1. Determine whether the following sets are convex or not.
   (a) $S = \{x \in \mathbb{R}^2 : x_2 \geqslant e^{x_1}\}$.
   (b) $S = \{x \in \mathbb{R}^2 : x_1 x_2 \geqslant 1, x_1 > 0, x_2 > 0\}$.

2. Determine whether the following functions are convex or concave or neither.
   (a) $f(x) = x_1 + x_2$
   (b) Let $S \triangleq \{x \in \mathbb{R}^2 : x_1 > 0, x_2 > 0\}$ and $f : S \to \mathbb{R}$ defined as $f(x) = 12x_1^{1/3}x_2^{1/2}$, Also check that if $g(x) = f(x) - p_1 x_1 - p_2 x_2$ is convex or concave or neither.
   (c) Let $f : \mathbb{R}^n \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be both convex but not necessarily differentiable, and define $h : \mathbb{R}^n \to \mathbb{R}$ as $h(x) = g(f(x))$ for each $x \in \mathbb{R}^n$. Show that h is convex or give a counter example.
   (d) Let $f(x) = x_1^3 + 2x_1^2 + 2x_1x_2 + \frac{x_2^2}{2} - 8x_1 - 2x_2 - 8$. Find a set $S \subseteq \mathbb{R}^2$ such that $f : S \to \mathbb{R}$ is convex.

3. Use the Lagrange multiplier theorem to solve the following problem for the two special cases

$$\min_{x \in \mathbb{R}^n : h(x) = 0} f(x).$$

   (a) $f(x) = \sum_{i=1}^n x_i, h(x) = \|x\|^2 - 1$.
   (b) $f(x) = \|x\|^2, h(x) = x^T QX$ where Q is known to be positive definite.

4. Write KKT conditions and find optimal solution and Lagrange multipliers for the below problem.

$$\min_{x^2+y^2 \leqslant 5, 3x+y \leqslant 6} 2x^2 + 2xy + y^2 - 10x - 10y.$$

   (a) Is the solution to the above problem optimal? Is it unique? Prove your claim.
   (b) Find the dual of the above problem.

5. Get the hyperplane and support vectors for the problem illustrated in the Figure 1.

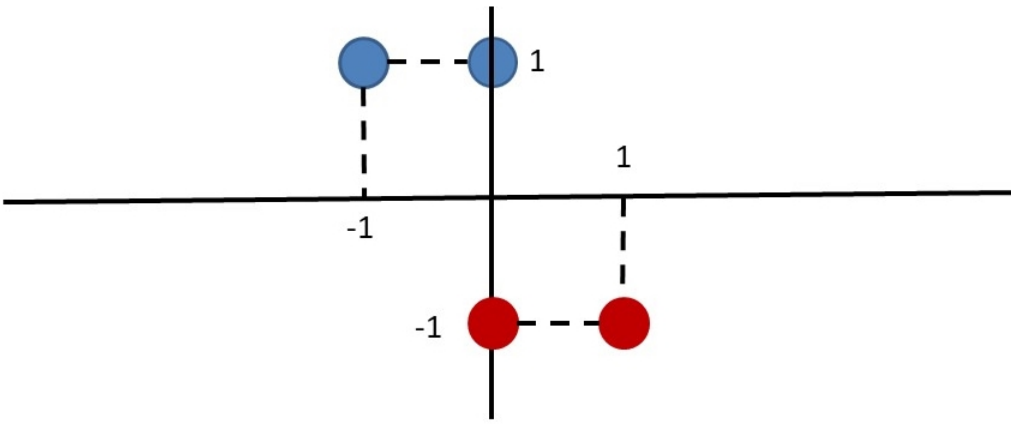

Figure 1: Two-dimensional data from two classes.

6. Epsilon-insensitive loss function $L_\varepsilon : \mathcal{Y} \times \mathcal{Y}' \to \mathbb{R}_+$ is defined as

$$L_\varepsilon(d, y) \triangleq \max\{|d - y| - \varepsilon, 0\}.$$

   Derive the dual problem from first principles for the SVM classifier that is not linearly separable with consideration to the ε-insensitive loss function.

7. Consider the data set `https://archive.ics.uci.edu/ml/datasets/iris`. We are interested in constructing a linear classifier for this data based on SVM.

   (a) Identify the classes that are linearly separable. From the first principles, write a code to obtain an SVM classifier for this result.

   (b) Comment on the accuracy of the approach.