1. Let a biased coin with probability of heads being p be tossed until the H,H,T,H,T appears in a discrete time setting. Prove that the number of times the coin needs to be tossed before H,H,T,H,T appears is $p^3(1-p)^2$.

2. Check whether the following functions $z : \mathbb{R}_+ \to \mathbb{R}_+$ are Direct Riemann Integrable (d.R.i.) or not, and justify.
   (a) The map $z(t) \triangleq \frac{1}{t}$ for $t \geqslant 1$.
   (b) The map $z(t) \triangleq e^{-t}$ for $t \geqslant 0$.

3. Suppose that customers arrive at a train dept according to a renewal process having a mean interarrival time μ. Whenever there are N customers waiting in the depot, a train leaves. If the depot incurs a cost at the rate of nc dollars per unit time whenever there are n customers waiting, what is the average cost incurred by the depot?

4. Let $B : \Omega \to \mathbb{R}^{\mathbb{R}_+}$ be a standard Brownian motionwith $B_0 = 0$. We define a random process $X : \Omega \to \mathbb{R}^T$ where $X_t \triangleq |B_t|$ for any $t \in T \subseteq \mathbb{R}$. For Brownian motion B, we define $\tau : \Omega \to (\mathbb{R} \cup \{\infty\})^{\mathbb{N}}$ be the sequence of hitting times to 0. That is, $\tau_n \triangleq \inf\{t > \tau_{n-1} : X_n = 0\}$. Show τ is a renewal sequence and the process X is regenerative with respect to the renewal sequence τ.

5. A truck driver regularly drives round trips from A to B and then back to A. Each time he drives from A to B, he drives at a fixed speed (in kmph) that is uniformly distributed between 40 and 60; each time he drives from B to A, he drives at a fixed speed that is equally likely to be either 40 or 60.
   (a) In the long run, what proportion of his driving time is spent going to B?
   (b) In the long run, for what proportion of his driving time is he driving at the rate 40 kmph?

6. The rate a certain insurance company charges its policy holders alternates between r_0 and $r_1 (> r_0)$. A new policy holder is initially charged at a rate of r_1 per unit time. If the holder doesn't make any claim for the most recent s time units, then the rate charged becomes r_0 per unit time. The rate charged remains at r_0 until a claim is made, at which time it reverts to r_1. Suppose that a given policy-holder lives forever and makes claims at times chosen according to a poisson process with rate λ, and find the proportion of time that the policy holder pays a rate $r_i, i \in \{0,1\}$.