1. An individual and a gambler play a game. Gambler choses a biased coin and tosses it. Indiviual wins if the toss is head. Else, the gambler wins. The probability of heads is $\frac{1}{3}$. Individual bets a $ 1 at the beginning. If he wins, he quits. Else, he doubles the previous bid amount and bets again. Assuming that the individual has infinite money and time, what are his net and average profits?

2. An airline knows that 5 percent of people making reservations on a certain flight will not show up. So, their policy is to sell 52 tickets for a flight that can hold only 50 passengers. Assuming that each person's showing up is independent of others, calculate the probability that there will be a seat available for every passenger shows up.

3. Let $X : \Omega \to \mathbb{R}_+$ be an exponential random variable with parameter μ. Find $\mathbb{E}[X \mid \{X > 1\}]$ and $\mathbb{E}[X \mid \{X \leqslant 1\}]$.

4. Let $X : \Omega \to \mathbb{Z}_+$ be a poisson random variable with rate Y, which is an exponential random variable with mean unity. What is the probability that X takes value n for some $n \in \mathbb{N}$?

5. The number of customers entering a store on a given day is Poisson distributed with rate 100. The amount of money spent by each customer is uniformly distributed over $(0,10)$. Find the mean and variance of the amount of money that the store makes on each day.

6. Let $(\mathcal{F}_n : n \in \mathbb{N})$ be an increasing sequence of sub-event spaces defined in the probability space $(\Omega, \mathcal{F}, P)$. Then, prove the following are sigma-algebra:
   (a) $\lim_{n\in\mathbb{N}} \mathcal{F}_n$
   (b) $\bigcap_{n\in\mathbb{N}} \mathcal{F}_n$
   (c) $\liminf_{n\in\mathbb{N}} \mathcal{F}_n$
   (d) $\limsup_{n\in\mathbb{N}} \mathcal{F}_n$

7. Two players decided to play a tug-of war. The game ends when either of their strengths fall below some constant c, or when the rope breaks. The rope's strength is k units. Assuming that the strength of players are exponentially distributed with rates r_1 and r_2 respectively and the rope breaking is independent of any other event, calculate the probability of ending the game in terms of r_1, r_2, c and k.

8. If X is uniform over $(0,1)$, calculate $\mathbb{E}[X^n]$ and $\text{Var}[X^n]$ for any $n \in \mathbb{N}$.