# On Real-Time Status Updates over Symbol Erasure Channels

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***Abstract*—As sensing, control, and actuation become further integrated into modern communication infrastructures, special consideration must be given to the type of traffic generated by associated devices. Real-time decision making relies on the availability of accurate data and, as such, delivering status updates in a timely fashion is of paramount importance. The topics of real-time status updates and low-delay communications have received much attention in recent years. Within this context, this article presents new results by looking at the interplay between average timeliness and design decisions made at the physical layer for unreliable communication channels. This study focuses on the natural tension between the protection afforded by additional redundancy and the decoding delay associated with longer codewords. The average timeliness is adopted as a performance criterion, and a framework to efficiently compute the performance of various transmission schemes for the binary erasure channel is developed. The problem formulation precludes the use of asymptotically long codewords typical of information theory. Yet, the presence of limited feedback does not seem to boost performance in the present context. Rather, having accurate channel estimates is key in minimizing average timeliness. Numerical examples are included in this article to further illustrate the applicability of the findings.**

***Index Terms*—Status updates, block codes, forward error correction, communication systems.**

## I. Introduction

The wide availability of wireless sensors, microcontrollers, and actuators is changing the profile of typical wireless traffic from links dominated by a small number of sustained connections to the addition of a myriad of packet updates. The evolving character of wireless systems is an important component of the Internet of Things, a moniker often employed to describe next-generation networks. As sensing and actuation progressively expand to the wireless world, they are imposing new and distinct service requirements on existing communication infrastructures. Cyber-physical systems depend on real-time status updates, relying on the latest telemetry data acquired by distributed devices for decision and control. Furthermore, mobile ad hoc networks need various kinds of status updates to know the state of their neighborhoods, elect routes, and coordinate transmissions.

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In recent years, researchers have introduced performance criteria to better understand the interplay between status update and communication systems. One of the guiding principles behind these new criteria is the fact that the timely delivery of information parcels is key in enabling smooth control and actuation. Stale information, on the other hand, can lead to incorrect decisions, greater residual errors, and instability. One specific performance criterion that has received attention in the present context is the average age of information at the destination. This criterion captures the essence of staleness while admitting tractable problem formulations. Owing to its popularity and ease of use, this is the performance criterion we adopt throughout.

An important consideration in the design of such communication systems pertains to the fact that wireless channels are often unreliable, with the realized quality of wireless links fluctuating over time. Traditionally, error correcting codes have been employed to protect sent data against channel impairments. With asymptotically long block lengths, it is possible to transmit data reliably at rates that approach the Shannon capacity. Yet, such coding techniques entail significant delays and, therefore, they may not be suitable for real-time status updates. The typical small payloads associated with status updates is another reason why such traffic may not be amenable to the asymptotic frameworks common to information theory. Rather, contemporary techniques based on coding schemes with short block lengths must be explored.

As mentioned above, we examine the tension between data protection and delay in the context of real-time status updates over erasure channels. We are especially interested in the topic of remote sensing over wireless communication links. Both the areas of real-time status updates [1]–[9] and coding for short block lengths [10]–[12] have received attention over the past several years. In addition, delay-sensitive communication has been investigated under error exponents, the normal approximation regime, and the moderate deviation regime [13], [14]. This high activity level points to the timeliness of the topic at hand. Our aim is to combine results from these areas by defining the communication channel at the symbol level and assessing the performance of the coded status update system using the average age of information. This perspective is new and provides insight into the design of wireless links for status updates.

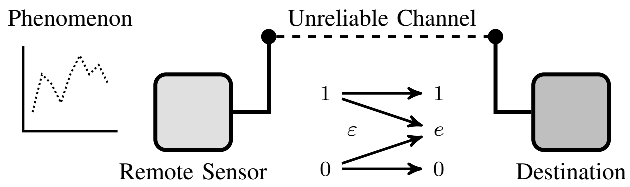


Fig. 1. This figure offers an illustration of the system model, which is composed of a random phenomenon, a remote sensing device, a communication channel, and a data aggregator that receives status updates.

## II. Background

Consider a scenario where a remote sensing device is monitoring a generic physical process. Every observation takes the form of a digital message containing exactly $K$ bits of information. The sensor can observe the physical process at any point in time; and we call the corresponding sample a status update. The remote sensor must transmit the collected message to a central entity over an unreliable link akin to a wireless channel. To protect the integrity of measurements, it is natural to employ forward error correction. As is customary, suitable coding strategies will improve the probability of accurately recovering the original message at the expense of additional redundancy bits in the transmitted codeword. In this article, we are especially interested in near real-time applications where the quality of a data sample is evaluated based on information staleness. This is a common setting for real-time status updates.

The need to deliver messages in a timely manner precludes the use of long codewords. Rather, the problem formulation demands the application of coding strategies with low latency. Thus, for a given payload, a natural tension arises between the protection afforded by longer blocks, which translates into small probabilities of failure, and the ability of short codewords to deliver information with low latency when successfully decoded. The balance between these opposing considerations hinges, partly, on the character of the underlying channel. A very noisy, unreliable channel favors the use of extra protection bits compared to an instance where corrupted symbols are infrequent or unlikely.

Throughout this early-stage study, we restrict our attention to coding schemes with short block lengths. Moreover, we explore partial feedback schemes such as hybrid automatic repeat request (ARQ) as means of gracefully adapting to channel realizations. These schemes are known to perform well for data transmission over unreliable channels in the context of delay-sensitive applications. As such, they form an attractive option for the problem at hand as well. Fig. 1 depicts the basic components of our envisioned system.

Our problem formulation differs from previous contributions on real-time status updates in that it defines the operation of the channel at the symbol level. This enables the study of various coding schemes tailored to this application scenario. Our objective is to provide guidelines on good coding schemes for this problem, and to compare the relative performance of different approaches. The specifics of our mathematical model are detailed in the following sections.

### A. System Model

The phenomenon being monitored is the simplest component of our system model. The sensing device is observing an abstract random process, and it can collect a measurement at any point in time. The size of an observation is $K$ information bits, irrespective of the past. After it is acquired, each observation must be communicated to a central location using a wireless link. In this paper, we do not consider source coding strategies such as joint source-channel coding, variable-length codes, or data compression based on differential encoding. The design and evaluation of such advanced schemes are typically tied to specific applications. The use of a generic source instead enables this work to focus on the tradeoff we wish to explore. It also offers a suitable mathematical framework that renders analysis tractable.

We adopt a channel model commonly found in the information theory literature, namely the binary symmetric erasure channel. In this model, every transmitted bit is received at the destination with probability $1-\varepsilon$ and it is erased with probability $\varepsilon$, independently of other symbols. Accordingly, the probability of experiencing $e$ erasures within a block of $N$ symbols is given by the binomial distribution

$$\Pr(|E| = e) = \binom{N}{e} \varepsilon^{e} (1-\varepsilon)^{N-e}, \tag{1}$$

where $E \subseteq [N]$ is the index set of erasure locations, with $|E|$ being its cardinality, and $[N] = \{1, \ldots, N\}$. We assume that the propagation delay between the transmission and reception of a symbol over the channel is negligible. As such, the receiver can attempt to decode a packet as soon as the sensing device completes the transmission of the last symbol associated with a codeword.

We also assume that forward error correction is implemented using a linear block code. We denote the generating matrix for the code by $G$, and the corresponding parity check matrix by $H$. The linear transformation $G : \mathcal{X}^K \to \mathcal{X}^N$ injectively maps $K$-bit information messages to $N$-bit codewords. In other words, when the sensor gathers observation $x$, it sends codeword $Gx$ over the erasure channel. The parity check matrix $H : \mathcal{X}^N \to \mathcal{X}^{N-K}$ is such that $HGx = 0$ for any source message $x$. A convenient way to express $H$ in terms of its column vectors $\{h_i \in \mathcal{X}^{N-K} : i \in [N]\}$ is

$$H = \sum_{i \in [N]} h_i e_i^T, \tag{2}$$

where $e_i$ represents the $i$th standard unit vector in $\mathcal{X}^N$.

We denote the output of the erasure channel by $y$ and define the erasure subset by $E = \{i \in [N] : y_i = e\}$. Let $\pi_E$ be the projection of $\mathcal{X}^N$ onto the span of $\{e_i : i \in E\}$, and $\pi_{E^c}$ be the corresponding annihilator. Then, we can express the channel output as

$$y = \sum_{i \in E} e e_i + \sum_{i \notin E} y_i e_i = e \pi_E 1 + \pi_{E^c} y. \tag{3}$$

By construction, we have $HGx = H\pi_E Gx + H\pi_{E^c} Gx = 0$. Furthermore, because operations take place over a binary field, we can rewrite this condition as $H\pi_E Gx = H\pi_{E^c} Gx$. We can therefore recreate erased symbols in $y$ provided that the equation

$$H\pi_E Gx = H\pi_{E^c} Gx = H\pi_{E^c} y$$

admits a unique solution. From this characterization, we deduce that the missing components of $y$ and, hence, the source message $x$ can be recovered faithfully whenever $H\pi_E$ has rank $|E|$ [15], [16]. That is, the column vectors $\{h_i : i \in E\}$ should be linearly independent. In view of this discussion, we define the set $E_{\mathrm{d}}$ of all erasure patterns that can be decoded under linear code $(G, H)$. With this notation, we can write the set indicator function of a decoding failure given erasure pattern $E$ as $\mathbb{I}_{E_{\mathrm{d}}^{\mathrm{c}}}(E)$. The motivation behind this definition will become manifest shortly.

### *B. Performance Criterion*

Our performance criterion is based on timeliness, computed in discrete symbol periods. The timeliness is defined as the difference between the current time and the time at which the most recent status update was observed by the sensing device; we denote the timeliness (or staleness) function at discrete time $t$ by $T(t)$. Our objective is to minimize average timeliness. To optimize mean timeliness, the remote device wishes to transmit observations as often as possible. Transmission opportunities, however, are governed by the operation of the communication unit. Therefore, the design of a suitable communication system is key to enabling rapid updates.

## III. Analysis and Numerical Computations

We provide a characterization of performance for different communication schemes. In the systems considered herein, the remote sensing device produces an observation containing exactly $K$ information bits, as explained above. The transmitter encodes this message and constructs a packet composed of $N$ binary symbols. Consequently, the earliest time at which the destination can decode a packet is immediately after these $N$ symbols are received. Following every decoding attempt, a feedback message informs the sensing device about the outcome of the decoding process, i.e., whether the message was recovered successfully or not. The feedback channel is assumed to be error-free and instantaneous. Codeword length $N$ is a design parameter that should be optimized.

### *A. Single Transmission Scheme*

The first communication scheme we explore is the simple strategy where, at every transmission opportunity, the sensing device collects an observation and encodes this sample into an $N$-bit codeword. This codeword is then sent over the communication channel, requiring $N$ successive channel uses to complete the task. Upon completion of a codeword transmission and irrespective of the decoding outcome, the sensing device initiates the transmission of the next observation. This renewal cycle repeats periodically, so long as the phenomenon is being monitored remotely.

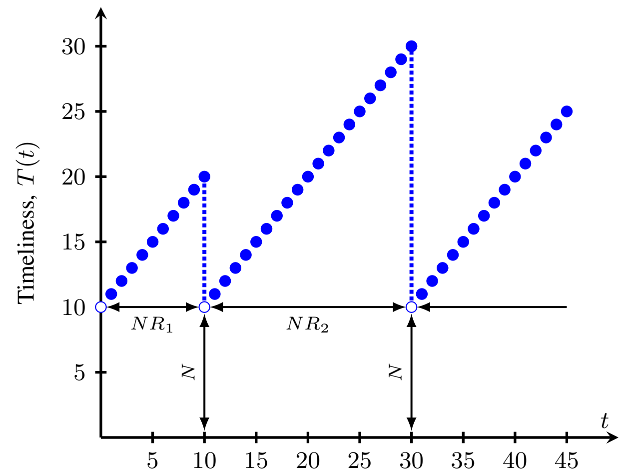


Fig. 2. This figure shows a sample path of the timeliness function with block length $N = 10$. In this example, the first message is decoded after one attempt ($R_1 = 1$), whereas the second message requires two transmissions ($R_2 = 2$) for a successful reception.

The analysis of this particular scheme is straightforward. The earliest time instant at which a message can be decoded is $N$ steps after it is collected by the remote sensing device. In between decoding successes, information staleness accumulates linearly with time. Since channel erasures are independent and identically distributed, packet decoding attempts form a Bernoulli process. For a particular code, the probability of decoding failure is given by

$$\rho = \mathbb{E}\left[\mathbb{I}_{E_{\mathrm{d}}^{\mathrm{c}}}(E)\right]. \tag{4}$$

The number of failed transmission attempts before a decoding success, which we call $M$, is geometrically distributed with

$$\Pr(M = m) = \rho^m (1 - \rho) \quad m \in \mathbb{N}_0. \tag{5}$$

Since the expected values for the inter decoding time and the experienced timeliness are finite, we can compute the mean timeliness using renewal theory,

$$\begin{aligned}\lim_{t\to\infty} \frac{\sum_{j=1}^{t} T(j)}{t} &= \frac{\sum_{m=0}^{\infty} \sum_{\tau=1}^{(m+1)N} (N+\tau) \Pr(M=m)}{\sum_{m=0}^{\infty} (m+1) N \Pr(M=m)} \\ &= \frac{\sum_{m=0}^{\infty} \left( (m+1)N^2 + \frac{(m+1)N((m+1)N+1)}{2} \right) \rho^m (1-\rho)}{\sum_{m=0}^{\infty} (m+1) N \rho^m (1-\rho)} \\ &= N + \frac{N(1+\rho)}{2(1-\rho)} + \frac{1}{2}.\end{aligned} \tag{6}$$

A notional diagram depicting the evolution of the timeliness for the single transmission scheme appears in Fig. 2.

### *B. Hybrid Automatic Repeat Requests*

The second paradigm we explore is a coding strategy based on hybrid ARQ. Under this scheme, the sensing device transmits additional redundancy bits (up to a limit) whenever decoding fails at the destination. Accurate feedback is crucial

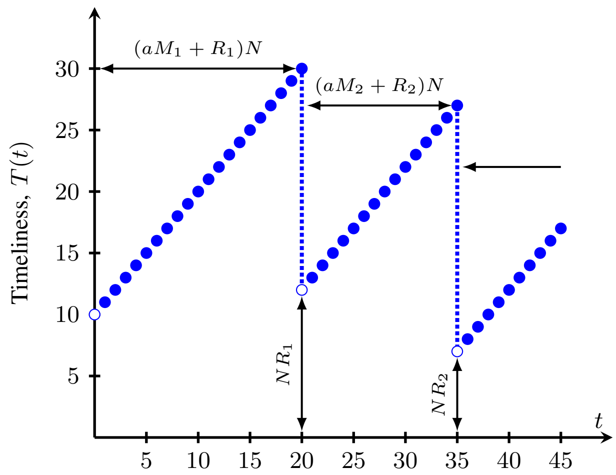


Fig. 3. This figure shows a sample path of the timeliness function for a generic hybrid ARQ coding scheme. The initial staleness of an information packet upon successful decoding depends on the depth of the current hybrid ARQ cycle; this creates variation in $NR$. Once decoded, timeliness accumulates linearly until the next successful decoding event. The graph above showcases three decoding points with variable initial timeliness.

to the success of this communication strategy. Hybrid ARQ is an enticing prospect for status updates because of its successful implementation in a number of delay-sensitive communication scenarios. This scheme is known to adapt gracefully to changing environmental conditions, and it performs well in various contexts.

The specific hybrid ARQ scheme we examine is also based on linear codes. In the envisioned framework, every source update is encoded in a codeword of length $aN$. During the first transmission attempt, the first $N$ bits of the codeword are sent over the communication channel. If decoding fails, the next $N$ bits are transmitted to the destination. This cycle continues until the codeword is successfully decoded at the destination, or until it reaches the maximum depth of $aN$ coded bits. Trailing bits are treated as erasures in early decoding attempts, in a manner akin to punctured codes. If the entire codeword has already been transmitted and the decoder still fails to recover the original message, the sensing device moves on to the next observation and information staleness continues to deteriorate at the destination. The evolution of timeliness for this hybrid ARQ scheme is illustrated in Fig. 3.

To analyze the performance of the hybrid ARQ scheme, we focus on the distribution of the timeliness function $T(t)$. Continuing with the notation introduced in Section II, we let $E_{\mathrm{d}}$ denote the set of erasure patterns that can be decoded by the aggregate linear code with codeword length $aN$. Let $E_{rN}$ be the set of erasure indices within the first $rN$ transmitted bits. The set of decodable events after the first block of data can be written as

$$D_1 := \{E_N : E_N \cup \{N+1, \dots, aN\} \subseteq E_{\mathrm{d}}\}. \tag{7}$$

Likewise, decodable sequences after $rN$ blocks are

$$D_r := \{E_{rN} : E_{rN} \cup \{rN+1, \dots, aN\} \subseteq E_{\mathrm{d}}\}. \tag{8}$$

We note that $D_1 \subseteq D_2 \subseteq \cdots \subseteq D_r$ because the communication channel is an erasure channel. Thus, a received codeword will be decoded at the $r$th attempt whenever the corresponding erasure pattern lies in $D_r$, but its truncated version does not belong to $D_{r-1}$. By transposition, if we define $F_r = D_r^{\mathrm{c}}$ for every possible $r$, we get $F_a \subseteq F_{a-1} \subseteq \cdots \subseteq F_1$.

From a mathematical perspective, we can consider the probability that the system fails to decode when it sends $r$ blocks, $\overline{f}_r = \mathbb{E}\left[\mathbb{I}_{F_r}(E_{rN})\right]$. Then, the probability that the hybrid ARQ scheme successfully decodes after transmitting exactly $r$ blocks, $r \leq a$ is

$$\overline{s}_r = \overline{f}_{r-1} - \overline{f}_r. \tag{9}$$

We notice that this probability of decoding success can also be expressed recursively as $\overline{s}_r = 1 - \overline{f}_r - \sum_{k=1}^{r-1} \overline{s}_k$.

Owing to the properties of the erasure channel, this system possesses a renewal structure. We begin our analysis of the hybrid ARQ case by focusing on the distribution of the number of codewords required to obtain a message at the destination. In this scheme, failure to decode a message implies $E_{aN} \in F_a$. The number of codeword failures before a decoding success, denoted by $M$, is geometrically distributed according to

$$\Pr(M = m) = \overline{f}_a^{\,m} \left(1 - \overline{f}_a\right) \quad m \in \mathbb{N}_0. \tag{10}$$

A second important aspect in assessing performance is the fact that the initial staleness immediately following a decoding success is no longer fixed in hybrid ARQ. Rather, it depends on the number of blocks used in the previous decoding task. When this process succeeds after receiving only one block, then the initial staleness is $N$. However, if additional blocks are required, the initial staleness jumps to $RN$. The expected value of the initial staleness is dictated by the conditional distribution of $R$, the number of blocks that was employed to recover the most recent observation in the hybrid ARQ decoding process. The mean of $R$, conditioned on a decoding success, is

$$\mathbb{E}[R | E_{aN} \in D_a] = \frac{1}{(1 - \overline{f}_a)} \sum_{r=1}^{a} r \overline{s}_r. \tag{11}$$

The average initial staleness upon decoding a message is therefore equal to $N\mathbb{E}[R|E_{aN} \in D_a]$. Counting from the beginning of a new transmission, the expected time to a successful decoding event is given by

$$\begin{aligned} \mathbb{E}[S] &= \sum_{m=0}^{\infty} \sum_{r=1}^{a} (maN + rN) \\ &\qquad \times \Pr(R = r | E_{aN} \in D_a) \Pr(M = m) \\ &= \frac{N}{(1 - \overline{f}_a)} \left( a\overline{f}_a + \sum_{r=1}^{a} r\overline{s}_r \right). \end{aligned} \tag{12}$$

In the limit, as time goes to infinity, the average timeliness can be computed using renewal theory. The mean timeliness, only considering discrete time instants, is equal to

$$
\begin{aligned}
&\lim_{t\to\infty} \frac{1}{t}\sum_{j=1}^{t} T(j) \\
&= \frac{1}{\mathbb{E}[S]} \sum_{m=0}^{\infty}\sum_{q=1}^{a}\sum_{r=1}^{a}\sum_{t=1}^{maN+rN} (qN+t) \\
&\qquad \times \Pr(R=q|E_{aN}\in D_a) \\
&\qquad \times \Pr(R=r|E_{aN}\in D_a)\Pr(M=m) \\
&= \frac{N}{(1-\overline{f}_a)}\sum_{r=1}^{a} r\overline{s}_r + \frac{1}{2} + \frac{1}{\mathbb{E}[S]}\Bigg( \sum_{m=0}^{\infty}\sum_{r=1}^{a} \frac{(maN+rN)^2}{2} \\
&\qquad \times \Pr(R=r|E_{aN}\in D_a)\Pr(M=m)\Bigg) \\
&= N\left(\frac{a\overline{f}_a + \sum_{r=1}^{a} r\overline{s}_r}{1-\overline{f}_a}\right) + \frac{1}{2} + \frac{N}{2}\left(\frac{a^2\overline{f}_a + \sum_{r=1}^{a} r^2\overline{s}_r}{a\overline{f}_a + \sum_{r=1}^{a} r\overline{s}_r}\right)
\end{aligned} \tag{13}
$$

Again, by ergodicity, this corresponds to mean timeliness in a stationary system. Specific values for $\overline{f}_r$ and $\overline{s}_r$ depend on the particulars of the coding scheme and decoding process.

### *C. Random Linear Codes*

While the derivations above apply to arbitrary linear codes, we turn to a specific coding scheme for performance evaluation. In particular, we adopt random linear coding as a suitable approximation for more pragmatic implementations. Random codes have a long history as a mathematical tool to obtain insights about communication problems [17], [18]. They are known to offer good performance over erasure channels [15]. As seen below, they are amenable to analysis and numerical simulations. In this article, random linear codes provide a foundation for the comparison between single transmissions and hybrid ARQ in the context of status updates.

As is customary, every message is associated with its own random code, independently of other messages. A code is created by first generating a random binary parity check matrix $H$. Each entry in this matrix is a zero or a one with equal probabilities, independently of other elements. The ensuing codebook corresponds to the nullspace of $H$, and we note that it necessarily contains at least $2^K$ codewords. The successful decoding of a codeword is independent from message to message, yet the decoding statistics are identical. This preserves the renewal structure of the system. Maximum-likelihood decoding is performed at the receiver.

By symmetry, probabilities of decoding failure for this scheme depend only on the number of erasures per codeword. The precise location of these erasures do not affect performance over a code ensemble. For a code with $K$ information bits, codeword length $N$, and $|E| = e$ erasures, the probability of decoding failure is equal to [15]

$$
P_f(N-K,e) = 1 - \prod_{i=0}^{e-1}\left(1 - 2^{i-(N-K)}\right). \tag{14}
$$

For the single transmission scheme of Section III-A, we can compute the probability of decoding as

$$
\begin{aligned}
\rho &= \mathbb{E}[P_f(N-K,|E|)] \\
&= \sum_{e=0}^{N} P_f(N-K,e)\binom{N}{e}\varepsilon^e(1-\varepsilon)^{N-e}.
\end{aligned} \tag{15}
$$

This equation, together with (6), completely characterizes the performance of single-transmission status updates with random linear coding.

Computations are slightly more involved for hybrid ARQ. In this scenario, the random linear code is generated for length $aN$, where $a$ is the maximum depth. The probability that decoding fails given $r$ blocks is equal to

$$
\overline{f}_r = \mathbb{E}\left[P_f(aN-K,|E_{rN}| + (a-r)N)\right] \tag{16}
$$

where $|E_{rN}|$ is the number of erasures within the first $r$ blocks. Based on this result, (9) yields the probability that the hybrid ARQ scheme with random linear coding successfully decodes after transmitting exactly $r$ blocks. The characterization above, together with (13) and (16), provides an algorithmic blueprint to numerically evaluate the performance of status updates with hybrid ARQ, random linear coding, and channel erasures.

## IV. System Simulations

To illustrate our results, we apply the techniques described in previous sections to specific scenarios. The length of all messages generated by the remote sensing device is 80 bits. This relatively low value reflects the fact that most of the information contained in scalar samples can be compressed to a small number of bits. Furthermore, it assumes the utilization of a header compression scheme, as protocols often feature significant overhead. The respective performances of our two schemes appear in Fig. 4. The probability of an erasure on the communication channel varies from $0.05$ to $0.3$, capturing a range of environmental conditions. Each value of $\varepsilon$ is associated with a different color. Since block length $N$ is the principal design parameter in this study, it is represented on the horizontal axis.

The specific values of the average timeliness depend heavily on channel parameter $\varepsilon$. The behavior of these curves, however, transcends channel conditions. The performance curves for the single transmission scheme are unimodal. The desired operating point for this strategy is the block length that corresponds to the unique minimum, optimally trading off the added protection afforded by extra redundancy with the decoding delay associated with transmitting more symbols.

In contrast, the performance curves associated with hybrid ARQ are bimodal. The first minimum occurs for values of $N$ that are smaller than $K$. This implies that the first decoding attempt always fails, triggering transmission of the second block. Once the two blocks are received at the destination, the decoding process becomes statistically equivalent to that of the single transmission scheme. Thus, the minimum value of a single transmission scheme matches exactly the first minimum on a depth-2 hybrid ARQ curve when the maximum

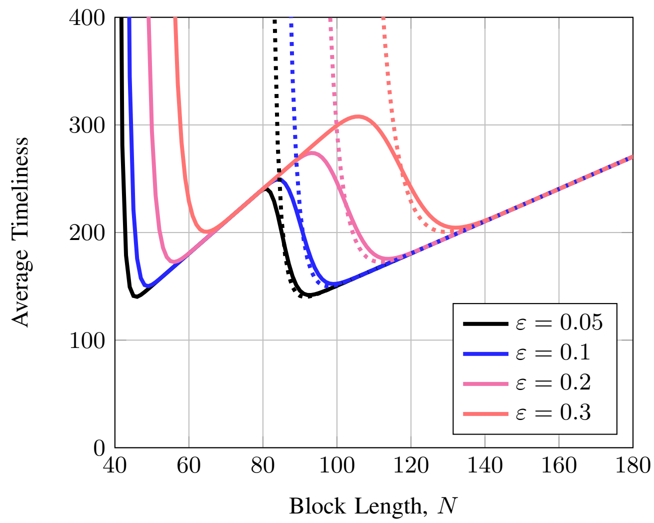


Fig. 4. This plot compares the performance of the single transmission scheme (dotted) and a depth-2 hybrid ARQ implementation (solid) for a real-time status update operating over a memoryless erasure channel. The number of information bits per message is fixed at $K = 80$ throughout, and every colored curve corresponds to a different erasure probability $\varepsilon$. Block length $N$ is a system parameter that should be optimized to match channel conditions.

length of the latter scheme equates the block length of the single transmission strategy. The second minimum of the hybrid scheme corresponds to a system where the message is typically decoded on first attempt. The dip in performance at the second minimum compared to the single transmission scheme is attributable to the fact that, for hybrid ARQ based on random linear coding, puncturing long codes leads to a small performance degradation.

Varying the block length of the system can have a very significant impact on performance for both the single transmission scheme and the hybrid ARQ implementation. Moreover, the penalty in selecting a block value that exceeds the optimum in the single transmission scheme is less severe than the catastrophic mistake of using too few bits for added protection. When $N$ is too close to $K$, an unreliable channel is likely to corrupt a succession of codewords and average timeliness greatly suffers. This points to the fact that having excellent channel estimates that enable the selection of a suitable block length, is of paramount importance for the problem at hand. In addition, our results hint at potential gains associated with hybrid ARQ implementations where block sizes change with the depth of the transmission cycle. In our current implementation, all blocks are bound to size $N$; this appears to be suboptimal. While having blocks of different lengths may render analysis more difficult, it will likely produce significant gains in terms of average timeliness.

## V. Conclusions

This article offers a framework to select suitable system parameters for real-time status updates over unreliable channels. Performance evaluation is conducted in terms of the elapsed time since the latest available observation was collected. For transmission over a binary erasure channel with random linear coding, efficient means to compute the average age of updates are provided. The results showcase a natural tradeoff between resilience against erasures and a timely delivery of the gathered data. Optimal code rates appear to be very sensitive to channel characteristics, and performance degrades rapidly if coding parameters are not selected adequately. The presence of limited feedback in the form of packet acknowledgements does not seem to improve performance significantly, at least for schemes with fixed block lengths. Our findings suggest several avenues of future research, including adaptive schemes with variable block lengths, transmissions over channels with memory, channel estimation schemes, and physical considerations for status updates in multiple access environments.

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