Ordered Transmissions for Energy-Efficient Detection in Energy Harvesting Wireless Sensor Networks

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***Abstract*— Ordered transmissions reduces the number of nodes that transmit in a wireless sensor network (WSN) and yet achieves the same performance as the conventional unordered transmissions scheme (UTS) in which all nodes transmit. However, it breaks down in energy harvesting (EH) WSNs because of missed transmissions by EH nodes that lack sufficient energy. For the Bayesian detection framework, we propose a novel scheme that addresses this challenge for the general case in which the log-likelihood ratio is bounded and has a continuous distribution function. Given the probability that a node misses its transmission, it reduces the average number of transmissions compared to UTS. For truncated Gaussian statistics, we then propose a novel refinement that requires even fewer transmissions and that simultaneously lowers the error probability. We also analyze its performance and show that it lends itself to a computationally-efficient Monte Carlo evaluation. When the time evolution of the battery energies of the nodes is tracked and the probability of a missed transmission becomes a function of the scheme itself, the proposed schemes achieve a markedly lower error probability than both UTS and sequential detection, except when the energy harvested is so less that very few nodes can transmit.**

***Index Terms*— Energy harvesting, wireless sensor networks, detection, ordered transmissions.**

I. Introduction

DISTRIBUTED detection using wireless sensor networks (WSNs) has many compelling applications in diverse fields such as transportation and logistics, environmental monitoring, military surveillance, and healthcare [2]. These networks consist of several easily deployable, low-cost sensor nodes that can perform sensing, data processing, and wireless communication. However, a node that is powered by a battery is rendered inactive once the energy stored in it is exhausted. Thus, energy-efficiency is a key issue in the design of such networks. Several techniques such as censoring, on-off keying, duty cycling or sleep scheduling, and clustering have been proposed to improve the energy-efficiency of the nodes by curtailing their transmissions [3]. However, this causes an unwelcome degradation in the performance of the WSN.

A notable exception that has attracted considerable interest is the ordered transmissions scheme (OTS), which reduces the number of transmissions without any degradation in performance [4]. In it, each node sets a timer that is a monotone non-increasing function of a locally computed non-negative real number called a metric. The metric of a node is the *absolute value* of its log-likelihood ratio (LLR); intuitively speaking, it captures how useful a node's observation is to the fusion node (FN) [4]. When a node's timer expires, it transmits a packet containing its LLR to the FN. Thus, the nodes transmit their LLRs one after another in the decreasing order of their metrics. This happens in a distributed manner without any node knowing any other node's metric. Every time the FN receives an LLR, it decides on a hypothesis or allows the nodes to further decrement their timers. Once the FN decides, it broadcasts a control signal to stop the other sensor nodes from transmitting further and draining their battery energies.

Compared to the scheme in which the FN receives LLRs from all the nodes, OTS provably reduces the number of transmissions by at least 50% without any increase in the error rate for the binary hypothesis testing problem [4]. It differs from sequential detection, in which the nodes transmit their LLRs one by one but in a random order [5, Ch. III.D].

Energy harvesting (EH) is an alternate solution that eliminates the problem of limited lifetime in WSNs. In it, the nodes are equipped with an EH circuitry that enables them to harvest energy from renewable sources, such as solar, vibration, and wind, to replenish their energy buffers, which are in the form of rechargeable batteries or supercapacitors [6]. An EH node is fundamentally different because it is not permanently rendered inactive once its battery energy is exhausted. The potential of an infinite lifetime makes EH WSNs very appealing for modern sensing applications.

A. Literature Survey on OTS and Detection in EH WSNs

We summarize the pertinent works on OTS and EH WSNs below.

1) OTS in Conventional WSNs: In [7], OTS is shown to require fewer transmissions if the LLRs at the sensor nodes are non-negative. In [8], it is shown that just one observation is sufficient for OTS to decide when the number of nodes is large. In [9], OTS is used for spectrum sensing in cognitive

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radio networks to maximize the weighted sum throughput of the primary and secondary users. A shared channel-based OTS for detecting a shift in the mean is studied in [10]. LLRs of the sensors are divided into intervals and the sensors whose LLRs belong to the same interval compete to transmit using the slotted Aloha protocol.

Fewer transmissions implies a better energy-efficiency. This can be understood using the following definition of energy-efficiency μ of a scheme. It is defined as the ratio of the average energy consumed by the conventional unordered transmissions scheme (UTS), in which all the N nodes transmit, to the average energy consumed by the scheme to make a decision. If the average number of transmissions of the scheme is $\bar{N}_{\text{tx}}$ and the energy consumed per transmission is E_{tx}, then

$$\mu = \frac{N E_{\text{tx}}}{\bar{N}_{\text{tx}} E_{\text{tx}}} = \frac{N}{\bar{N}_{\text{tx}}}. \tag{1}$$

The larger the value of μ, the more energy-efficient is the scheme. All the above OTS schemes are more energy-efficient than UTS. For example, $\mu \geq 2$ for the scheme in [4]. Note that such a comparison of the average number of transmissions is meaningful only if the error probability is not compromised, as is the case in OTS. In [11], not only do the sensor nodes transmit in the decreasing order of their channel power gains, but they also adapt their transmit powers. Dynamic programming is used to determine the LLR-based thresholds and minimize the expected transmission energy.

2) Detection in EH Sensor Networks: Detection in EH WSNs is a separate problem by itself due to the randomness in the energy harvested. Now, the EH sensor nodes can occasionally have insufficient energy to transmit, which affects the design and performance of the network. In [12], the optimal power allocation and detection scheme that minimizes the worst case detection delay in a WSN with only one EH node is studied. In [13], a transmission policy based on the Bhattacharya distance is proposed to minimize the error probability for the binary hypothesis testing problem. The Neyman-Pearson detection framework is instead studied in [14]. The goal is to optimize a node-specific decision threshold that maximizes the Kullback-Leibler distance between the probability distributions, conditioned on the hypotheses, of the received signals at the FN.

3) Detection in EH Cognitive Radio Networks: In [15], the primary detection threshold that maximizes the average throughput of an EH secondary transmitter, which is subject to energy causality and collision probability constraints, is optimized. In [16], the average throughput of an EH secondary user is maximized by jointly optimizing the primary detection threshold, sensing duration, and the fraction of time spent on energy harvesting. In [17], the sensing time of relays, which either sense the primary transmissions or harvest energy from them, is optimized to minimize the average energy they consume during data transmissions. For a given probability of miss-detection, a throughput-maximizing policy that determines when a secondary user should sense its channel and when it should transmit is proposed in [18]. For a hybrid cognitive radio network that consists of battery-powered data sensors and EH-enabled spectrum sensors, [19] studies the problem of scheduling the sensors to maximize the average time available for channel access by the data sensors. In [20], the detection threshold is optimized to minimize the error probability in sensing the primary when the secondary users harvest energy from the primary signals. In [21], the primary detection threshold is optimized and a sub-channel allocation algorithm is proposed to maximize the average throughput of the secondary transmitter.

B. Focus and Contributions

We see that OTS has been studied for conventional WSNs [7], [8], [10], [11], but not for EH WSNs. A fundamental new challenge arises in an EH WSN. An EH sensor node that needs to transmit might not do so if it lacks sufficient energy at that time instant. Consider, for example, the following scenario for OTS in which the FN has received LLRs from the best $k-1$ among N nodes and the k^{th} best node lacks sufficient energy and misses its transmission. The FN ends up waiting for the LLR by the $(k+1)^{\text{th}}$ best node. OTS then breaks down because while the FN thinks that $N-k$ nodes remain to transmit, only $N-k-1$ nodes remain to transmit. Thus, the randomness in the energy harvested presents a unique challenge for OTS.

In this manuscript, we present novel ordered transmission schemes for EH WSNs. To the best of our knowledge, these are the first schemes that solve the problem of missed transmissions, which disrupt the sequence of LLRs available at the FN. We first modify the transmission scheme as follows. When an EH node that does not have sufficient energy to transmit a packet containing its LLR to the FN, it transmits a low energy pilot signal, which carries no data. As we shall see, due to the ordering of the metrics, even such transmissions provide valuable information to the FN about the missing LLRs. For the general case of LLRs that follow any continuous probability distribution and are bounded, we present a new scheme called LLR-based EH OTS (LL-EH-OTS) and derive novel decision rules for it.

We then address the special, but widely studied, case of Gaussian statistics [5, Ch. III]. Here, as above, the magnitude of a node's measurement is bounded above by a threshold [22]. This is motivated by the fact that a node's measurement saturates in practice. We note that the infinite support assumed in the literature for Gaussian statistics is itself an idealization that is made to ensure tractability. We show that the metric for each node can be expressed in terms of an affine, monotonically increasing function of its LLR. This removes the LLR sign ambiguity that arises in LL-EH-OTS in which the metric is the absolute value of the LLR. Based on this metric, we then propose a non-negative metric-based EH OTS (NN-EH-OTS) and derive new decision rules for it.

To gain quantitative insights into these algorithms and benchmark their performance, we make two comparisons. In the first one, we compare the schemes for a given probability ρ that an EH node has insufficient energy to transmit. We present two novel findings. Firstly, both LL-EH-OTS and NN-EH-OTS require fewer sensor nodes to transmit on an average compared to conventional UTS. Since the energy consumed is directly proportional to the average number of transmissions, this also implies that both these schemes are more energy-efficient than UTS. We observe that NN-EH-OTS requires fewer transmissions than LL-EH-OTS because it exploits the aforementioned monotonic relationship between the metric and the LLR. Secondly, while the error probability of LL-EH-OTS is equal to that of conventional UTS, the error probability of NN-EH-OTS is lower than both of them. Thus, NN-EH-OTS provides a surprising double benefit of lowering the error probability and also reducing the average number of transmissions.

For NN-EH-OTS, we use results from order statistics and derive a compact analytical expression for the average number of transmissions as a function of ρ. We show that its special form lends itself to a new computationally-efficient Monte Carlo-based analysis.

In the second comparison, we simulate an EH WSN by using a time-driven simulator that explicitly models the energy harvesting, storage and consumption, and the evolution and coupling of the battery energies of the nodes. These processes collectively determine the value of ρ, which becomes scheme-dependent. We observe that LL-EH-OTS and NN-EH-OTS achieve a lower error probability than both UTS and sequential detection, and we characterize the regime in which this happens.

C. Organization and Notations

Section II presents the system model. Section III specifies LL-EH-OTS and NN-EH-OTS and their new decision rules, and analyzes NN-EH-OTS. Simulation results are presented in Section IV, and are followed by our conclusions in Section V.

Notations: The probability of an event A is denoted by $\Pr[A]$. The conditional probability of A given event B is denoted by $\Pr[A|B]$. The joint probability of events A and B is denoted by $\Pr[A, B]$. The probability density function (PDF) and the cumulative distribution function (CDF) of a random variable (RV) X are denoted by $f_X(\cdot)$ and $F_X(\cdot)$, respectively. We denote the expectation with respect to X by $\mathbb{E}_X[\cdot]$. Similarly, the expectation conditioned on an event A is denoted by $\mathbb{E}_X[\cdot|A]$. Matrices and vectors are denoted using boldface characters. For a set $\mathcal{B}$, its cardinality is denoted by $|\mathcal{B}|$. The indicator function $1_{\{a\}}$ equals 1 if a is true and is 0 otherwise. We shall extensively employ the order statistics notation [23], which is as follows. For N RVs, $X_1, \dots, X_N$, $X_{[r]}$ denotes the r^{th} largest value and $[r]$ denotes the index of this RV. Therefore, $X_{[1]} > X_{[2]} > \cdots > X_{[N]}$.

II. System Model

Consider a WSN that consists of N EH sensor nodes and an FN. Time is divided into slots. At the beginning of a slot, the sensor node i makes a measurement y_i. Within each slot, the FN collects measurements from some or all of the sensor nodes and makes a decision about the hypotheses. The models for node measurements, EH, and energy consumption are as follows.

Measurement Model: We consider the binary hypothesis testing framework, which is a fundamental problem in signal detection theory [10], [13], [14], and has several practical applications such as target detection [7], spectrum sensing [9], fingerprint detection [24], and landmine detection [25, Ch. 1]. In this setting, the LLR of node i, denoted by L_i, is given by

$$L_i = \log\left(\frac{f_{Y_i}(y_i|H_1)}{f_{Y_i}(y_i|H_0)}\right), \tag{2}$$

where $f_{Y_i}(y_i|H_h)$ denotes the PDF of the measurement y_i conditioned on hypothesis H_h, where $h \in \{0,1\}$. For example, in spectrum sensing, H_0 models the absence of the primary signal, whereas H_1 models its presence; in target detection, H_0 models the absence of the target, whereas H_1 models its presence. We assume that $|L_i| \le \Psi$, for $1 \le i \le N$, and that it has a continuous PDF. The measurements conditioned on the hypotheses are independent and identically distributed (i.i.d.) across the nodes [5, Ch. III.B], [13].

For Gaussian statistics [14], [26], y_i is given by

$$y_i = \begin{cases} s_i + n_i, & \text{under hypothesis } H_1, \\ n_i, & \text{under hypothesis } H_0. \end{cases} \tag{3}$$

Here, s_i is the random signal component with zero mean and variance σ_s^2 that is present under H_1 and is absent under H_0, and n_i is the measurement noise with zero mean and variance σ_0^2. The assumption that the LLR is bounded is equivalent to the condition that the magnitude of the measurement by a node is bounded above by a threshold τ, i.e., $|y_i| \le \tau$. This is justified practically since the measurements by a sensor inevitably saturate. The PDFs of the measurement $y_i \in [-\tau, \tau]$ conditioned on the two hypotheses are then given by

$$f_{Y_i}(y_i|H_0) = \frac{1}{\sqrt{2\pi}\sigma_0\left[1 - 2Q\left(\frac{\tau}{\sigma_0}\right)\right]} \exp\left(-\frac{y_i^2}{2\sigma_0^2}\right), \tag{4}$$

$$f_{Y_i}(y_i|H_1) = \frac{1}{\sqrt{2\pi}\sigma_1\left[1 - 2Q\left(\frac{\tau}{\sigma_1}\right)\right]} \exp\left(-\frac{y_i^2}{2\sigma_1^2}\right), \tag{5}$$

where $\sigma_1 = \sqrt{\sigma_s^2 + \sigma_0^2}$ and $Q(\cdot)$ denotes the Gaussian Q-function [27, Ch. 26.2]. Substituting (4) and (5) in (2) yields the following expression for the LLR of node i:

$$L_i = \log\left(\frac{\sigma_0}{\sigma_1}\right) + \log\left(\frac{1 - 2Q(\tau/\sigma_0)}{1 - 2Q(\tau/\sigma_1)}\right) + \frac{y_i^2}{2}\frac{\sigma_s^2}{\sigma_1^2\sigma_0^2}. \tag{6}$$

Hence, it follows that the relation $|L_i| \le \Psi$ can be expressed as $y_i^2 \le \frac{2\sigma_1^2\sigma_0^2}{\sigma_s^2}\left[\Psi + \log\left(\frac{\sigma_1}{\sigma_0}\right) + \log\left(\frac{1-2Q(\tau/\sigma_1)}{1-2Q(\tau/\sigma_0)}\right)\right]$. Therefore, the relationship between τ and Ψ is $\tau = \sqrt{\frac{2\sigma_1^2\sigma_0^2}{\sigma_s^2}\left[\Psi + \log\left(\frac{\sigma_1}{\sigma_0}\right) + \log\left(\frac{1-2Q(\tau/\sigma_1)}{1-2Q(\tau/\sigma_0)}\right)\right]}$.

EH and Consumption Model: An EH node harvests energy at the beginning of a slot, which is stored in an energy buffer for use in the current slot and subsequent slots. The EH node can transmit a measurement packet to the FN, which consumes an energy E_{tx}, only if the energy stored in its energy buffer at the beginning of the slot exceeds E_{tx}. In such a case, we say that the sensor node is *energy-sufficient* in that slot. Else, we say that it is *energy-deficient*. Note that a node can be energy-sufficient in some slots and energy-deficient in others.

III. Ordered Transmission Schemes

We first present LL-EH-OTS for the general case of bounded LLRs. Before we do so, we recap the decision rules and OTS for the Bayesian detection framework.

Let c_{uv} be the cost incurred if hypothesis H_u is chosen when hypothesis H_v is true, and let ζ_h be the prior probability of the hypothesis H_h, for $h \in \{0, 1\}$. The LLR-based Bayesian hypothesis test, which minimizes the error probability P_{E} is given by [5, Ch. II.B]

$$\sum_{i=1}^{N} L_i \underset{H_0}{\overset{H_1}{\gtrless}} \beta, \tag{7}$$

where $\beta = \log\left(\frac{(c_{10}-c_{00})\zeta_0}{(c_{01}-c_{11})\zeta_1}\right)$.[1]

As per the order statistics notation, let $[k]$ denote the index of the node with the k^{th} largest absolute value of LLR. Then, the decision rules for OTS are as follows [4]:

$$\text{Decide } H_1 \text{ if}: \sum_{i=1}^{k} L_{[i]} > \beta + (N-k)|L_{[k]}|, \tag{8}$$

$$\text{Decide } H_0 \text{ if}: \sum_{i=1}^{k} L_{[i]} < \beta - (N-k)|L_{[k]}|, \tag{9}$$

Wait for the next transmission, otherwise.

These decision rules lead to the same decision as UTS, in which the FN has the LLRs of all the N nodes. The measurements are assumed to be received at the FN with a negligible probability of decoding error [4], [26], [28].

A. LL-EH-OTS

1) Specification: As in OTS [4], the *metric* of node i is $|L_i|$. Each node sets a timer that is a monotone non-increasing function of its metric [29]. Once the timer expires, if the node is energy-sufficient, it transmits its LLR to the FN, which consumes an energy E_{tx}. Else, it sends a low-energy pilot signal to the FN. The sensor nodes transmit over orthogonal channels. Thus, the data or pilot transmissions of different sensor nodes do not collide [4], [26]. The FN receives on all the channels. The timer scheme ensures that the nodes transmit in the decreasing order of their metrics, without any node knowing the metric of any other node a priori.

Every time the FN receives an LLR, it decides between the hypotheses H_0 and H_1, or waits for the next measurement packet. If it senses a pilot signal, it does not get to know the metric but it does know that a sensor node has missed its turn to transmit. It then waits for the next measurement packet or it makes a decision if no more sensor nodes are left to transmit. Once the FN has made a decision, it broadcasts a control signal to all the sensor nodes to halt their timers for the rest of the slot. Else, the nodes continue to decrement their timers. The process starts afresh at the beginning of every slot when the new set of measurements arrive at the nodes.

Unlike a data packet that contains several bytes of headers that enable time and frequency synchronization and accurate channel estimation in addition to the data and higher layer headers [30], the pilot signal does not carry any information. All that is needed is that the FN can detect it. Therefore, it is shorter in length and consumes much less energy to transmit. Let it be qE_{tx}, where $q \ll 1$. This also implies that the node can maintain a small energy reserve to transmit pilots even if it is energy-deficient for several measurement slots. For example, even a storage capacity of wE_{tx}, for $w \geq 1$, is sufficient to support $w/q \gg 1$ pilot transmissions.

[1] P_E is a weighted combination of the probabilities of false alarm and miss-detection [5, Ch. II.B].

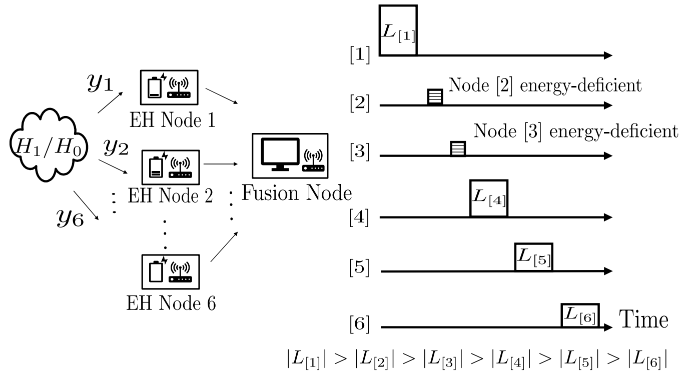


Fig. 1. System model and illustration of the order statistics notation used to track energy-sufficient nodes.

To derive the decision rules for LL-EH-OTS, we define the following important notations that are based on order statistics. Let $[k]$ denote the node with the k^{th} largest metric. Thus, $|L_{[1]}| > |L_{[2]}| > \cdots > |L_{[N]}|$. When the time for node $[k]$ to transmit comes, let the nodes $[m_1], [m_2], \ldots, [m_j]$ be energy-deficient, where $1 \leq j < k$. These nodes have, thus, missed their transmissions. Let p_l be the largest integer that is less than m_l such that the sensor node $[p_l]$ is energy-sufficient. If the sensor nodes $[1], \ldots, [m_l]$ are all energy-deficient, then $p_l \triangleq 0$. Similarly, let n_l be the smallest integer that is greater than m_l such that the sensor node $[n_l]$ is energy-sufficient. *Example:* To understand the notations, consider an EH WSN with $N = 6$ EH nodes. Let the second best $[2]$ and third best $[3]$ nodes be energy-deficient. Hence, $m_1 = 2$ and $m_2 = 3$. For node $[m_1]$, $p_1 = 1$ and $n_1 = 4$. Similarly, for node $[m_2]$, $p_2 = 1$ and $n_2 = 4$. The example is illustrated in Fig. 1.

2) Decision Rules With Missing Transmissions: The following key result, which we derive in Appendix A, compactly specifies the new decision rules for this scenario. Let $|L_{[0]}| \triangleq \Psi$.

Result 1: Let the FN have received the LLR from node $[k]$, where $1 \leq k \leq N$, and let the j EH nodes $[m_1], [m_2], \ldots, [m_j]$, for $1 \leq j < k$, be energy-deficient. Then, the decision rules are as follows:

$$\text{Decide } H_1 \text{ if}: \sum_{i=1, i\notin\{m_1,\ldots,m_j\}}^{k} L_{[i]} > \beta + \sum_{l=1}^{j} |L_{[p_l]}| + (N-k)|L_{[k]}|, \tag{10}$$

$$\text{Decide } H_0 \text{ if}: \sum_{i=1, i\notin\{m_1,\ldots,m_j\}}^{k} L_{[i]} < \beta - \sum_{l=1}^{j} |L_{[p_l]}| - (N-k)|L_{[k]}|, \tag{11}$$

Wait for the next transmission, otherwise.

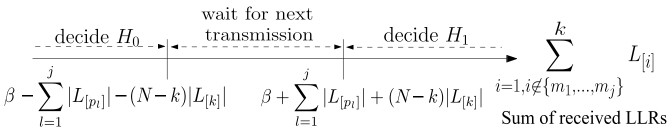


Fig. 2. Decision regions for LL-EH-OTS when j sensor nodes $[m_1], [m_2], \dots, [m_j]$ have missed their transmissions and the most recent LLR received by the FN is from sensor node $[k]$.

If no more energy-sufficient nodes remain, i.e., $k = N$ or $m_j = N$, and the FN has not made a decision, then it decides H_1 if

$$\sum_{i=1, i\notin\{m_1,\dots,m_j\}}^{N} L_{[i]} > \beta, \tag{12}$$

and decides H_0, otherwise. In the extreme case in which all the N nodes are energy-deficient, the FN declares the hypothesis with the larger prior probability as its decision.

Remark: As shown in Appendix A, if either (10) or (11) holds, then the decision is the same as the FN knowing all the N LLRs, including even the missed ones. The decision rules are illustrated in Fig. 2. The FN can identify the scenario where no more energy-sufficient nodes remain by waiting until the maximum time duration budgeted for the timer scheme. The decision rule in this case, which is shown in (12), is the same as that of UTS and ensures that the error probability never exceeds that of UTS.

B. NN-EH-OTS

We now consider Gaussian statistics. We first show in Lemma 1 that the detection test in (7) can be expressed in terms of an affine non-negative function of the nodes' LLRs.

Lemma 1: Let $\Theta_i = y_i^2$. The optimal detection test that minimizes the error probability and is equivalent to (7) is given by

$$\sum_{i=1}^{N} \Theta_i \underset{H_0}{\overset{H_1}{\gtrless}} \lambda, \tag{13}$$

where $\lambda = \frac{2\sigma_0^2\sigma_1^2}{\sigma_s^2}\left[\beta + N\log\left(\frac{\sigma_1}{\sigma_0}\right) + N\log\left(\frac{1-2Q(\tau/\sigma_1)}{1-2Q(\tau/\sigma_0)}\right)\right]$.

Proof: The proof is relegated to Appendix B. ■

We define Θ_i as the *metric* of sensor node i. It is an affine, monotonically increasing function of its LLR since it can be written as

$$\Theta_i = \frac{2\sigma_1^2\sigma_0^2}{\sigma_s^2}\left[L_i + \log\left(\frac{\sigma_1}{\sigma_0}\right) + \log\left(\frac{1-2Q(\tau/\sigma_1)}{1-2Q(\tau/\sigma_0)}\right)\right] \tag{14}$$

Using order statistics notation, let $[i]$ denote the index of the sensor node with the i^{th} largest value of Θ_i, for $1 \le i \le N$. Hence, $\Theta_{[1]} > \Theta_{[2]} > \cdots > \Theta_{[N]}$. Then, (13) is equivalent to

$$\sum_{i=1}^{N} \Theta_{[i]} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda. \tag{15}$$

1) Specification: A sensor node i sets its timer as a monotone non-increasing function of its metric Θ_i. When the timer expires, it transmits a packet containing its metric to the FN if it is energy-sufficient. Note that this is equivalent to the node transmitting its LLR itself given the affine relationship between the LLR and the metric. Else, if it is energy-deficient, it transmits a low-energy pilot signal. As before, nodes with larger metrics automatically transmit before those with smaller metrics. In NN-EH-OTS, the nodes transmit in the decreasing order of their LLRs. This is unlike LL-EH-OTS where a node with a large positive value of the LLR or a large negative value of the LLR is likely to transmit earlier, since the metric is the absolute value of the LLR.

Every time the FN receives a metric, it applies the decision rules, which we present next, to decide between H_0 and H_1. If it senses a pilot signal, it waits for the next transmission or it makes a decision if no more sensor nodes are left to transmit. Once the FN has made a decision, it broadcasts a control signal to all the nodes to halt their timers for the rest of the slot.

2) Decision Rules Without Missing Transmissions: Note that in OTS [4] and LL-EH-OTS, the metric of node i is $|L_i|$, whereas it is Θ_i in NN-EH-OTS. Due to this, even the simpler case where there are no missing transmissions requires revisiting the decision rules. They are derived in Appendix C, and are as follows.

Result 2: Let the first k ordered metrics $\Theta_{[1]}, \Theta_{[2]}, \dots, \Theta_{[k]}$, for $1 \le k \le N$, be available at the FN. The decision rules with no missing transmissions are as follows:

$$\text{Decide } H_1 \text{ if}: \sum_{i=1}^{k} \Theta_{[i]} > \lambda, \tag{16}$$

$$\text{Decide } H_0 \text{ if}: \sum_{i=1}^{k} \Theta_{[i]} < \lambda - (N-k)\Theta_{[k]}, \tag{17}$$

Wait for the next transmission, otherwise.

Remark: By design, when either (16) or (17) is satisfied, it leads to the same decision as when all the N metrics are available at the FN. Once all the N nodes transmit, (16) or (17) will always hold. Hence, the FN will always make a decision. We note that the decision rules in (16) and (17) hold even if the LLR is negative. This is different from [7] where the LLRs are assumed to be non-negative.

3) Decision Rules With Missing Transmissions: The following key result, which is derived in Appendix D, provides a compact representation of the new decision rules for NN-EH-OTS for the general case with missing transmissions. Let $\Theta_{[0]} \triangleq \tau^2$.

Result 3: Let the FN have received the metric from node $[k]$, where $1 \le k \le N$, and let the j EH nodes $[m_1], [m_2], \dots, [m_j]$ be energy-deficient, where $1 \le j < k$. The decision rules for NN-EH-OTS with missing transmissions are as follows:

$$\text{Decide } H_1 \text{ if}: \sum_{i=1, i\notin\{m_1,\dots,m_j\}}^{k} \Theta_{[i]} > \lambda - \sum_{l=1}^{j} \Theta_{[n_l]}, \tag{18}$$

$$\text{Decide } H_0 \text{ if}: \sum_{i=1, i\notin\{m_1,\dots,m_j\}}^{k} \Theta_{[i]} < \lambda - \sum_{l=1}^{j} \Theta_{[p_l]} - (N-k)\Theta_{[k]}, \tag{19}$$

Wait for the next transmission, otherwise.

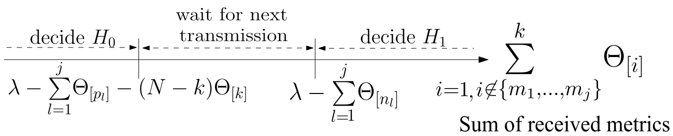


Fig. 3. Decision regions for NN-EH-OTS when j sensor nodes $[m_1], [m_2], \ldots, [m_j]$ have missed their transmissions and the most recent metric is received by the FN from sensor node $[k]$.

If no more energy-sufficient nodes remain, i.e., $k = N$ or $m_j = N$, and the FN has not made a decision, then it decides H_1 if

$$\sum_{i=1, i \notin \{m_1,\ldots,m_j\}}^{N} \Theta_{[i]} > \lambda - \frac{2j\sigma_0^2\sigma_1^2}{\sigma_s^2} \log\left(\frac{\sigma_1}{\sigma_0}\right) - \frac{2j\sigma_0^2\sigma_1^2}{\sigma_s^2} \log\left(\frac{1-2Q(\tau/\sigma_1)}{1-2Q(\tau/\sigma_0)}\right), \quad (20)$$

and H_0, otherwise. If all the N nodes are energy-deficient, then the FN just decides the hypothesis that has a higher prior probability.

Remark: By design, if either (18) or (19) holds, then the decision is the same as that when the FN knows all the N metrics, including the missed ones. The decision rules are illustrated in Fig. 3. In the derivations of Results 2 and 3, the underlying logic, which bounds the sum LLR, is similar to that for LL-EH-OTS in Result 1. However, the resultant decision rules are different.

To understand the above decision rules, recall the previous example with $N = 6$ EH nodes, where the nodes [2] and [3] are energy-deficient. After node [1] transmits, the FN will decide H_0 if $\Theta_{[1]} < \lambda - 5\Theta_{[1]}$, and it will decide H_1 if $\Theta_{[1]} > \lambda$. Otherwise, it waits for the next transmission. Since nodes [2] and [3] are energy-deficient, they both send low-energy pilot signals to the FN. Node [4], being energy-sufficient, then sends its metric $\Theta_{[4]}$ to the FN. The FN will decide H_0 if $\Theta_{[1]} + \Theta_{[4]} < \lambda - 2\Theta_{[1]} - 2\Theta_{[4]}$, and will decide H_1 if $\Theta_{[1]} + \Theta_{[4]} > \lambda - 2\Theta_{[4]}$. Else, the algorithm proceeds further.

C. Average Number of Transmissions in NN-EH-OTS

For NN-EH-OTS, we now derive an expression for the average number of transmissions, $\bar{N}_{\text{tx}}$, as a function of ρ. Its other performance measure, which is the error probability, is already guaranteed to be less than or equal to that of UTS, and is not analyzed below. For a given ρ, the average energy consumed per slot for UTS is $E_{\text{tx}}N(1-\rho)$. Therefore, from the definition of energy-efficiency in Section I-A, μ for NN-EH-OTS is given by

$$\mu = \frac{E_{\text{tx}}N(1-\rho)}{E_{\text{tx}}\bar{N}_{\text{tx}}} = \frac{N(1-\rho)}{\bar{N}_{\text{tx}}}. \quad (21)$$

Note that this comparison is meaningful because the error probability of NN-EH-OTS is no more than that of UTS.

This analysis applies to the general model in which the EH process at a node is stationary and ergodic with mean $\bar{E}_h > 0$ per slot, and is i.i.d. across nodes. This encompasses several models studied in the literature, such as the Bernoulli [13] and Markovian EH models [31]. The event that a node is energy-deficient is taken to be independent of the energy states of the other nodes.

Let Ω be the set of all possible subsets of the elements $[1], \ldots, [N]$. Consider one such subset $\mathcal{S}_i = \{[s_1], [s_2], \ldots, [s_{|\mathcal{S}_i|}]\}$, where $1 \le s_1 < s_2 < \cdots < s_{|\mathcal{S}_i|} \le N$ and $1 \le i \le 2^N$. It shall track the set of energy-sufficient nodes. These nodes transmit in the order $[s_1], [s_2], \ldots, [s_{|\mathcal{S}_i|}]$. Let $F_\Theta(\theta_k|H_h)$ represent the CDF of the metric Θ_k at node k conditioned on the hypothesis H_h, for $h \in \{0,1\}$. From (4) and (5), it is given by

$$F_\Theta(\theta_k|H_0) = \frac{1}{1-2Q\left(\frac{\tau}{\sigma_0}\right)}\left[1-2Q\left(\frac{\sqrt{\theta_k}}{\sigma_0}\right)\right], \quad (22)$$

$$F_\Theta(\theta_k|H_1) = \frac{1}{1-2Q\left(\frac{\tau}{\sigma_1}\right)}\left[1-2Q\left(\frac{\sqrt{\theta_k}}{\sigma_1}\right)\right], \quad (23)$$

where $0 \le \theta_k \le \tau^2$ and σ_1 is defined in Section II.

Result 4: The average number of transmissions $\bar{N}_{\text{tx}}$ is given by

$$\begin{aligned}\bar{N}_{\text{tx}} = {} & \left(1-\rho^N\right) + \sum_{t=2}^{N} \sum_{\substack{\mathcal{S}_i \subseteq \Omega: \\ |\mathcal{S}_i| \ge t}} c_0(|\mathcal{S}_i|) \\ & \times \mathbb{E}_{\boldsymbol{\Theta}_{t-1}}\left[I_0(\mathcal{S}_i, \boldsymbol{\Theta}_{t-1})\, 1_{\{\boldsymbol{\Theta}_{t-1} \in \mathcal{R}(\mathcal{S}_i)\}} | H_0, \mathcal{S}_i\right] \\ & + \sum_{t=2}^{N} \sum_{\substack{\mathcal{S}_i \subseteq \Omega: \\ |\mathcal{S}_i| \ge t}} c_1(|\mathcal{S}_i|) \\ & \times \mathbb{E}_{\boldsymbol{\Theta}_{t-1}}\left[I_1(\mathcal{S}_i, \boldsymbol{\Theta}_{t-1})\, 1_{\{\boldsymbol{\Theta}_{t-1} \in \mathcal{R}(\mathcal{S}_i)\}} | H_1, \mathcal{S}_i\right], \quad (24)\end{aligned}$$

where $\boldsymbol{\Theta}_{t-1} = (\Theta_1, \Theta_2, \ldots, \Theta_{t-1})$, the coefficient $c_h(l)$ is given by

$$c_h(l) = (1-\rho)^l \rho^{N-l} \zeta_h, \text{ for } h \in \{0,1\}, \quad (25)$$

and the function $I_h(\mathcal{S}_i, \boldsymbol{\Theta}_{t-1})$ is given by

$$\begin{aligned}& I_h(\mathcal{S}_i, \boldsymbol{\Theta}_{t-1}) \\ & = \frac{N!}{(N-s_{t-1})!(s_1-1)! \prod_{n=2}^{t-1}(s_{t-n+1}-s_{t-n}-1)!} \\ & \quad \times [F_\Theta(\Theta_{t-1}|H_h)]^{N-s_{t-1}} [1-F_\Theta(\Theta_1|H_h)]^{s_1-1} \\ & \quad \times \prod_{n=2}^{t-1} [F_\Theta(\Theta_{t-n}|H_h) - F_\Theta(\Theta_{t-n+1}|H_h)]^{s_{t-n+1}-s_{t-n}-1}, \quad (26)\end{aligned}$$

and $h \in \{0,1\}$.

The region $\mathcal{R}(\mathcal{S}_i)$ is a $(t-1)$-dimensional hyperplane that is formed by the intersection of the three half-spaces $\mathcal{H}, \mathcal{L}(\mathcal{S}_i)$, and $\mathcal{U}(\mathcal{S}_i)$ that are given by

$$\mathcal{H} = \left\{\boldsymbol{\theta}_{t-1} : \tau^2 > \theta_1 > \cdots > \theta_{t-1}\right\},$$

$$\mathcal{L}(\mathcal{S}_i) = \left\{\boldsymbol{\theta}_{t-1} : s_1\theta_1 + \sum_{j=2}^{t-1}(s_j - s_{j-1})\,\theta_j < \lambda\right\},$$

$$\mathcal{U}(\mathcal{S}_i) = \left\{\boldsymbol{\theta}_{t-1} : (s_1-1)\,\tau^2 + \sum_{j=2}^{t-1}(s_j - s_{j-1})\,\theta_{j-1} + (N - s_{t-1} + 1)\,\theta_{t-1} > \lambda\right\}, \quad (27)$$

where $\boldsymbol{\theta}_{t-1} = (\theta_1, \theta_2, \ldots, \theta_{t-1})$.

Proof: The proof is relegated to Appendix E. ■

To evaluate $\bar{N}_{\text{tx}}$ in (24), we need to compute

$$\mathbb{E}_{\boldsymbol{\Theta}_{t-1}}\left[I_h(\mathcal{S}_i, \boldsymbol{\Theta}_{t-1})\, 1_{\{\boldsymbol{\Theta}_{t-1} \in \mathcal{R}(\mathcal{S}_i)\}} | H_h, \mathcal{S}_i\right], \quad (28)$$

for $h = 0$ and 1. This requires evaluating a $(t-1)$-dimensional integral over the region $\mathcal{R}(\mathcal{S}_i)$, which is analytically intractable.

Computationally-Efficient Monte Carlo Evaluation: For a given t, $\mathcal{S}_i$, and H_h, we generate M *i.i.d. realizations* of $\Theta_1, \Theta_2, \ldots, \Theta_{t-1}$. Here, $\Theta_1, \Theta_2, \ldots, \Theta_{t-1}$ each follow the CDF in (22) for $h = 0$, and the CDF in (23) for $h = 1$. Let θ_i be the realization of Θ_i. For a given realization, we compute the function $I_h(\mathcal{S}_i, \boldsymbol{\theta}_{t-1})\, 1_{\{\boldsymbol{\theta}_{t-1} \in \mathcal{R}(\mathcal{S}_i)\}}$ from (26) and (27). The empirical mean of this function over M i.i.d. realizations of the metrics gives $\mathbb{E}_{\boldsymbol{\Theta}_{t-1}}\left[I_h(\mathcal{S}_i, \boldsymbol{\Theta}_{t-1})\, 1_{\{\boldsymbol{\Theta}_{t-1} \in \mathcal{R}(\mathcal{S}_i)\}} | H_h, \mathcal{S}_i\right]$. The error in the Monte Carlo evaluation decreases as $O\left(1/\sqrt{M}\right)$ [32, Ch. 2]. To achieve an accuracy of two decimal places, we have found $M = 10^4$ to be sufficient for all values of ρ.

Fast Computation of $\bar{N}_{tx}$: The next step is to sum over all subsets for a given t and h to obtain $\sum_{\mathcal{S}_i \subseteq \Omega:\, |\mathcal{S}_i| \geq t} c_h(|\mathcal{S}_i|)\, \mathbb{E}_{\boldsymbol{\Theta}_{t-1}}\left[I_h(\mathcal{S}_i, \boldsymbol{\Theta}_{t-1})\, 1_{\{\boldsymbol{\Theta}_{t-1} \in \mathcal{R}(\mathcal{S}_i)\}} | H_h, \mathcal{S}_i\right]$. This requires summing over $\sum_{m=t}^{N} \binom{N}{m}$ terms. However, we show below that it is sufficient to sum over $\binom{N-1}{t-1}$ terms. This reduces the computation complexity significantly. Consider, for example, $N = 10$. Then, for $t = 2$, instead of summing over $\sum_{m=2}^{10} \binom{10}{m} = 1013$ terms, only $\binom{9}{1} = 9$ terms need to be summed over. The corresponding number of terms for $t = 3$ is 36 instead of 968, and for $t = 4$ is 84 instead of 848.

The key observation that enables this is that $I_h(\mathcal{S}_i, \boldsymbol{\Theta}_{t-1})$ in (26) and $\mathcal{R}(\mathcal{S}_i)$ in (27) depend only on $[s_1], \ldots, [s_{t-1}]$ and not on $[s_t], \ldots, [s_{|\mathcal{S}_i|}]$. Given the set $\mathcal{S}_i$, such that $|\mathcal{S}_i| \geq t$, let the set $\mathcal{Q}$ consist of the first $(t-1)$ elements of $\mathcal{S}_i$. Thus, $\mathcal{Q} = \{[s_1], \ldots, [s_{t-1}]\}$. Then,

$$\begin{aligned}&\mathbb{E}_{\boldsymbol{\Theta}_{t-1}}\left[I_h(\mathcal{S}_i, \boldsymbol{\Theta}_{t-1})\, 1_{\{\boldsymbol{\Theta}_{t-1} \in \mathcal{R}(\mathcal{S}_i)\}} | H_h, \mathcal{S}_i\right] \\ &\quad = \mathbb{E}_{\boldsymbol{\Theta}_{t-1}}\left[I_h(\mathcal{Q}, \boldsymbol{\Theta}_{t-1})\, 1_{\{\boldsymbol{\Theta}_{t-1} \in \mathcal{R}(\mathcal{Q})\}}\right]. \quad (29)\end{aligned}$$

Note that $\mathcal{Q}$ cannot contain $[N]$ since it consists of the first $(t-1)$ elements of $\mathcal{S}_i$ and $|\mathcal{S}_i| \geq t$.

Therefore, it follows from (29) that

$$\begin{aligned}&\sum_{\substack{\mathcal{S}_i \subseteq \Omega: \\ |\mathcal{S}_i| \geq t}} c_h(|\mathcal{S}_i|)\, \mathbb{E}_{\boldsymbol{\Theta}_{t-1}}\left[I_h(\mathcal{S}_i, \boldsymbol{\Theta}_{t-1})\, 1_{\{\boldsymbol{\Theta}_{t-1} \in \mathcal{R}(\mathcal{S}_i)\}} | H_h, \mathcal{S}_i\right] \\ &= \sum_{\substack{\mathcal{Q} \subseteq \Omega \setminus \{[N]\}: \\ |\mathcal{Q}| = t-1}} \mathbb{E}_{\boldsymbol{\Theta}_{t-1}}\left[I_h(\mathcal{Q}, \boldsymbol{\Theta}_{t-1})\, 1_{\{\boldsymbol{\Theta}_{t-1} \in \mathcal{R}(\mathcal{Q})\}}\right] \\ &\quad \times \left(\sum_{\substack{\mathcal{S}_i \subseteq \Omega: \\ \mathcal{Q} \subset \mathcal{S}_i}} c_h(|\mathcal{S}_i|) \right). \quad (30)\end{aligned}$$

The number of nodes that missed their transmissions before node $[s_{t-1}]$ transmits is $s_{t-1} - (t-1)$. Thus, $t \leq |\mathcal{S}_i| \leq N - (s_{t-1} - (t-1))$. Furthermore, the number of possible sets of $\mathcal{S}_i$ of cardinality j, where $\mathcal{Q} \subset \mathcal{S}_i$ and $t \leq j \leq N + t - 1 - s_{t-1}$, is $\binom{N - s_{t-1}}{j-t+1}$. Hence,

$$\sum_{\substack{\mathcal{S}_i \subseteq \Omega: \\ \mathcal{Q} \subset \mathcal{S}_i}} c_h(|\mathcal{S}_i|) = \sum_{j=t}^{N+t-1-s_{t-1}} c_h(j) \binom{N - s_{t-1}}{j - t + 1}. \quad (31)$$

Substituting (31) in (30) yields the final expression to be computed for the Monte Carlo evaluation.

IV. Numerical Results

We present two numerical comparisons that provide different insights. We first specify ρ and compare the average number of transmissions and the error probability as a function of ρ. Thereafter, in Section IV-B, we instead specify the energy harvesting and energy storage processes, and simulate the time evolution of the battery energy of each node. We benchmark LL-EH-OTS and NN-EH-OTS with the following schemes:

- *UTS*: In this scheme, the FN makes a decision in a slot only after it receives the LLRs from all the energy-sufficient nodes. The decision rule is given by

$$\sum_{i \in \mathcal{S}} L_i \underset{H_0}{\overset{H_1}{\gtrless}} \beta, \quad (32)$$

  where $\mathcal{S}$ is the set of all energy-sufficient nodes in a slot.
- *Sequential Detection:* In this scheme, the nodes transmit their LLRs to the FN in a random order. Let $\mathcal{T}$ denote the set of nodes that have transmitted their LLRs to the FN thus far. The decision rules are as follows [5, Ch. III.D]:

$$\text{Decide } H_1 \text{ if}: \sum_{i \in \mathcal{T}} L_i > B, \quad (33)$$

$$\text{Decide } H_0 \text{ if}: \sum_{i \in \mathcal{T}} L_i < A, \quad (34)$$

  Wait for the next transmission, otherwise.

  If no more energy-sufficient EH nodes remain and a decision is yet to be made, the FN decides the hypothesis along the lines of UTS. As per Wald's formulae, the thresholds A and B are chosen to achieve a target false alarm probability P_{FA} and a detection probability P_{D} as follows [5, Ch. III.D]:

$$A = \log\left(\frac{1 - P_{\text{D}}}{1 - P_{\text{FA}}}\right) \quad \text{and} \quad B = \log\left(\frac{P_{\text{D}}}{P_{\text{FA}}}\right). \quad (35)$$

  In order to ensure a fair comparison, P_{FA} and P_{D} are set to be the same as those observed numerically for NN-EH-OTS for each parameter choice.

  These formulae are exact only when the number of measurements available to the FN is sufficiently large. Therefore, we include sequential detection in the benchmarking comparisons only in Section IV-B, in which the time-dynamics of an EH WSN are simulated and the performance measure is the error probability itself. These simulations also account for the inaccuracy of the Wald's formulae in specifying the decision thresholds for a limited number of transmissions.

We simulate the EH WSN for a duration of 10^6 slots. We consider the uniform cost model with $c_{uv} = 1$, if $u \neq v$,

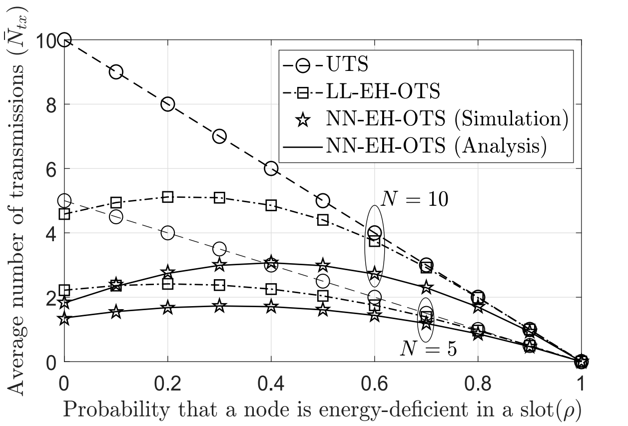


Fig. 4. Comparison of the average number of transmissions as a function of the probability ρ that a node is energy-deficient in a slot for different values of N $\left(\sigma_s^2/\sigma_0^2 = 10 \text{ dB}\right)$.

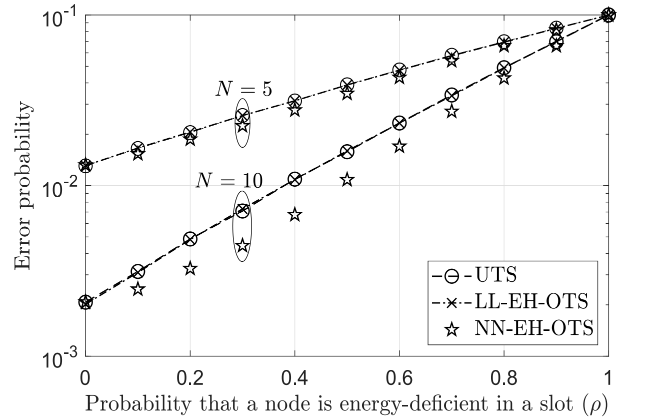


Fig. 5. Comparison of the error probability as a function of the probability ρ that a node is energy-deficient in a slot for different values of N $\left(\sigma_s^2/\sigma_0^2 = 10 \text{ dB}\right)$.

and $c_{uv} = 0$, if $u = v$ [4], [5, Ch. II.B]. We set $\tau = 3\sqrt{\sigma_s^2 + \sigma_0^2}$. Unless stated otherwise, $\zeta_1 = 0.1$.

A. Simulation Results for a Given ρ

Fig. 4 plots the average number of transmissions, $\bar{N}_{\text{tx}}$, of UTS, LL-EH-OTS, and NN-EH-OTS as a function of ρ for different numbers of EH sensor nodes N. For NN-EH-OTS, also shown are the results from the analytical expression in (24). We observe a good match between it and the simulations. As N increases, the average number of transmissions increases, since more nodes are available to transmit their measurements to the FN. When $\rho = 0$, i.e., when all the nodes are energy-sufficient, LL-EH-OTS, which in this case is equivalent to the OTS scheme in [4], reduces the average number of transmissions significantly compared to UTS. Notably, NN-EH-OTS reduces the average number of transmissions even more. For example, when $N = 10$, $\bar{N}_{\text{tx}}$ is 4.56 for LL-EH-OTS, and is just 1.83 for NN-EH-OTS. Hence, from (21), NN-EH-OTS is significantly more energy-efficient than LL-EH-OTS. This happens because in NN-EH-OTS, the transmissions occur in the decreasing order of the LLRs and not in the decreasing order of the absolute value of the LLRs, which creates a sign ambiguity at the FN about the yet-to-be received LLRs. Thus, the decision rules for NN-EH-OTS can bound these LLRs in a narrower range and require fewer transmissions to decide.

For LL-EH-OTS and NN-EH-OTS, as ρ increases, $\bar{N}_{\text{tx}}$ first increases. This is because the FN will require more transmissions from other nodes, which have less informative, smaller metrics since the transmissions occur in the decreasing order of the metrics. However, for larger ρ, $\bar{N}_{\text{tx}}$ decreases because more nodes are energy-deficient and cannot transmit. Both these schemes outperform UTS. For UTS, $\bar{N}_{\text{tx}}$ decreases linearly as ρ increases. When $\rho = 1$, all the nodes are energy-deficient. Hence, $\bar{N}_{\text{tx}} = 0$ for all the schemes.

Fig. 5 plots the error probability of the three schemes as a function of ρ for different values of N. As N increases, the error probability for all the schemes decreases since the FN receives measurements from more nodes. As ρ increases, the error probability of all schemes increases because more nodes miss their transmissions. LL-EH-OTS has the same error probability as UTS. However, NN-EH-OTS yields a lower error probability for all N and for all $0 < \rho < 1$. This is because the decision rules for it in (18) and (19) take into account the fact that the metric of an EH node that missed its transmission lies between those previously received and those that will be received. On the other hand, the decision rules for LL-EH-OTS, in which the metric is the absolute value of the LLR, only utilizes the information conveyed by the previously received metric. Only for $\rho = 0$ and $\rho = 1$ is the error probability of all schemes the same.

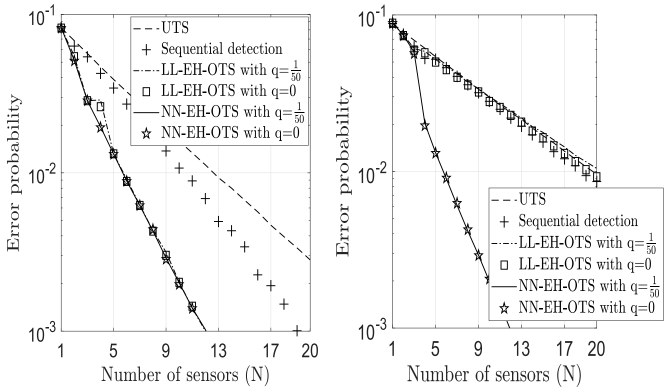
B. Simulation Results Based on Tracking the Battery Energy Evolution

We now present results for an alternate and more physically realistic simulation model in which the energy harvesting and storage models are specified as per Section II and in which the evolution of the battery energy in each node for each slot is tracked. We can no longer pre-specify ρ. Let $B_i(t)$ be the battery capacity of node i at the end of slot t, $B_{\max}$ be the maximum battery capacity of a node, and $H_i(t)$ be the energy harvested by the i^{th} node at the start of slot t. Then,

$$B_i(t) = \min\{B_i(t-1) + H_i(t) - E_{\text{tx}} 1_{\{B_i(t-1)+H_i(t) \geq E_{\text{tx}}\}} 1_{\{T_i(t)\}}, B_{\max}\}, \quad (36)$$

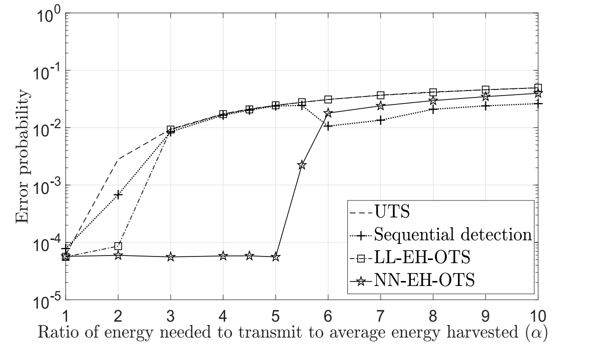
where the event $T_i(t)$ is true if node i has to transmit in slot t and is false otherwise. A node transmits and consumes an energy E_{tx} only if $1_{\{B_i(t-1)+H_i(t) \geq E_{\text{tx}}\}}$ and $1_{\{T_i(t)\}}$ are both one. Note that the event that a node transmits, which depends on and affects its battery energy, is affected by the battery energies and metrics of all the other nodes in the network. Thus, the battery energies of the different nodes are coupled.

The Bernoulli EH model is simulated [13], $B_{\max}$ is set to $25E_{\text{tx}}$, and the simulations are carried over 10^6 time slots to measure the steady-state results. Let $\alpha \triangleq E_{\text{tx}}/\bar{E}_h$. Thus, on an average, an EH node requires the energy harvested in α slots to transmit one measurement packet to the FN. Unlike Section IV-A, the error probability in this

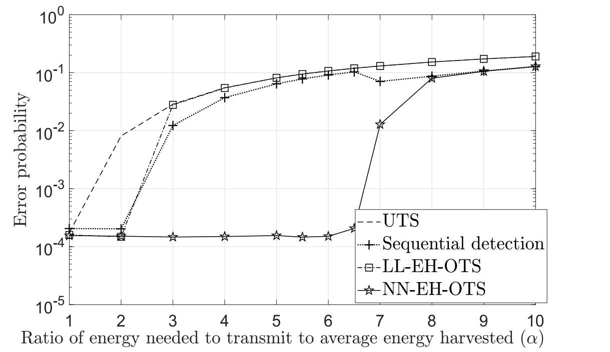


(a) $\alpha = 2$ (b) $\alpha = 3$

Fig. 6. Comparison of the error probability of different schemes as a function of N for different values of α $\left(\sigma_s^2/\sigma_0^2 = 10 \text{ dB}\right)$.



(a) $\zeta_1 = 0.1$



(b) $\zeta_1 = 0.5$

Fig. 7. Comparison of the error probability of different schemes as a function of α for different values of ζ_1 $\left(N = 20, \sigma_s^2/\sigma_0^2 = 10 \text{ dB}, \text{ and } q = 0\right)$.

case is the performance measure of interest since ρ, which affects the average number of transmissions, itself becomes scheme-dependent.

Figs. 6(a) and 6(b) plot the error probability as a function of N for $\alpha = 2$ and $\alpha = 3$, respectively, for LL-EH-OTS, NN-EH-OTS, UTS, and sequential detection. They also show the impact of the energy consumed in the pilot transmissions. The replenishment of the pilot energy reserve eventually reduces the energy available for transmitting data packets. Therefore, (36) is modified to account for this in our simulations. For this, results for $q = 1/50$ for LL-EH-OTS and NN-EH-OTS are shown and compared with those for the ideal $q = 0$ case. Recall that pilot transmissions are not required in UTS and sequential detection. In both figures, as N increases, the error probability decreases, since more nodes are available to transmit their LLRs to the FN. NN-EH-OTS and LL-EH-OTS require far fewer sensor nodes in the WSN to achieve a target error probability as compared to UTS and sequential detection. For example, for a target error probability of 0.01 and $\alpha = 2$, $N = 6$ nodes are sufficient for both LL-EH-OTS and NN-EH-OTS while 13 nodes are needed by UTS and 11 nodes are needed by sequential detection.

As N increases, the error probability for NN-EH-OTS is one to two orders of magnitude lower than UTS and sequential detection for both values of α. Furthermore, the pilot transmission energy has a negligible impact on its performance. LL-EH-OTS behaves differently. For $\alpha = 2$, its error probability is markedly lower than that of UTS and sequential detection and is the same as that of NN-EH-OTS for $N > 4$. However, for $\alpha = 3$, its error probability is marginally higher than that of UTS and sequential detection. This is because the replenishment of the energy reserve for pilots reduces the energy available for data transmissions.[2] The significant reduction in the error probability of NN-EH-OTS and LL-EH-OTS as α decreases occurs because the following effect reinforces itself. The odds that the nodes are energy-sufficient increase as α decreases. As we saw in Fig. 4, this, in turn, reduces the number of nodes that need to transmit and increases the probability that a node is energy-sufficient.

Figs. 7(a) and 7(b) study the effect of α and the prior probability ζ_1 on the error probability for $N = 20$. They bring out an interesting new trend. Consider first $\zeta_1 = 0.1$. For $\alpha = 1$, all the nodes are energy-sufficient with probability one in all schemes. Therefore, the difference in performance between them is negligible. As α increases, the schemes behave differently. For $1 < \alpha \leq 5$, the error probability of NN-EH-OTS is insensitive to α and is up to two orders of magnitude lower than that of UTS and sequential detection. LL-EH-OTS also yields a lower error probability compared to UTS and sequential detection for $1 < \alpha \leq 2$. However, for $\alpha > 5$ for NN-EH-OTS and for $\alpha > 2$ for LL-EH-OTS, the error probability increases and approaches that of UTS. This is because so few nodes have sufficient energy to transmit that these schemes require all of them to transmit for the FN to make a decision; this is also what UTS does.

For sequential detection, the error probability is as high as that of UTS for $\alpha \leq 5.5$. This is because its two thresholds, which are chosen to match the P_{FA} and P_{D} of NN-EH-OTS, are large enough to require all the energy-sufficient nodes to transmit with a high probability. For larger α, its error probability surprisingly decreases. This happens because its two thresholds A and B now become so close to each other that transmissions by only a subset of the energy-sufficient nodes are sufficient for it to decide. This increases the odds

[2]Note that when $\alpha = 2$, the error probability of LL-EH-OTS is marginally higher than that of NN-EH-OTS at $N = 4$. This happens because the average number of transmissions for LL-EH-OTS is almost the same as that of UTS. Hence, the probability that a node is energy-sufficient is less than that for NN-EH-OTS.

that a node is energy-sufficient in subsequent slots. However, this happens only in the regime in which the error probability is two to three orders of magnitude larger than the lowest error probability achievable. For $\zeta_1 = 0.5$, the trends are similar except that the error probability of all the schemes is greater. NN-EH-OTS achieves a markedly lower error probability for a larger range of α compared to the other schemes.

V. Conclusion and Future Work

We proposed two novel ordered transmissions schemes for an EH WSN, namely, LL-EH-OTS and NN-EH-OTS. We derived new decision rules for them that tackled the problem of some energy-deficient EH nodes not transmitting. Given the probability that a node was energy-deficient, LL-EH-OTS reduced the average number of nodes that transmitted while achieving the same error probability as UTS. On the other hand, NN-EH-OTS markedly reduced both average number of transmissions and error probability as it avoided the sign ambiguity in LL-EH-OTS about the LLRs of nodes that missed their transmissions or were yet to transmit. We also derived an expression for the average number of transmissions for NN-EH-OTS. Its special form lent itself to a computationally-efficient Monte Carlo-based evaluation. When the time evolution of the battery energies of the nodes was tracked and the probability of a missed transmission became a function of the scheme itself, the proposed schemes achieved a much lower error probability than both UTS and sequential detection.

Interesting avenues for future work include modeling the effect of deep channel fades, due to which transmissions from the sensor nodes may not be decoded at the FN, and quantization and correlation of the LLRs. Other avenues include adapting the transmit power as a function of the channel gain between the sensor nodes and the FN to improve the energy-efficiency, and optimizing the decision thresholds to reduce the error probability.

Appendix

A. LL-EH-OTS: General Decision Rules When Multiple Transmissions are Missed

As per our notation, the transmissions of j sensor nodes $[m_1], [m_2], \ldots, [m_j]$ are missed, where $1 \le m_1 < \cdots < m_j < k < N$. As in (7), the FN should decide H_0 if $\sum_{i=1}^{N} L_{[i]} < \beta$. Splitting the summation $\sum_{i=1}^{N} L_{[i]}$ in terms of the received, missed, and yet-to-be-received LLRs, we get

$$\eta \triangleq \sum_{i=1, i\notin\{m_1,\ldots,m_j\}}^{k} L_{[i]} + \sum_{l=1}^{j} L_{[m_l]} + \sum_{i=k+1}^{N} L_{[i]} < \beta. \quad (37)$$

In LL-EH-OTS, the metrics are ordered as $|L_{[1]}| > |L_{[2]}| > \cdots > |L_{[N]}|$. Recall that p_l is the largest integer that is less than m_l such that the sensor node $[p_l]$ is energy-sufficient. Therefore, $|L_{[m_l]}| < |L_{[p_l]}|$, for $1 \le l \le j$. Furthermore, $|L_{[i]}| < |L_{[k]}|$, for $k < i \le N$. Hence, we have

$$-|L_{[p_l]}| < L_{[m_l]} < |L_{[p_l]}|, \text{ for } 1 \le l \le j, \quad (38)$$

$$-|L_{[k]}| < L_{[i]} < |L_{[k]}|, \text{ for } k < i \le N. \quad (39)$$

Note that (38) includes the case where $p_l = 0$, in which case $-\Psi \le L_{[m_l]} \le \Psi$.

Hence, from (38) and (39), the left hand side of (37) can be bounded as follows:

$$\eta < \sum_{i=1, i\notin\{m_1,\ldots,m_j\}}^{k} L_{[i]} + \left(\sum_{l=1}^{j} |L_{[p_l]}|\right) + (N-k)|L_{[k]}|, \quad (40)$$

and

$$\eta > \sum_{i=1, i\notin\{m_1,\ldots,m_j\}}^{k} L_{[i]} - \left(\sum_{l=1}^{j} |L_{[p_l]}|\right) - (N-k)|L_{[k]}|. \quad (41)$$

Therefore, if

$$\sum_{i=1, i\notin\{m_1,\ldots,m_j\}}^{k} L_{[i]} < \beta - \left(\sum_{l=1}^{j} |L_{[p_l]}|\right) - (N-k)|L_{[k]}|, \quad (42)$$

then it follows from (37) and (40) that $\sum_{i=1}^{N} L_{[i]} < \beta$. Thus, the FN should decide H_0. Else, if

$$\sum_{i=1, i\notin\{m_1,\ldots,m_j\}}^{k} L_{[i]} > \beta + \left(\sum_{l=1}^{j} |L_{[p_l]}|\right) + (N-k)|L_{[k]}|, \quad (43)$$

then from (37) and (41), it follows that $\sum_{i=1}^{N} L_{[i]} > \beta$. Therefore, the FN should decide H_1. A key point to note here is that this leads to the same decision as when the FN receives the LLRs of all the N nodes. If neither (42) and (43) is satisfied, then the FN waits for the next transmission.

Boundary Case: If $k = N$ or $m_j = N$, which corresponds to the case that no more energy-sufficient nodes remain, the FN will not receive any more LLRs. Hence, it needs to make a decision on the basis of the $(N-j)$ LLRs it has received, just as UTS would. Therefore, proceeding as (7), it decides H_1 if $\sum_{i=1, i\ \notin\{m_1,\ldots,m_j\}}^{N} L_{[i]} > \beta$, and decides H_0, otherwise.

B. Detection Test for Truncated Gaussian Statistics

From (6), the LLR L_i of node i is given by $L_i = \log\left(\frac{\sigma_0}{\sigma_1}\right) + \log\left(\frac{1-2Q(\tau/\sigma_0)}{1-2Q(\tau/\sigma_1)}\right) + \frac{y_i^2}{2}\frac{\sigma_s^2}{\sigma_1^2\sigma_0^2}$. Substituting this in (7), we get

$$N\log\left(\frac{\sigma_0}{\sigma_1}\right) + N\log\left(\frac{1-2Q(\tau/\sigma_0)}{1-2Q(\tau/\sigma_1)}\right) + \frac{\sigma_s^2}{2\sigma_1^2\sigma_0^2}\sum_{i=1}^{N} y_i^2 \underset{H_0}{\overset{H_1}{\gtrless}} \beta. \quad (44)$$

Rearranging terms and substituting $\Theta_i = y_i^2$ yields (13).

C. NN-EH-OTS: Decision Rules When No Transmission is Missed

In NN-EH-OTS, the metrics are ordered as $\Theta_{[1]} > \Theta_{[2]} > \cdots > \Theta_{[N]}$. From (15), the FN should decide hypothesis H_0 if $\sum_{i=1}^{N} \Theta_{[i]} < \lambda$. Splitting the summation $\sum_{i=1}^{N} \Theta_{[i]}$ in terms of the received metrics $\Theta_{[1]}, \ldots, \Theta_{[k]}$ and the yet-to-be-received metrics $\Theta_{[k+1]}, \ldots, \Theta_{[N]}$, we get

$$\sum_{i=1}^{k} \Theta_{[i]} + \sum_{i=k+1}^{N} \Theta_{[i]} < \lambda. \tag{45}$$

Since $\Theta_{[i]} < \Theta_{[k]}$, for $k < i \le N$, we get

$$\sum_{i=1}^{k} \Theta_{[i]} + \sum_{i=k+1}^{N} \Theta_{[i]} < \sum_{i=1}^{k} \Theta_{[i]} + (N-k)\Theta_{[k]}. \tag{46}$$

Thus, if $\sum_{i=1}^{k} \Theta_{[i]} < \lambda - (N-k)\Theta_{[k]}$, then it implies that $\sum_{i=1}^{N} \Theta_{[i]} < \lambda$. In that case, the FN will decide H_0. Note that this leads to the same decision as receiving the metrics of all the N nodes. The rule in (16) for H_1 can be similarly derived.

D. NN-EH-OTS: Decision Rules When One or More Transmissions are Missed

The transmissions of j sensor nodes $[m_1], [m_2], \ldots, [m_j]$ are missed, where $1 \le m_1 < \cdots < m_j < k < N$. As in (15), the FN should decide hypothesis H_0 if $\sum_{i=1}^{N} \Theta_{[i]} < \lambda$. Splitting the summation $\sum_{i=1}^{N} \Theta_{[i]}$ in terms of the received, missed, and yet-to-be received metrics, we get

$$\sum_{i=1, i \notin \{m_1,\ldots,m_j\}}^{k} \Theta_{[i]} + \sum_{l=1}^{j} \Theta_{[m_l]} + \sum_{i=k+1}^{N} \Theta_{[i]} < \lambda. \tag{47}$$

Recall that p_l is the largest integer that is less than m_l such that the sensor node $[p_l]$ is energy-sufficient. Thus, $\Theta_{[m_l]} < \Theta_{[p_l]}$, for $1 \le l \le j$. Note that if $p_l = 0$, then $\Theta_{[m_l]} \le \Theta_{[0]} \triangleq \tau^2$. Furthermore, $\Theta_{[i]} < \Theta_{[k]}$, for $k < i \le N$. Hence, it follows that

$$\begin{aligned}\sum_{i=1, i \notin \{m_1,\ldots,m_j\}}^{k} \Theta_{[i]} + \sum_{l=1}^{j} \Theta_{[m_l]} + \sum_{i=k+1}^{N} \Theta_{[i]} \\ < \sum_{i=1, i \notin \{m_1,\ldots,m_j\}}^{k} \Theta_{[i]} + \sum_{l=1}^{j} \Theta_{[p_l]} + (N-k)\Theta_{[k]}. \end{aligned} \tag{48}$$

Thus, if $\sum_{i=1, i \notin \{m_1,\ldots,m_j\}}^{k} \Theta_{[i]} < \lambda - \sum_{l=1}^{j} \Theta_{[p_l]} - (N-k)\Theta_{[k]}$, then (47) and (48) imply that $\sum_{i=1}^{N} \Theta_{[i]} < \lambda$. Therefore, the FN should decide H_0.

Similarly, the FN should decide H_1 if $\sum_{i=1, i \notin \{m_1,\ldots,m_j\}}^{k} \Theta_{[i]} + \sum_{l=1}^{j} \Theta_{[m_l]} + \sum_{i=k+1}^{N} \Theta_{[i]} > \lambda$. Recall that n_l is the smallest integer that is greater than m_l such that the sensor node $[n_l]$ is energy-sufficient. Hence, $\Theta_{[m_l]} > \Theta_{[n_l]}$, for $1 \le l \le j$. Substituting this and $\Theta_{[i]} > 0$, we get

$$\begin{aligned}\sum_{i=1}^{N} \Theta_{[i]} &= \sum_{i=1, i \notin \{m_1,\ldots,m_j\}}^{k} \Theta_{[i]} + \sum_{l=1}^{j} \Theta_{[m_l]} + \sum_{i=k+1}^{N} \Theta_{[i]} \\ &> \sum_{i=1, i \notin \{m_1,\ldots,m_j\}}^{k} \Theta_{[i]} + \sum_{l=1}^{j} \Theta_{[n_l]}. \end{aligned} \tag{49}$$

Therefore, if $\sum_{i=1, i \notin \{m_1,\ldots,m_j\}}^{k} \Theta_{[i]} > \lambda - \sum_{l=1}^{j} \Theta_{[n_l]}$, then it follows from (49) that $\sum_{i=1}^{N} \Theta_{[i]} > \lambda$ and the FN decides H_1. If $\lambda - \sum_{l=1}^{j} \Theta_{[p_l]} - (N-k)\Theta_{[k]} < \sum_{i=1, i \notin \{m_1,\ldots,m_j\}}^{k} \Theta_{[i]} < \lambda - \sum_{l=1}^{j} \Theta_{[n_l]}$, then the FN waits for the next transmission.

Boundary Case: As before, if $k = N$ or $m_j = N$, the FN cannot receive any more metrics. Hence, it decides on the basis of the $(N-j)$ metrics it has received just as UTS would. Hence, the FN decides H_1 if

$$\begin{aligned}\sum_{i=1, i \notin \{m_1,\ldots,m_j\}}^{N} \Theta_{[i]} > \lambda - \frac{2j\sigma_0^2\sigma_1^2}{\sigma_s^2} \log\left(\frac{\sigma_1}{\sigma_0}\right) \\ - \frac{2j\sigma_0^2\sigma_1^2}{\sigma_s^2} \log\left(\frac{1-2Q(\tau/\sigma_1)}{1-2Q(\tau/\sigma_0)}\right), \end{aligned} \tag{50}$$

and decides H_0, otherwise.

E. NN-EH-OTS: Average Number of Transmissions

Let N_{tx} denote the number of transmissions in a slot. Since it is a positive integer-valued RV, its average value $\bar{N}_{\text{tx}}$ is given by

$$\bar{N}_{\text{tx}} = \Pr[N_{\text{tx}} \ge 1] + \sum_{t=2}^{N} \Pr[N_{\text{tx}} \ge t]. \tag{51}$$

From the definition of ρ, we have $\Pr[N_{\text{tx}} \ge 1] = 1 - \Pr[N_{\text{tx}} = 0] = 1 - \rho^N$. For the other terms in (51), we condition on H_0 and H_1 and use the law of total probability to get

$$\begin{aligned}\bar{N}_{\text{tx}} = \left(1-\rho^N\right) + \sum_{t=2}^{N} \Pr[N_{\text{tx}} \ge t | H_0] \Pr[H_0] \\ + \sum_{t=2}^{N} \Pr[N_{\text{tx}} \ge t | H_1] \Pr[H_1]. \end{aligned} \tag{52}$$

If at least t transmissions are needed to make a decision, then the number $|\mathcal{S}_i|$ of energy-sufficient nodes should at least be t. Thus, $\Pr[N_{\text{tx}} \ge t | H_h] = 0$ if $|\mathcal{S}_i| < t$. Therefore,

$$\Pr[N_{\text{tx}} \ge t | H_h] = \sum_{\substack{\mathcal{S}_i \subseteq \Omega: \\ |\mathcal{S}_i| \ge t}} \Pr[N_{\text{tx}} \ge t | \mathcal{S}_i, H_h] \Pr[\mathcal{S}_i | H_h] \Pr[H_h], \tag{53}$$

where $h \in \{0, 1\}$ and from the definition of ρ, we have $\Pr[\mathcal{S}_i | H_h] = (1-\rho)^{|\mathcal{S}_i|} \rho^{N - |\mathcal{S}_i|}$. Furthermore, $N_{\text{tx}} \ge t$ if and only if the FN cannot decide after receiving 1 or 2 or ... or $(t-1)$ transmissions. Lemma 2 below presents a compact set of conditions to characterize this scenario.

Lemma 2: When the set of energy-sufficient nodes is $\mathcal{S}_i = \left\{[s_1], [s_2], \ldots, \left[s_{|\mathcal{S}_i|}\right]\right\}$, where $1 \le s_1 < s_2 < \cdots < s_{|\mathcal{S}_i|} \le N$, the FN cannot decide H_0 or H_1 after receiving 1 or 2 or... or $(t-1)$ transmissions if

$$\begin{aligned}(s_1 - 1)\tau^2 + \sum_{j=2}^{t-1} (s_j - s_{j-1}) \Theta_{[s_{j-1}]} \\ + (N - s_{t-1} + 1) \Theta_{[s_{t-1}]} > \lambda, \end{aligned} \tag{54}$$

and

$$s_1 \Theta_{[s_1]} + \sum_{j=2}^{t-1} (s_j - s_{j-1}) \Theta_{[s_j]} < \lambda, \tag{55}$$

Proof: We discuss the cases of deciding H_0 and H_1 separately below.

1) Deciding H_0: We map the decision rules for NN-EH-OTS in (19) in terms of the metrics of the nodes $[s_1], [s_2], \ldots, [s_{t-1}]$ and derive the condition that the FN does not decide H_0 after it has received them.

Let r nodes have missed their transmissions. Among these nodes, the metrics of (s_1-1) nodes are greater than $\Theta_{[s_1]}$ and less than τ^2, metrics of (s_2-s_1-1) nodes are greater than $\Theta_{[s_2]}$ and less than $\Theta_{[s_1]}$, and proceeding similarly, metrics of $(s_{t-1}-s_{t-2}-1)$ nodes are greater than $\Theta_{[s_{t-1}]}$ and less than $\Theta_{[s_{t-2}]}$. Furthermore, for the yet-to-be-received metrics, $\Theta_{[s_{t-1}]} > \Theta_{[v]} > 0$ for $s_{t-1} < v \leq N$. Thus, from (48) and (49), we have

$$\sum_{i=1}^{N} \Theta_{[i]} < (s_1-1)\,\tau^2 + \sum_{j=2}^{t-1} (s_j - s_{j-1})\,\Theta_{[s_{j-1}]} + (N - s_{t-1} + 1)\,\Theta_{[s_{t-1}]}, \quad (56)$$

$$\sum_{i=1}^{N} \Theta_{[i]} > s_1 \Theta_{[s_1]} + \sum_{j=2}^{t-1} (s_j - s_{j-1})\,\Theta_{[s_j]}. \quad (57)$$

From (15) and (56), the FN is not able to decide H_0 if $(s_1-1)\,\tau^2 + \sum_{j=2}^{t-1}(s_j - s_{j-1})\,\Theta_{[s_{j-1}]} + (N - s_{t-1}+1)\,\Theta_{[s_{t-1}]} > \lambda$, which is the condition in (54).

We can rewrite (54) as

$$\begin{aligned}(s_1-1)\,\tau^2 &+ \sum_{j=2}^{t-2} (s_j - s_{j-1})\,\Theta_{[s_{j-1}]} \\ &+ (N - s_{t-2} + 1)\,\Theta_{[s_{t-2}]} + (s_{t-1} - s_{t-2})\,\Theta_{[s_{t-2}]} \\ &+ (N - s_{t-1} + 1)\,\Theta_{[s_{t-1}]} - (N - s_{t-2} + 1)\,\Theta_{[s_{t-2}]} > \lambda. \end{aligned} \quad (58)$$

As $s_{t-2} < s_{t-1}$ and $\Theta_{[s_{t-2}]} > \Theta_{[s_{t-1}]} > 0$, we get $(s_{t-1}-s_{t-2})\,\Theta_{[s_{t-2}]} + (N - s_{t-1} + 1)\Theta_{[s_{t-1}]} - (N - s_{t-2}+1)\,\Theta_{[s_{t-2}]} < 0$. Thus, (58) implies $(s_1-1)\,\tau^2 + \sum_{j=2}^{t-2}(s_j - s_{j-1})\,\Theta_{[s_{j-1}]} + (N - s_{t-2}+1)\,\Theta_{[s_{t-2}]} > \lambda$. This is the condition that the FN does not decide H_0 after $(t-2)$ transmissions. Using this condition and proceeding as before, it can be shown that the FN has not decided H_0 after receiving 1 or 2 or… or $(t-3)$ transmissions.

2) Deciding H_1: The proof for this has a similar flavor as above and is skipped. ∎

Using Lemma 2, we now evaluate $\Pr[N_{\text{tx}} \geq t | \mathcal{S}_i, H_h]$. For the set $\mathcal{S}_i$, let $\theta_1, \theta_2, \ldots, \theta_{t-1}$ denote a realization of the metrics for the nodes $[s_1]$, $[s_2]$, …, $[s_{t-1}]$, respectively, which satisfies (54) and (55) in Lemma 2. Let $\boldsymbol{\theta}_{t-1} = (\theta_1, \theta_2, \ldots, \theta_{t-1})$ and the region in which these tuples lie be $\mathcal{J}(\mathcal{S}_i)$. Then,

$$\Pr[N_{\text{tx}} \geq t | \mathcal{S}_i, H_h] = \int_{\mathcal{J}(\mathcal{S}_i)} f_{\Theta_{[s_1]},\ldots,\Theta_{[s_{t-1}]}}(\theta_1, \ldots, \theta_{t-1} | H_h, \mathcal{S}_i)\, d\theta_1 \ldots d\theta_{t-1}, \quad (59)$$

where $f_{\Theta_{[s_1]},\ldots,\Theta_{[s_{t-1}]}}(\theta_1, \ldots, \theta_{t-1} | H_h, \mathcal{S}_i)$ is the joint PDF of the ordered metrics $[s_1]$, $[s_2]$, …, $[s_{t-1}]$ conditioned on hypothesis H_h and the set of energy-sufficient nodes $\mathcal{S}_i$. It is given in closed form as [23]

$$\begin{aligned} f_{\Theta_{[s_1]},\ldots,\Theta_{[s_{t-1}]}}&(\theta_1, \ldots, \theta_{t-1} | H_h, \mathcal{S}_i) \\ = &\frac{N!}{(N - s_{t-1})!(s_1-1)! \prod_{n=2}^{t-1}(s_{t-n+1} - s_{t-n} - 1)!} \\ &\times \prod_{n=2}^{t-1} \left[F_\Theta(\theta_{t-n}|H_h) - F_\Theta(\theta_{t-n+1}|H_h)\right]^{s_{t-n+1}-s_{t-n}-1} \\ &\times \left[F_\Theta(\theta_{t-1}|H_h)\right]^{N-s_{t-1}} \left[1 - F_\Theta(\theta_1|H_h)\right]^{s_1-1} \\ &\times \left[\prod_{i=1}^{t-1} f_\Theta(\theta_i|H_h)\right] 1_{\{\theta_1 > \cdots > \theta_{t-1}\}}. \end{aligned} \quad (60)$$

Substituting (60) in (59) and rearranging terms, we get

$$\begin{aligned}\Pr[N_{\text{tx}} \geq t | \mathcal{S}_i, H_h] = &\int_{\boldsymbol{\theta}_{t-1}} I_h(\mathcal{S}_i, \boldsymbol{\theta}_{t-1})\, 1_{\{\boldsymbol{\theta}_{t-1} \in \mathcal{R}(\mathcal{S}_i)\}} \\ &\times \left[\prod_{i=1}^{t-1} f_\Theta(\theta_i|H_h)\right] d\theta_1 \ldots d\theta_{t-1}, \end{aligned} \quad (61)$$

where $I_h(\mathcal{S}_i, \boldsymbol{\theta}_{t-1})$ is given in (26) and the region $\mathcal{R}(\mathcal{S}_i)$ is specified in (27). Using the law of total expectation, the integral in (61) can be written as

$$\begin{aligned}&\int_{\boldsymbol{\theta}_{t-1}} I_h(\mathcal{S}_i, \boldsymbol{\theta}_{t-1})\, 1_{\{\boldsymbol{\theta}_{t-1} \in \mathcal{R}(\mathcal{S}_i)\}} \\ &\qquad \times \left[\prod_{i=1}^{t-1} f_\Theta(\theta_i|H_h)\right] d\theta_1 \ldots d\theta_{t-1} \\ &= \mathbb{E}_{\boldsymbol{\Theta}_{t-1}}\left[I_h(\mathcal{S}_i, \boldsymbol{\Theta}_{t-1})\, 1_{\{\boldsymbol{\Theta}_{t-1} \in \mathcal{R}(\mathcal{S}_i)\}} | H_h, \mathcal{S}_i\right]. \end{aligned} \quad (62)$$

Substituting (62) in (61) and then in (53) yields (24).

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