp. 721–734, Feb. 2013.

[4] H. Huang, Z. Li, J. Si, and R. Gao, "Outage analysis of underlay cognitive multiple relays networks with a direct link," *IEEE Commun. Lett.*, vol. 17, no. 8, pp. 1600–1603, Aug. 2013.

[5] M. Xia and S. Aissa, "Underlay cooperative AF relaying in cellular networks: Performance and challenges," *IEEE Commun. Mag.*, vol. 51, no. 12, pp. 170–176, Dec. 2013.

[6] X. Lin, J. G. Andrews, and A. Ghosh, "Spectrum sharing for device-to-device communication in cellular networks," *IEEE Trans. Wireless Commun.*, vol. 13, no. 12, pp. 6727–6740, Dec. 2014.

[7] S. Majhi and A. Banerjee, "Asymptotic outage analysis of incremental decode and forward cognitive radio relay network," in *Proc. Int. Conf. Commun. Syst. Netw.*, Jan. 2015, pp. 1–8.

[8] Z. El-Moutaouakkil, K. Tourki, and S. Saoudi, "Spectrally-efficient SIMO relay-aided underlay communications: An exact outage analysis," in *Proc. ICC*, Jun. 2014, pp. 1531–1536.

[9] W. C. Ao, S. M. Cheng, and K. C. Chen, "Power and interference control with relaying in cooperative cognitive radio networks," in *Proc. ICC*, May 2010, pp. 1–5.

[10] S. S. Ikki and M. H. Ahmed, "Performance analysis of cooperative diversity with incremental-best-relay technique over Rayleigh fading channels," *IEEE Trans. Commun.*, vol. 59, no. 8, pp. 2152–2161, Aug. 2011.

[11] Y. Zhang *et al.*, "Maximizing achievable rate strategies for incremental-relay multicarrier transmission," *IEEE Trans. Veh. Technol.*, vol. 65, no. 7, pp. 5820–5825, Jul. 2016.

[12] K. Tourki, H.-C. Yang, and M.-S. Alouini, "Accurate outage analysis of incremental decode-and-forward opportunistic relaying," *IEEE Trans. Wireless Commun.*, vol. 10, no. 4, pp. 1021–1025, Apr. 2011.

[13] L. Musavian, S. Aïssa, and S. Lambotharan, "Effective capacity for interference and delay constrained cognitive radio relay channels," *IEEE Trans. Wireless Commun.*, vol. 9, no. 5, pp. 1698–1707, May 2010.

[14] M. Xia and S. Aissa, "Cooperative AF relaying in spectrum-sharing systems: Performance analysis under average interference power constraints and nakagami-m fading," *IEEE Trans. Commun.*, vol. 60, no. 6, pp. 1523–1533, Jun. 2012.

[15] R. Zhang, "On peak versus average interference power constraint for protecting primary users in cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 8, no. 4, pp. 2112–2120, Apr. 2009.

[16] P. Das, N. B. Mehta, and G. Singh, "Novel relay selection rules for average interference-constrained cognitive AF relay networks," *IEEE Trans. Wireless Commun.*, vol. 14, no. 8, pp. 4304–4315, Aug. 2015.

[17] S. Kashyap and N. B. Mehta, "SEP-optimal transmit power policy for peak power and interference outage probability constrained underlay cognitive radios," *IEEE Trans. Wireless Commun.*, vol. 12, no. 12, pp. 6371–6381, Dec. 2013.

[18] P.-Y. Chen, S.-M. Cheng, W. C. Ao, and K.-C. Chen, "Multi-path routing with end-to-end statistical QoS provisioning in underlay cognitive radio networks," in *Proc. IEEE Conf. Comput. Commun. Workshop*, Apr. 2011, pp. 7–12.

[19] M. Ridouani, A. Hayar, and A. Haqiqk, "Relaxed constraint at cognitive relay network under both the outage probability of the primary system and the interference constraint," in *Proc. Eur. Conf. Netw. Commun.*, Jun. 2015, pp. 311–316.

[20] S. I. Hussain, M. Alouini, K. Qaraqe, and M. Hasna, "Reactive relay selection in underlay cognitive networks with fixed gain relays," in *Proc. ICC*, Jun. 2012, pp. 1784–1788.

[21] S. I. Hussain, M. S. Alouini, M. Hasna, and K. Qaraqe, "Partial relay selection in underlay cognitive networks with fixed gain relays," in *Proc. VTC (Spring)*, May 2012, pp. 1–5.

[22] J. Si, Z. Li, J. Chen, P. Qi, and H. Huang, "Performance analysis of adaptive modulation in cognitive relay networks with interference constraints," in *Proc. WCNC*, Apr. 2012, pp. 2631–2636.

[23] J. Lee, H. Wang, J. G. Andrews, and D. Hong, "Outage probability of cognitive relay networks with interference constraints," *IEEE Trans. Wireless Commun.*, vol. 10, no. 2, pp. 390–395, Feb. 2011.

[24] Z. Yan, X. Zhang, and W. Wang, "Exact outage performance of cognitive relay networks with maximum transmit power limits," *IEEE Commun. Lett.*, vol. 15, no. 12, pp. 1317–1319, Dec. 2011.

[25] T. Yi, L. Guo, and J. Lin, "Outage probability of cognitive relay network with transmit power and interference constraints," in *Proc. WPMC*, Sep. 2012, pp. 133–137.

[26] P. Das and N. B. Mehta, "Direct link-aware optimal relay selection and a low feedback variant for underlay CR," *IEEE Trans. Commun.*, vol. 63, no. 6, pp. 2044–2055, Jun. 2015.

[27] J. F. Kurose and K. W. Ross, *Computer Networking: A Top-Down Approach Featuring the Internet*. Pearson Education, 2005.

[28] J. A. Hussein, S. S. Ikki, S. Boussakta, and C. C. Tsimenidis, "Performance analysis of opportunistic scheduling in dual-hop multiuser underlay cognitive network in the presence of cochannel interference," *IEEE Trans. Veh. Technol.*, vol. 65, no. 10, pp. 8163–8176, Oct. 2016.

[29] M. Vu, N. Devroye, and V. Tarokh, "On the primary exclusive region of cognitive networks," *IEEE Trans. Wireless Commun.*, vol. 8, no. 7, pp. 3380–3385, Jul. 2009.

[30] K. Woradit, T. Q. S. Quek, W. Suwansantisuk, H. Wymeersch, L. Wuttisittikulkij, and M. Z. Win, "Outage behavior of selective relaying schemes," *IEEE Trans. Wireless Commun.*, vol. 8, no. 8, pp. 3890–3895, Aug. 2009.

[31] A. J. Goldsmith, *Wireless Communications*, 3rd ed. Cambridge, U.K.: Cambridge Univ. Press, 2005.

[32] S. Sagong, J. Lee, and D. Hong, "Capacity of reactive DF scheme in cognitive relay networks," *IEEE Trans. Wireless Commun.*, vol. 10, no. 10, pp. 3133–3138, Oct. 2011.

[33] S. Bharadwaj and N. B. Mehta, "Accurate performance analysis of single and opportunistic AF relay cooperation with imperfect cascaded channel estimates," *IEEE Trans. Commun.*, vol. 61, no. 5, pp. 1764–1775, May 2013.

[34] V. Shah, N. B. Mehta, and R. Yim, "Optimal timer based selection schemes," *IEEE Trans. Commun.*, vol. 58, no. 6, pp. 1814–1823, Jun. 2010.

[35] C. Luo, F. R. Yu, M. Chen, and L. T. Yang, "Centralized scheme for joint relay selection and channel access in partially-sensed cognitive radio cooperative networks," in *Proc. IEEE GLOBECOM*, Dec. 2011, pp. 1–5.

[36] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*, 4th ed. San Francisco, CA, USA: Academic, 1980.

[37] M. Abramowitz and I. Stegun, *Handbook of Mathematical Functions With Formulas, Graphs, and Mathematical Tables*, 9th ed. New York, NY, USA: Dover, 1972.

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