A Subcarrier Allocation Algorithm for OFDMA using Buffer and Channel State Information

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Abstract— **We propose a new subcarrier allocation algorithm for Orthogonal Frequency Division Multiple Access (OFDMA) that gives fair allocation of capacity to multiple users with different channel and traffic characteristics. This is achieved by utilizing buffer state information and measured traffic statistics in addition to channel state feedback. Multiuser diversity gains are achieved in the proposed algorithm by scheduling across both time (buffering) and frequency (subcarriers). Simulation results are shown to illustrate (a) improved capacity allocation and throughput, and (b) larger admissible traffic when compared to the existing algorithms.**

Keywords - OFDMA; Scheduling; Subcarriers; Buffering; Throughput; Delay; Fairness.

I. Introduction

Joint subcarrier and power allocation in OFDMA is a complex problem [1]. Usually, the problem is simplified by separating subcarrier allocation and power allocation [2], [3]. Subcarrier allocation provides more gain than power allocation [3], [4]. In this paper, we focus on subcarrier allocation. Maximum capacity is provided by the Max-Rate subcarrier allocation algorithm (MR-SAA) which allocates each subcarrier to the user with the best channel conditions [5], [6]. However, it is not fair. The Rhee-Cioffi subcarrier allocation algorithm (RC-SAA) [2] achieves proportional fairness amongst the users. However, the overall capacity achieved is much lower. The capacity achieved by these subcarrier allocation algorithms that are solely based on channel conditions does not necessarily translate into throughput when the input traffic is bursty. Furthermore, [7] shows that proportionally-fair scheduling can lead to unstable queues even for low arrival rates since it does not utilize the buffer state.

Buffering of bursty traffic can take advantage of multiuser diversity across time and improve throughput performance by trading delay for throughput [8]. Recently, subcarrier allocation in the presence of buffer and channel information has been studied in [10], [9]. In [9], the number of subcarriers for each user is determined prior to choosing the specific subcarrier allocation scheme. In [10], improved delay performance is achieved by using the mean waiting times in the sub-carrier allocation algorithm. The Modified-Largest Weighted Delay First (M-LWDF) rule [8] for time slot allocation in dynamic TDMA is a *throughput optimal* rule[1]. We propose a subcarrier allocation algorithm (SAA) which extends the M-LWDF idea to schedule users on each subcarrier of an OFDMA system during every time slot. Results are shown to illustrate that the proposed SAA: (a) allocates capacity fairly amongst the users, and (b) supports traffic load very close to the maximum capacity (achieved by the MR-SAA). The proposed SAA chooses the user for each subcarrier based on a combination of the following: (a) current channel conditions, (b) current buffer state (delay), and (c) the measured ratio of arrival rate to throughput for each user.

The paper is organised as follows. We describe the system model in section II. We propose our subcarrier allocation algorithm in section III. Simulation specifications, results, and discussions follow in section IV. Conclusions are drawn in section V.

II. System Model

Fig. 1 shows the system model. A downlink OFDMA system with N users and K carriers is considered. The randomly arriving incoming packets are buffered in a first-in-first-out (FIFO) queue with the buffer size for each user determined by the absolute delay bound for the corresponding user [11]. We transmit bits at the maximum rate for reliable communication over the channel in any time slot, the outage (number of bits dropped) is only due to buffer overflow. if the buffer overflows. A single user is scheduled on each sub-carrier. Therefore, the problem of subcarrier allocation is essentially to choose an user for each subcarrier. We define the set of all carriers as $A = \{1, 2, \ldots, K\}$. Let $\Omega_n(t)$ be the set of carriers assigned to user n in symbol period t. Since each OFDM carrier can be assigned only to one user, we have for all t

$$\Omega_n(t) \bigcap \Omega_m(t) = \phi \quad \forall m \neq n, \text{ and } \bigcup_{n=1}^{N} \Omega_n(t) = A. \qquad (1)$$

The wireless multipath fading channel is assumed constant over one OFDM symbol transmission and noise is assumed to be i.i.d. and AWGN with variance σ^2 for all carriers and all users. The received OFDM symbol $Y_{n,k}(t)$ can be written in terms of the frequency-domain channel

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[1] It renders the queues at the base-station stable if any other rule can do so [8].

gain $H_{n,k}(t)$, the noise at the receiver $\eta_{n,k}(t)$ for user n and the transmitted symbol $X_k(t)$ on carrier k in t^{th} symbol period, as:

$$Y_{n,k}(t) = H_{n,k}(t)X_k(t) + \eta_{n,k}(t), n \in \{1, 2, \dots N\}, k \in \Omega_n(t).$$

The normalized channel gains seen by k^{th} carrier of user n in symbol period t is $\gamma_{n,k}(t) = (|H_{n,k}(t)|^2/\sigma^2)$. We assume that the transmitter has knowledge of the channels of all the users. Since channel conditions are time-varying, this information is assumed to be updated periodically with the help of feedback channels.

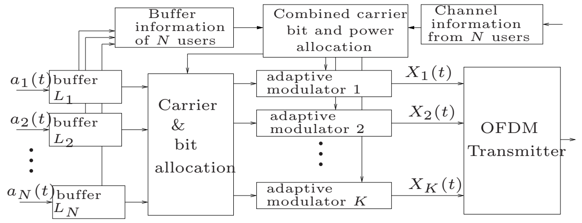

Fig. 1. Orthogonal Frequency Division Multiple Access

For power allocation, we use water-pouring [12]. The power allocated to subcarrier k in time slot t is given by

$$P_k(t) = (\nu(t) - (1/\gamma_{n,k}(t)))^+, \tag{2}$$

where $(x)^+ = \max(x, 0)$ and $\nu(t)$ is chosen to satisfy the absolute power constraint $\sum_{k=1}^{K} P_k(t) = P$. Therefore, the maximum achievable rate for reliable communication in a time-slot t for a user n by any SAA is

$$\mu_n(t) = \sum_{k \in \Omega_n(t)} \mu_{n,k}(t), \tag{3}$$

$$\text{where } \mu_{n,k}(t) = \log_2 (1 + \gamma_{n,k}(t)P_k(t)) \tag{4}$$

is the maximum rate that can be reliably supported on the carrier k for that user.

We denote the buffer length for user n by L_n. In each OFDM symbol period t, one packet of random size arrives per user. The number of bits in the packet for n^{th} user is $a_n(t)$, distributed uniformly in $[0, M_n]$.

A FIFO queue is implemented where $Q_n(t)$ shows the buffer state of user n at time slot t in terms of $p_{n,i}(t)$, which denotes the enqueued bits of user n that arrived in the $(t-i)^{th}$ time slot. The absolute delay bound D_n is the maximum number of symbol periods a bit can be in the queue, after which it is dropped. The buffer size is taken as $L_n = M_n D_n$ to ensure the delay bound D_n for user n [11]. The queue, in terms of $p_{n,i}(t)$ and D_n, is given by

$$Q_n(t) = \begin{bmatrix} p_{n,D_n}(t) & p_{n,D_{n-1}}(t) & \dots & p_{n,1}(t) \end{bmatrix}. \tag{5}$$

Bits are transmitted with priority order from left to right. The total number of bits in the queue $q_n(t)$ and the delay of the Head-of-Line (HoL) packet $W_n(t)$ for user n in symbol period t are given by

$$q_n(t) = \sum_{i=1}^{D_n} p_{n,i}(t) \tag{6}$$

$$W_n(t) = \max\{j : p_{n,j}(t) \neq 0\}, \tag{7}$$

The problem of resource allocation for a limited total available power P is divided into subcarrier allocation, i.e., finding a set of carriers $\Omega_n(t)$ assigned to user n and power allocation, i.e., finding $P_k(t), \quad k \in A$ over the total set of carriers A. The total throughput $d_n(t)$ for any user n in a time slot t is the minimum of the achievable rate $\mu_n(t)$ and the total number of packets in the buffer $q_n(t)$;

$$d_n(t) = \min(q_n(t), \mu_n(t)). \tag{8}$$

III. SUBCARRIER ALLOCATION ALGORITHM

Subcarrier allocation is the assignment of a disjoint set of carriers $\Omega_n(t)$ to each user n (for $n = 1, 2, \dots, N$) in time slot t. It can be seen from (8) that maximizing the supportable rate $\mu_n(t)$ for a user n in any time slot t does not maximize the throughput $d_n(t)$. This underlines the importance of using buffer state information $Q_n(t)$ in subcarrier allocation.

Ideas used in time-slot allocation in dynamic TDMA can be extended to subcarrier allocation in OFDMA. In dynamic TDMA, buffers are used to achieve fairness over time. In OFDMA, we have freedom of allowing multiple users to share the channel simultaneously during each time slot. Therefore, we can schedule both across time slots and subcarriers (frequency).

Resource allocation algorithms in [2], [3] schedule users across subcarriers to achieve fairness during every symbol period. These algorithms do not schedule across time slots. We incorporate buffers for a multiuser OFDMA system and propose a subcarrier allocation algorithm that utilizes the buffer and channel state information and measured traffic statistics available at transmitter. We achieve higher multiuser diversity gain by scheduling both across time and across the subcarriers.

The Modified-Largest Weighted Delay First (M-LWDF) [8] rule for time slot allocation in dynamic TDMA schedules the user with largest weighted delay, where the weight is proportional to the supportable rate of a user n on the channel in that time slot t. It is a *throughput optimal* rule. so We propose a subcarrier allocation algorithm that extends the M-LWDF idea for scheduling users on each subcarrier in every time slot. We also use a weighted delay measure for subcarrier allocation. However, the weight is proportional to the normalized channel gain and not the supportable rate used in the M-LWDF rule for time slot allocation.

It has been shown in [3], [4] that power allocation does not offer substantial gains at high SNRs. Therefore, if we assume that every subcarrier gets equal power, then we can consider the subcarriers as parallel channels. Since we know MLWDF rule is throughput optimal for each individual channel, we can select best user for each subcarrier using a rule similar to it. Once the subcarriers have been allocated we do water-pouring to maximize the capacity region in every time-slot. Other power allocation methods given in [3] and [13] can also be combined with our subcarrier allocation algorithm.

Let $\bar{a}_n(t)$ and $\bar{d}_n(t)$ denote mean windowed arrival and mean windowed throughput respectively for user n, averaged over a "sliding-window" T_w. Given the normalized channel gains $\gamma_{n,k}(t)$, k^{th} carrier of user n in symbol period t, $\bar{a}_n(t)$, $\bar{d}_n(t)$ and buffer state $Q_n(t)$, we determine the subcarrier set $\Omega_n(t)$ allocated to user n by the following algorithm.

1. Initialize: $\Omega_n(t) = \phi \ \ , n = 1, 2, \ldots, N$
2. For each subcarrier $k = 1$ to K
  (a) Select a user for subcarrier k:

$$j = \arg\max_{i} \left[\frac{\bar{a}_i(t)}{\bar{d}_i(t)} \gamma_{i,k}(t) W_i(t) \right] \quad i = 1, 2, \ldots, N. \qquad (9)$$

  (b) Allocate subcarrier k to user j: $\Omega_j(t) = \Omega_j(t) \bigcup \{k\}$.

This algorithm equalizes between queue and channel conditions when subcarriers are allocated to users. Instead of allocating carriers solely based on channel conditions, this algorithm also considers measured traffic statistics and buffer state. If a user has a bad channel persistently and/or if it is a high rate user (relatively more bits to transmit per slot), it will be preferred over others with better channel conditions but relatively less bits to send. Since the available capacity is allocated to the users who require it most, the throughput and delay performance are very good (as shown in the simulation results in Section IV).

IV. Simulation Results and Discussions

A 9 user, 64 subcarrier OFDMA system is considered for a 6-tap multipath fading channel with an exponentially decaying multipath profile. The channel for each user is assumed to be independent and identically distributed in every OFDM symbol. We have simulated two important cases, (a) users with i.i.d. channels, (channel variance being identically 1 for all users) and (b) users with scaled channel profiles, i.e. all the subcarriers are assumed to be fading independently with variances for different users multiplied with [.75 .5 .5 1 1 .5 .75 1 1] respectively. Simulation results can also be obtained when the channels are correlated. The 9 users have linearly increasing rates in the ratio $1 : 2 : \ldots : 9$ with an uniform arrival distribution. The total power available at the base-station is fixed.

The capacity allocated to n^{th} user ($\lim_{T\to\infty} \frac{1}{T}\mu_n(t)$), is the average maximum achievable rate over all the subcarriers for reliable communication under the given resource allocation policy. Fig. 2 shows the capacity allocated to each user by the following algorithms: MR-SAA, RC-SAA, the SAA in [10] and the proposed SAA for case (a). The required capacity for each user (equal to the average arrival rate) is also plotted for reference. The total traffic load is chosen to be 22.5 units (bits/OFDM symbol). Since the users have i.i.d. channels, the MR-SAA allocates approximately the same capacity to each user. The MR-SAA achieves the maximum overall capacity (sum of the capacity allocated to all users) of 26.2, but the capacity is not allocated to the users who require it. The RC-SAA algorithm can allocate capacity proportional to the arrival rates. However, the total capacity achieved by the RC-SAA (≈ 14) is not enough to support the required rate. The SAA proposed in [10] is simulated as (corresponding to $\gamma = 2$ in [10])

$$j = \arg\max_{i} \left[\frac{\bar{Q}_i(t)}{[\bar{a}_i(t)]^2} \log_2 \left\{ 1 + \frac{P}{K}\gamma_{i,k}(t) \right\} \right], \qquad (10)$$

assuming equal power allocation initially to obtain the values of the achievable transmission rate in each subcarrier for each user at time t.

Fig. 3 shows the capacity allocated to each user by above-mentioned SAA's for case (b). MR-SAA allocates maximum capacity to the best channel users. Therefore, even when the overall capacity is maximum, it is not allocated according to the users' requirements. RC-SAA attempts to take arrivals into account, however achieves much less overall capacity, since it doesn't take into account buffer state. The proposed SAA and SAA in [10] take buffer and channel state into account in addition to arrivals and achieve higher diversity gains.

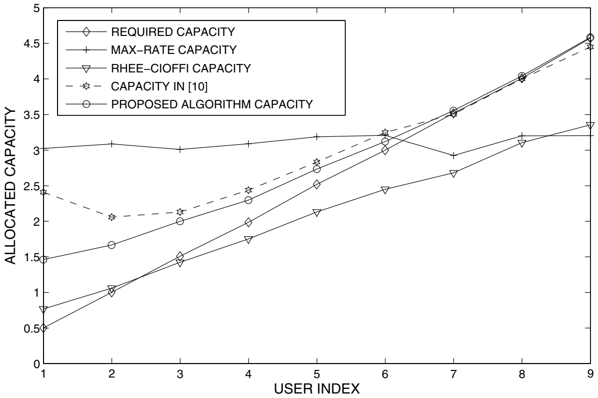

Fig. 2. Allocated capacity and the required capacity for heterogeneous rate users with i.i.d channels (Case (a))

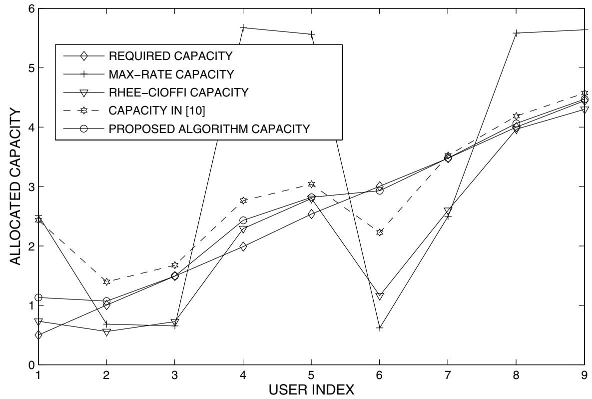

Fig. 3. Allocated capacity and the required capacity for heterogeneous rate users with scaled channel profiles (Case (b))

Fig. 4 plots the outage as a percentage of total arrived bits for the maximum outage user as a function of the total traffic for case (a). It is evident that the proposed SAA can support significantly higher traffic (≈ 25) than the RC-SAA (≈ 15) while maintaining stable queues. In fact, the proposed SAA can support traffic upto 91% of the maximum achievable capacity of 26.2 without any outage, i.e., dropped traffic. The MR-SAA can support traffic only upto $26.2 * (5/9) = 14.55$. SAA in [10] supports a total traffic of 20 approximately. We can also infer for case (b), the maximum traffic supported by different SAA's without any outage from Fig. 5. The proposed SAA supports higher maximum traffic (≈ 22) compared to that achieved by SAA in [10] (≈ 16).

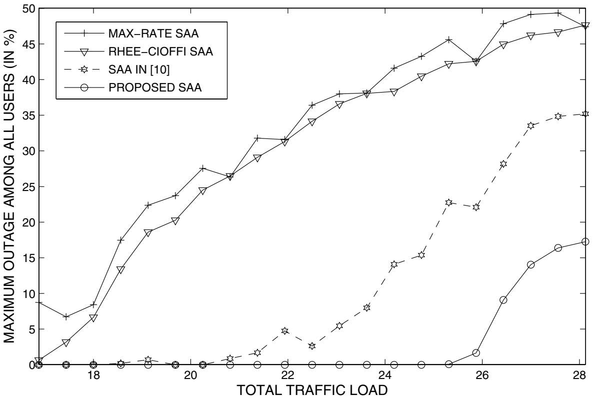

Fig. 4. Maximum outage among all users (in %) vs. Traffic for heterogeneous rate users with i.i.d. channels (Case (a))

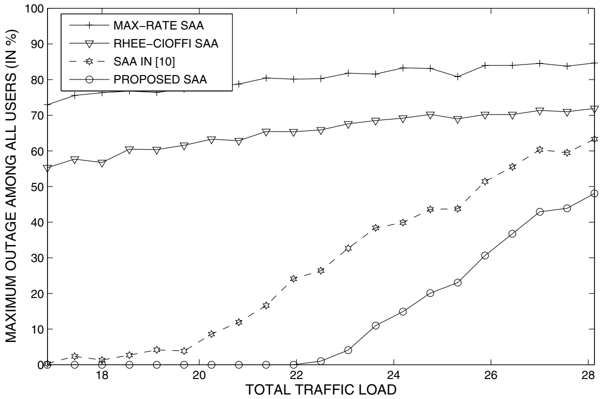

Fig. 5. Maximum outage among all users (in %) vs. Traffic for heterogeneous rate users with scaled channel profiles (Case (b))

The proposed SAA uses the HoL packet delay instead of the average queue delay used in [10] and provides a better allocation of capacity. The Greedy Reassignment policy in [10] is not used in the simulation. The Greedy Reassignment policy can be used to improve our proposed SAA as well as the SAA in [10]. In summary, the proposed SAA is able to allocate capacity to the users based on their requirements and keep the queues stable. Furthermore, the proposed SAA uses measured arrival rates and does not need prior knowledge of the actual arrival rates.

V. Conclusion

Buffering bursty traffic can provide a good delay-throughput trade-off and improve the overall throughput of the system. Therefore, subcarrier allocation based on queue and channel information can perform significantly better than MR-SAA and RC-SAA which do not use queue information. A good SAA should be close to the MR-SAA in total (sum) capacity while being as fair as possible. The proposed algorithm performs better than the existing subcarrier allocation algorithms like [2], [10] in terms of throughput improvement and fair capacity allocation. The proposed SAA: (a) allocates capacity fairly amongst the users, and (b) supports traffic load very close to the maximum capacity.

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