1. Determine whether the following sets are convex or not.
   (a) $S = \{x \in \mathbb{R}^2 : x_1^2 + x_2^2 \leqslant 1, x_1 + x_2 \leqslant 0\}$.
   (b) $S = \{x \in \mathbb{R}^2 : x_1^2 + x_2^2 \leqslant 1, x_1 \leqslant 0\}$.
2. Determine whether the following functions are convex or concave or neither.
   (a) Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x) \triangleq x_1^2 + 2x_2^2 - 4x_1 + 3x_2 + 5$.
   (b) Consider the function $g : \mathcal{X} \to \mathbb{R}$ defined by $g(x) \triangleq \ln(x_1) + x_2^2 + 3x_3 - x_1x_2$ where $\mathcal{X} \triangleq \{x \in \mathbb{R}^3 : x_1 > 0\}$.
   (c) Consider the function $h : \mathbb{R} \to \mathbb{R}$ defined by $h(x) \triangleq -x^2 + 4$.
3. Solve the following constrained optimization problems.
   (a) Minimize $f(x,y) = (x-1)^2 + (y-2)^2$, where $x^2 + y^2 \leqslant 4$.
   (b) Minimize $f(x) = \frac{1}{2}x^\top Qx + c^\top x$, where $Q = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \quad c = \begin{bmatrix} -2 \\ -5 \end{bmatrix}$ and $x_1 + x_2 \leqslant 3, x_1, x_2 \geqslant 0$.
   (c) Minimize $f(x,y) = x^2 + 4y^2$, where $x + 2y - 4 = 0$.
4. Write KKT conditions and find the optimal solution for the below problem
   (a) Minimize $f(x,y) = x^2 + y^2$ where $x + y - 1 = 0, x - 2 = 0, y - 2 = 0$.
   (b) Maximize $f(x,y) = 3x + 4y$, where $x^2 + y^2 - 1 = 0, x = 0, y = 0$.
5. Consider the following data set.
   - Class 0: $(1,2), (2,3), (3,3)$
   - Class 1: $(5,5), (6,7), (7,8)$

   (a) Plot the data on a two dimensional plane and determine if they are linearly separable.
   (b) If the data is separable, describe how you would find the optimal hyperplane using SVM.
6. Consider a binary classification problem with the following data points.
   - Class 1: $(1,2), (2,3)$
   - Class -1: $(2,1), (3,2), (4,4)$

   (a) Explain why this data set may not be linearly separable.
   (b) Describe how you would apply a non-separable SVM to classify these points.