1. An individual has a radio that works on a single battery. As soon as the battery in use fails, he immediately replaces with a new one. If the lifetime of a battery is distributed uniformly over the interval $(30,60)$ in hours, then, what is the limiting rate does he has to change the batteries?
   (a) Suppose, if he does not keep any surplus batteries in hand. So, each time a failure occurs, he should go and buy a new one. If the amount of time it takes to buy a new one is uniformly distributed over $(0,1)$ in hours, then, what is the average rate that he changes the batteries?

2. Consider a manufacturing process that sequentially produce items, each of them is either defective or acceptable. The following type of sampling scheme is often employed in an attempt to detect and elimiante mose of the defective items. Initially, each item is inspected and this continues until there are k consecutive acceptable items. At this point this 100% inspection ends and each successive item is independently inspected with probability α. This partial inspection continues until a defective item is encountered, at which time 100% inspection is reinstuted, and the process begins anew. The probability of an item being defective is $q \neq \alpha$, for some $q \in (0,1)$ and is independent of others. Then find,
   (a) What proportion of items are inspected?
   (b) If defective items are removed when detected, what proportion of the remaining items are defective?

3. A machine in use is replaced by a new machine either when it fails or when it reaches the age of T years. If the lifetimes of successive machines are independent with a common distribution F having density f, then show that the long run rate at which machines are replaced equals,

$$\left[\int_0^T xf(x)dx + T(1-F(T))\right]^{-1}.$$

4. Two players are playing a sequence of games, which begins with one of the player serves. Suppose that player 1 wins each game she serves with probability p_1 and wins each game her opponent serves with probability p_2. Further, suppose that the winner of a game becomes the server of the next game. Find the number of games that are won by player 1.

5. Let $(U_i)_{i\in\mathbb{N}}$ be an *i.i.d.* uniform $(0,1)$ sequence defined on $(\Omega,\mathcal{F},P)$. Let $\mathcal{F}_\bullet$ be a sequence of increasing sigma-algebras spanned by U_i's. Let N_1 and N_2 defined as,

$$N_1 \triangleq \min\{n : U_n > 0.5\} \qquad N_1 \triangleq \max\{n : U_n > 0.5\}.$$

   Are N_1 and N_2 stopping times? Compute $\mathbb{E}\sum_{i=1}^{N_1} U_i$.

6. An individual's car buying policy is always buy a new car, repair all the breakdowns that occur during the first T time units of ownership and then junk the car and buy a new one at the first breakdown that occurs after the car has reached the age T. Suppose that the time until the first breakdown of a new car is exponential with rate λ, and that each time a car is repaired the time until the next breakdown is exponential with rate μ. Find the rate does the individual buys a new car.