Real-Time Status Updates for Correlated Source

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***Abstract*—For timely sensor update, the traditional approach is to send new information at every available opportunity. Recent research has shown that with limited receiver feedback, sensors can improve the update timeliness by transmitting differential information for slowly varying correlated sources. In general, correlated sources can elect to transmit actual or differential state information depending on the current state. This encoding scheme generalizes the actual and differential updates schemes. Using this generalized scheme, we quantify the timeliness gains for some example sources. Further, we show a stochastic ordering among the actual update, the differential update, and the generalized update schemes.**

***Index Terms*—age of information, renewal theory, Markov source, erasure channel, differential encoding, block codes.**

I. Introduction

In the past few years, there has been a tremendous interest in the Internet of things which envisions a world of connected devices. Apart from being able to communicate, these devices may have additional functionalities including sensing and actuation. For applications monitoring a physical process such as weather, traffic, pollution, etc., the process state is sensed and communicated to the cloud for further analytics. For real-time actuation such as selection of less congested/polluted route, the timeliness of data is crucial. This metric is subtly different than the traditional communication metric of reliability, throughput, and latency. Data timeliness is also important in applications such as news and social media updates, distributed system updates, and route updates in ad-hoc networks.

In [1], the authors define a metric called 'age' to measure timeliness or staleness of information. This is the metric we adopt to measure communication timeliness in this article. To improve timeliness performance in real-time communication systems, new information is sent at every available opportunity. This update scheme is referred to as *true update*. Most of the existing literature on communication timeliness [2]–[6], study a queue theoretic abstraction where the update generation and delivery times are stochastic. In these works, the update is always received correctly at the receiver, and the channel uncertainty is captured in the random reception time of an update. Contrastingly, we model the channel unreliability by an information theoretic binary erasure channel. In our setup, the update may not be decodable and can be dropped. Further, we focus on the effect of coding on the update timeliness in a setup similar to that studied in [7], [8], where the source sends an update packet of n bits that includes coded information bits.

For a temporally correlated source, one can transmit the difference between the last correctly decoded state and the current state, instead of the actual current state. Indeed, it is shown in [7], that sending differential update improves source timeliness considerably, in the presence of limited receiver feedback. We will refer to this scheme as *differential update with feedback* or *incremental update* for brevity. The timeliness gains in this scheme result from the exploitation of temporal correlation across messages. The authors inherently assume that the source can always send differential updates. However, temporal correlation can vary between two transmission opportunities for a general source, and the number of bits needed to represent differential message may depend on the actual realization of the states.

For a finite state source, we assume that the number of bits needed to represent any actual state is m. In this article, the source sends a differential update only when the differential information can be represented by k bits, for some $k \leq m$. The number of bits k is fixed as a design parameter. The source transmits a true update in either of the following two cases. Either the source is unable to encode a differential update in k bits, or the receiver fails to decode an update. When the last correctly decoded state is i, the source can encode the differential information in k bits with *differential encoding probability* denoted as $p_3(i)$. That is, we allow randomness in the source's ability to encode the differential information. For simplicity of analysis, we assume that this randomness is uniform across states, i.e. $p_3(i) = p_3$ for all states i in the message set. This generalizes the incremental update scheme for structured sources, and we refer to this as the *generalized incremental update* scheme. In Section IV, we will see examples of sources where the possibility of incremental update is identical for all states.

We note that as we increase the threshold k, the number of additional parity bits $n-k$ for the incremental update reduces as compared to the $n-m$ for the true update. On the other hand, if we reduce k, only a handful of state differences can be sent as a differential update. Both these scenarios limit the performance gain of incremental update over true updates. This points to a natural trade-off in selection of threshold k for timely updates.

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A. Main Contribution

We show that the true and the differential updates are two special cases of the generalized incremental update scheme. We illustrate the trade-off between additional coding gains and the coding opportunities for two structured sources. Further, we demonstrate a stochastic ordering among performance of the true update, incremental update, and the generalized incremental update schemes.

II. System Model

In this section, we describe in detail the communication model shown in Fig. 1, and associated transmission protocol.

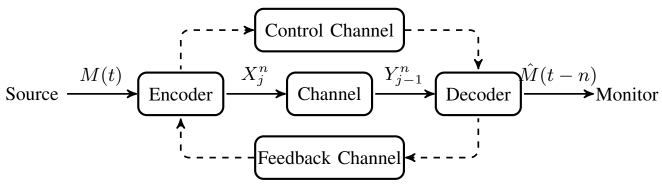

Figure 1. We show an abstract discrete time communication model for a source with state $M(t)$ at time $t = (j-1)n+1$ for $n \in \mathbb{N}$.

A. Encoder

At discrete instants $t \in \{(j-1)n+1 : j \in \mathbb{N}\}$, the source encodes the update message $M(t)$ into an n bit codeword X_j^n and sends over an unreliable channel to the monitor.

B. Source

We assume a correlated physical source with realizations from a finite field $\mathbb{F}_{2^m}$, such that the source state $M(t)$ can be represented by m bits at every discrete time sample $t \in \mathbb{N}$. We assume that the source process $\{M(t) : t \in \mathbb{N}\}$ is Markov. Further, we can define a set of symbols

$$\Delta_k \triangleq \{-2^{k-1}, \dots, 2^{k-1}-1\}.$$

Hence, if the difference $\delta_M(t) \triangleq M(t)-M(t-n) \in \Delta_k$, then this difference can be represented by k bits. We call this event as *differential encoding success* and denote the probability of this event conditioned on the source state at time $t-n$ as

$$p_3(i) \triangleq \Pr\{M(t)-M(t-n) \in \Delta_k | M(t-n) = i\}.$$

We consider the simple case where $p_3(i)$ does not depend on the state i. That is, $p_3(i) = p_3$ for all states i. In terms of transition matrix P for the Markov source $M(t)$, it means

$$\sum_{j:j-i\in\Delta_k} P_{ij} = p_3, \quad \text{for each } i \in \mathbb{F}_{2^m}.$$

C. Channel

We assume that each bit-transmission requires single channel use, and each transmitted bit can be erased independently and identically with probability ϵ. Therefore, the encoder gets to transmit the update codeword X_j^n only at periodic instants $\{(j-1)n+1 : j \in \mathbb{N}\}$, which is received as a channel output Y_j^n after n channel uses at instants $\{jn+1 : j \in \mathbb{N}\}$. Let E_j be the number of erasures in an n-length received codeword Y_j^n. Since the bit-wise erasure channel is *iid*, it follows that the number of erasures $\{E_j : j \in \mathbb{N}\}$ in received codewords are *iid* Binomial random variables with parameters (n, ϵ).

D. Decoder

From the channel output Y_j^n at time $t = jn+1$, the decoder finds an estimate $\hat{M}(t-n)$ of the message transmitted n channel uses ago. For binary erasure channels, the estimate is able to decode the transmitted message $M(t-n)$ perfectly or declare a decoding failure. For a permutation invariant coding scheme the event of decoding failure depends solely on the number of erasure E in a codeword, and not their locations. We denote the probability of decoding failure for an n-length codeword with $n-r$ parity bits and E erasures, as $P(n, n-r, E)$ for a permutation invariant code. From *iid* nature of the channel, it follows that the codeword decoding failure events are also independent and Bernoulli with probability $\mathbb{E}P(n, n-r, E)$, where the expectation is taken over the binomial random variable E with parameters (n, ϵ).

E. Transmission Protocol

After the codeword reception, we have following two possibilities at the receiver. First, the receiver is able to decode the transmitted message correctly, leading to a successful status update at the monitor. Alternatively, receiver declares a decoding failure and sends an immediate and accurate negative feedback to the source. The source always responds to an event of decoding failure by sending its true state as the following update.

1) True Updates: For the true update scheme, the update message from the source is always its m-bit current state $M(t)$ at time t. Hence, the probability of decoding failure for a true update is $p_1 = \mathbb{E}P(n, n-m, E)$.

2) Incremental Updates: For the differential update scheme, the update message is the k-bit difference between its current state $M(t)$ and the last transmitted state $M(t-n)$, when $M(t-n)$ is correctly decoded. Underlying assumption here is that the source is slowly varying such that $\delta_M(t) \in \Delta_k$ almost surely. In this case, the probability of decoding failure for an incremental update is $p_2 = \mathbb{E}P(n, n-k, E)$.

3) Generalized Incremental Updates: For the generalized incremental update, the source uses the following algorithm at each transmission instant t. If the update sent at time $t-n$ was successfully decoded and the incremental message can be represented by $k \le m$ bits, then the update message is a differential update. Else, the true state update is encoded and transmitted. That is, if $\hat{M}(t-n) = M(t-n)$ and the difference $\delta_M(t) \in \Delta_k$, then this difference of k bits is encoded as an n-length incremental update codeword. Recall, we denote the probability of this event as p_3, and assumed it to be independent of the source state $M(t-n)$. This assumption simplifies the analysis greatly, and captures the inability of source to represent incremental message using pre-specified k bits at all times. In addition, this case generalizes both the previously studied update schemes. The values of encoding success probability p_3 being 0 and 1 correspond to the true and the incremental update schemes, respectively.

F. Control Channel

We also assume existence of a separate control channel [9] that allows the decoder to distinguish between a true and an incremental update.

G. Performance Metric

We adopt the *age of information* metric defined in [1] as our primary performance metric. Let $U(t)$ denote the generation time of last correctly decoded update at time t. Then the age of information $A(t)$ at time t, is given by $t - U(t)$. In [10], the authors define a metric called the *peak age of information*. The peak age is the information age just before the monitor successfully decodes an update. Suppose the ith successful status update is received by the monitor at time V_i, then $A(V_i - 1)$ is the peak age corresponding to the ith successful update. The peak age is related to the receiver waiting time for a successful update.

III. Computation of Limiting Average Age

We will denote the age of the true, the incremental, and the generalized updates by $A, \hat{A}$, and $\tilde{A}$, respectively. Next, we compute the limiting average age $\mathbb{E}\tilde{A}$ for the generalized incremental updates, from which the expressions for $\mathbb{E}A$ and $\mathbb{E}\hat{A}$ can be obtained as special cases.

A. Generalized Incremental Update

We will use a similar approach as used in [7] to compute the limiting average age for this scheme. We denote $R_0 = 0$ and let R_i be the time instant of the ith correctly decoded true update. Due to the *iid* nature of the underlying channel, it follows that the time duration $T_i = R_i - R_{i-1}$, between consecutive decoding successes of true updates are *iid*. These form inter-renewal times for a counting process that counts the number of decoding successes of true updates. Let Z_i and W_i respectively denote the number of true and incremental updates sent in the ith renewal interval $[R_{i-1}, R_i - 1]$. In this renewal cycle, we have W_i incremental updates followed by Z_i true updates. Hence,

$$T_i = nZ_i + nW_i.$$

Lemma 1. *The number of true updates $\{Z_i : i \in \mathbb{N}\}$ and the number of incremental updates $\{W_i : i \in \mathbb{N}\}$ are independent processes. Both processes are* iid *with the distribution of number of true updates Z_i being geometric with the success parameter $(1 - p_1)$.*

Proof. Independence of $\{Z_i : i \in \mathbb{N}\}$ and $\{W_i : i \in \mathbb{N}\}$ follows from the independence of erasure channel. Each decoding failure of a true update is *iid* Bernoulli with probability p_1, and $Z_i - 1$ is the number of true updates failures before a successful reception. Hence, the result follows. ■

The number of incremental updates W_i in the ith renewal interval, is composed of number of decoding successes and failures denoted by W_i^s and W_i^f respectively.

Theorem 2. *The number of successful incremental updates in the ith renewal interval $\{W_i^s : i \in \mathbb{N}\}$ is* iid *with distribution*

$$\Pr\{W_i^s = j\} = r^j(1 - r), \quad j \in \mathbb{N}_0.$$

The number of incremental update failures $\{W_i^f : i \in \mathbb{N}\}$ is an iid *Bernoulli process with probability $\Pr\{W_i^f = 0\} = \frac{(1-p_3)}{(1-r)}$, where $r = p_3(1-p_2)$. Further, W_i^s and W_i^f are independent.*

Proof. The monitor receives an incremental update successfully, when the source can encode the difference in k bits and the incremental update is successfully decoded. Decoding successes of incremental updates and encoding of incremental packets are *iid* due to the channel and the source structure respectively. Further, they are independent due to the source and the channel independence. Hence, the successful reception of incremental updates are *iid* Bernoulli with probability $r = p_3(1 - p_2)$.

In any renewal interval i, the failure to encode or decode an incremental update leads to transmission of a true update, and hence the number of successful incremental updates W_i^s is geometrically distributed with incremental update success probability r. A sequence of j consecutive incremental updates can be interrupted by either incremental encoding failure at the source or decoding failure at the receiver, corresponding to W_i^f being 0 or 1 respectively. Hence,

$$\Pr\{W_i^s = j, W_i^f = 0\} = r^j(1 - p_3), \ j \in \mathbb{N}_0.$$

From this we can obtain the marginal distribution of W_i^f, and verify the independence of W_i^f and W_i^s. ■

In the ith renewal interval $[R_{i-1}, R_i - 1]$, the age is reset to n at times $R_{i-1} + (j-1)n$ for $j \in \{1, \dots, W_i^s\}$, corresponding to the instants when the source state can be successfully decoded from the incremental updates. Age grows linearly at all other points in the interval. We illustrate the evolution of the age process and the peak ages for this scheme with an example in Fig. 2. Let $S(T_i) = \sum_{j=0}^{T_i - 1} \tilde{A}(R_{i-1} + j)$ be the accumulated age over renewal period T_i. Then, as an application of renewal reward theorem [11], the limiting average age is

$$\mathbb{E}\tilde{A} \triangleq \lim_{t \to \infty} \frac{1}{t} \sum_{s=1}^{t} \tilde{A}(s) = \frac{\mathbb{E}S(T_i)}{\mathbb{E}T_i}.$$

Theorem 3. *The limiting average age of information for the generalized incremental update scheme is given by*

$$\mathbb{E}\tilde{A} = n - \frac{1}{2} + \frac{n\mathbb{E}(W_i^s)^2 + n\mathbb{E}(W_i^f + Z_i)^2}{2(\mathbb{E}W_i^s + \mathbb{E}W_i^f + \mathbb{E}Z_i)}.$$

Proof. The mean of cumulative age $\mathbb{E}S(T_i)$ and $\mathbb{E}T_i$ are

$$\mathbb{E}S(T_i) = \mathbb{E}\left[\sum_{j=1}^{W_i^s}\sum_{k=0}^{n-1}(n+k) + \sum_{k=0}^{T_i - nW_i^s - 1}(n+k)\right],$$

$$\mathbb{E}T_i = n(\mathbb{E}Z_i + \mathbb{E}W_i^s + \mathbb{E}W_i^f).$$

Using Lemma 1 and Theorem 2, we can compute the average age. ■

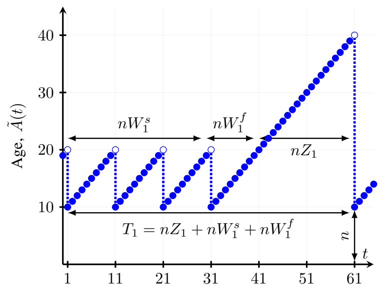

Figure 2. This plot shows a sample path of the age process in one renewal interval for generalized incremental update for codeword length $n = 10$. In this example, $W_1^s = 3$ incremental updates are successfully decoded. Fourth incremental update sent at time $3n+1$ fails to get decoded and hence $W_1^f = 1$. The source starts sending the true updates from the next transmission opportunity at $4n+1$. True update sent at instant $4n+1$ fails to get decoded and the update is successfully decoded after $Z_1 = 2$ transmissions. Age process has peaks at in for $i \in \{1, \dots, W_1^s\} \cup \{W_1^f + W_1^s + Z_1\}$.

The limiting average ages $\mathbb{E}A$ and $\mathbb{E}\hat{A}$ for the true update and the incremental update scheme can be found from the expression of $\mathbb{E}\tilde{A}$ in Theorem 3. Specifically, setting $p_3 = 0$ and $p_3 = 1$ in the expression of $\mathbb{E}\tilde{A}$ results in $\mathbb{E}A$ and $\mathbb{E}\hat{A}$ respectively. From Lemma 1 and Theorem 2, we can see that $\mathbb{E}W_i^s$, $\mathbb{E}W_i^f$ are functions of (p_2, p_3) and $\mathbb{E}Z_i$ is a function of p_1. Therefore, $\mathbb{E}\tilde{A}$ is a function of (p_1, p_2, p_3). It follows from Theorem 3, that $\mathbb{E}\tilde{A}$ is increasing in p_2 and decreasing in p_3.

IV. Some Example Sources

In this section, we consider two examples where we use the analysis from Section III to compute the limiting average age. First, we consider a source generating *iid* messages, while in the second example we consider a Markov source. In both cases, the source message set is $\mathbb{F}_{2^m}$ which can be represented by m bits. It is clear that, the differential encoding probability p_3 increases with the threshold k. We also notice that the decoding failure probability p_2 for incremental updates, increases with k. Since mean age for generalized incremental update is decreasing in p_3 and increasing in p_2, it indicates a choice of k that can optimally trade-off between these two competing considerations.

For numerical studies, we adopt a random coding scheme [12], for which the probability of decoding failure in an n-length codeword with $n-r$ parity bits and E erasures is given by

$$P(n, n-r, E) = 1 - \prod_{i=0}^{E-1} \left(1 - 2^{i-(n-r)}\right).$$

We plot the mean age for the three update schemes as a function of the number of differential information bits k for two example sources considered below. Probability of incremental update p_3 is 0 and 1 respectively for the true and incremental update schemes, bounding the performance of the differential update scheme. In practice, a source would not be able to send differential information at time t if $M(t) - M(t-n) \notin \Delta_k$.

A. *The* iid *case*

In this case, the source generates messages uniformly at random from $\mathbb{F}_{2^m}$. The probability p_3, that the source message is encoded as an incremental update is $2^k/2^m$. For the *iid* case plotted in Fig. 3, we see that the limiting average age for generalized incremental update is close to that of the true update, with the optimal number of differential information bits, $k^* \approx m$. Since an *iid* source has no temporal correlation, the generalized differential updates do not reduce the average age significantly. It is clear that even though incremental update promise timeliness gains, they can't be achieved for *iid* sources.

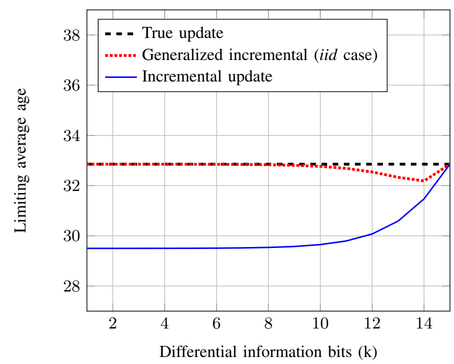

Figure 3. This plot shows the variation of limiting average age as the number of differential information bits k increases in generalized incremental update, for an *iid* source. We have chosen code length $n = 20$, the number of information bits $m = 15$, and the erasure probability $\epsilon = 0.1$. The optimal number of differential information bits for generalized incremental update is $k^* = 14$.

B. *Markov source*

We now consider the source $M(t)$ to be Markov with the associated transition probability matrix P such that for all $i, j \in \mathbb{F}_{2^m}$, we have

$$P_{ij} \triangleq \Pr\{M(t+1) = j | M(t) = i\} = \begin{cases} 1-\alpha, & j = i, \\ \alpha/2, & |j-i| = 1. \end{cases}$$

Thus, given past state $M(t-n) = i$, the probability that the current state $M(t) = j$ is P_{ij}^n, where P^n is the n-step transition probability matrix for the Markov chain described above. Parameter α captures the source correlation, with smaller values corresponding to more correlated sources. We see in Fig. 4, that the generalized updates offer significant timeliness gains for such Markov sources. We also observe that the gains increase with the source correlation, and the optimal number of differential information bits k^* for correlated Markov sources is much smaller than m. This is in contrast to the *iid* case, where the optimal k is closer to m. We observe that the differential encoding is better for the Markov source when compared to the *iid* source, since this update scheme is able to exploit the temporal correlation. Further, we observe that the timeliness gains reduce as the source correlation decreases. In particular, the average age for the Markov source with $\alpha = 0.1$ is smaller than average age for the Markov source with relatively less temporal correlation having $\alpha = 0.7$.

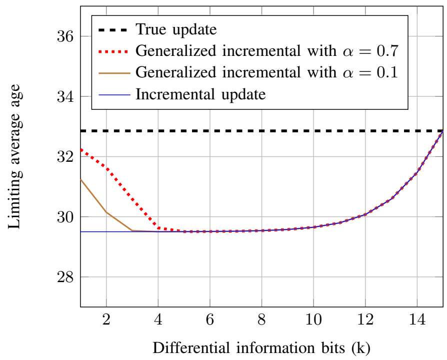

Figure 4. This plot shows the variation of limiting average age as the number differential information bits k increases in generalized incremental update for Markov sources. We have chosen code length $n = 20$, the number of information bits as $m = 15$, and the erasure probability as $\epsilon = 0.1$.

V. Stochastic Ordering of Peak Age

Let us denote the peak age for the true, incremental, and generalized incremental updates by $A_M, \hat{A}_M, \tilde{A}_M$ respectively.

Lemma 4. *The distribution of peak age $\tilde{A}_M$ is given by*

$$\Pr\{\tilde{A}_M = jn - 1\} = \begin{cases} s & j = 2, \\ (1-s)p_1^{j-3}(1-p_1) & j \geq 3. \end{cases}$$

where $s = p_3(1-p_2) + (1-p_3)(1-p_1)$.

Proof. Following an update decoding success, the next update is an incremental or true state update with probability p_3 and $1 - p_3$ respectively. Probability of decoding success for the corresponding updates are $1 - p_2$ and $1 - p_1$ respectively. If this update succeeds, the peak age $\tilde{A}_M$ is $2n - 1$. Combining these we get $\Pr\{\tilde{A}_M = 2n - 1\} = s$.

If the first update fails, the source starts sending true updates till one of them is successfully decoded. The number of true updates Z, sent between successful updates is geometric with success parameter $(1 - p_1)$. Thus, the peak age, in the event of the first incremental update failure is $nZ + 2n - 1$. Thus, $\Pr\{\tilde{A}_M = jn - 1\} = (1 - s)p_1^{j-3}(1 - p_1)$, for $j \geq 3$. ■

We can find the distribution for A_M and $\hat{A}_M$ from Lemma 4, by setting differential encoding probability p_3 as 0 and 1 for true and incremental updates respectively. We now compare the timeliness performance of the three encoding schemes.

Theorem 5. *The mean age of information for the generalized incremental update scheme satisfies*

$$\mathbb{E}\hat{A} \leq \mathbb{E}\tilde{A} \leq \mathbb{E}A. \tag{1}$$

Proof. When p_3 is 0, $\mathbb{E}\tilde{A} = \mathbb{E}A$ and when p_3 is 1, $\mathbb{E}\tilde{A} = \mathbb{E}\hat{A}$. Result follows if we prove $\mathbb{E}\tilde{A}$ is decreasing in p_3. To this end, we observe that as $p_1 > p_2$, $\frac{d\mathbb{E}\tilde{A}}{dp_3} = \frac{-(p_1-p_2)}{2(p_3(p_2-p_1)+1)^2} < 0$. ■

Theorem 6. *The peak age for three update schemes satisfy the following stochastic ordering*

$$A_M \geq_{st} \tilde{A}_M \geq_{st} \hat{A}_M. \tag{2}$$

where $X \geq_{st} Y$ implies X is stochastically larger than Y.

Proof. The minimum peak age for all the three schemes is $2n - 1$. Hence, $\Pr\{A_M \geq 2n - 1\} = \Pr\{\hat{A}_M \geq 2n - 1\} = \Pr\{\tilde{A}_M \geq 2n - 1\} = 1$. For $j \geq 3$, it follows from Lemma 4,

$$\frac{\Pr(A_M \geq jn - 1)}{p_1} = \frac{\Pr(\hat{A}_M \geq jn - 1)}{p_2} = \frac{\Pr(\tilde{A}_M \geq jn - 1)}{(1 - s)}.$$

The result follows as $p_2 \leq (1 - s) \leq p_1$. ■

The above theorem shows that for any cost function which is a non-decreasing function of the peak age, the average cost will be maximum for the true updates scheme and minimum for the incremental updates with feedback scheme [13].

VI. Conclusion and Future Work

In this article, we considered a generalized incremental update scheme for real time status updates which exploits temporal correlation between consecutive messages. We showed that this scheme generalizes true and incremental update schemes. We demonstrated a trade-off between probabilities of decoding failure and differential encoding, for two example sources. In addition, we showed a stochastic ordering between the three update schemes with respect to the peak age metric. This was illustrated through some numerical examples. An interesting direction for future work would be to consider sources with state dependent differential encoding probabilities which would further generalize our current work.

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