1. **Rademacher identities.** Let $X : \Omega \to \{-1, 1\}$ be an *i.i.d.* Rademacher random sequence, and $S : \Omega \to \mathbb{Z}^{\mathbb{N}}$ is defined as $S_n \triangleq \sum_{i=1}^{n} X_i$ for all $n \in \mathbb{N}$. Derive the moment generating function $M_n : \mathbb{R}_+ \to \mathbb{R}_+$ for S_n defined as $M_n(t) \triangleq \mathbb{E}[e^{tS_n}]$ for each $n \in \mathbb{N}$, and find its limit as $n \to \infty$.

2. **Growth functions.** Consider label set $\mathcal{Y} \triangleq \{0, 1\}$, the input set $\mathcal{X} \triangleq \mathbb{R}$, and the hypothesis set $H \subseteq \mathcal{Y}^{\mathcal{X}}$ be the family of threshold functions over the real line $\mathbb{R}$ defined as

$$H \triangleq \left\{x \mapsto \mathbb{1}_{\{x \leqslant \theta\}} : \theta \in \mathbb{R}\right\} \cup \left\{x \mapsto \mathbb{1}_{\{x \geqslant \theta\}} : \theta \in \mathbb{R}\right\}.$$

   Give an upper bound on the growth function $\Pi_m(H)$. Use that to derive an upper bound on the Rademacher complexity $\mathcal{R}_m(H)$.

3. **Affine classifiers.** Consider binary label set $\mathcal{Y} \triangleq \{0, 1\}$, input space $\mathcal{X} \triangleq \mathbb{R}^2$, and a labeled sample $z \in (\mathcal{X} \times \mathcal{Y})^m$ Consider the hypothesis set $H \triangleq \{x \mapsto \langle w, x\rangle + b : w \in \mathcal{X}, b \in \mathbb{R}\}$ of affine classifiers. Find VC-dim(H).

4. **Closed balls.** Consider a closed ball in $\mathbb{R}^n$ with center $x_0 \in \mathbb{R}^n$ and radius $r \geqslant 0$, written as $B(x_0, r) \triangleq \{x \in \mathbb{R}^n : \|x - x_0\| \leqslant r\}$. Define binary set $\mathcal{Y} \triangleq \{0, 1\}$ and consider the hypothesis set that classifies based on the membership of a closed ball, defined as

$$H \triangleq \left\{x \mapsto \mathbb{1}_{\{x \in B(x_0, r)\}} : x_0 \in \mathbb{R}^n, r \in \mathbb{R}_+\right\} \subseteq \mathcal{Y}^{\mathbb{R}^n}.$$

   Show that VC-dim$(H) \leqslant n + 2$.

5. **Axis aligned rectangles.** What is the VC-dimension of axis-aligned rectangles in three dimensional space, and how does it differ from the VC-dimension in two dimensional space?