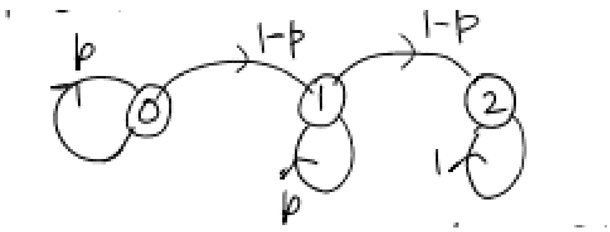
1. A player has two umbrellas that he uses while commuting between playing ground and home. If it rains and an umbrella is available at her location, he takes it and if it is not raining, he always forgets to carry an umbrella. Suppose that probability of raining is p each time he commutes, independent of other times. What is the steady state probability that he gets wet during a commute?

2. Consider a two state DTMC $X_k, k \geq 0, X_k \epsilon (0,1)$ with $P_{00} = (1-p)$ and $P_{11} = (1-q)$ Where $0 < p < 1$ and $0 < q < 1$. Define $Y = min(k \geq 1 : X_k \neq X_0)$, i.e. Y denotes the time spent in the initial state.
   (a) obtain $P[Y = t \mid X_0 = 0]$ and $P[Y = t \mid X_0 = 1]$
   (b) obtain $P[T_0 < \infty]$ and $P[T_1 < \infty]$

3. Consider a DTMC with TPD shown below



   Let $T_i(i = 0,1,2)$ denote the recurrence time of state i. Obtain distribution of T_i and $E[T_i]$ for $(i = 0,1,2)$

4. Consider a discrete time wireless channel model $X_k, k > 0, X_k \epsilon (0,1)$ are i.i.d. Bernoulli (μ). If $X_k = 1$, one can transmit 1 packet in slot k otherwise one cannot transmit any packet in slot k. Arrivals in each slot are Bernoulli (μ) when the queue length is less than or equal to Bernoulli (μ) and otherwise it is Bernoulli λ).
   (a) Identify the condition under which the queue length DTMC is positive recurrent
   (b) Find the distribution and mean of the queue length in steady state for the above condition.

5. An urn contains two balls red and blue. A ball is randomly picked up and is replaced with another ball of same colour with probability of 0.8 and it is replaced with a ball of different colour with probability of 0.2 and this process is repeated many times. Find the probability that fourth picked ball is red.

   [Hint: let X_0 be the initial number of red balls and X_k, $k > 1$ be the number of red balls after k^{th} replacement. Observe that $X_k, k > 0$ is a DTMC]