

1. Consider a two-link network. Let  $S1 = [1, 0]$ ,  $S2 = [0, 1]$ ,  $S3 = [0.25, 0.75]$ , and  $S4 = [0.75, 0.25]$  be the four possible schedules in the network. Draw the link layer capacity region for this network.
2. Consider the two-link network where node 1 and 2 are only origin and destination respectively. Rate  $R$  from the origin is divided into 2 paths flows  $x_1$  and  $x_2$  to minimize a cost function based on M/M/1 approximation  $D(x) = D_1(x_1) + D_2(x_2)$  where for  $i = 1, 2$ , and  $D_i(x_i) = x_i / (C_i - x_i)$  and  $C_i$  is the capacity of the link  $i$ . Find the optimum solution for the network to sustain and plot optimal path flows for this routing.
3. Consider the optimal routing of one unit of input flow in a single OD-pair, three-link network for the cost function  $D(x) = 1/2 \times [x_1^2 + 2x_2^2 + x_3^2] + 0.7 * x_3$ . Mathematically the problem is to minimize  $D(x)$  subject to  $x_1 + x_2 + x_3 = 1; x_1, x_2, x_3 > 0$ . Show that  $x_1^* = 2/3$ ,  $x_2^* = 1/3$  and  $x_3^* = 0$  is the optimal solution.
4. Consider an output link with  $c = 1$  bps and shared by two flows, flow  $i$  and flow  $j$ . The link maintains one queue for each flow; both queues are empty at the beginning ( $t = 0$ ). As shown in Figure, flow  $i$  have two packets, one arriving at time  $t = 0$  and the other arriving at time  $t = 2$ , and flow  $j$  has two packets, one arriving at the beginning of time slot 1 and the other arriving at time  $t = 7$ . The numbers in the boxes are packet sizes and the numbers above the boxes are the finish tags. The evolution of the virtual time is also illustrated in Figure. Find the transmission rates for the two queues under Weighted Fair Queuing (WFQ) and Generalized Processor Sharing (GPS).

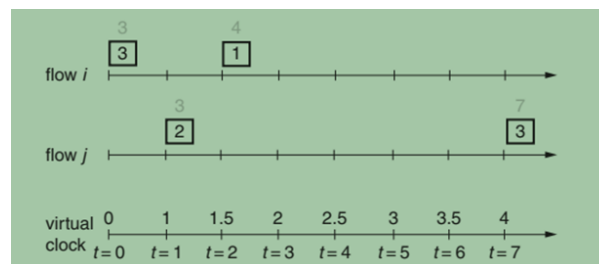


Figure 1: Finish tags and the virtual time

5. Let,  $X_i$  be i.i.d. Bernoulli random variables with mean  $\mu$ . Prove that, for  $x > 0$ ,  $Pr(\sum_{i=1}^n X_i \geq n(\mu + x)) \leq \exp(-nD((\mu + x)||\mu))$  where  $D((\mu + x)||\mu) = (\mu + x) \times \log(\frac{\mu+x}{\mu}) + (1 - \mu - x) \times \log(\frac{1-\mu-x}{1-\mu})$